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Hierarchical Adaptive Controller for a Nonlinear Aeroelastic Wing Section with Multiple Control Surfaces

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Abstract

This paper concerns the design of an adaptive controller for suppressing aeroelastic vibrations on a nonlinear wing section with leading- and trailing-edge flaps as the actuators. While a single flap under adaptive control can suppress vibrations, the response rate is limited by the system zero dynamics. The adaptive controller pursued here synergistically utilizes both leading- and trailing-edge flaps to completely determine the closed-loop system response. The proposed algorithm addresses the problem of designing a singularity-free adaptive controller when the control inputs are coupled via an input gain matrix whose parameters are uncertain. The stability result that is achieved here is global asymptotic tracking. Simulation results demonstrate the efficacy of the MIMO control toward suppressing flutter and limit cycle oscillations (LCOs) as well as reducing the vibrational level in the subcritical flight speed range. Pertinent conclusions have been outlined.

1 Introduction

Conventional methods of examining aeroelastic behavior have relied on a *linear approximation* of the governing equations of the flowfield and the structure [1, 2]. However, aerospace systems inherently contain *structural* and *aerodynamic nonlinearities* [3, 4] and it is well known that with these nonlinearities present, an aeroelastic system may exhibit a variety of responses that are typically associated with nonlinear regimes of response including Limit Cycle Oscillation (LCO), flutter, and even chaotic vibrations [5]. To prevent aeroelastic instabilities and improve system performance, aeroelasticians have focused their studies on various forms of control. Early studies have shown that the flutter instability can be postponed and consequently the flight envelope can be expanded via implementation of a *linear feedback control* capability. However, the conversion of the catastrophic type of flutter boundary into a benign one requires the incorporation of a *nonlinear feedback* capability given a nonlinear aeroelastic system [4, 6, 7, 8].

For many years, the analysis of 2-D lifting surfaces has provided a basis for the dynamic analysis of a flexible structure, including nonlinear effects, as well as control of instabilities. In particular, several control strategies have been examined for a wing section with a single control surface and have been verified experimentally [8, 9, 10]. In particular, we would like to mention the effort by NASA on the Active Flexible Wing (AFW) [11, 12] where a single trailing edge control surface has been used to suppress aeroelastic instability. Since then, a great deal of research activity devoted to the aeroelastic active control and flutter suppression of flight vehicles has been accomplished. The state-of-the-art of advances in these areas is presented in [13] where a number of recent contributions related to the active control of aircraft wing are discussed at length.

In [6], [14]-[16], linear control theory, feedback linearizing technique, and adaptive and robust control strategies have been derived to account for the effect of nonlinear structural stiffness. Adaptive methods were introduced in [7, 8, 9] to account for uncertainties in the parameters. In these methods, the pitching degree of freedom is usually chosen as the primary variable to control. First, the system is feedback linearized via adaptive cancellation of the nonlinearities introduced by the torsion stiffness – next, conventional linear methods like LQR or pole placement are applied to enhance the aeroelastic response. The eventual controller designed here is full-state feedback. In [17, 18], an adaptive control scheme was implemented using only the feedback for the pitching variable — its performance toward suppressing flutter and LCOs as well as reducing the vibrational level in the subcritical flight speed range was demonstrated.

However, a problem with these single input adaptive controllers is that the aeroelastic response is limited by the system zeros which end up in the system characteristic polynomial due to feedback linearization. Another practical problem is that the velocity at which adaptive control worked well was limited up to approximately 23% greater than the velocity of LCO onset. Improvement are possible through the development of the structured model reference adaptive controller [19]. A better way of improving the performance of the adaptive control scheme was proposed in [20] via an extension to a wing section with two control surfaces. Here, a MIMO adaptive controller was designed but the uncertainty in the coupling between the control inputs was not taken into account — instead, a simple inversion of a (non-singular and non-symmetric) nominal input gain matrix is utilized to decouple the control inputs. Once that is accomplished, adaptive feedback linearization and subsequently, linear control techniques can be applied to complete the design. The aeroelastic response here is better than with a single flap because the MIMO system is flat, *i.e.*, there are no zero dynamics.

In this paper, we extend the work in [20, 21] to develop a hierarchical adaptive control strategy that compensates for uncertainty in both the drift vector field and the input gain matrix which is non-symmetric and has all non-zero leading principal minors. The strategy utilized here applies a matrix decomposition that results in an upper triangular input gain matrix with +1 or -1 on the diagonal which is then utilized to hierarchically design the control inputs. This process ensures that the eventual control design is singularity free and does not have any algebraic loops. This strategy can be utilized to suppress the pitching and plunging vibrations independently and achieve exactly a desired response rate. The controller developed here is proven globally asymptotically stable via a Lyapunov based analysis.

The rest of this paper is organized as follows. In Section 2, the system dynamics are introduced. In Section 3, we lay down the problem statement. In Section 4 we propose the control design and stability analysis is carried out. Section 5 shows some simulation results. Conclusions are presented in Section 6.

2 Configuration of the 2-D Wing Aeroelastic Model

Figure 1 shows a schematic for a plunging-pitching typical wing-flap section that is considered in the present analysis. It is basically a two degrees-of-freedom system with moving leading (LECS) and trailing edges (TECS) control surfaces; $h(t)$ denotes the plunge displacement (positive downward), $\alpha(t)$ the pitch angle (measured from the horizontal at the elastic axis of the airfoil, positive nose-up); $\beta(t)$ and $\gamma(t)$ are the trailing edge and leading edge control surfaces deflections (measured from the axis created by the airfoil at the control flap hinge, positive flap-down). The plunging (h) and pitching (α) displacements are restrained by a pair of springs attached to the elastic axis of the airfoil (EA) with spring constants k_h and $k_\alpha(\alpha)$, respectively. Here, $k_\alpha(\alpha)$ denotes a continuous, linear parameterizable nonlinearity, *i.e.*, the aeroelastic system has a continuous nonlinear restoring moment in the pitch degree of freedom. We also consider the structural damping in the form of the parameters c_h and c_α. Since the high-frequency dynamics of the control surfaces could be neglected, aerodynamic forces and moments created by the control surface deflections are modeled as loads acting at the elastic axis [20]. The governing equations of motion for the aeroelastic system under consideration are given by [8, 20, 22]

$$\begin{bmatrix} m_T & m_w x_\alpha b \\ m_w x_\alpha b & I_\alpha \end{bmatrix} \begin{bmatrix} \ddot{h} \\ \ddot{\alpha} \end{bmatrix} + \begin{bmatrix} c_h & 0 \\ 0 & c_\alpha \end{bmatrix} \begin{bmatrix} \dot{h} \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} k_h & 0 \\ 0 & k_\alpha(\alpha) \end{bmatrix} \begin{bmatrix} h \\ \alpha \end{bmatrix} = \begin{bmatrix} -L \\ M \end{bmatrix} \quad (1)$$

where $k_\alpha(\alpha)$ denotes the nonlinear pitch spring stiffness a particular choice for which is presented in Section 5 based on a model in [20]. In (1), the quasi-steady aerodynamic lift and moment, denoted by $L(\dot{h}, \dot{\alpha}, h, \alpha, \beta, \gamma)$ and $M(\dot{h}, \dot{\alpha}, h, \alpha, \beta, \gamma)$, respectively, are modeled as

$$\begin{aligned} L &= \rho U^2 bsC_{l\alpha}(\alpha + \frac{\dot{h}}{U} + (\frac{1}{2} - a)b\frac{\dot{\alpha}}{U}) \\ &\quad + \rho U^2 bsC_{l\beta}\beta + \rho U^2 bsC_{l\gamma}\gamma \\ M &= \rho U^2 b^2 sC_{m\alpha-eff}(\alpha + \frac{\dot{h}}{U} + (\frac{1}{2} - a)b\frac{\dot{\alpha}}{U}) \\ &\quad + \rho U^2 b^2 sC_{m\beta-eff} + \rho U^2 b^2 sC_{m\gamma-eff} \end{aligned} \quad (2)$$

where $C_{m\alpha-eff}, C_{m\beta-eff}$, and $C_{m\gamma-eff}$ are the effective control moment derivatives, about the elastic axis, due to angle of attack, trailing, and leading edge control surface deflections, respectively, and are defined as follows [20]

$$\begin{aligned} C_{m\alpha-eff} &= (\frac{1}{2} + a)C_{l\alpha} + 2C_{m\alpha} \\ C_{m\beta-eff} &= (\frac{1}{2} + a)C_{l\beta} + 2C_{m\beta} \\ C_{m\gamma-eff} &= (\frac{1}{2} + a)C_{l\gamma} + 2C_{m\gamma} \end{aligned} . \quad (3)$$

The model parameters utilized in (1)-(3) have been defined in the nomenclature presented in Appendix A. By utilizing (1) and (2), an input output form can be derived for the system of Figure 1 as follows

$$\frac{d^{(2)}y}{dt} = f_\mu\left(\alpha, \dot{\alpha}, h, \dot{h}, \theta_1\right) + G_\mu(\theta_2)\, u \quad (4)$$

where $y(t) \triangleq \begin{bmatrix} \alpha(t) & h(t) \end{bmatrix}^T \in \Re^2$ denotes the output vector, $u(t) \triangleq U^2 \begin{bmatrix} \beta(t) & \gamma(t) \end{bmatrix}^T \in \Re^2$ denotes the control input vector, $f_\mu \in \Re^2$ is defined as follows

$$f_\mu(\cdot) = \begin{bmatrix} -k_3 h - \left[k_4 U^2 + q(\alpha)\right]\alpha - c_3\dot{h} - c_4\dot{\alpha} \\ -k_1 h - \left[k_2 U^2 + p(\alpha)\right]\alpha - c_1\dot{h} - c_2\dot{\alpha} \end{bmatrix}$$

while $G_\mu \in \Re^{2\times 2}$ is a constant non-singular (but sign-indefinite), non-symmetric, gain matrix

$$G_\mu = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$$

where the constant matrix entries g_{jk} , the constants k_i, c_i $\forall$ $i = 1, \cdots, 4$, and the functions $q(\alpha), p(\alpha)$ have been explicitly defined in Appendix A. In (4), $\theta_1 \in \Re^{p_1}, \theta_2 \in \Re^{p_2}$ denote unknown model parameter vectors with p_1 and p_2 being positive integers. Since both the leading principal minors g_{11} and $\Delta \triangleq \det(G_\mu)$ of the matrix G_μ are non-zero, we utilize a matrix decomposition introduced in [23] to obtain $G = ST$ where S is a symmetric, positive-definite matrix while T is an upper triangular matrix with its diagonal elements belonging to the set $\{+1, -1\}$. From the solution of a nonlinear system of equations, S and T are obtainable as follows

$$\begin{aligned} S(\theta_2) &= \begin{bmatrix} |g_{11}| & \text{sign}(g_{11})g_{21} \\ \text{sign}(g_{11})g_{21} & S_{22} \end{bmatrix} \\ S_{22}(\theta_2) &= \text{sign}(g_{11})\text{sign}(\Delta)\,(g_{22} \\ &\quad - g_{11}^{-1} g_{21}(g_{12} - g_{21}\text{sign}(\Delta))) \\ T(\theta_2) &= \begin{bmatrix} \text{sign}(g_{11}) & \left|g_{11}^{-1}\right|(g_{12} - g_{21}\text{sign}(\Delta)) \\ 0 & \text{sign}(g_{11})\text{sign}(\Delta) \end{bmatrix} \end{aligned} \quad (5)$$

where the notation sign$(\cdot)$ denotes the standard signum function. Based on the matrix decomposition above, we can now rewrite (4) as follows

$$\begin{aligned} M(\theta_2)\frac{d^2y}{dt} = &\ f\left(\alpha, \dot{\alpha}, h, \dot{h}, \theta_1, \theta_2\right) \\ &+ \det(S(\theta_2))T(\theta_2)\, u \end{aligned} \quad (6)$$

where $M(\cdot) \triangleq \text{adj}(S(\cdot)) \in \Re^{2\times 2}$ is symmetric p.d., while $f(\cdot) \triangleq M(\cdot) f_\mu(\cdot) \in \Re^2$ is an auxiliary vector.

3 Problem Statement

Provided the structure of the aeroelastic model and the pitch spring constant nonlinearity is known, our control objective is to drive the pitching and plunging motions to the origin (possibly along desired trajectories generated by a reference model) in the presence of uncertainty in the knowledge of the model parameters θ_1, θ_2 of (6). It is assumed that $\alpha(t), \dot{\alpha}(t), h(t)$, and $\dot{h}(t)$ are measurable. Towards achieving our control objective, we first define a tracking error $e(t) \in \Re^2$ as

$$e = y_d - y \quad (7)$$

where $y_d(t) \in \mathcal{C}^2$ is a reference trajectory. In order to achieve our desired objective, we define

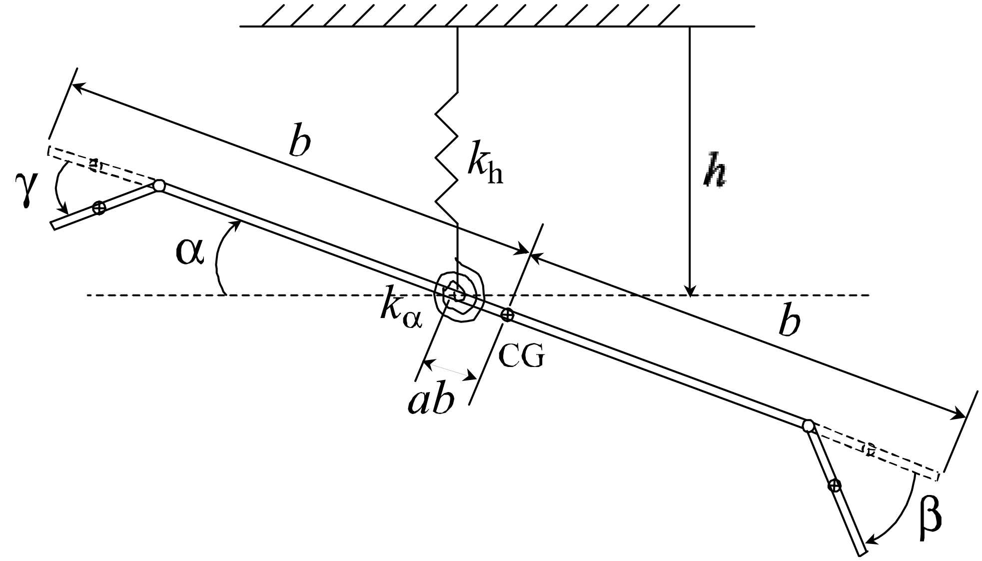

Figure 1: Structure of 2-D Aeroelastic Lifting Surface

a sliding surface $r(t) \in \Re^2$

$$r(t) \triangleq \dot{e} + \Lambda_1 e \qquad (8)$$

where $\Lambda_1 \in \Re^{2\times 2}$ is a constant, positive definite matrix chosen such that the resulting characteristic polynomial is Hurwitz. Motivated by the subsequent stability analysis, we also define parameter estimation errors

$$\tilde{\theta} = \theta - \hat{\theta} \qquad \tilde{\theta}_s(t) = \theta - \hat{\theta}_s \qquad (9)$$

where $\theta \in \Re^{p_3}$ and $\theta_s \in \Re^{p_4}$ contain the unknown system parameters while $\hat{\theta}(t) \in \Re^{p_3}$ and $\hat{\theta}_s(t) \in \Re^{p_4}$ denote their yet to be designed estimates, respectively.

4 Control Design and Stability Analysis

In this Section, we motivate and present the design of the control inputs $\beta(t)$ and $\gamma(t)$. After taking the time derivative of (8) and utilizing (6), the open loop tracking error system can be determined as

$$M\dot{r} = -\frac{1}{2}\dot{M}r + Y\theta - y_s\theta_s D_2 u \qquad (10)$$

where $Y(\cdot) \in \Re^{2\times p_3}$ and $y_s(\cdot) \in \Re^{1\times p_4}$ are measurable regressors that are explicitly defined as

$$Y(\cdot)\theta \triangleq \left[M\left(\ddot{y}_d + \Lambda_1\dot{e}\right) - f - \det(S)\bar{T}u + \frac{1}{2}\dot{M}r\right] \qquad (11)$$

$$y_s(\cdot)\theta_s \triangleq \det(S) \qquad (12)$$

while θ, θ_s have been previously defined in (9). Here, we have taken advantage of the fact that the model in (6) is linear parameterizable. Furthermore, we define a *strictly* upper triangular matrix $\bar{T}(\theta_2)$ as follows

$$\bar{T}(\theta_2) \triangleq T(\theta_2) - D_2 \qquad (13)$$

where $D_2 \triangleq \operatorname{diag}\{\operatorname{sign}(g_{11}), \operatorname{sign}(g_{11})\operatorname{sign}(\Delta)\}$ is a diagonal matrix which is assumed to be known. Notice that this strict upper triangularity allows one to design control in a hierarchical fashion without the possibility of any static loops, *i.e.*, one can first design $\gamma(t)$ independently of $\beta(t)$ and $\beta(t)$ can be designed next to depend on $\gamma(t)$. From the form of (10), we can design the following adaptive control law

$$u = D_2\left(y_s\hat{\theta}_s\right)^{-1}\left(Y\hat{\theta} + Kr\right) \qquad (14)$$

where $K \in \Re^{2\times 2}$ is a constant, positive definite gain matrix while the estimates $\hat{\theta}(t)$, $\hat{\theta}_s(t)$ are dynamically generated as follows

$$\dot{\hat{\theta}} = \Gamma Y^T r \qquad (15)$$

$$\dot{\hat{\theta}}_s = \operatorname{Proj}\{\Gamma_s\mu\}, \quad \mu \triangleq -y_s^T\left(Y\hat{\theta} + Kr\right)^T\left(y_s\hat{\theta}_s\right)^{-1} r, \qquad (16)$$

where $\Gamma \in \Re^{p_3\times p_3}, \Gamma_s \in \Re^{p_4\times p_4}$ are constant, positive definite gain matrices while $\operatorname{Proj}\{\cdot\}$ is a standard parameter projection operator used to ensure that $y_s\hat{\theta}_s > 0$ for all time [24]. After substituting (14) into (10) and performing some algebraic manipulations, one arrives at the closed-loop error system as follows

$$\begin{aligned} M\dot{r} &= -\frac{1}{2}\dot{M}r - Kr + Y\tilde{\theta} \\ &\quad - \left(y_s\hat{\theta}_s\right)^{-1}\left(Y\hat{\theta} + Kr\right)y_s\tilde{\theta}_s \end{aligned} \tag{17}$$

where $\tilde{\theta}(t)$ and $\tilde{\theta}_s(t)$ are parameter estimation errors that have been previously defined in (9) and we have utilized the fact that $D_2D_2 = I_2$ with I_2 denoting the 2×2 identity matrix.

To prove stability, we define a non-negative function $V(t)$ as follows

$$V = \frac{1}{2}r^T Mr + \frac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta} + \frac{1}{2}\tilde{\theta}_s^T\Gamma_s^{-1}\tilde{\theta}_s. \tag{18}$$

After differentiating (18) along (17) and (15), we obtain

$$\begin{aligned} \dot{V} &= -r^T Kr - \tilde{\theta}^T\Gamma^{-1}\dot{\hat{\theta}} - \tilde{\theta}_s^T\Gamma_s^{-1}\dot{\hat{\theta}}_s \\ &\quad + r^T\left[Y\tilde{\theta} - \left(y_s\hat{\theta}_s\right)^{-1}\left(Y\hat{\theta} + Kr\right)y_s\tilde{\theta}_s\right] \\ &= -r^T Kr + \tilde{\theta}_s^T\left(\mu - \Gamma_s^{-1}\dot{\hat{\theta}}_s\right) \end{aligned} \tag{19}$$

where the definition of $\mu(t)$ in (16) has been utilized. Standard results can be used to show how the projection operator ensures that $\tilde{\theta}_s^T\left(\mu - \Gamma_s^{-1}\dot{\hat{\theta}}_s\right) \leq 0$ [24]; hence, the left hand side of (19) can be upperbounded as

$$\dot{V} \leq -\lambda_{\min}\{K\}\left\|r\right\|^2 \tag{20}$$

where $\lambda_{\min}\{\cdot\}$ denotes the minimum eigenvalue operator. From (18) and (20), one can conclude that $r(t) \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ while $\tilde{\theta}(t), \tilde{\theta}_s(t) \in \mathcal{L}_\infty$ which implies directly that $e(t), \dot{e}(t) \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ and $\hat{\theta}(t), \hat{\theta}_s(t) \in \mathcal{L}_\infty$. Since the reference trajectory and its first two time derivatives are bounded, one can conclude that $\alpha(t), \dot{\alpha}(t), h(t)$, and $\dot{h}(t) \in \mathcal{L}_\infty$. From the previous boundedness assertions and the fact that the projection operator ensures that $y_s\theta_s \neq 0$, the second row of the input vector $u(t)$ of (14) can be shown to be bounded. Given the boundedness of $\gamma(t)$ and that of the parameter estimates, the system states, and the reference trajectories, one can now prove the boundedness of $\beta(t)$. By virtue of all previous assertions, the right hand side of the expression of (17) is boundedness which implies that $\dot{r}(t) \in \mathcal{L}_\infty$. One can now apply Barbalat's Lemma [24] to show that $\lim_{t\to\infty} r(t) = 0$; hence, $e(t)$ and $\dot{e}(t)$ on the sliding surface go to zero in the limit.

5 Simulations and Results

See Appendix B.

6 Conclusions

Results related to the adaptive control of a nonlinear plunging-pitching wing section operating in an incompressible flight speed have been presented here. The MIMO control strategy is implemented via leading- and trailing-edge control surfaces. A fully adaptive strategy is used here via employment of full-state feedback. The performance of the adaptive control is analyzed and simulations show that the control strategy effectively controls the pitching and plunging even after the system is allowed to evolve into LCO. Future work would involve implementing the control under output feedback.

References

[1] R.L. Bisplinghoff, H. Ashley, and R.L. Halfman, *Aeroelasticity*, Dover Publications, Inc., New York, NY, 1996.

[2] R.H. Scanlan and R. Rosenbaum, *Introduction to the Study of Aircraft Vibration and Flutter*, The Macmillian Co., 1951.

[3] E.H. Dowell, "A Modern Course in Aeroelasticity", Sijthoff and Noordhoff, 1978.

[4] P. Marzocca, L. Librescu, and W. A. Silva, "Nonlinear Open/Closed-Loop Aeroelastic Analysis of Airfoils via Volterra Series," *AIAA Journal*, Vol. 42, No. 4, April 2004, pp 673-686.

[5] E.H. Dowell, J. Edwards, T. Strganac, "Nonlinear Aeroelasticity," *Journal of Aircraft*, Vol. 40, No. 5, 2003, pp. 857-874.

[6] J. Ko, A.J. Kurdila, and T.W. Strganac, "Nonlinear Control of a Prototypical Wing Section with Torsional Nonlinearity", *Journal of Guidance, Control, and Dynamics*, Vol. 20, No. 6, 1997, pp. 1181-1189.

[7] J. Ko, T.W. Strganac, and A.J. Kurdila, "daptive Feedback Linearization for the Control of a Typical Wing Section With Structural Nonlinearity", *Nonlinear Dynamics*, Vol. 18, No. 3, 1999, pp. 289-301.

[8] T.W. Strganac, J. Ko, and D.E. Thompson, "Identification and Control of Limit-Cycle Oscillations in Aeroelastic Systems", *Journal of Guidance, Control, and Dynamics*, Vol. 23, No. 6, 2000, pp. 11271133.

[9] J. Ko, T.W. Strganac, and A.J. Kurdila, "Stability and Control of Structurally Nonlinear Aeroelastic System", *Journal of Guidance, Control, and Dynamics*, Vol. 21, No. 5, 1998, pp. 718-725.

[10] J.S. Vipperman, R.L. Clark, M.D. Conner, and E.H. Dowell, "Investigation of the experimental active control of a typical section airfoil using a trailing edge flap", *Journal of Aircraft*, Vol.35, No.2, 1998, pp.224-229.

[11] M. Wazak and S. Srinathkumar, "Design and Experimental Validation of a Flutter Suppression Controller for the Active FlexibleWing", *NASATM 4381*, Washington, DC, 1992.

[12] V. Mukhopadhyay, "Flutter Suppression Digital Control Law Design and Testing for the AFWWind Tunnel Model", *NASA TM 107652*, Hampton, VA, 1992.

[13] V. Mukhopadhyay, Ed., "Benchmark Active Control Technology", *Journal of Guidance, Control, and Dynamics*, Part I Vol.23, No.5, 2000,pp.913-960. Part II, Vol.23, No.6, 2000, pp.1093-1139. Part III Vol.24, No.1, 2001, pp.146-192.

[14] W. Xing and S.N. Singh, "Adaptive Output Feedback Control of a Nonlinear Aeroelastic Structure", *Journal of Guidance, Control, and Dynamics*, Vol. 23, No. 6, 2000, pp. 1109-1116.

[15] R. Zhang and S.N. Singh, "Adaptive Output Feedback Control of an Aeroelastic System with Unstructured Uncertainties", *Journal of Guidance, Control, and Dynamics*, Vol. 24, No. 3, 2001, pp. 503-509.

[16] R. Zhang, and S.N. Singh, "Adaptive Output Feedback Control of an Aeroelastic System with Unstructured Uncertainties," *Journal of Guidance, Control, and Dynamics*, Vol.24, No. 3, 2001, pp.502-509.

[17] A. Behal, P. Marzocca, V.M. Rao, and A. Gnann, "Nonlinear Adaptive Control of an Aeroelastic 2-D Lifting Surface", *Journal of Guidance, Control and Dynamics*, in press, 2005.

[18] A. Behal, P. Marzocca, D.M. Dawson, and A. Lonkar, "Nonlinear Adaptive Model Free Control of an Aeroelastic 2-D Lifting Surface," *AIAA Guidance, Navigation, and Control Conference and Exhibit*, Providence, Rhode Island, 16-19 Aug 2004.

[19] T.W. Strganac, J. Ko, D.E. Thompson, and A.J. Kurdila, "Investigations of Limit Cycle Oscillations in Aeroelastic Systems," *CEAS/AIAA/ICASE/NASA Langley, International Forum on Aeroelasticity*, Williamsburg, Virginia, June 22-25, 1999.

[20] G. Platanitis and T.W. Strganac, "Control of a NonlinearWing Section Using Leading- and Trailing-Edge Surfaces", *Journal of Guidance, Control and Dynamics*, Vol. 27, No. 1, 2004, pp. 52-58.

[21] X. Zhang, D.M. Dawson, M. de Queiroz, B. Xian, "Adaptive Control for a Class of MIMO Nonlinear Systems with Non-Symmetric Input Matrix", *Proceedings of the 2004 IEEE International Conference on Control Applications,* Taipei, Taiwan, pp. 1324-1329, Sept. 2004.

[22] J.J. Block and T.W. Strganac, "Applied Active Control for a Nonlinear Aeroelastic Structure", *Journal of Guidance, Control, and Dynamics*, Vol. 21, No. 6, 1998, pp. 838-845.

[23] A.S. Morse, "A Gain Matrix Decomposition and Some of Its Applications," *Systems and Control Lett.*, Vol. 21, pp. 1-10, 1993.

[24] M. Krstic, I. Kanellakopoulos, and P. Kokotovic, *Nonlinear and Adaptive Control Design*, New York: John Wiley & Sons, 1995.

Appendices available upon request.