

# Linear Parameter-Varying Lean Burn Air-Fuel Ratio Control

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**Abstract**—Maximization of the fuel economy of the lean burn SI engine strongly depends on precise air-fuel ratio control. A great challenge associated with the air-fuel ratio feedback control is the large variable time delay in the exhaust system. In this paper, a systematic development of an air-fuel ratio controller based on post-LNT UEGO sensor feedback using linear parameter-varying (LPV) control is presented. Satisfactory stability and disturbance rejection performance is obtained in the face of the variable time delay. The LPV controller is simplified to an explicit parameterized gain scheduled 1st order controller form for the ease of implementation. A Ford F-150 truck with a V8 4.6 Liter lean burn engine was used to demonstrate the LPV air-fuel ratio control design. Both simulation and experimental results demonstrate that the designed controller regulates the tailpipe air-fuel ratio to the preset reference for the full engine operating range.

## I. INTRODUCTION

In 2003, U.S. consumed about 20 million barrels of oil per day. The gasoline for cars and light trucks accounts for 45% of the total oil consumption. Lean burn technology for gasoline engines has drawn great attention during the past decade, largely due to its potential for improving fuel economy and reducing CO<sub>2</sub> emissions [1]. A lean burn engine is designed to operate at high intake manifold pressure with an air-fuel ratio greater than 10 and less than 23. Consequently, combustion efficiency can be improved through reduced pumping losses and enhanced thermodynamic efficiency. Compared to the conventional port fuel injection (PFI) engine, the gasoline lean burn engine represents a new set of challenges to the engine control community. The main challenge for lean burn technologies is that, under lean operating conditions, the conventional three-way catalyst (TWC) system is no longer effective in reducing NO<sub>x</sub> pollutants. A special TWC with NO<sub>x</sub> trapping and conversion capabilities, known as lean NO<sub>x</sub> trap (LNT), has to be used downstream of the conventional TWC to meet the government emission standards. During the lean operation, NO<sub>x</sub> in the feedgas is stored in the LNT. When the stored NO<sub>x</sub> reaches a certain threshold, the trap must be purged by switching to rich operation for a short period of time to regenerate the storage capacity and recover the efficiency. The NO<sub>x</sub> released from the LNT during the purge period is converted into non-polluting nitrogen by the rich air-fuel mixture [2] [3] [4]. Properly managing the

storage and purge cycles is critical for achieving the fuel economy and NO<sub>x</sub> emission control targets of the lean burn gasoline engine.

In this paper, the design of the "outer-feedback loop" air-fuel ratio controller is considered. A linear universal exhaust gas oxygen (UEGO) sensor is used downstream the LNT to measure the tailpipe air-fuel ratio. The air-fuel ratio controller to be designed is used to generate the commanded air-fuel ratio for the fuel injection system. During the storage phase when the engine is operating under lean conditions, the air-fuel ratio is selected to (i) meet the driver's demand, (ii) maximize the fuel economy benefits and (iii) satisfy other constraints, such as lean burn limit [5]. These requirements often dictate the set-point selection, and the optimal choice for the air-fuel ratio in the storage phase is usually a constant set-point.

A number of publications have described various designs of air-fuel ratio controllers for the stoichiometric feedgas air-fuel control [6] [7] [8] [9]. Related work in the air-fuel control for lean burn is limited. In [10], an adaptive-feedforward model-based feedgas air-fuel ratio controller was developed for a 4 cylinder, 2.2 Liter Mercedes-Benz lean burn engine. The control performance is largely dependent on the control-oriented engine model. However, it is very difficult to establish an accurate "outer-feedback loop" emission model covering all operating conditions over the engine life cycle for systems involving TWC and LNT. In addition, there is significant open-loop uncertainty in the fuel injection and exhaust system, such as canister purge, which cannot be handled by the feedforward control. Therefore, feedback control is necessary in order to maintain accurate air-fuel ratio control.

The biggest challenge associated with the air-fuel ratio feedback control stems from the variable time delay in the exhaust system. Since the UEGO sensor is positioned after the LNT, a significant time delay occurs between the UEGO sensor signal and the effective change of the engine feedgas air-fuel ratio. In addition, the time delay is largely dependent on the engine operating condition defined by the engine speed and the air mass flow. Throughout the engine operating envelop, the time delay can change significantly. For example, according to the experimental vehicle data collected on a Ford truck, the delay in this engine varied from 0.3 sec to 2.7 sec. Such a large variable time delay is the main hurdle to achieve the desired disturbance rejection performance using a single controller for the full engine operating range. Therefore, gain scheduled control must be applied to obtain the desired performance for the full engine operating envelope. Linear parameter varying (LPV)

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gain-scheduling control [11] [12] [13] has recently received significant attention because it provides a systematic way of computing gain scheduled controllers for nonlinear and parameter dependent systems with stability and performance guarantees.

In this work, we will use the LPV gain-scheduling method to design the air-fuel ratio controller. The scheduling parameter will be the time delay in the exhaust system, which is considered to be a function of the engine operating point. The relation between the time delay and the engine operating point defined by the engine speed and air mass flow is identified off-line. On-line identification of the time delay in the exhaust system can be found in [14] for stoichiometric burn engines. However, on-line identification is too slow and gain-scheduling based on the on-line identification is impossible. Hence, in the present work a parametric expression is used to estimate the delay based on measurements of engine speed and air mass flow. The experimental vehicle data used for the time delay identification were collected from a Ford F-150 truck with a V8 4.6 Liter lean burn engine. The designed LPV controller was tested on the same truck model. All the experimental data presented in the paper were provided by Ford Motor Company.

The paper is organized as follows. Section 2 describes the engine model for the air-fuel ratio subsystem in the lean burn mode. The time delay in the exhaust system is identified and a simplified engine model for the control design is proposed. In section 3, the LPV controller design for the simplified engine model is discussed. The LPV controller implementation and discretization are discussed in section 4. Finally, the experimental results are presented in section 5. Section 6 concludes the paper.

## II. ENGINE MODEL FOR AIR-FUEL RATIO DYNAMICS IN THE LEAN BURN MODE

The lean burn aftertreatment system with commonly used sensors and the "outer-feedback" control system configuration is shown in Figure 1. It consists of a conventional TWC, an LNT, a HEGO sensor downstream the TWC and two UEGO sensors downstream the engine and LNT. The measurement signal of the controller is the air-fuel ratio measured by the UEGO sensor downstream the LNT. The controller regulates the air-fuel ratio of the air-fuel mixture entering the engine to follow the reference air-fuel ratio using the information from the engine, such as air mass flow (MAF) and engine speed (RPM). The commanded fuel for the fuel injector is calculated based on the air-fuel ratio command and the cylinder air charge per engine cycle. Thus, the input of the open loop engine model is the commanded air-fuel ratio at the fuel injector and the output is the measured air-fuel ratio by the post-LNT UEGO sensor.

During the lean burn mode, the TWC eventually becomes saturated by  $O_2$ , therefore, the complex oxygen dynamics introduced by the TWC and the LNT can be approximated by a pure time delay. Due to the feedback control bandwidth limitation introduced by the time delay, the dynamics of the UEGO sensor is above the control bandwidth and

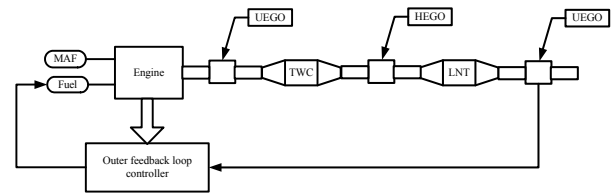


Fig. 1. Aftertreatment system and "outer-feedback" loop AFR control system configuration in the lean burn mode

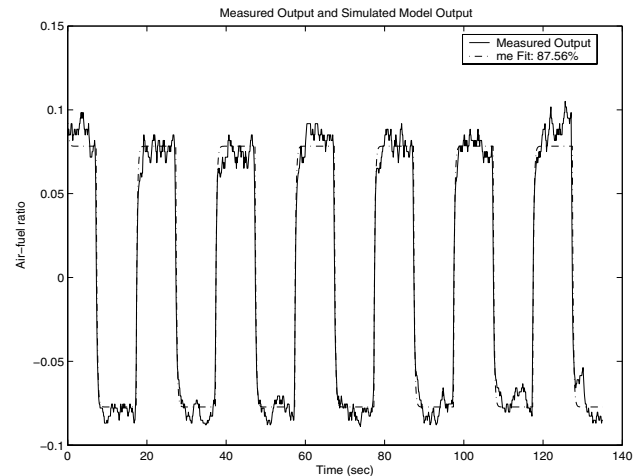


Fig. 2. System identification example (Mean Engine Speed = 1137 rpm; Mean Air Mass Flow = 3.0047 lb/min; Mean AFR = 1.1)

can be neglected for feedback control design. Hence, the simplified engine "outer-feedback loop" air-fuel ratio model can be considered as an pure variable time delay for the feedback control design. The time delay is identified using the available experimental vehicle data. A first order ARX model and time domain least squares estimation method are used for model identification. Figure 2 shows a representative experimental and identified response. The solid line is the measured air-fuel ratio and dash-dot line is the estimated model output. The mean air-fuel ratio is removed from the data for identification purposes. It can be seen that the identified model matches the measurement data well.

Since the overall time delay  $\tau$  is identified using the experimental data, the relation between the time delay and the engine operating condition is approximated as follow. The overall time delay  $\tau$  of the air-fuel ratio from the fuel injection command to tailpipe UEGO sensor consists mainly of two parts: the cycle delay  $\tau_c$  and the exhaust gas transport delay  $\tau_s$ . The cycle delay  $\tau_c$  is due to the four stokes of the engine and it is approximately one engine cycle. Hence, the cycle delay is given by

$$\tau_c = \frac{720}{(360/60)n} = \frac{120}{n}$$

where  $n$  is the engine speed. The transport delay  $\tau_s$  is due to the exhaust gas flowing from the exhaust valve to the tailpipe UEGO sensor and it varies inversely with the air flow rate

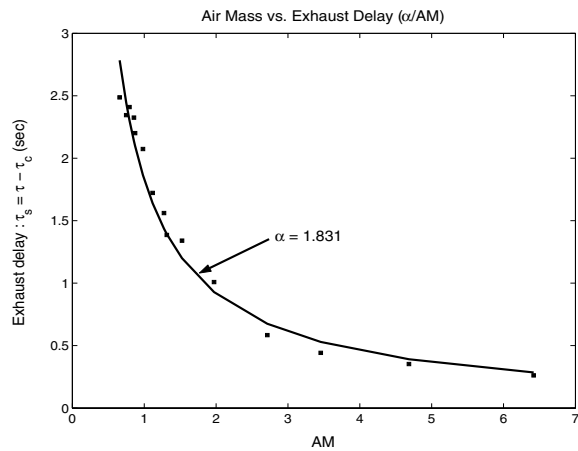


Fig. 3. Air Mass vs. Exhaust Delay

$\dot{m}_a$  assuming an average exhaust temperature, i.e.

$$\tau_s = \frac{\alpha}{\dot{m}_a}$$

where  $\alpha$  is determined based on the experimental data and time delay identification results. Thus, an approximate estimate for the overall time delay of the feedback system is given by  $\tau = \tau_c + \tau_s$ . Since one engine cycle delay has a fixed relation with the engine speed, we will remove the engine cycle delay part from the identified time delay. The residue is the exhaust delay which is considered to be solely dependent on the air mass flow. Based on the time delay identification results, we obtain an average value  $\alpha = 1.831$  for the F-150 V8 4.6L engine. Hence, the approximate estimation of the time delay is given by:

$$\tau = \left[ \frac{120}{n} + \frac{1.831}{\dot{m}_a} \right] \text{ (sec)} \quad (1)$$

Figure 3 shows the relation between the air mass flow and the exhaust delay. The squares represent the identified exhaust delay ( $\tau - \tau_c$ ) and the solid line is the estimated exhaust delay  $\tau_s = \frac{\alpha}{\dot{m}_a}$ . The average estimation error is 0.115 sec. It is worth mentioning that the sampling time for the experimental data is 0.1 sec. Therefore, the delay estimation using equation (1) approximates the identified time delay very well. In the following, the time delay estimation based on the engine speed and the air mass flow is used to schedule the designed LPV controller. Notice that the engine speed alone is not sufficient to characterize the time delay variability. The proposed time delay estimation is based on both engine speed and air mass flow measurements. Therefore, the LPV controller is scheduled based on the real-time measurement of these parameters.

### III. LPV CONTROLLER DESIGN FOR A SIMPLIFIED ENGINE MODEL

#### A. Controller design objectives and challenges

During a  $\text{NO}_x$  purge cycle, the engine is running at a rich air-fuel ratio to purge  $\text{NO}_x$  from the trap. Then the engine is subsequently placed in a lean air-fuel ratio mode, i.e., the

engine is commanded to run at a predetermined lean air-fuel ratio, typically 1.1 or 1.4 times nominal stoichiometry. A feedback controller is required to regulate the target lean air-fuel ratio in the presence of disturbances such as canister purge, open loop uncertainties and unmodelled dynamic effects. The design objectives are the following:

*Tracking performance:* The controller should control the engine to reach the reference air-fuel ratio as quickly as possible and the final steady state error should tend to zero.

*Disturbance rejection:* For the system subject to uncertain disturbances, such as canister purge and measurement noise, the air-fuel ratio excursion should be as low as possible.

*Transient response:* The percentage overshoot should be minimized. Zero percent overshoot is desired. There is a trade-off between this requirement and the disturbance rejection requirement.

*Implementation:* The designed LPV controller should have a simple structure for the ease of implementation and calibration.

Furthermore, the above design objectives need to be achieved for the full engine operating envelope. From Section 1, it can be seen that the variation of the time delay due to the engine operating condition change is large. It is known that the time delay results in a bandwidth limitation for the feedback control. Thus, the main challenge here is to handle the large time delay variation so that satisfactory feedback control performance is achieved regardless of the delay. In addition, the controller design method should be generic, i.e., the same design procedure should be carried out to obtain controllers for different models of the vehicles.

#### B. LPV control design

The general system interconnection for the LPV controller design in the lean burn mode is shown in Figure 4, where  $r(t)$  is the commanded air-fuel ratio input for the fuel injection system,  $y(t)$  is the tailpipe air-fuel ratio measurement and  $G_{sys}$  is the simplified engine model, which is considered to be a pure variable time delay. For the LPV control design,  $G_{sys}$  is approximated by a 1st order parameter dependent Pade approximation given by

$$G_{sys}(s) \cong \frac{1 - \frac{\tau}{2}s}{1 + \frac{\tau}{2}s} \quad (2)$$

where  $\tau$  is the time delay in the engine model. Higher order Pade approximation can be used for the LPV design at the cost of increasing the order of the designed controller, which is undesirable for implementation. In order to achieve zero steady state error corresponding to an air-fuel ratio set-point, the LPV controller is selected to have an explicit integrator term. Hence,  $K_{LPV}$  is the parameter varying part of the LPV controller to be designed, which is scheduled using the time delay estimation based on the engine speed and the air mass flow.

The weighting functions play an important role in the LPV control design. For improved performance, higher order weighting functions should be applied. However, this will increase the order of the dynamic controller. In practical

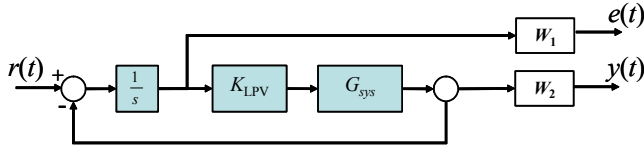


Fig. 4. Weighted Control Structure for LPV Synthesis

applications, a high order controller is undesirable. In the present application, a constant performance weight and a 1st order robustness weight will be used for the design. The selected weighting functions are given by

$$W_1 = 0.1 \quad (3)$$

$$W_2 = \frac{s}{s/400 + 1/1.4} \quad (4)$$

The state space realization of the 1st order Pade approximation is parameterized by  $\rho = \frac{1}{\tau}$  affinely. Therefore, for the LPV controller design,  $\rho$  is chosen as the design parameter and will be used to schedule the LPV controller in real-time. It can be calculated based on the measured engine speed and the air mass flow. For details about the LPV control design, see [11], [15] for reference.

The designed LPV controller provides satisfactory time-domain and frequency domain performance for the full engine speed operating region. However, this controller has a complicated structure that does not allow its implementation in automotive applications. In the next section, we will reduce the controller order and we will formulate all vertex controllers in a simple analytical parameter-dependent format with coefficients represented as functions of the time delay, which greatly simplifies its practical implementation.

### C. Controller order reduction and explicit parameterization

Each vertex controller of the designed LPV controller has 3 poles and 2 zeros. However, there is only 1 dominant pole and 1 dominant zero at every vertex controller. The additional 2 poles and 1 zero only characterize the high frequency dynamics of the controller outside the frequency region of interested. A simplified controller structure can be achieved by removing the high frequency dynamics of the vertex controllers and keeping only the corresponding steady state gain. Figure 5 shows the LPV controller model reduction results. It can be seen that the low frequency characteristics of the controller are preserved very well.

As a result, the scheduled controller obtains the following simple parameter dependent structure:

$$K(s) = K_g \left( \frac{\frac{s}{T_z} + 1}{\frac{s}{T_p} + 1} \right) \quad (5)$$

where the parameters  $K_g$ ,  $T_z$  and  $T_p$  are functions of the time delay, which is a function of the engine speed  $n$  and the air mass flow  $\dot{m}_a$ . To simplify the controller implementation, we obtain analytical expressions of the parameters  $K_g$ ,  $T_z$  and  $T_p$  as functions of the delay using polynomial interpolation

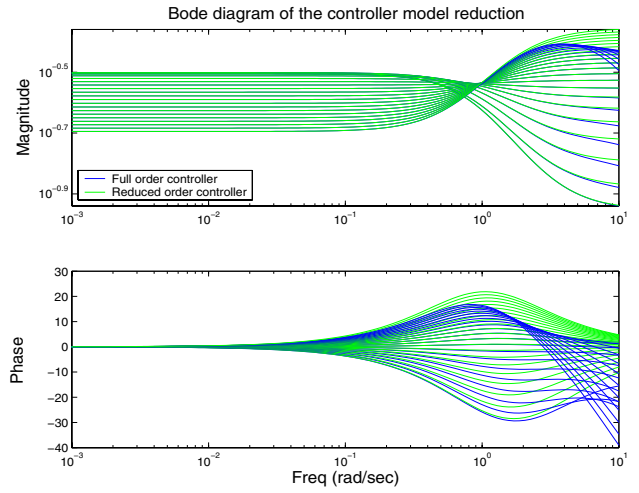


Fig. 5. LPV controller model reduction results

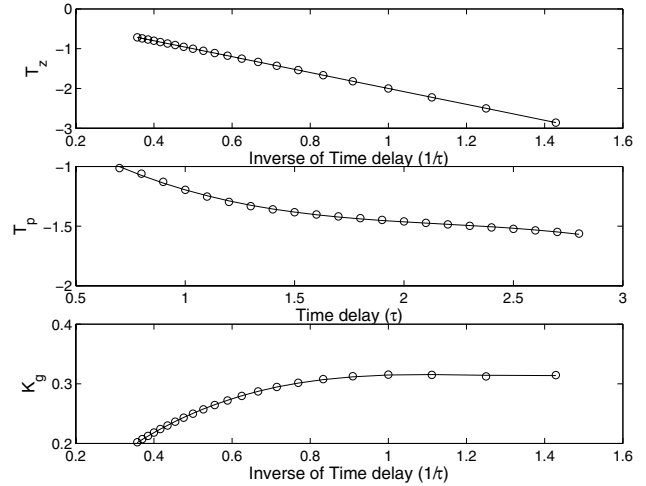


Fig. 6. Polynomial fitting for the coefficient of the LPV controller

(see figure 6). The results of the polynomial fitting are the followings:

$$\begin{aligned} K_g &= 0.1807 \left( \frac{1}{\tau} \right)^3 - 0.6701 \left( \frac{1}{\tau} \right)^2 + 0.8173 \left( \frac{1}{\tau} \right) \\ &\quad - 0.0133 \\ T_z &= 2.0010 \left( \frac{1}{\tau} \right) - 0.0005 \\ T_p &= 0.1030\tau^3 - 0.6736\tau^2 + 1.5683\tau + 0.1984 \end{aligned} \quad (6)$$

**Remark:** The expressions in (6) provide a simple functional dependence of the parameters  $K_g$ ,  $T_z$  and  $T_p$  with respect to the delay  $\tau$  that define a simple explicit parameterization of the LPV controller. Such an explicit functional dependence cannot be obtained from single point  $H_\infty$  designs.

**Remark:** The proposed simplified gain scheduled controller has a 1st order lead-lag compensator form where the controller parameters  $K_g$ ,  $T_z$  and  $T_p$  are given from explicit analytical formulas as a function of the engine time delay

(or the engine speed  $n$  and the air mass flow  $\dot{m}_a$ ). Hence the complexity of the controller is low allowing efficient implementation.

**Remark:** The LPV controller design does not use any vehicle information other than the time delay estimation result and the design is solely based on the time delay range. This results in a general implementation framework because the designed controller can be implemented on any other vehicle with same time delay range without any controller redesign. However, the time delay estimation used for scheduling needs to be tailored for different vehicles.

#### D. Simulation results

The designed LPV controller is validated through time-domain simulations and experimental vehicle tests. An experimental speed and air mass flow profile from the Federal Test Procedure (FTP) drive circle (see Figure 7) is used for simulation. The engine model for the simulation is a variable time delay component that uses the estimated time delay. In order to characterize the open loop disturbances in the engine operation, such as fuel injector uncertainty and canister purge, the disturbance profile in Figure 8 is added to the air-fuel ratio input of the engine.

In practice, the estimated time delay will not match the actual time delay in the engine and exhaust system exactly. Therefore, simulations with delay estimation errors have also been carried out. Figure 9 shows the simulation results for 3 cases: (i) exact time delay estimation, i.e., the LPV controller uses the same variable time delay as the actual time delay in the engine model; (ii) 20% time delay underestimation; (iii) 20% time delay overestimation. The dashed line is the preset air-fuel reference signal. The solid line is the simulated tailpipe air-fuel ratio output when the estimation of the time delay is exact. The corresponding delay underestimated case is shown by the dash-dot line and the delay overestimated case is shown by the dotted line. It can be seen that the LPV controller successfully regulates the tailpipe air-fuel ratio to a preset reference in the face of the large variable delay in the engine model. The robustness of the LPV controller to delay estimation errors is indeed very good. It can be seen that the three corresponding time responses are almost overlapping. A more detailed view of the simulation results is shown in Figure 10. It can be observed that for the underestimated delay case the response speed is a bit faster and there is small increase in the percentage overshoot. For the overestimated delay case, the response speed is a bit slower. This is as expected because the phase margin of the closed-loop system will increase when the time delay is overestimated.

#### IV. EXPERIMENTAL RESULTS

Finally, the controller was implemented on F-150 trucks with V8 4.6 Liter lean burn engines. Figure 11 shows the experimental results for the low speed driving situation using the designed LPV controller, which characterizes the long delay case. Figure 12 shows the highway driving experimental results using the LPV controller, which represents the short delay case. It can be seen that the settling time is always less

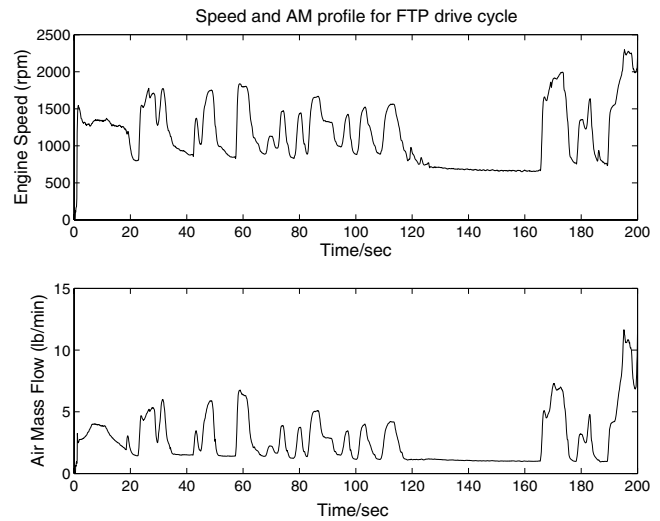


Fig. 7. Speed and air mass flow profile

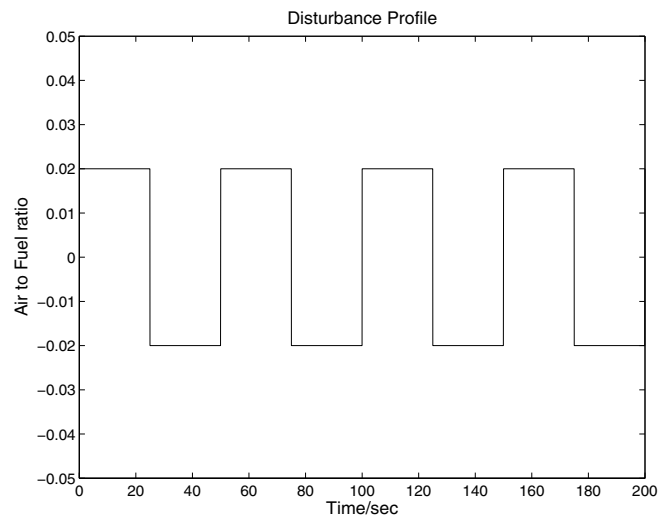


Fig. 8. Disturbance profile

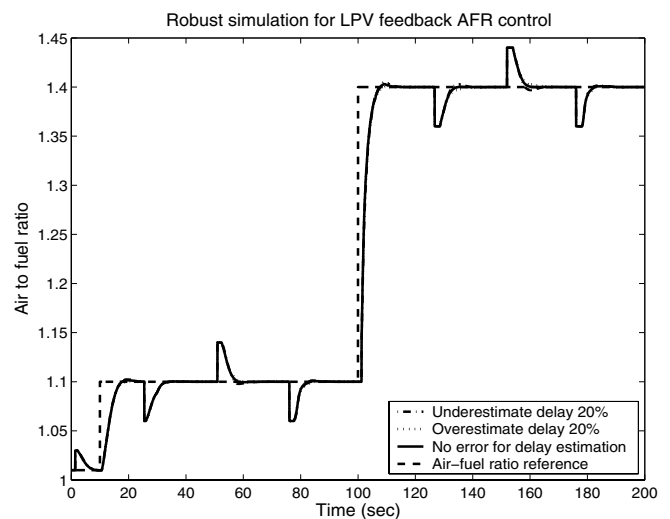


Fig. 9. Air-fuel ratio regulation simulation results

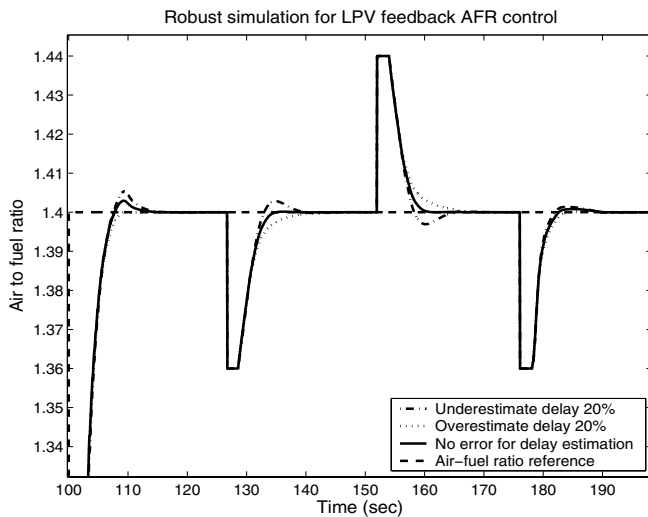


Fig. 10. Air-fuel ratio regulation simulation results (Zoom In)

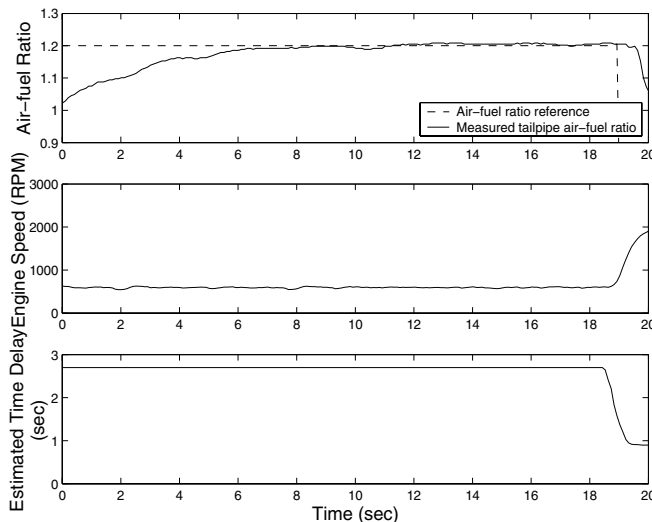


Fig. 11. Low engine speed driving result (LPV controller)

than 10 sec despite the engine speed changes. No significant overshoot is observed. Clearly, the LPV controller provides satisfactory performance at both low and high engine speeds.

## V. CONCLUSION

An LPV controller is designed for precise air-fuel ratio control in lean burn SI engines. The controller is scheduled based on the variable time delay of the feedback loop to accommodate the engine speed and air mass flow variability. A 1st order simplified gain-scheduled controller that includes polynomial forms of the controller gains as a function of the control loop delay is obtained to satisfy desired transient and steady-state response characteristics. Both simulations and experimental results in Ford F-150 trucks with V8 4.6 Liter lean burn engines demonstrate the efficiency of the LPV controller to regulate the tailpipe air-fuel ratio for the full engine operating envelope.

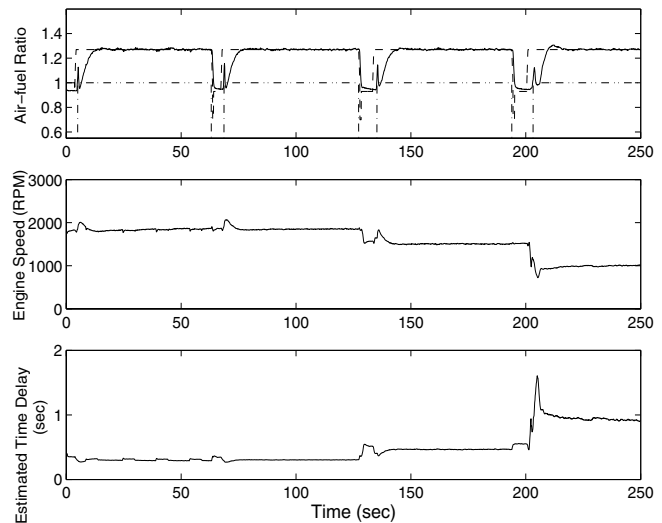


Fig. 12. Highway driving experimental results (LPV Controller)

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