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State Estimation for Repetitive Processes Using Iteratively Improving Moving Horizon Observers

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Abstract— **This paper considers the problem of state estimation for repetitive nonlinear systems. Taking the repetitive nature of the process into account a new state estimation scheme is proposed, which from repetition to repetition iteratively improves the estimate. The scheme combines ideas from iterative learning control and moving horizon state estimation. The state estimate during every repetition is based on approximately minimizing the deviation between the measured and estimated output. Stability and iterative improvements of the state estimates are ensured by enforcing a sufficient contraction of the deviation between the measured and estimated output over the considered estimation window. As shown, under the contraction constraints the state estimation scheme ensures asymptotic convergence of the state estimation error in the nominal case, provided that the system satisfies an uniform reconstructability condition.**

I. INTRODUCTION

Many processes are inherently repetitive, i.e. the same process happens over and over again. Typical examples for repetitively operating processes are:

- Industrial robot operations for welding, cutting, etc. .
- Batch processes in the chemical or the pharmaceutical industry.
- Synchrotrons for particle acceleration.
- Rail vehicles operated on a specific track over and over again.

The industrial importance of these processes has lead to significant research interest with respect to the control, modeling and analysis over the past decades [5, 7, 8].

In comparison to the control of standard/non repetitive continuous time systems systems the control of repetitive processes offers several challenges. Most of these challenges arise from the fact that one has to consider two distinct time scales. The time elapsing in every repetition and the iteration or repetition number, which is naturally discrete valued. Due to the additional degree of freedom in the time, repetitive systems are sometimes also referred to as 2D systems or two degree of freedom systems.

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By now several results with respect to the control of repetitive/iterative systems have been established. We do not go into details here, see for example [5, 7, 8].

The promising results in the area of repetitive, iterative and learning control schemes for repetitive processes naturally lead to the question if the same algorithms can be extended to the state estimation problem for repetitive processes. State estimation for repetitive processes is of practical as well as theoretical interest, since the estimated state can be used for various purposes such as process monitoring and state feedback control. The interest in the state estimation problem is also driven by the fact that very often the development of new control methods has given birth to analogous observers schemes. Thus, the focus of this paper is on the state estimation problem for repetitive processes, for which, surprisingly, only limited results are available by now, see for example [13].

In this paper we propose to combine ideas from iterative learning control with ideas from moving horizon observers [1, 6, 10, 11, 14]. Specifically we outline an extension of the contraction based moving horizon state estimation scheme proposed in [6] for repetitive processes.

In moving horizon state estimation the state estimate is obtained by (approximately) minimizing a cost functional, typically the deviation of the estimated and measured output, over a past measurement window. Stability of moving horizon estimation schemes for non repetitive continuous time systems is typically achieved by either employing an upper bound on the “initial weight”, the so called arrival cost [10], or by the application of a contraction constraint that enforces a sufficient decrease of the cost function. The approach proposed here is based on a double contraction constraint on the cost resulting from the approximate solution of the moving horizon problem, ensuring contraction/convergence of the estimation error in the run of each repetition as well as from repetition to repetition.

The paper is structured as follows: Section II introduces the considered problem setup, the considered system structure, and the objective of the paper. Section III outlines the basic idea behind the proposed moving horizon observer strategy for repetitive systems. The stability of the proposed approach in the sense of convergence of the state estimation error is established in Section IV. Section V contains a small application example of the outlined state estimation strategy considering a repetitively operated batch reactor. The paper is concluded in Section VI with some final discussions.

In the following $\|\cdot\|$ denotes the Euclidean vector norm. $\mathbb{N}_0$ denotes the nonnegative numbers including 0, while $\mathbb{N}$

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denotes the nonnegative numbers without 0.

II. Problem Setup

A. System Class and Assumptions

Throughout the paper we use the word repetition for the subsequent instances of the repetitive process. Specifically, the considered nonlinear time-invariant system that operates repetitively over the finite-time interval $[0,T]$ is given by

$$\dot{x}^j(t) = f(x^j(t), u^j(t)), \qquad x^j(0) = x_0^j \tag{1a}$$
$$y^j(t) = g(x^j(t)), \tag{1b}$$
$$0 \le t \le T, \tag{1c}$$

where:

$t \in [0,T]$ denotes the time during each repetition
$T > 0$ is the repetition/batch time
$(\cdot)^j$ denotes the discrete repetition index $j \in \mathbb{N}_0$
$x^j(t) \in \mathbb{R}^n$ is the state of the system at time t at the repetition j
$u^j(t) \in \mathbb{R}^m$ denotes the system input at time t at the repetition j. It is assumed to be measurable and bounded over the interval $[0,T]$, i.e. $u^j(\cdot) \in \mathscr{L}^\infty_{[0,T]}$
$f(\cdot)$, $g(\cdot)$ are vector valued functions of appropriate dimension that are locally Lipschitz continuous in all their arguments.

Remark 1: Even so that only the state estimation problem is considered, we consider from repetition to repetition varying, but known inputs. This allows the application of the derived state estimation method for state feedback stabilization of repetitive processes.

One can distinguish various forms of repetitivity depending on the values that change from repetition to repetition. For simplicity of presentation it is assumed that the system is repetitive in the initial conditions, which will be denoted as initial condition repetitive, or shortly IC-repetitive:

Definition 1: (IC-repetitive process)
The process (1) is called *IC-repetitive* if for all repetitions $j \in \mathbb{N}_0$ $x^j(0) = x_0^j = x_0$.

Note that this definition allows for varying inputs from repetition to repetition, which are, however, assumed to be provided externally since we focus on the state estimation problem. For existence and uniqueness of solutions we assume that:

Assumption 1: (Existence and uniqueness of solutions)
The functions f and g of the system (1) are locally Lipschitz continuous with respect to x and u, and $f(0,0) = 0$. Furthermore, for any initial conditions x_0, any repetition time $T \in (0,\infty)$, and any control $u^j(\cdot) \in \mathscr{L}^\infty_{[0,T]}$, (1) has a finite solution.

We also assume that:

Assumption 2: (Nominal System and Exact Knowledge of the Input) There is no model plant mismatch and noise present. Furthermore, the input $u^j(\cdot)$ during every repetition is known exactly up to the current time t.

For any $(w,t) \in \mathbb{R}^n \times [0,T]$, let $x^j(\cdot; w,t)$ denote the solution of (1), due to the externally provided input $u^j(\cdot)$, which passes through state w at time t. We will use the abbreviation $x^j(t)$ to shortly denote the real system state at repetition j at time t, i.e. $x^j(t) = x^j(\cdot; x_0, t)$. Figure 1 clarifies the used notation and the appearing two time scales, the repetition number j and the time t during each repetition.

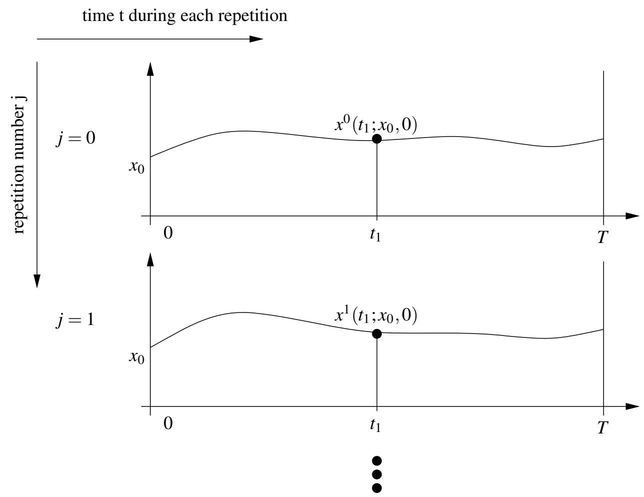

Fig. 1. Two time scales of repetitive processes.

B. Objective

We assume that not the full system state is accessible via the output measurements (1b) and that the true initial state x_0 is unknown. The objective is to design an iterative learning observer for the repetitive process such that the observer state, in the following denoted by $\hat{x}$ becomes smaller with an increasing number of repetitions, i.e.:

$$\|x^j(t) - \hat{x}^j(t)\| \to 0 \quad \text{as} \quad j \to \infty. \tag{2}$$

Remark 2: As outlined in Section III we will actually only achieve the convergence of the observer error at "discrete" sampling times t_i in the interval $[0,T]$, i.e.

$$\|x^j(t_i) - \hat{x}^j(t_i)\| \to 0 \quad \text{as} \quad j \to \infty \quad \forall i \in \{0,1,\dots N\}. \tag{3}$$

Here t_i are the sampling instants and N is the number of sampling times per repetition, compare Section III. Considering discrete sampling instants is necessary due to the usage of a moving horizon state estimation strategy.
To achieve this objective it is necessary to require certain observeability assumptions on the system. For this purpose let $W^j(w_1, w_2; t_1, t_2)$, with $t_1 < t_2$, $t_1, t_2 \in (0,T]$, and w_1, $w_2 \in \mathbb{R}^n$, denote the L_2 norm of the difference between the system outputs of (1) corresponding to trajectories at repetition j passing through the points (w_1, t_1) and (w_2, t_2), respectively, i.e.:

$$W^j(w_1, w_2; t_1, t_2) := \int_{t_1}^{t_2} \|g(x^j(s; w_1, t_1)) - g(x^j(s; w_2, t_2))\|^2 ds. \tag{4}$$

Specifically, it is required that the following uniform reconstructability condition, which is similar to the one used in [6], holds:

Assumption 3: (Uniform reconstructability)
There exists a horizon $T^E \in (0,T)$ and a constant $\gamma \in (0,\infty)$ such that for any two states w_1, $w_2 \in \mathbb{R}^n$, any $t \in (T_E, T)$, any repetition j and any control u^j satisfying Assumption (2).

$$W^j(w_1, w_2; t-T^E, t) \geq \gamma \|w_1 - w_2\|^2. \quad (5)$$

Remark 3: Assumption 3 is an uniform reconstructability condition, similar to the one for continuous time non repetitive processes assumed in [6]. For repetitive processes it implies that there exists a time horizon T^E such that the operator mapping the current value of the state into the corresponding system outputs (defined on an interval $[t-T^E, t]$ in the past) is injective. Thus, any control u^j and output function on the interval $[t-T^E, t]$ uniquely determines the current state of the system. Note that for repetitive/batch system this assumption only ensures that the state can be uniquely determined from the measured output and the known input after the time T^E, since the system is not defined for time $t < 0$, for which thus no output informations are available.

III. Basic Idea

We propose to use an iteratively improving moving horizon estimator to estimate the unknown system states. Basically in moving horizon state estimation the state estimate is given by minimizing the distance between the measured output and the estimated output over a window of past measurements [1, 6, 10, 11, 14] utilizing the system model. The degree of freedom in the minimization is the unknown initial state at the beginning of the estimation window.

Since the numerical solution of the resulting dynamic optimization problem, which must be solved on-line, typically requires a non negligible time, moving horizon observers typically provide state estimates only at "discrete" sampling instant t_i. For this purpose we assume that the interval $[0,T]$ is partitioned in a finite series of sampling instants t_i, $i \in \{1,\ldots,N\}$, such that $t_0 = 0$, $t_1 \in [T^E, T]$, $t_i < t_{i+1} < T$ and $t_{i+1} - t_1 < T_E$ $\forall i \in \{2,\ldots,N-1\}$, and $t_N = T$.

We will shortly denote values at the sampling instants by a subscript, i.e. x_i^j denotes the real state of the system during repetition j at the sampling instant t_i, i.e. $x_i^j := x^j(t_i)$.

At every sampling instant t_i for in ideal moving-horizon observer the state estimate would be obtained from the solution of the following minimization problem:

$$\min_{w_i^j \in \mathbb{R}^n} V_E(w_i^j; t_i - T_E, t_i). \quad (6)$$

Here $V_E(w^j; t_1, t_2)$ denotes the following measure of the observer error over the interval $[t_1, t_2]$:

$$V_E(w; t_1, t_2) = \int_{t_1}^{t_2} \|g(x^j(s; w, t_1)) - y^j(s)\|^2 ds, \quad (7)$$

where $y^j(\cdot)$ denotes the measured output at the repetition j, i.e. $y^j(\cdot) := g(x^j(\cdot; x_0, 0))$, compare also Figure 2. Thus, one would ideally minimize the error between the estimated

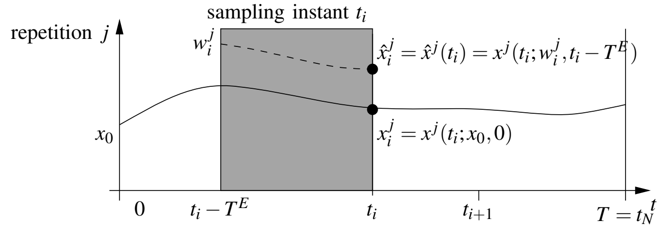

Fig. 2. Moving horizon state estimation for repetitive processes.

output and the measured output over the horizon T^E in the past.

Remark 4: Note that Assumption 3 guarantees that the minimum value for V_E is 0, i.e. that one ideally could estimate the state perfectly based on the information provided over the moving window $[t_i - T_E, t_i]$. However, this would involve to find the exact solution of (6), which is typically not possible in finite time since one has to find the global minima of an (often) non-convex optimization problem. Thus it was proposed in [6] to replace the exact minimization (6) by an approximated solution requiring only that the value of V_E rather only decreases from sampling instant to sampling instant by enforcing a certain contraction constraint. This does, in the non-repetitive case lead to asymptotic convergence of the observer error. This, however, can not be directly transfered to repetitive processes, since the repetition time T is finite.

To achieve convergence of the error in the repetitive case we rather propose a double contraction constraint, requiring that the error between the estimates from sampling instant to sampling instant in one repetition as well as in between repetitions decreases by a certain amount. The resulting scheme that provides state estimates at the sampling instants t_i can be describe by the following algorithm:

Strategy 1:
Initialization: Pick an arbitrary initial guess w_0^0 for x_0. Choose two contraction parameters α, $\beta \in (0,1)$.
Repetition $j = 0$, initialization of the state estimation:

1. *Continuous estimates for $t \in [0, t_1)$:* The state estimate for $t \in [0, t_1)$ is given by $\hat{x}^0(t) = x^0(t; w_0^0, 0)$ (open-loop simulation).

2.a *Improving at the first sampling instant t_1:* At time t_1 find a $w_1^0 \in \mathbb{R}^n$ (improved initial guess) such that

$$V_E(w_1^0; t_1 - T_E, t_1) \quad (8)$$

becomes as small as possible. The point $w_1'^0 = w_0^0$ is used as an initial guess for this calculation.

2.b *Improving at the sampling instants t_i:* At any time t_i, $i = 2, \ldots N$ an (improved) state estimate w_i^0 is calculated such that

$$V_E(w_i^0; t_i - T^E, t_i) \leq \beta V_E(w_{i-1}^0; t_{i-1} - T^E, t_{i-1}) \quad (9)$$

is satisfied. For this the point $w_i'^0 = x^0(t_i - T^E; w_{i-1}^0, t_{i-1})$ is used as initial guess.

3. *Continuous estimates for $t \geq t_1$:* At any time $t \in [t_i, t_{i+1})$, $i = 1, \ldots N-1$, the estimate of the state $x^0(t)$ is given by

forward simulation of the system from w_i^0, i.e. $\hat{x}^0(t) = x^0(t; w_i^0, t_i - T^E)$.

Repetition $j = 1,2,3,\ldots$, iteratively improving the estimates:

1. *Continuous estimates for $t \in [0,t_1)$:* The state estimate for $t \in [0,t_1)$ is given by $\hat{x}^j(t) = x^0(t; w_0^{j-1}, 0)$ (open-loop simulation). Here w_0^j is given by setting $w_0^j = w_1^{j-1}$.

2.a *Improving at the first sampling instant t_1:* At time t_1 an (improved) state estimate w_1^j is calculated such that

$$V_E(w_1^j; t_1 - T^E, t_1) \leq \alpha V_E(w_1^{j-1}; t_1 - T^E, t_1) \quad (10)$$

is satisfied. For this the point $w_i'^j = w_1^{j-1}$ can be used as initial guess.

2.b *Improving at the sampling instants t_i:* At any time t_i, $i = 2,\ldots N$ an (improved) state estimate w_i^j is calculated such that

$$V_E(w_i^j; t_i - T^E, t_i) \leq \min\Big\{\beta V_E(w_{i-1}^j; t_{i-1} - T^E, t_{i-1}), \alpha V_E(w_i^{j-1}; t_i - T^E, t_i)\Big\} \quad (11)$$

is satisfied. For this the point $w_i'^j = x^j(t_i - T^E; w_{i-1}^j, t_{i-1})$ can be used as initial guess.

3. *Continuous estimates for $t \geq t_1$:* At any time $t \in [t_i, t_{i+1})$ $i = 1,\ldots N-1$, the estimate of the state $x^j(t)$ is given by forward simulation of the system from w_i^j, i.e. $\hat{x}^j(t) = x^j(t; w_i^j, t_i - T^E)$.

Remark 5: For the repetitions $j = 1,2,\ldots$ we enforce a double contraction to ensure that the estimate is improving in the repetition from sampling instant to sampling instant and also from repetition to repetition, see (11). The contraction parameter α ensures improvement from repetition to repetition (excluding the first sampling interval $[t_0^j, t_1^j)$). The contraction parameter β ensures contraction from sampling instant to sampling instant similarly to the work presented in [6] for non repetitive systems.

Remark 6: It is necessary to distinguish the first repetition (j=0) from the following repetitions, since in the first repetition no informations from the previous repetition can be used. Furthermore, one has to distinguish the first and second sampling instant.

For t_0 this is necessary since no past state informations, which would span over the interval $[-T_E, 0)$, are available. Rather one can only use state informations from the previous run. Different possibilities exist for transferring the old state estimates to the new repetition. We propose here the most simplest case, using w_1^{j-1} as initial guess for w_0^j. One could in principle also backward simulate the last predicted state, i.e. setting $w_0^j = x^{j-1}(0; w_N^{j-1}, t_N)$. However, for practical applications this might be not of advantage due unconsidered model plant mismatch, noise, and external disturbances.

Remark 7: Under the assumption of no model mismatch and no measurement noise the outlined strategy is well defined and produces always an improving sequence w_i^j. This is due to the fact that under nominal conditions there exists always an improved estimate w_i^j in Step 2.a and 2.b since $V_E(x^j(t_i) - T^E; t_i - T^E, t_i) = 0 \ \forall i = 1,\ldots,N$. Thus $w_i^j = x^j(t_i - T_E)$ always satisfies (9) or respectively (11). In practice satisfying (9) or (11) might be difficult due to measurement noise, model plant mismatch or numerical optimization problems for chosen values of α and β. Practically such problems can be overcome by monitoring the decrease in V_E and, if necessary, increase the value of V_E or increase the values for α or β.

Remark 8: For the first repetition $j = 0$ the outlined scheme does not provide a corrected state estimate until the time T_E. This can be problematical if T_E is rather large to ensure observability. This problem could be overcome by employing a so called batch state estimator with increasing horizon length or an extended Kalman filter until the time T_E [9, 10].

Remark 9: The main advantage of the outlined scheme lays in the fact that the solution of the global optimization problem of minimizing V_E exactly at every sampling instant t_i is distributed over multiple sampling instants and repetitions, thus leading to asymptotic convergence of the estimation error as shown in the next section. The decrease/improvement in the state estimate as required by (9) or respectively (11) can for example be achieved by the application of optimization algorithm that guarantee feasibility in every sub-iteration, such as for example the scheme describe in [4] for static optimization. Similar approaches are also employed in suboptimal nonlinear model predictive control strategies [3, 12].

Note that the speed of convergence of the observer can be directly adjusted by decreasing the values of β, which corresponds to a decrease in the speed of convergence in between sampling instants, and decreasing of the value of α, which corresponds to a decrease in the speed of convergence in between repetitions.

IV. Stability

The properties of he observer/state estimator as outlined in the algorithm above can be summarized in the following theorem, which is closely related to the non repetitive results as presented in [6]:

Theorem 1: Suppose that Assumptions 1-3 are satisfied. Then Strategy 1 produces for any initial state $x_0 \in \mathbb{R}^n$ and any initial guess $w_0^0 \in \mathbb{R}^n$ an infinite sequence of state estimates at the sampling instances $\{\hat{x}_i^j\}_{j\in\mathbb{N}}$, where $\hat{x}_i^j = \hat{x}^j(t_i)$ such that $\hat{x}_i^j \to x_i^j$, $x_i^j = x^j(t_i)$, as $j \to \infty$, i.e. Strategy 1 is globally convergent. Furthermore, the convergence rate is exponential in that there exist a constant $M \in (0,\infty)$ such that

$$\|\hat{x}_i^j - x_i^j\| \leq M e^{-\zeta j} \|\hat{x}_i^0 - x_i^0\| \quad \text{for all} \quad i \in \{1,\ldots N\}, \quad j \in \mathbb{N}, \quad (12)$$

where $\zeta \in (0,\infty)$ is given by $\zeta = -0.5\ln(\alpha)$.

Proof: The proof is similar to the one presented in [6] for non repetitive systems. First, note that from the assumption on f and g as well as due to the uniform reconstructability Assumption 3 and Assumption 1 it immediately follows that $V_E(w_i; t_i - T^E, t_i)$ exists and is finite.

Furthermore, the existence of the improved initial conditions w_i as required in (9) and (11) respectively is guaranteed, see Remark 7. Hence, Strategy 1 produces for any sampling time t_i an infinite sequence $\{\hat{x}_i^j\}_{j\in\mathbb{N}}$. By virtue of (9), (11), and the fact that $0<\beta<1$ and $0<\alpha<1$ it follows that

$$V_E(w_i^j; t_i - T^E, t_i) \to 0 \quad \text{as} \quad j \to \infty. \tag{13}$$

Furthermore, by the definition of the functions W^j (see (4)) and V_E (see (7)), uniqueness of solutions due to Assumption (1), and Assumption 3 we have that

$$\gamma \|x_i^j - \hat{x}_i^j\| \leq W_E^j(x_i, \hat{x}_i^j, t_i - T^E, t_i). \tag{14}$$

This together with (13) implies that

$$\|x_i^j - \hat{x}_i^j\| \to 0 \quad \text{as} \quad j \to \infty, \tag{15}$$

which establishes global convergence of the state estimation Strategy 1. Condition (11) furthermore implies (at least) that

$$V_E(w_i^j; t_i - T^E, t_i) \leq \alpha V_E(w_i^{j-1}; t_i - T^E, t_i), \quad \forall i \in \{1, \dots N\}. \tag{16}$$

Since $V_E(w_i^0; t_i - T^E, t_i)$, $\forall i \in \{1, \dots N\}$ is finite, there exists a constant M_1 such that

$$V_E(w_i^0; t_i - T^E, t_o) \leq M_1 \|\hat{x}_i^0 - x_i^0\|^2. \tag{17}$$

Combining equation (16) and (17) yields

$$\|x_i^j - \hat{x}_i^j\| \leq \alpha^{0.5j} M_1 \|\hat{x}_i^0 - x_i^0\| \leq M e^{-\zeta j} \|\hat{x}_i^0 - x_i^0\|, \tag{18}$$

where $M := (M_1)^{1.5} - \gamma^{-0.5}$, and $\zeta = -0.5\ln(\beta) \in (0, \infty)$ (so that $e^{-\zeta} = \beta^{0.5}$). ■

Note that condition (9) and (11) also ensure a decrease of the observer error during every repetition. However, due to the finite repetition interval $[0, T]$ and due to the fact that only asymptotic convergence is achieved, an exact state estimation in single repetitions can normally not be achieved if V_E is not exactly minimized at every sampling instant.

V. Application Example

We consider the application of the outlined ILMHE to a feed-batch polymerization process. The simplified 4th order model is given by:

$$\dot{m} = F \tag{19}$$

$$\dot{m}_r = F - \mu \cdot V \cdot k_{po} \cdot e^{-\frac{E_A}{RT}} \cdot M(m, m_r) \tag{20}$$

$$\dot{T} = \frac{1}{\rho_m C_p V} \Big[(-\Delta H)\mu \cdot V \cdot k_{po} \cdot e^{-\frac{E_A}{RT}} \cdot M(m, m_r) \tag{21}$$

$$+ U_A(T_j - T) + F \cdot MW_m \cdot C_p(T_f - T) \Big] \tag{22}$$

$$\dot{V} = \frac{MW_m}{\rho_m} F. \tag{23}$$

Here m is the total number of monomer in mole, m_r is the total number of residual monomer (mole), T is the reactor temperature (degree C), and V (cm^3) is the reactor volume. M is the concentration of monomer in the polymer which is given by:

$$M(m, m_r) = \frac{m_r}{MW_m \left(\frac{m - m_r}{\rho_p} - \frac{m_r}{\rho_m} \right)}. \tag{24}$$

The manipulated inputs are the jacket temperature T_j and the monomer input flow rate F. They are given as pre-specified fixed profiles over the batch time T of 80 min as shown in Figure 3. Details about the parameters and all other variables can be found in [2]. As measurements

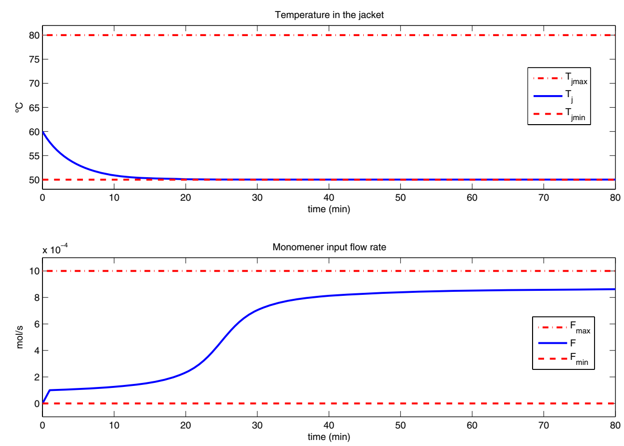

Fig. 3. Pre-specified batch control profiles of the manipulated variables T_j and F

the total number of monomer m, the reactor temperature T, and the reaction volume V are available. The objective for the ILMHE is to estimate the number of residual monomer m_r. The sampling instants at which new measurements are available are equidistant, i.e. $t_i = i \cdot \delta$, where δ is the fixed sampling time of 1 min.

The window length of the moving horizon estimator is set to $T_E = 4$min, and the ILMHE parameters are chosen to $\alpha = \beta = 0.95$.

Figure 4 shows the evolution of the estimated residual monomer concentration from batch to batch starting with incorrect initial conditions. As can be seen, as expected the ILMHE is improving its estimates from iteration to iteration

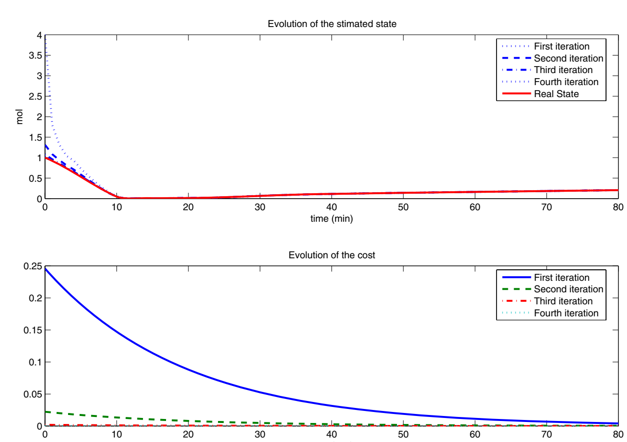

Fig. 4. Evolution of the real and the estimated residual monomer concentration m_r and of the MHE cost function over the time and over subsequent batches.

Already after four batch iterations a neligible estimation error is achieved. This underpins the increasing performance of the ILMHE from iteration to iteration taking the informations from previous runs into account. Further examinations will consider changing input signals as well as the influence of unconsidered external disturbances and noise.

VI. Conclusions

Many processes are inherently repetitive, i.e. the same operation is repeated over and over again. Even so that the control of repetitive processes have received significant academic and industrial interest over the recent years, only limited results with respect to iteratively improving state estimation strategies are available by now [13]. In this paper we have proposed an optimization based moving horizon strategy for the state estimation problem of nonlinear continuous time repetitive processes. The outlined strategy combines ideas from iterative learning control and moving horizon state estimation. Specifically, the state estimate during each repetition is based on the approximate minimization of the deviation between the estimated and measured output. Asymptotic convergence and improvement of the state estimate is ensured by enforcing a double contraction constraint on the deviation between the measured and estimated output during each repetition and in between repetitions. As shown, the outlined iteratively improving state estimation strategy achieves asymptotic convergence of the state estimation error, at least at the sampling instants, provided that the system satisfies an uniform reconstructabilty condition. Further research will investigate the application of the outlined strategy to derive optimization based output-feedback control strategies for nonlinear repetitive processes.

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