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Robust Interpolation using Interval Structures

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Abstract— **This paper considers the problem of robust interpolation for plants subject to parametric uncertainty. With mixed time/frequency domain noiseless data and the plant structure (model order and uncertain independent valued parameters), intervals for parameters that interpolate the given set of data points are computed using interpolation theory and the LFT description of the model set.**

I. INTRODUCTION

Uncertainty in models which describe physical systems has deserved a great deal of attention. Robust Control and Identification techniques have worked with this hypothesis in the '80s and '90s respectively. Uncertainty has been considered both as unstructured (or dynamical) and structured (either parametric or dynamic). The area of Control oriented Identification considered initially frequency domain or time domain unstructured uncertainty in a deterministic worst-case approach ([1], Chap. 10 of [2]). Further research led to combinations of time and frequency domain measurements ([3]) as well as parametric and non–parametric models ([4]). Recent research in the area of parametric uncertainty has produced both analysis and synthesis tools to deal with these type of model structures ([5], [6]). Therefore, identification techniques which can deal with hard bounds in the parameters are needed to compute more realistic models from experimental system data.

In the literature two main approaches can be found, namely: the interpolation interval-based approach and the set-membership approach. In the case of the set-membership approach ([7]), given a set of frequencies and the corresponding measured frequency response with unknown but bounded measurement errors, the set of feasible parameter values are computed. These are defined as the set of parameters for which the model provides an estimated frequency response that belongs to the set of measured frequency responses ([8]).

On the other hand, in the interpolation interval-based approach the aim is to find an interval of parameters, such that the estimated worst-case frequency response contains all the measured frequency responses. In this case, a single or multiple measured system trajectory is considered. In [6] (pp. 582-592), [9], [10], [11], [12] interval model identification algorithms are proposed using a single and a set of measured frequency responses, respectively, either, for SISO or MIMO systems. A possible extension of such approach is considered in [13]. In this case, an interval for frequency measurements is considered which produces a family of interval models with hard bounds in the parameters, using results based on Kharitonov's theorem. Nevertheless, none of these results are stated in the Robust Identification framework or use combinations of time and frequency domain measurements.

The goal of this paper is, to present a robust interpolation method that preserves the known structure of the plant. This solves exactly the problem stated in [6] (pp. 582-592) and [9], [10], [11], [12], due to the fact that it computes all interval models that interpolate **all** data points. Furthermore it generalizes this result to simultaneous time and frequency noiseless information. We consider the set of models as the Linear Fractional interconnection between a known plant $P(z)$, and an unknown vector of parameters $\delta \in \mathbb{R}^{n_p}$. Exploiting the properties of LFTs and constrained interpolation theory, we obtain a LFT description of this set and some ways to obtain a "maximal" set $\mathcal{P} \subset \mathbb{R}^{n_p}$ such that the interconnection $\{(P, \delta), \delta \in \mathcal{P}\}$ includes all fixed order interpolatory models. Next section introduces some necessary background for this work. Section III presents the main result: an LFT description of the set of models with interval structure which includes those that interpolate the given set of noiseless data points. An example is presented in Section IV and concluding remarks and future research directions end this presentation in Section V.

II. BACKGROUND

The main result is based on the rational interpolation theory developed in [14]:

Theorem 2.1: There exists a transfer function $f(z) \in \mathcal{BH}_\infty$ ($\overline{\mathcal{B}}\mathcal{H}_\infty$) such that:

$$\sum_{z_o \in \mathcal{D}} \mathrm{Res}_{z=z_o} f(z) C_-(zI - A)^{-1} = C_+ \tag{1}$$

if and only if the following discrete time Lyapunov equation has a unique positive (semi) definite solution.

$$M = A^* M A + C_-^* C_- - C_+^* C_+ \quad (2)$$

where A, C_- and C_+ are constant complex matrices of appropriate dimensions. If $M > 0$ then the solution $f(z)$ is non–unique and the set of solutions can be parameterized in terms of $q(z)$, an arbitrary element of $\overline{\mathcal{B}}\mathcal{H}_\infty$, as follows:

$$\begin{aligned} f(z) &= \frac{T_{11}(z)q(z) + T_{12}(z)}{T_{21}(z)q(z) + T_{22}(z)} && (3) \\ &= F_\ell\left[L(z), q(z)\right] && (4) \\ T(z) &= \begin{bmatrix} T_{11}(z) & T_{12}(z) \\ T_{21}(z) & T_{22}(z) \end{bmatrix} && (5) \end{aligned}$$

where system $T(z)$ has the following state space realization:

$$\begin{aligned} T(z) &\equiv \left[\begin{array}{c|c} A_t & B_t \\ \hline C_t & D_t \end{array}\right] \\ A_t &= A \\ B_t &= M^{-1}\left(A^* - I\right)^{-1}\left[\begin{array}{cc} -C_+^* & C_-^* \end{array}\right] \\ C_t &= \begin{bmatrix} C_+ \\ C_- \end{bmatrix}(A - I) \\ D_t &= I + \begin{bmatrix} C_+ \\ C_- \end{bmatrix} M^{-1} * X \\ X &= \left(A^* - I\right)^{-1}\left[\begin{array}{cc} -C_+^* & C_-^* \end{array}\right] \end{aligned}$$

The classical frequency response (Nevanlinna-Pick) and time response (Carathéodory-Fejér) interpolation results can be obtained as special cases of the above, by a suitable selection of the matrices A, C_+, C_-.

A practical characterization of set $\overline{\mathcal{B}}\mathcal{H}_\infty$ which provides the general class of candidate models that could interpolate the data is as follows (see [3]):

$$\mathcal{S}(K,\rho) = \left\{ f \in \mathcal{H}_\infty, \sup_{|z|<\rho} \|f(z)\|_\infty < K \right\},$$

with $K < \infty$ and $\rho > 1$.

III. Main Results

In this section we present the result which computes the set of models with interval structure which includes those that interpolate the given set of noiseless data points. This solves exactly the problem stated in [6] (pp. 582-592) and [9], [10], [11], [12], because it computes all interval models that interpolate **all** data points. These results are based on two key facts:

- Interval plants can be considered as a LFT of the unknown parameters.
- The set of all models that interpolate time and/or frequency response measurements is a LFT of a given transfer matrix and a free parameter in $\mathcal{B}\mathcal{H}_\infty$, according to the previous results (eqn. (4)).

Specifically:

- Given the interval model $\tilde{f}(z, \Delta_p)$, there is a $\tilde{P}(z)$ such that:

$$\begin{aligned} \tilde{f}(z,\Delta_p) &= F_\ell\left[\tilde{P}(z), \Delta_p\right] \\ &= a_n + \frac{a_{n-1}z^{n-1} + \cdots + a_1 z + a_0}{z^n + b_{n-1}z^{n-1} + \cdots + b_1 z + b_0} \end{aligned} \quad (6)$$

  with $\Delta_p = \operatorname{diag}\left[a_0 \cdots a_n \, b_0 \cdots b_{n-1}\right]$.
  Equivalently, a nominal set of parameters $\{\bar{a}_i\}$, $i = 0, \ldots, n$ and $\{\bar{b}_j\}$, $j = 0, \ldots, n-1$ can be introduced in:

$$\begin{aligned} a_i &= \bar{a}_i + \delta_{i+1}, & i &= 0, \ldots, n \\ b_j &= \bar{b}_j + \delta_{j+n+2}, & j &= 0, \ldots, n-1 \end{aligned}$$

  which replaced in $\tilde{f}(z, \Delta_p)$ produce the interval model $f(z, \Delta) = F_\ell\left[P(z), \Delta\right]$, for a certain $P(z)$ and the interval structure $\Delta = \operatorname{diag}\left[\delta_1 \cdots \delta_{2n+1}\right]$. The latter corresponds to the uncertainty vector $\delta = \left[\delta_1 \cdots \delta_{2n+1}\right]^T$.
- Given the *a posteriori* experimental measurements, the set of all interpolating models (of all possible orders) can be represented by (eqn. (4)):

$$\mathcal{M} = \left\{F_\ell\left[L(z), q(z)\right], \; \|q(z)\|_\infty < 1\right\}$$

- There always exists some $R(z)$ such that

$$P(z) = F_\ell\left[L(z), R(z)\right] \quad (7)$$

  Moreover, as we will show in Section III-C, if $P(z)$ is stable, then $L(z)$ and $R(z)$ can always be chosen to be stable.
- Replacing the free parameter $q(z)$ by

$$q_R(z) = F_\ell\left[R(z), \Delta\right] \quad (8)$$

  and closing the LFT yields:

$$\begin{aligned} F_\ell\left[P(z), \Delta\right] &= F_\ell\left\{F_\ell\left[L(z), R(z)\right], \Delta\right\} \\ &= F_\ell\left\{L(z), F_\ell\left[R(z), \Delta\right]\right\} \\ &= F_\ell\left[L(z), q_R(z)\right] \end{aligned} \quad (9)$$

  as shown in Fig. 1. In particular we also need that the *central* model of both, the interval structure ($\Delta \equiv 0$) and the interpolatory set ($q(z) \equiv 0$) are coincident.
- It can be proven (see Lemma 3.1) that the subset of fixed order $\{F_\ell\left[P(z), \Delta\right], \delta \in \delta_D\}$ that interpolates the experimental data, needs to have related parameters $(\delta_1 \cdots \delta_{2n+1})$. Hence there is no independent parameter structure for the set of interpolators (see [15]) and $\delta_D \subset \mathbb{R}^{n_p}$ (strictly), with $n_p = 2n+1$.

Based on the previous facts, it follows that the elements of the set

$$\mathcal{P} \doteq \left\{\delta \in \mathbb{R}^{n_p} : F_\ell(R, \Delta) \in \mathcal{B}\mathcal{H}_\infty\right\} \quad (10)$$

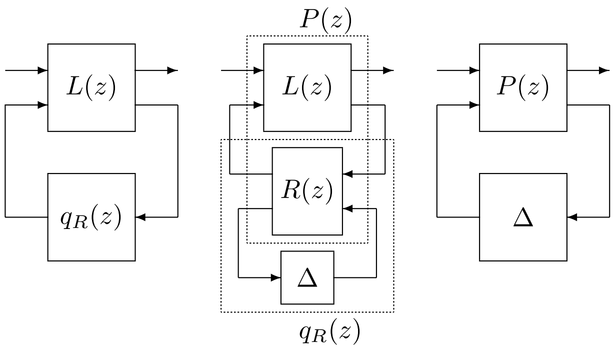

Fig. 1. Parameterization of all interpolating interval models.

include all models with the required structure which interpolate the given data, due to the fact that we have considered $\mathcal{P}$ as an independent parameter set, i.e. $\delta_D \subset \mathcal{P} \subset \mathbb{R}^{n_p}$. Hence, the following problems need to be solved:

1) From the experimental data points, find the interpolatory system $L(z)$, which produces the set in equation (4), for all $q(z) \in \overline{\mathcal{B}}\mathcal{H}_\infty$.
2) Given the interval plant $f(z,\Delta)$, with nominal value $f_o(z) = f(z,0)$, find $P(z)$ achieving $f(z,\Delta) = F_\ell\left[P(z),\Delta\right]$ and

$$f_o(z) = F_\ell\left[P(z),0\right] = F_\ell\left[L(z),0\right] \tag{11}$$

3) Find $R(z)$ as a function of $P(z)$ and $L(z)$, that satisfies equation (7). Note that with the choice of $P(z)$ and $L(z)$ in step (2), the *central* model of both, the interval model ($\Delta \equiv 0$) and the interpolatory set ($q(z) \equiv 0$) coincide, and thus $R_{11} = 0$.
4) Find the maximal hyperbox $\mathcal{P} \subseteq \mathcal{S}(K,\rho)$.

Step 1 is the standard Nevanlinna-Pick and/or Carathedory-Fejer interpolation theory with frequency and/or time experimental data points. In Steps 2 and 3, $P(z)$ and $R(z)$ can be found by standard procedures relating LFT's. In particular $R(z)$ is a function of $P(z)$ and $T(z)$ (or $L(z)$), where the latter is part of the parameterization of all interpolating models in equations (3)–(4). System $P(z)$ is the element of this problem representing the interval model structure and $T(z)$ (or $L(z)$) the experimental data.

In step 4 we may add a scaling function $W(s)$ to the Δ structure in order to accommodate the maximum interval lengths according to pre-specified physical constraints. This determines the form the maximal box will have, as a function of frequency.

In the next subsections we present the solutions to these problems, but first we prove the following result.

Lemma 3.1: The model set $\{f(z,\Delta),\ \delta \in \delta_D\}$ that interpolates the experimental data, needs to have related parameters $(\delta_1 \cdots \delta_{2n+1})$. Here $f(z,0) = F_\ell\left[P(z),0\right]$ is the nominal interpolator.

Proof:
Consider any interpolating point $f(z_i,0) = w_i \in \mathbb{C}$, hence $f(z_i,\delta^j) \neq w_i \in \mathbb{C}$ for $\delta^j = [0 \cdots \delta_j \cdots 0]^T$, $\delta_j \neq 0$. Otherwise $\delta_j z_i^j = 0$, which is not the case in general for the class of interval models. Therefore, δ_D is a strict subset of $\mathbb{R}^{n_p}$.

□

A. Measurement information $L(z)$

According to the interpolation results presented in section II, ([14]) and to further developments in the area of mixed *a posteriori* information ([3]), the state space realization of $T_o(z)$ and $L_o(z)$ in equations (3)–(4) can be found as functions of both, time and frequency experimental information and measurement bounds.

Due to the fact that equation (9) should be satisfied and that the interpolation theory described previously considers systems in the ball $\mathcal{B}\mathcal{H}_\infty$, system $L(z)$ should be scaled so that interpolates $f(z,\Delta) \in \mathcal{S}(K,\rho)$. Therefore we transform it to $L(z) = KL_o(\frac{z}{\rho})$, which in state space form is as follows:

$$L(z) \equiv \left[\begin{array}{c|c} \rho A_\ell & KB_\ell \\ \hline \rho C_\ell & KD_\ell \end{array}\right],\ L_o \equiv \left[\begin{array}{c|c} A_\ell & B_\ell \\ \hline C_\ell & D_\ell \end{array}\right]$$

as a consequence we should verify $q_R(z) = F_\ell\left[R(z),\Delta\right] \in \mathcal{S}(K,\rho)$.

We also produce a nominal set of parameters Δ_0 which interpolates the set of data using the procedure in [3] and put the resulting nominal model in numerator/denominator form (interval format). The values of these coefficients of z are the nominal parameters, i.e. $f_o(z) = F_\ell\left[L(z),0\right]$. Note that the value of $L(z)$ corresponding to the minimum value of K is unique (except in degenerate cases)[1].

B. Structural information $P(z)$

From well known results from state space theory, we can always find a canonical state space realization for the interval model $\tilde{f}(z,\Delta_p)$ in equation (6):

$$\left[\begin{array}{cccc|c} 0 & 1 & \dots & 0 & 0 \\ 0 & 0 & \ddots & 0 & 0 \\ & & & & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & 0 & 1 & 0 \\ -b_0 & -b_1 & \dots & -b_{n-1} & 1 \\ \hline a_0 & a_1 & \dots & a_{n-1} & a_n \end{array}\right] \tag{12}$$

[1]In an Identification framework ([3]), since K enters the analysis LMIs linearly, one can find the minimum value of K such that the *a priori* and the *a posteriori* experimental information are consistent. From a design standpoint, finding this K is desirable, since it leads to smaller consistency sets and therefore to smaller worst-case identification error.

Based on the previous result and the definition of the uncertainty structure Δ_p we find the state space realization of $\tilde{P}(z)$. Take $P_{ss} = \begin{bmatrix} a_p & b_p \\ c_p & d_p \end{bmatrix}$:

$$
\begin{aligned}
a_p &= \begin{bmatrix} 0 & & \\ 0 & \mathbf{I}_n & \\ \vdots & & \\ 0 & \cdots & 0 \end{bmatrix} \in \mathbb{R}^{(n+1)\times(n+1)} \\
b_p &= \begin{bmatrix} & & \mathbf{O}_{(n-1)\times(2n+1)} & & & \\ 0 & \cdots & 0 & -1 & \cdots & -1 \\ 1 & \cdots & 1 & 0 & \cdots & 0 \end{bmatrix} \\
c_p &= \begin{bmatrix} \mathbf{I}_{n+1} & \\ & 0 \\ \mathbf{I}_n & \vdots \\ & 0 \end{bmatrix} \in \mathbb{R}^{(2n+1)\times(n+1)} \\
d_p &= \mathbf{O}_{(2n+1)\times(2n+1)}
\end{aligned}
$$

with $b_p \in \mathbb{R}^{(n+1)\times(2n+1)}$. The state space matrices for $\tilde{P}(z)$ are obtained from P_{ss} as follows:

$$
\left[\begin{array}{c|c} \tilde{A}_p & \tilde{B}_p \\ \hline \tilde{C}_p & \tilde{D}_p \end{array}\right] = \left[\begin{array}{c|c} P_{ss}(i,j) & P_{ss}(i,\ell) \\ \hline P_{ss}(k,j) & P_{ss}(k,\ell) \end{array}\right]
$$
$$
i,j = 1,\cdots,n \quad k,\ell = n+1,\cdots,3n+2
$$

Replacing the uncertainty structure $\Delta_p = \Delta_0 + \Delta$, where Δ_0 is the set of the nominal parameters, we may obtain $P(z)$ as the interconnection between $\tilde{P}(z)$ and Δ_0, as shown in Fig. 2. This guarantees that $P(z)$ is stable when $f(z)$ is stable and $f_o(z) = F_\ell\left[\tilde{P}(z), 0\right]$, which proves to be useful in computing $R(z)$ in the next subsection.

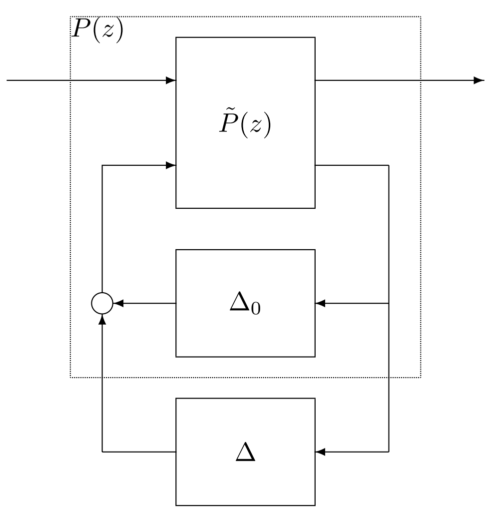

Fig. 2. Structural model $P(z)$.

C. Interconnection $R(z)$

Manipulating equation (4) with $q(z)$ replaced by $F_\ell\left[R(z), \Delta\right]$ produces the following equations to find $R(z)$:

$$
\begin{aligned}
R &= \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \\
&= \begin{bmatrix} \Phi\left(P_{11} - L_{11}\right) & \Phi P_{12} L_{21} \\ \Phi P_{21} L_{12} & P_{22} - \Phi P_{21} P_{12} L_{22} \end{bmatrix} \\
&= \begin{bmatrix} 0 & P_{12} L_{12}^{-1} \\ P_{21} L_{21}^{-1} & P_{22} - R_{21} R_{12} L_{22} \end{bmatrix}
\end{aligned}
$$

with $\Phi = \left[L_{12}L_{21} + \left(P_{11} - L_{11}\right)L_{22}\right]^{-1}$.The second equation is the simplification produced in the case equation (11) is satisfied, i.e. $f_o(z) = P_{11}(z) = L_{11}(z)$, and both, the interpolation and the interval *centers* coincide. This is a better solution to determine $R(z)$, from the numerical point of view as well.

D. Interval bounds computation

Once L and R are obtained using the factorization proposed in the previous section, then bounds on the values of the parameters guaranteeing that $F_\ell(R,\Delta) \in \mathcal{BH}_\infty$ can be obtained via a mixed-μ procedure, considering a scalar complex block in the first input–output pair of $R(z)$ and $(2n+1)$ independent parameter uncertainty in the remaining pairs. A fixed weighting function $W(s)$ can be added to the uncertainty structure to accommodate possible physical restrictions or scalings of the problem. Note however, that in order to use mixed-μ, R must be "open loop" stable. From the constrained interpolation theory introduced in section II, it follows that the LFT $F_\ell(L, F_\ell(R,\Delta))$ is stable as long as $F_\ell(R,\Delta) \in \mathcal{BH}_\infty$. However, this does not guarantee stability of R due to the (possible) existence of unstable pole/zero cancellations. To avoid these cancellations and obtain a stable R, start by considering inner-outer factorizations:

$$
\begin{aligned}
&L_{21} = L_{21}^o L_{21}^i \\
&L_{12} = L_{12}^o L_{12}^i \\
&L_{i,j}^o, (L_{i,j}^o)^{-1} \in \mathcal{H}_\infty, \quad L_{i,j}^{i\,\sim} L_{i,j}^i = 1
\end{aligned} \tag{13}
$$

where $L(z)^\sim \doteq L^T(\frac{1}{z})$. Next, absorb the inner factors of L_{12} and L_{21} into R_{12} and R_{21} leading to the structures:

$$
\begin{aligned}
\hat{L} &= \begin{bmatrix} L_{11} & L_{12}^o \\ L_{21}^o & L_{22} \end{bmatrix} \\
\hat{R} &= \begin{bmatrix} 0 & P_{12}(L_{12}^o)^{-1} \\ P_{21}(L_{21}^o)^{-1} & P_{22} - \hat{R}_{21}\hat{R}_{12}L_{22} \end{bmatrix}
\end{aligned} \tag{14}
$$

which are both stable. Finally, note that from the fact that $R_{11} = 0$ it follows that $\|F_\ell(R,\Delta)\|_\infty \leq 1 \iff \|F_\ell(\hat{R},\Delta)\|_\infty \leq 1$. It follows that a bound on the size of the unknown parameters can be obtained by performing a μ analysis using $\hat{R}$, instead of R.

index	0	1	2	3	4
a	0.455	-0.317	-0.156	0.166	0.835
b	-1.675	0.175	0.206	-0.105	1

TABLE I

INTERVAL MODEL PARAMETERS.

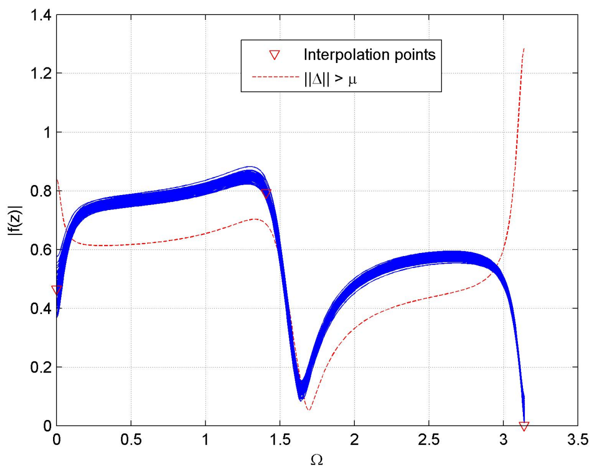

Fig. 3. (full) Interval models which interpolate all data, i.e. $\|\Delta\| < \mu$. (dashed) Model outside this set, i.e. $\|\Delta\| \geq \mu$.

IV. EXAMPLE

The following example illustrates the method. The input data are 3 points obtained from the frequency domain representation of the model:

$$f(z) \quad = \quad a_4 + \frac{a_3 z^3 + a_2 z^2 + a_1 z + a_0}{z^4 + b_3 z^3 + b_2 z^2 + b_1 z + b_0}$$

whose nominal set of parameters are indicated in Table I. This discrete time model belongs to set $\mathcal{S}(K, \rho)$ with $K = 1$ and $\rho = 1.05$. This is the case because all its poles have magnitude greater that ρ and its frequency response is bounded by $K = 1$ as illustrated by the full line curves in Fig. 3. Recall that the actual model should be represented with the standard z–transform, that is $\tilde{f}(z) = f(1/z)$. The resulting value of the structured singular value, considering mixed–μ is 0.014. This corresponds to a robust performance problem defined over the first input/output pair of the LFT of $R(z)$ and Δ such that $\|F_\ell\,[R(z), \Delta]\|_\infty < 1$, when the parametric uncertainty $\|\Delta\| < 0.014$. Fig. 3 presents the frequency response of the set of interval models which interpolate the data, and also belong to the set defined by K and ρ, represented with full lines. In the same figure, the plot of other model whose parameters have magnitude greater than μ, has also been represented (dashed line), which clearly does not belong to the *a priori* set $\mathcal{S}(K, \rho)$ nor interpolates the data points.

V. CONCLUSIONS

A computationally tractable algorithm has been developed which produces a set of interval models that cover all models which interpolate given time and/or frequency noiseless experiments. This result can be used as a first step in identification for Robust control design and/or Fault diagnosis. Nevertheless, the practical significance of these results are limited because of the following reasons:

- They apply only to noiseless data,
- the number of data points places a lower bound on the model order,
- the set of models are *included* in the consistency set.

To produce practical results which could be used as an identification tool in Fault diagnosis, as well as for Robust control, the previous limitations need to be lifted. The authors are presently working in this research direction ([16]).

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