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Optimal Design of a Feedback Controller that Achieves Output Regulation in the Presence of Actuator Saturation

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Abstract— **In this paper, we show a design method of a feedback control law that achieves output regulation in the presence of actuator saturation. It is well-known that it is impossible to obtain a bounded feedback control law that achieves global output regulation in the case where the plant has unstable poles. For such systems, we show a method of constructing the regulatable region based on the notion of the invariant set theory. Furthermore, we show that we can design a feedback control law that minimizes the LQ type cost function by using the proposed method. The design problem is reduced to a convex optimization problem with linear matrix inequality (LMI) constraints. The effectiveness of the proposed method is shown through a numerical example and an experimental result.**

I. Introduction

In many practical control systems, there exist saturation limitations on controller outputs. To design control systems which achieve high control performance (e.g., tracking performance, disturbance attenuation), design methods of a controller that explicitly take into account the actuator limitation are required. In this paper, we consider a design method of a feedback controller that achieves output regulation in the presence of the actuator limitation. The problem dealt with in output regulation is to design a feedback controller that internally stabilizes a given plant such that the output of the resulting closed-loop system converges to a certain reference signal [10]. The output regulation problem in the presence of input constraints has been investigated in [7], [10], [11]. In [10], a design method of a feedback controller that achieves the semi-global output regulation in the case where the plant has poles in the closed-left half plane. On the other hand, in [7], [11], design methods of a controller that can be applied to the unstable systems. However, in these literature, the optimality of the transient response is not considered.

In this paper, we will show a design method of a feedback controller that can be applied to the unstable systems and optimizes the transient response. By using the proposed controller, the output regulation can be achieved if the initial value of the state variable belongs to a set referred to as the regulatable region. Furthermore, we show that, by using the proposed method, we can design a feedback control law that minimizes the LQ type cost function. The design problem is reduced to a convex optimization problem with linear matrix inequality (LMI) [3] constraints. The effectiveness of the proposed method is shown through a numerical example and an experimental result.

Notations: For a vector $v \in \mathcal{R}^n$, we define the standard multivariable saturation function as $\Phi(v) := (\phi(v_1), \cdots, \phi(v_n))^T$, where

$$\phi(v_i) := \begin{cases} \mathrm{sgn}(v_i), & |v_i| > 1 \\ v_i, & |v_i| \le 1 \end{cases}$$

For a vector $v \in \mathcal{R}^n$, we denote its Euclidean norm as $\|v\|_2 := (v^T v)^{1/2}$. For a positive definite matrix $P \in \mathcal{R}^{n\times n}$, we denote $\mathcal{E}(P,\rho) := \{x \in \mathcal{R}^n : x^T P x \le \rho\}$. For a matrix $F \in \mathcal{R}^{m\times n}$, we denote the ith row of F as $F^{(i)}$. Furthermore, we define $\mathcal{L}(F,\rho) := \{x \in \mathcal{R}^n : |F^{(i)}x| \le \rho_i, i = 1,\cdots,m\}$, where $\rho = \mathrm{diag}[\rho_1,\cdots,\rho_m]$. Let $\mathcal{V}$ be the set of $m \times m$ diagonal matrices whose diagonal element are either 1 or 0. We suppose that each element of $\mathcal{V}$ is labeled as $\mathbf{E}_i, i = 1,2,\cdots,2^m$, and denote $\mathbf{E}_i^- := I - \mathbf{E}_i$.

II. Output Regulation

Let us consider the system described by

$$\begin{aligned} x(t+1) &= Ax(t) + Bu(t) + Ew(t) & (1)\\ z(t) &= Cx(t) + Du(t) + D_w w(t) & (2)\\ u(t) &= \Phi(v(t)) & (3) \end{aligned}$$

where $x \in \mathcal{R}^n, u \in \mathcal{R}^m, v \in \mathcal{R}^m, w \in \mathcal{R}^p, z \in \mathcal{R}^q$. We suppose that the signal w is generated by the following dynamics.

$$w(t+1) = Sw(t) \quad (4)$$

Further, we suppose that the system (4) is neutrally stable and $\|w(t)\|_2 \le w_{\max}, \forall t \ge 0$.

We consider the following problem.

Problem 1: Consider the system (1)–(4). Design a state feedback control law

$$v(t) = Fx(t) + Mw(t) \quad (5)$$

that guarantees $\lim_{t\to\infty} z(t) = 0$ and minimizes $J := \sum_{t=0}^{\infty} \|z(t)\|_2^2$.

For this problem, we can obtain the following result.

Theorem 1: Consider the system (1)–(5). We suppose that there exist matrices $\Pi \in \mathcal{R}^{n\times p}, \Gamma \in \mathcal{R}^{m\times p}$ that satisfy

$$\begin{aligned} \Pi S &= A\Pi + B\Gamma + E & (6)\\ 0 &= C\Pi + D\Gamma + D_w & (7) \end{aligned}$$

Further, we suppose that $\max_{t\ge 0} |\Gamma^{(l)} w(t)| < 1, \forall l \in [1,m]$. For given positive definite matrices $Q_i, (i =$

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$1,\cdots,2^m$) and a positive scalar α, assume that there exist matrices G, Y, Z and a positive scalar γ that satisfy

$$\begin{bmatrix} G+G^T-Q_i & * & * \\ CG+D(\mathbf{E}_iY+\mathbf{E}_i^-Z) & \gamma I & * \\ AG+B(\mathbf{E}_iY+\mathbf{E}_i^-Z) & 0 & Q_j \end{bmatrix} > 0 \quad \forall i,j \in [1,2^m] \tag{8}$$

$$\begin{bmatrix} G+G^T-Q_i & * \\ Z^{(l)} & \rho_l^2/\alpha \end{bmatrix} \geq 0 \quad \forall i \in [1,2^m],\ \forall l \in [1,m] \tag{9}$$

where $\rho_l := 1 - \max_{t\geq 0}|\Gamma^{(l)}w(t)|$ and the symbol $*$ stands for symmetric block in matrix inequalities. Further, we suppose that $\xi(0) \in \mathcal{E}(P(\lambda(0)),\alpha)$. Then, by applying the feedback control law (5) with $F = YG^{-1}$ and $M = \Gamma - F\Pi$ to the system (1)–(4), the relations $\xi(t) \in \mathcal{E}(P(\lambda(t)),\alpha), \forall t \geq 0$, $\lim_{t\to\infty} z(t) = 0$ and $J < \gamma\alpha$ hold. Here we denote $\xi := x - \Pi w$ and $P(\lambda) := \sum_{i=1}^{2^m}\lambda_i P_i,\ P_i := Q_i^{-1}$.

Proof: From eqs.(1),(3)–(6), we obtain

$$\xi(t+1) = A\xi(t) + B\Psi(F\xi(t)) \tag{10}$$

where $\Psi(F\xi) := \Phi(F\xi + \Gamma w) - \Gamma w$ (see Fig.1). In the following, we first show that if $\xi \in \mathcal{L}(H,\rho)$ and $\max_{t\geq 0}|\Gamma^{(l)}w(t)| < 1, \forall l \in [1,m]$, the nonlinearity $\Psi(F\xi)$ can be represented as $\Psi(F\xi) = \sum_{i=1}^{2^m}\lambda_i\{\mathbf{E}_iF + \mathbf{E}_i^-H\}\xi$, where $H \in \mathcal{R}^{m\times n}$ and $\rho := \mathrm{diag}[\rho_1,\cdots,\rho_m]$. If $\xi \in \mathcal{L}(H,\rho)$ and $\max_{t\geq 0}|\Gamma^{(l)}w(t)| < 1, \forall l \in [1,m]$, then $|H^{(l)}\xi + \Gamma^{(l)}w| \leq 1, \forall l \in [1,m]$. Hence, in this case, the relation $\Phi(F\xi+\Gamma w) = \sum_{i=1}^{2^m}\lambda_i\{\mathbf{E}_i(F\xi+\Gamma w)+\mathbf{E}_i^-(H\xi+\Gamma w)\}$ holds [6] (see Lemma 1 in Appendix). Therefore, we can show that

$$\begin{aligned} \Psi(F\xi) &= \sum_{i=1}^{2^m}\lambda_i\{\mathbf{E}_i(F\xi+\Gamma w)+\mathbf{E}_i^-(H\xi+\Gamma w)\} \\ &\qquad - \sum_{i=1}^{2^m}\lambda_i\{\mathbf{E}_i+\mathbf{E}_i^-\}\Gamma w \\ &= \sum_{i=1}^{2^m}\lambda_i\{\mathbf{E}_iF+\mathbf{E}_i^-H\}\xi \end{aligned} \tag{11}$$

By using this relation, if $\xi(t) \in \mathcal{L}(H,\rho)$ and $\max_{t\geq 0}|\Gamma^{(l)}w(t)| < 1, \forall l \in [1,m]$, the close-loop system (1), (3) and (5) can be rewritten as

$$\xi(t+1) = \mathcal{A}(\lambda(t))\xi(t) \tag{12}$$

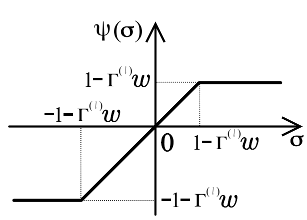

Fig. 1. Nonlinearity Ψ

where $\mathcal{A}(\lambda) := \sum_{i=1}^{2^m}\lambda_i\mathcal{A}_i,\ \mathcal{A}_i := A + B\{\mathbf{E}_iF + \mathbf{E}_i^-H\}$. Similarly, from eqs.(2), (3), (5) and (7), if $\xi(t) \in \mathcal{L}(H,\rho)$ and $\max_{t\geq 0}|\Gamma^{(l)}w(t)| < 1, \forall l \in [1,m]$, the controlled output $z(t)$ can be represented as

$$z(t) = \mathcal{C}(\lambda(t))\xi(t) \tag{13}$$

where $\mathcal{C}(\lambda) := \sum_{i=1}^{2^m}\lambda_i\mathcal{C}_i,\ \mathcal{C}_i := C + D\{\mathbf{E}_iF + \mathbf{E}_i^-H\}$. From eqs.(12) and (13), in the ξ-coordinate system, the feedback system can be regarded as the system without exogenous input. In the following, we prove Theorem 1 based on this representation.

Since $G + G^T - Q_i > 0$ and $Q_i > 0$ from eq.(8), the matrix G is nonsingular. In the following, we first show that the condition (9) implies that $\mathcal{E}(P(\lambda),\alpha) \subseteq \mathcal{L}(H,\rho)$, where $H = ZG^{-1}$. Since $Q_i > 0$, we have $(G-Q_i)^TQ_i^{-1}(G-Q_i) \geq 0$[8]. Hence, the following inequality holds.

$$G^TQ_i^{-1}G \geq G + G^T - Q_i \tag{14}$$

Then, from eq.(14) and $Z^{(l)} = H^{(l)}G$, we can conclude that eq.(9) implies

$$\begin{bmatrix} G^TQ_i^{-1}G & * \\ H^{(l)}G & \frac{\rho_l^2}{\alpha} \end{bmatrix} \geq 0,\ \forall i \in [1,2^m], \forall l \in [1,m] \tag{15}$$

By multiplying eq.(15) from the left by block-diag$[G^{-T},1]$ and from the right by block-diag$[G^{-1},1]$, and substituting $Q_i^{-1} = P_i$ for the resulting inequality, and multiplying each inequality by λ_i, and summing them up for $i = 1,\cdots,2^m$, and applying Schur complement, we have

$$\frac{1}{\rho_l^2}H^{(l)T}H^{(l)} \leq \frac{1}{\alpha}P(\lambda),\ \forall l \in [1,m] \tag{16}$$

Eq.(16) implies that $\mathcal{E}(P(\lambda),\alpha) \subseteq \mathcal{L}(H,\rho)$.

Then, we show that the relations $\xi(t) \in \mathcal{E}(P(\lambda(t)),\alpha), \forall t \geq 0$, $\lim_{t\to\infty} z(t) = 0$ and $J < \gamma\alpha$ hold. By substituting $Z = HG$ and $Y = FG$ for eq.(8), we obtain

$$\begin{bmatrix} G+G^T-Q_i & * & * \\ \mathcal{C}_iG & \gamma I & * \\ \mathcal{A}_iG & 0 & Q_j \end{bmatrix} > 0, \forall i,j \in [1,2^m] \tag{17}$$

By taking account of eq.(14), we can state that eq.(17) implies

$$\begin{bmatrix} G^TQ_i^{-1}G & * & * \\ \mathcal{C}_iG & \gamma I & * \\ \mathcal{A}_iG & 0 & Q_j \end{bmatrix} > 0, \forall i,j \in [1,2^m] \tag{18}$$

Then, by multiplying eq.(18) from the left by block-diag$[G^{-T}, I, Q_j^{-1}]$ and from the right by block-diag$[G^{-1}, I, Q_j^{-1}]$, and substituting $Q_j^{-1} = P_j,\ Q_i^{-1} = P_i$ for the resulting inequality, we obtain

$$\begin{bmatrix} P_i & * & * \\ \mathcal{C}_i & \gamma I & * \\ P_j\mathcal{A}_i & 0 & P_j \end{bmatrix} > 0, \forall i,j \in [1,2^m] \tag{19}$$

Then, by multiplying eq.(19) by $\lambda_i(t)$, and summing them up for $i = 1,\cdots,2^m$, and multiplying each inequality by

$\lambda_j(t+1)$ and summing them up for $j = 1, \cdots, 2^m$, and by multiplying the resulting inequality from both sides by $\mathrm{diag}[I, I, P(\lambda(t+1))^{-1}]$, we obtain

$$\begin{bmatrix} P(\lambda(t)) & * & * \\ \mathcal{C}(\lambda(t)) & \gamma I & * \\ \mathcal{A}(\lambda(t)) & 0 & P(\lambda(t+1))^{-1} \end{bmatrix} > 0 \tag{20}$$

By applying Schur complement to eq.(20), and multiplying the resulting inequality from the left by $\xi(t)^T$ and from the right by $\xi(t)$, and using eqs.(12), (13), we have

$$V(\xi(t+1)) - V(\xi(t)) < -\frac{1}{\gamma}\|z(t)\|_2^2 \tag{21}$$

where $V(\xi(t)) := \xi(t)^T P(\lambda(t))\xi(t)$. From eq.(21), we can conclude that if $\xi(0) \in \mathcal{E}(P(\lambda(t)), \alpha)$ then

$$V(\xi(t)) < V(\xi(0)) \le \alpha, \ \ \forall t \ge 0 \tag{22}$$

Eq.(22) implies that $\xi(t) \in \mathcal{E}(P(\lambda(t)), \alpha)$, $\forall t \ge 0$. By the way, the time varying nonlinearity $\Psi(F\xi(t))$ can be represented as $\Psi(F\xi(t)) = \sum_{i=1}^{2^m} \lambda_i(t)\{\mathbf{E}_i F + \mathbf{E}_i^- H\}\xi(t)$ if $\xi(t) \in \mathcal{L}(H, \rho)$ and $\max_{t\ge 0} |\Gamma^{(l)} w(t)| < 1, \forall l \in [1, m]$. From eqs.(16) and (22), we can state that if the conditions in Theorem 1 hold, the relation $\xi(t) \in \mathcal{L}(H, \rho), \forall t \ge 0$ holds. From eq.(21), since $\xi(t) \to 0, (t \to \infty)$, $z(t) \to 0, (t \to \infty)$. Further, since the inequality $\sum_{i=0}^{t} \|z(t)\|_2^2 < \gamma\alpha$ holds from eqs.(21) and (22), we have $J < \gamma\alpha$. ∎

Based on Theorem 1, the following optimization problem can be obtained.

Problem 2: Compute the matrices Γ and Π by solving eqs.(6) and (7). Then, compute the matrix F by solving $\min_{G,Z,Y,Q_i>0} \gamma$ s.t. eqs.(8) and (9) for given α.

Remark 1: Eqs.(6) and (7) are the conditions for the output regulation problem is solvable in the case of linear systems [4], [10].

Remark 2: The parameter dependent Lyapunov function used in Section was introduced in [8] and applied to nonlinear $\mathcal{H}_\infty$ control problem in [12].

III. An Extension

The objective of the design problem presented in the previous section is only to design a control law that make the signal z converge to zero as soon as possible. Therefore, the obtained feedback control law may produce an extremely large control signal as v. In this section, to prevent such a problem, we show a method of introducing a term into the cost function J that imposes a penalty on the signal v. We can obtain the following result.

Corollary 1: Consider the system (1)–(5). We suppose that there exist matrices $\Pi \in \mathcal{R}^{n\times p}, \Gamma \in \mathcal{R}^{m\times p}$ that satisfy eqs.(6) and (7). Further, we suppose that $\max_{t\ge 0} |\Gamma^{(l)} w(t)| < 1, \forall l \in [1, m]$. For given positive definite matrices $Q_i, (i = 1, \cdots, 2^m)$, $\mathbf{R}$ and a positive scalar α, assume that there exist matrices G, Y, Z and a positive scalar γ that satisfy eq.(9) and

$$\begin{bmatrix} G + G^T - Q_i & * & * \\ \begin{bmatrix} CG \\ \mathbf{R}^{\frac{1}{2}} Y \end{bmatrix} + \mathbf{D}(\mathbf{E}_i Y + \mathbf{E}_i^- Z) & \gamma I & * \\ AG + B(\mathbf{E}_i Y + \mathbf{E}_i^- Z) & 0 & Q_j \end{bmatrix} > 0$$
$$\forall i, j \in [1, 2^m] \tag{23}$$

where $\mathbf{D} := [D^T, 0]^T$ and $\rho_l := 1 - \max_{t\ge 0} |\Gamma^{(l)} w(t)|$. Further, we suppose that $\xi(0) \in \mathcal{E}(P(\lambda(0)), \alpha)$. Then, by applying the feedback control law (5) with $F = YG^{-1}$ and $M = \Gamma - F\Pi$ to the system (1)–(4), the relations $\xi(t) \in \mathcal{E}(P(\lambda(t)), \alpha), \forall t \ge 0$, $\lim_{t\to\infty} \mathbf{z}(t) = 0$ and $J = \sum_{t=0}^{\infty} \|\mathbf{z}(t)\|_2^2 < \gamma\alpha$ hold, where $\mathbf{z} := [z^T, u_e^T \mathbf{R}^{1/2}]^T$, $\xi := x - \Pi w$, $u_e := v - \Gamma w$ and $P(\lambda) := \sum_{i=1}^{2^m} \lambda_i P_i$, $P_i := Q_i^{-1}$.

Proof: From the definition, the signal $\mathbf{z}$ can be rewritten as

$$\begin{aligned} \mathbf{z} &= \begin{bmatrix} z \\ \mathbf{R}^{\frac{1}{2}} u_e \end{bmatrix} \\ &= \mathbf{C}x + \mathbf{D}u + \mathbf{D_w} w \end{aligned} \tag{24}$$

where

$$\mathbf{C} := \begin{bmatrix} C \\ \mathbf{R}^{\frac{1}{2}} F \end{bmatrix}, \ \mathbf{D_w} := \begin{bmatrix} D_w \\ \mathbf{R}^{\frac{1}{2}}(M - \Gamma) \end{bmatrix}$$

It can easily be verified that the matrices Π and Γ that satisfy eqs.(6) and (7) also satisfy

$$\mathbf{C}\Pi + \mathbf{D}\Gamma + \mathbf{D_w} = 0 \tag{25}$$

Therefore, by applying the procedure of the proof of Theorem 1 to the system (1),(3)–(5) and (24), we can show that the results in Corollary 1 hold. ∎

Remark 3: The cost function utilized in Corollary 1 is $J = \sum_{t=0}^{\infty}\{z(t)^T z(t) + u_e(t)^T \mathbf{R} u_e(t)\}$. The signal u_e represents the deviation between the controller output v and the signal Γw that is control signal in the steady state. Therefore, by designing a control law that minimizes this cost function, we can obtain a controller that makes the signal z converge to zero rapidly and prevents the signal v from being excessively large.

Based on Corollary 1, the following optimization problem can be obtained.

Problem 3: Compute the matrices Γ and Π by solving eqs.(6) and (7). Then, compute the matrix F by solving $\min_{G,Z,Y,Q_i>0} \gamma$ s.t. eqs.(9) and (23) for given α.

IV. Estimation of the Regulatable Region

Since the system matrix of (1) is not assumed to be stable, the initial set of the state variables of the feedback system (1)-(5) on which the output regulation can be achieved becomes local in general. Such a local set of the initial state variables is referred to as the regulatable region. In this section, we show a method for estimating the regulatable region based on Theorem 1. From Theorem 1, a subset of the regulatable region is given by $\{(x_0^T, w_0^T)^T \in \mathcal{R}^{n+p} | \mathcal{E}(P(\lambda(0))) \cap \mathcal{W}_0\}$, where $\mathcal{E}(P(\lambda(0))) = \{\xi_0 \in$

$\mathcal{R}^n | \xi_0^T P(\lambda(0))\xi_0 \le \alpha\}$, $\mathcal{W}_0 := \{w_0 \in \mathcal{R}^p : |\Gamma^{(l)} S^t w_0| < 1, \forall t \ge 0, \forall l \in [1, m]\}$, $\xi_0 = x_0 - \Pi w_0$. As we will see in the next numerical examples and experimental result, there exist cases where the set $\mathcal{W}_0$ is obvious from the structure of the matrix S. More generally, the set $\mathcal{W}_0$ can be obtained by computing the invariant set [2]. On the other hand, the inequality $\xi^T P(\lambda(0))\xi \le \alpha$ depends on the parameter $\lambda(0)$, and hence it is inconvenient for computing the regulatable region. This condition can be replaced by $\xi^T \mathcal{P} \xi \le \alpha$, where $\mathcal{P}$ is a positive definite matrix and satisfies $P_i \ge \mathcal{P}, \forall i \in [1, 2^m]$.

V. Numerical Example

Let us consider the system (1)-(4) with the following coefficient matrices [10].

$$A = \begin{bmatrix} 0 & -1 & -1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, E = \begin{bmatrix} 0 & 2 \\ -2 & -2 \\ -1 & 2 \\ -2 & -1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, D = 0, D_w = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$S = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

For this system, we solve Problem 2 with $\alpha = 1$, and obtain the following feedback controller.

$$v = \begin{bmatrix} -0.5965 & 0.1880 & 0.6002 & 0.1947 \\ -0.2142 & 0.5432 & -0.2213 & -0.5473 \end{bmatrix} x + \begin{bmatrix} 0.9963 & -0.3827 \\ 0.4355 & 1.0040 \end{bmatrix} w \quad (26)$$

Fig.2 and Fig.3 show the responses of the feedback system for $x(0) = [1, 1, 1, 1]^T, w(0) = [0.25, -0.25]^T$. Although the controller output v exceeds the limit ± 1, the signal z converges to zero rapidly.

VI. Experiment

Let us consider a mass damper system described by the following equation of motion (see Fig.4).

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{c}{M} \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{a}{M} \end{bmatrix} u, \quad y = \begin{bmatrix} 1 & 0 \end{bmatrix} x \quad (27)$$

where $x = [x_1, x_2]^T \in \mathcal{R}^2$ is the state variable, x_1[m] and x_2[m/s] represent position and speed of the body respectively, M[Kg] is the mass of the body, c[Ns/m] is the damping coefficient, a[N/V] denotes a conversion factor, u[V] represents a control input. The physical parameters of our experimental setup are $a/M = 17, c/M = 13$. We select the sampling time $T_s = 0.001$[sec] and discretize the plant using the zero-order hold. We choose $z = y - w$ as the controlled output. Further, we select $S = 1$ as the coefficient of the system (4), and set $w(0) = 0.25$. Thus, the objective of the design problem is to design a control law that makes the position of the mass converge to the step reference signal whose magnitude is 0.25[m] as soon

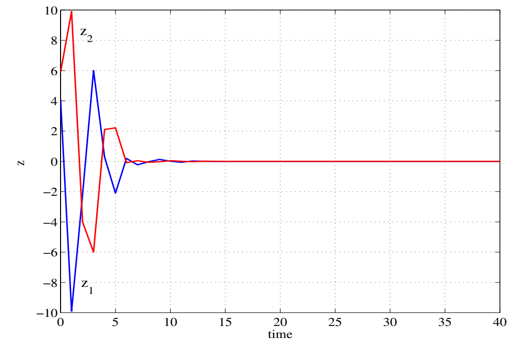

Fig. 2. Response of z

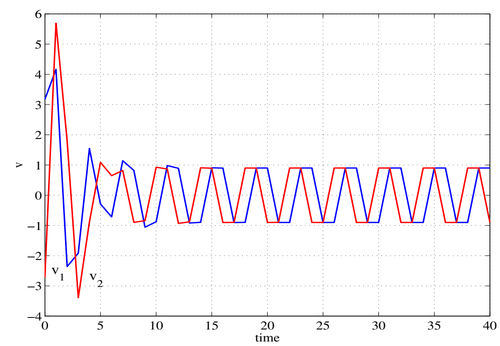

Fig. 3. Response of v

as possible. In the following numerical simulations and experiments, we design feedback control laws by solving Problem 3 with $\alpha = 1$. Further, we choose $x(0) = 0$ as the initial value of the state variable.

Fig.5 and Fig.6 show the responses of z and v of the system with the following feedback control law that is design by solving Problem 3 with $\mathbf{R} = 10^{-8}$.

$$v = \begin{bmatrix} -26.0455 & -1.5545 \end{bmatrix} x + 26.0455w \quad (28)$$

In these figures, the solid lines show the results of the experiment and the dashed lines show those of the numerical simulation. The results of the experiment are quite similar to those of the numerical simulation. It can be seen that the controlled output z converges to zero quickly.

Fig.7 and Fig.8 show the responses of z and v of the system with the following feedback control law that is design by solving Problem 3 with $\mathbf{R} = 10^{-3}$.

$$v = \begin{bmatrix} -3.9897 & -0.1881 \end{bmatrix} x + 3.9897w \quad (29)$$

In these figures, the solid lines show the results of the experiment and the dashed lines show those of the numerical

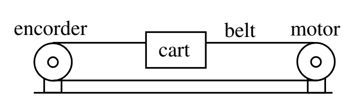

Fig. 4. Experimental setup

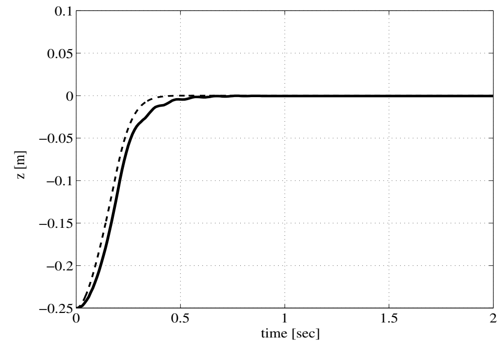

Fig. 5. Responses of z (solid:experiment, dashed:numerical simulation)

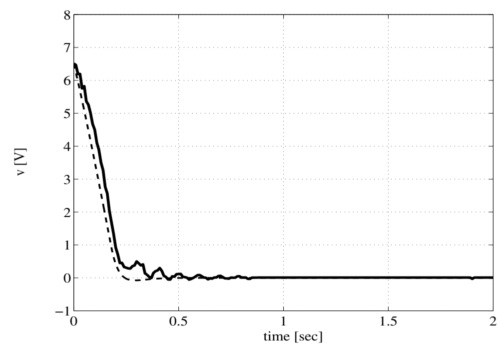

Fig. 6. Responses of v (solid:experiment, dashed:numerical simulation)

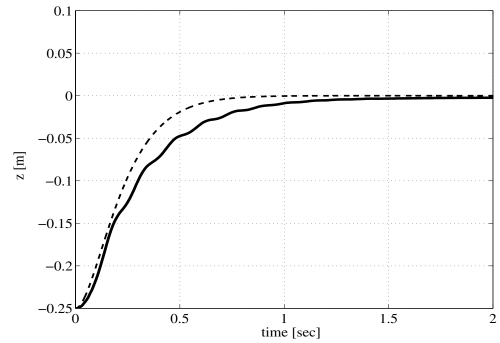

Fig. 7. Responses of z (solid:experiment, dashed:numerical simulation)

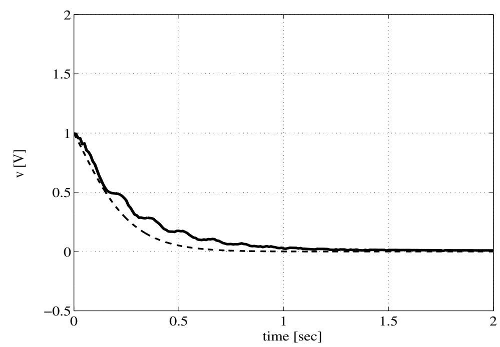

Fig. 8. Responses of v (solid:experiment, dashed:numerical simulation)

simulation. In this case, the magnitude of the controller output v is suppressed as compared with the case of (28). As a result, the response of the output z of the system with (29) becomes slower. Hence, we can expect that the speed of the transient response of the feedback system can be tuned by the matrix $\mathbf{R}$.

VII. Conclusions

We have shown a design method of a state feedback control law that achieves the output regulation in the presence of the actuator saturation. The design problem is reduced to a convex optimization problem with LMI constraints. We have applied the proposed design method to a numerical example of the multivariable system and an experiment of a position control problem of a mass damper system. Through the numerical simulation and the experiment, we have shown that, by using the proposed design method, we can obtain the feedback control laws that make the controlled output converge to zero quickly.

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Appendix

Lemma 1: [6] Let $v, f \in \mathcal{R}^m$. Suppose that $|f_i| \leq 1, \forall i \in [1, m]$, then $\Phi(v)$ can be represented as $\Phi(v) = \sum_{i=1}^{2^m} \lambda_i(\mathbf{E}_i v + \mathbf{E}_i^- f)$, where $0 \leq \lambda_i \leq 1,\ \sum_{i=1}^{2^m} \lambda_i = 1$.