

# Optimal Multivariable Control for Wind Energy Conversion System - A comparison between $H_2$ and $H_\infty$ controllers

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**Abstract**— Considering that a Wind Energy Conversion System is a non-linear system highly dependent of a stochastic variable and subjected to several kinds of inputs, including a periodical disturbance due to operational phenomena, it represents an interesting problem in viewpoint of control theory. In this paper, it is realized a comparison between  $H_2$  and  $H_\infty$  controllers. Simulation results are presented to verify the performance of both controllers.

## I. INTRODUCTION

Wind Energy Conversion System (WECS) has attracted a growing interest in the last few years since it can offer, from a renewable and pollution-free source, a significant contribution to support electric energy demand. Its basic configuration is a wind turbine (WT) coupled to an electric generator, directly or through gearbox. In spite of its simplicity, this system can represent an interesting problem in viewpoint of control theory. Wind is simultaneously energy source and disturbance signal with a stochastic behavior and subject to sudden variations, such as impulses and steps. The aerodynamic characteristic of a WT is nonlinear and highly dependent on wind speed. Operational aerodynamic phenomena such as tower shadow, wind shear, yaw misalignment and shaft tilt cause cyclical variations in WT, introducing a periodical ripple torque on WECS [1].

The main control objective in a WECS is the regulation of speed and power of a WT under varying wind conditions. In many WECS, WT speed must be controlled to restrict speed variations since the generator is directly connected to fixed-frequency electrical load. However, advances on Power Electronics and Control Systems technologies have made interesting the use of variable-speed WECS, which offers important advantages, such as the increase of energy captured from the wind, reduction of fatigue damage on rotor blades and drive train, reduction of aerodynamic acoustic noise level and operational flexibility [2], [3]. Simplest WECS configuration uses a stall regulated WT, where control speed is realized by stall effect through of adjusts on the electric load connected to generator [2]. In some WT configurations, it is possible to vary the pitch angle of blades aiming to

regulate its aerodynamic efficiency and, consequently, its speed and power [4]. Although the yaw misalignment and shaft tilt can be used to control WT speed and power, these methods are rarely used [1].

The basic control requirements for a WECS are to reduce detrimental dynamic loads and to maximize efficiency on energy conversion, shaping system dynamics to satisfy the performance and stability specifications [5]. Another important control objective is to reduce the influence of wind fluctuation and ripple torque on WECS at any rotation speed, which introduce unavoidable vibrations along the drive-line with detrimental effects on WECS, resulting in large power fluctuations available to costumers and contributing greatly to the fatigue of various mechanical components [6]. Since an exact WECS model is a complex problem due to difficult characterization of physical phenomenon by means of experimental investigation, the control system has to deal with several system uncertainties such as parameter variations, non-linearities, noise or unmodeled dynamics.

Although the classical methods are traditionally utilized to design controllers for WECS [5], [6], [7], they do not offer a completely satisfactory solution, since they do not assure the robustness for both stability and performance, resulting in controllers that are not able to provide a good tolerance to WECS uncertainties. Considering that WECS objectives can be easily specified in terms of maximum allowable gain in the disturbance-to-output transfer functions,  $H_2$  and  $H_\infty$  methodologies consist in good options to control design for WECS [8]. The  $H_2$  philosophy is particularly appropriate in situations where disturbance rejection and noise suppression are important, while  $H_\infty$  is usually preferred when the robustness to plant uncertainties is the dominant issue. Both methodologies offer the capability of stabilizing the plant disturbance and a trade-off between performance and control effort, combining several specifications such as disturbance attenuation, asymptotic tracking, bandwidth limitation and robust stability.

The objective of this work is to compare performance between multivariable controllers for a WECS designed using  $H_2$  and  $H_\infty$  methodologies. A simple and linearized WECS model is developed in section II, which is augmented to weight control objectives.  $H_2$  controller design for WECS is presented in the section III while section IV describes  $H_\infty$  controller design. Using a complex nonlinear WT model presented in [4], where are considered several operational phenomena (such as blade dynamics and wind shears effects), the dynamic response of close-loop system for both controllers is simulated. From simulation results,

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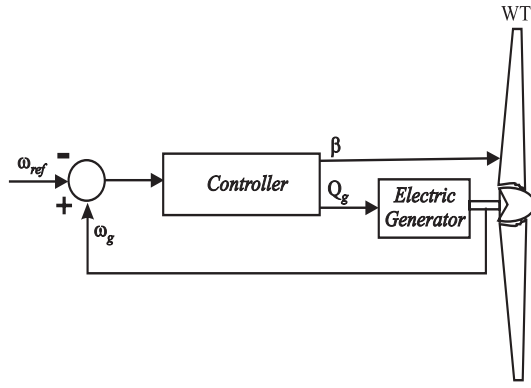


Fig. 1. Speed feedback control system structure

performances of both resulting controllers are analyzed and compared.

## II. WECS NOMINAL MODEL AND AUGMENTED MODEL

The structure of a multivariable speed feedback control system for a WECS with variable pitch angle is shown in fig. 1. An optimal feedback control problem consists of finding a multivariable controller  $\mathbf{K}$ , using optimization problems involving  $H_2$  or  $H_\infty$  norms, for a generalized system  $\mathbf{G}(s)$  shown in fig. 2, which includes the nominal model and weighting functions that are used in the manipulation of control objectives to obtain a satisfactory trade-off between stability, performance and robustness requirements. This generalized system  $\mathbf{G}(s)$  can be described by:

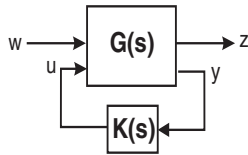


Fig. 2. Generalized system

$$\mathbf{G}(s) = \begin{cases} \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}_1\mathbf{w} + \mathbf{B}_2\mathbf{u} \\ \mathbf{z} = \mathbf{C}_1\mathbf{x} + \mathbf{D}_{11}\mathbf{w} + \mathbf{D}_{12}\mathbf{u} \\ \mathbf{y} = \mathbf{C}_2\mathbf{x} + \mathbf{D}_{21}\mathbf{w} + \mathbf{D}_{22}\mathbf{u} \end{cases} \quad (1)$$

where  $\mathbf{x}$  = state vector,  $\mathbf{u}$  = plant inputs,  $\mathbf{w}$  = exogenous inputs,  $\mathbf{z}$  = control objectives outputs and  $\mathbf{y}$  = measured outputs.

Aiming to facilitate the controller design, a WECS nominal model must be simple. Usually, a WECS is modeled according to schematic diagram shown in fig. 3 [1], [2]. The dynamics of blades and tower are considered as unmodeled uncertainties. The aerodynamic behavior of the wind turbine is nonlinear, depends on wind speed and may change over time due to contamination of blade surfaces. However, dimensionless coefficients that define the WT ability to convert kinetic energy of moving air into mechanical power ( $C_p$ ) or torque ( $C_q$ ) are usually used to evaluate the aerodynamic torque  $Q_a$  of a WT. Both coefficients  $C_p$  and  $C_q$  are

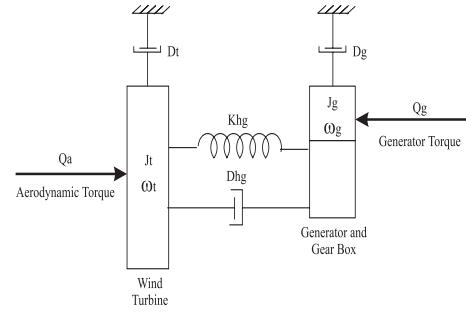


Fig. 3. WECS model

nonlinear functions of pitch angle  $\beta$ , yaw angle  $\theta$  and tip-speed ratio  $\lambda$ , which is defined as:

$$\lambda = \frac{R\omega_t}{V} \quad (2)$$

where  $R$ = turbine radius,  $\omega_t$ = turbine speed and  $V$ = wind speed. Admitting that WT is always aligned with wind direction (i.e.,  $\theta = 0^\circ$ ), the aerodynamic torque  $Q_a$  is given by:

$$Q_a = \frac{1}{2}\rho AR \frac{C_p(\lambda, \beta)}{\lambda} V^2 = \frac{1}{2}\rho AR C_q(\lambda, \beta) V^2 \quad (3)$$

where  $\rho$ = air density and  $A$ = rotor area.

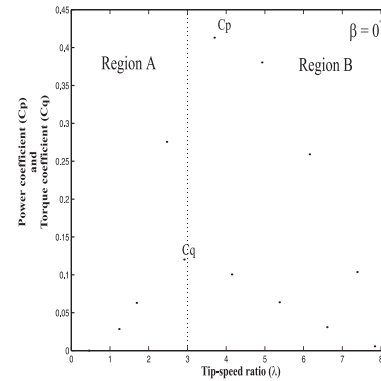


Fig. 4. Aerodynamic characteristics of a WT

The  $C_p$  and  $C_q$  characteristics for  $\beta = 0^\circ$  are shown in Fig. 4. It is noted two distinct regions in the WT operation: the unstable stall region (A), characterized by a positive slope, where occurs a suddenly and significantly drop on aerodynamic torque, and the stable operation region (B), characterized by a negative slope, which corresponds to normal operation of WT where the aerodynamic torque  $Q_a$  can be linearized for control design without degeneration of the results as [2], [9]:

$$\dot{Q}_a = \alpha \dot{V} + \gamma \dot{\omega}_t + \kappa \dot{\beta} \quad (4)$$

where  $\alpha$  is only the scaling factor of torque disturbance due to wind variations  $\dot{V}$ ,  $\gamma$  denotes the speed-feedback coefficient from drive-train and  $\kappa$  represents the pitch control gain. In steady state,  $\dot{V}$  is the wind fluctuation, which can be assumed as white noise with zero mean [4].

Since it is desirable to obtain the maximum  $C_p$ , operating at specified  $\lambda_{opt}$  which is always situated at normal operation region with  $\beta = 0^\circ$ , the aerodynamic torque linearization can be performed in this point. Considering  $\bar{V}$  as the nominal wind speed on WECS location, the coefficients  $\alpha$ ,  $\gamma$  and  $\kappa$  can be easily computed from wind turbine data as:

$$\alpha = \left. \frac{\partial Q_a}{\partial V} \right|_{\lambda_{opt}} = \frac{3}{2} \rho A R \frac{C_{p_{opt}} \bar{V}}{\lambda_{opt}} \quad (5)$$

$$\gamma = \left. \frac{\partial Q_a}{\partial \omega_t} \right|_{\lambda_{opt}} = -\frac{1}{2} \rho A R^2 \frac{C_{p_{opt}} \bar{V}}{\lambda_{opt}^2} \quad (6)$$

$$\kappa = \left. \frac{\partial Q_a}{\partial \beta} \right|_{\lambda_{opt}} = \frac{1}{2} \rho A R \bar{V}^2 \left. \frac{\partial C_q}{\partial \beta} \right|_{\lambda_{opt}} \quad (7)$$

The turbine speed  $\omega_t$  is obtained from drive train modeling. Although a real mechanical drive train has rigid disks, flexible shaft elements with distributed mass and stiffness, an approximated 2-mass model is enough to fit the dynamic behavior of the WT drive train. Admitting an ideal gearbox and reducing all quantities to primary side, the mechanical couple can be described using classical rotational dynamic:

$$J_t \dot{\omega}_t + D_t \omega_t = Q_a - Q_m - D_{hg} (\omega_t - \omega_g) \quad (8)$$

$$J_g \dot{\omega}_g + D_g \omega_g = D_{hg} (\omega_t - \omega_g) + Q_m - Q_g \quad (9)$$

$$Q_m = K_{hg} \int (\omega_t - \omega_g) dt \quad (10)$$

where  $\omega_g$ = generator speed,  $J_t$ = turbine inertia,  $J_g$ = generator inertia,  $D_t$ = turbine damping,  $D_g$ = generator damping,  $D_{hg}$ = shaft damping,  $K_{hg}$ = shaft stiffness,  $Q_a$ = aerodynamic torque,  $Q_g$ = generator torque and  $Q_m$ = shaft torque.

Since electrical generator is the interface between mechanical energy and electricity, the generator torque  $Q_g$  represents the electric load connected to generator seen by mechanical system. Considering that electric load is adjustable and virtually independent from WECS dynamics [2], the generator torque  $Q_g$  consists of one control inputs. Another control input is the pitch angle  $\beta$ . Although pitch actuator can have a great impact over control system, its dynamics are also considered as unmodeled uncertainty in the nominal model, since few details about this components was found in literature. The exogenous inputs on WECS are the speed reference  $\omega_{sp}$  and wind fluctuation  $\dot{V}$ . Due to practical constraint relatives to assembler, cost and maintenance of sensor for shaft torque and turbine speed measurements, this work considers the generator speed  $\omega_g$  as the only measured output  $y$ .

The main control requirement of a WECS is to reduce detrimental dynamic loads on shaft, which is obtained from minimizing the shaft torque variations over all bandwidth. In this context, the first control objective output  $z_1$  is obtained weighting the difference  $\Delta\omega = \omega_t - \omega_g$  with a fixed gain  $K_\delta$ . Another important control requirement is the turbine speed control, which can be defined by reducing speed error  $e_t = \omega_{sp} - \omega_t$ . Thus, the second control objective output  $z_2$

is generated weighting  $e_t$  with a PI function:

$$K_e(s) = \frac{z_2(s)}{e_t(s)} = K_p + \frac{K_i}{s} \quad (11)$$

which implicates in augmenting the original nominal model with an integrator. The control design has to minimize the effects of wind fluctuation and ripple torque over the energy delivered to electric load, and the third control objective output  $z_3$  is the generator torque  $Q_g$  weighted by a fixed gain  $K_q$ . Finally, considering all uncertainties related to pitch actuator, it is necessary to limit the bandwidth of  $\beta$ , weighting it by a fixed gain  $K_\beta$ . Admitting  $\mathbf{u} = [Q_g \ \beta]'$ ,  $\mathbf{w} = [\omega_{sp} \ \dot{V}]'$  and  $\mathbf{x} = [Q_a \ \omega_t \ \omega_g \ Q_m \ \int e_t dt]'$ , the nominal linearized state model of the pitch regulated WT, augmented with weighting functions, is described as:

$$\dot{\mathbf{x}} = \begin{bmatrix} \frac{\gamma}{J_t} & -\frac{\gamma(D_t+D_{hg})}{J_t} & \frac{\gamma D_{hg}}{J_t} & -\frac{\gamma}{J_t} & 0 \\ \frac{1}{J_t} & -\frac{D_t+D_{hg}}{J_t} & \frac{D_{hg}}{J_t} & -\frac{1}{J_t} & 0 \\ 0 & \frac{D_{hg}}{J_g} & -\frac{D_{hg}+D_g}{J_g} & \frac{1}{J_g} & 0 \\ 0 & K_{hg} & -K_{hg} & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 & \alpha \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} \mathbf{w} + \begin{bmatrix} 0 & \kappa \\ 0 & 0 \\ -\frac{1}{J_g} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \mathbf{u} \quad (12)$$

$$\mathbf{z} = \begin{bmatrix} 0 & K_\delta & -K_\delta & 0 & 0 \\ 0 & -K_p & 0 & 0 & K_i \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 & 0 \\ K_p & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \mathbf{w} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ K_q & 0 \\ 0 & K_\beta \end{bmatrix} \mathbf{u} \quad (13)$$

$$\mathbf{y} = \begin{bmatrix} 0 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \mathbf{w} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \mathbf{u} \quad (14)$$

TABLE I  
NORMALIZED PARAMETERS: BASE VALUE: 2.5 MVA AND 1.84 RAD/SEC

Drive Train Parameters		
$J_t = 37,413$	$D_t = 2,024 \times 10^{-2}$	$J_g = 2,091$
$D_g = 3,01 \times 10^{-2}$	$K_{hg} = 28,4$	$D_{hg} = 1,831$
Linearized Aerodynamic Parameters		
$\alpha = 7.6949$	$\gamma = -16.1814$	$\kappa = -144.5773$
Weighting Functions		
$K_\delta = 5$	$K_q = 1$	$K_\beta = 10$
$K_e(s) = 1.5 + \frac{1}{s}$		

Normalized parameters of a 2.5 MW Horizontal Axis WT connected to four pole generator via a gearbox [4] are presented in the table I. The frequency response of open-loop WECS is shown in fig. 5. Although high frequency wind fluctuations are well rejected, WECS is much affected by low frequency wind disturbances. It is noted that control input

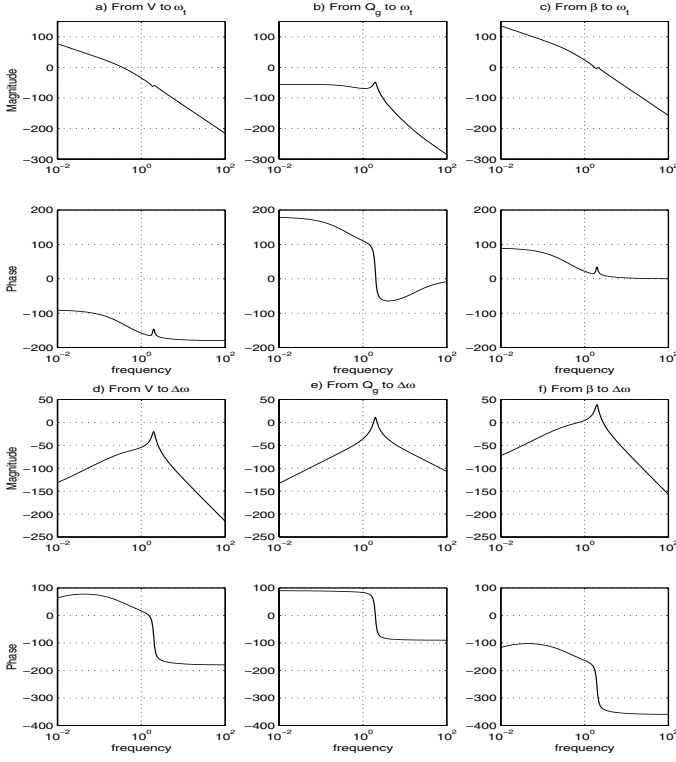


Fig. 5. Bode plots of linearized open-loop WECS model

$\beta$  is more effective on WT regulation than  $Q_g$ , although it reduces the efficiency on energy conversion. Torsional modes can be excited by sudden wind variations and/or operational disturbances since this WECS presents a resonance peak on:

$$\omega_{res} = \sqrt{K_{hg} \left( \frac{1}{J_t} + \frac{1}{J_g} \right)} = 1.9752 \text{ rad/s} \quad (15)$$

### III. $H_2$ CONTROLLER DESIGN

The  $H_2$  controller design can be formalized as an optimization problem, where the goal is to find a controller  $\mathbf{K}_2$  which stabilizes internally the system  $\mathbf{G}(s)$  so that  $H_2$  norm:

$$\|\mathbf{H}_{zw}\|_2 = \sqrt{\frac{1}{2} \int_{-\infty}^{+\infty} [\mathbf{H}_{zw}(j\omega)\mathbf{H}_{zw}^H(-j\omega)] d\omega} \quad (16)$$

is minimized, where  $\mathbf{H}_{zw}$  denotes the transfer function matrix from exogenous inputs  $\mathbf{w}$  to objective outputs  $\mathbf{z}$ . This  $H_2$  optimization problem is equivalent to conventional problem LQG [10], involving a cost function:

$$J = \int_0^{\infty} \begin{bmatrix} \mathbf{x}' & \mathbf{u}' \end{bmatrix} \begin{bmatrix} \mathbf{C}'_1 \mathbf{C}_1 & \mathbf{C}'_1 \mathbf{D}_{12} \\ \mathbf{D}'_{12} \mathbf{C}_1 & \mathbf{D}'_{12} \mathbf{D}_{12} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{u} \end{bmatrix} dt \quad (17)$$

with correlated white noises  $\xi$  (states) and  $\eta$  (measurements) entering in the system via  $\mathbf{w}$  channel associated with correlation function:

$$E \left\{ \begin{bmatrix} \xi(\tau)\xi'(\tau) & \xi(t)\eta'(\tau) \\ \eta(\tau)\xi'(t) & \eta(\tau)\eta'(\tau) \end{bmatrix} \right\} = \begin{bmatrix} \mathbf{B}_1 \mathbf{B}'_1 & \mathbf{B}_1 \mathbf{D}'_{12} \\ \mathbf{D}_{12} \mathbf{B}'_1 & \mathbf{D}_{12} \mathbf{D}'_{12} \end{bmatrix} \delta(t - \tau) \quad (18)$$

This problem can be realized through of resolution of two Riccati equations:

$$\mathbf{Y}_2 \mathbf{A}' + \mathbf{A} \mathbf{Y}_2 - \mathbf{Y}_2 \mathbf{C}'_2 \mathbf{C}_2 \mathbf{Y}_2 + \mathbf{B}_1 \mathbf{B}'_1 = 0 \quad (19)$$

$$\mathbf{A}' \mathbf{X}_2 + \mathbf{X}_2 \mathbf{A} - \mathbf{X}_2 \mathbf{B}_2 \mathbf{B}'_2 \mathbf{X}_2 + \mathbf{C}_1 \mathbf{C}'_1 = 0 \quad (20)$$

resulting in  $H_2$  optimal controller  $\mathbf{K}_2$  given by:

$$\mathbf{K}_2(s) = \begin{cases} \dot{\hat{\mathbf{x}}} = (\mathbf{A} - \mathbf{B}_2 \mathbf{B}'_2 \mathbf{X}_2 - \mathbf{Y}_2 \mathbf{C}'_2 \mathbf{C}_2) \hat{\mathbf{x}} + \mathbf{Y}_2 \mathbf{C}'_2 \mathbf{y} \\ \mathbf{u} = -\mathbf{B}'_2 \mathbf{X}_2 \hat{\mathbf{x}} \end{cases} \quad (21)$$

Considering the augmented WECS model developed in section II and data presented in the table I, this design procedure results in the following  $H_2$  controller:

$$\mathbf{K}_2(s) = \begin{bmatrix} \frac{0.07592s^4 - 0.6462s^3 - 3.923s^2 - 2.761s - 0.7238}{s^5 + 4.504s^4 + 11.09s^3 + 13.27s^2 + 7.454s + 7.547 \times 10^{-6}} \\ \frac{-0.2345s^4 - 0.6949s^3 - 1.269s^2 - 0.5969s - 0.07599}{s^5 + 4.504s^4 + 11.09s^3 + 13.27s^2 + 7.454s + 7.547 \times 10^{-6}} \end{bmatrix}$$

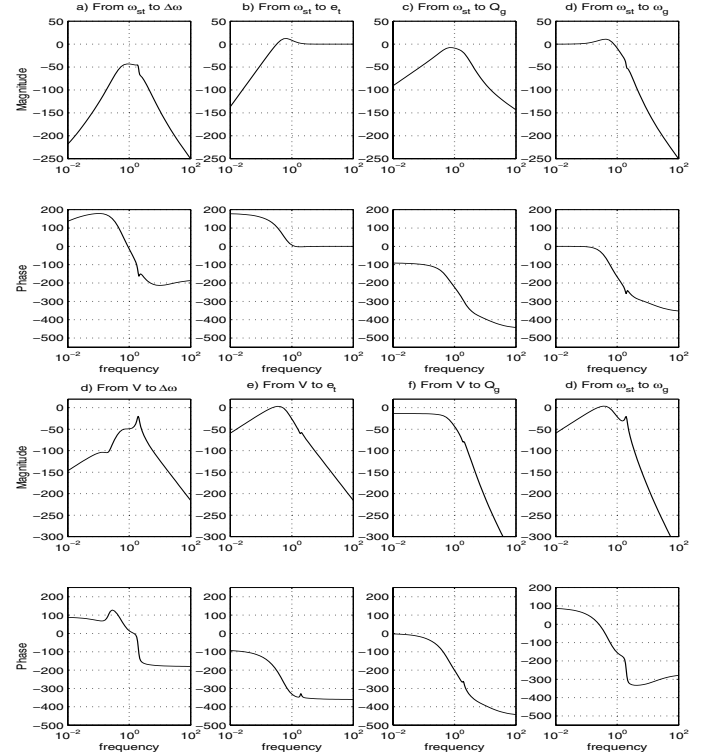


Fig. 6. Bode plots of close-loop WECS with  $H_2$  controller

The frequency response of closed-loop WECS with  $H_2$  controller is shown in fig. 6. High frequency wind fluctuations are submitted to a strong attenuation. For frequencies below 0.7 rad/s, sensitivity function, given by transfer function  $e_t/\omega_{sp}$ , decays rapidly when the frequency tends to zero, as shown in its Bode plots, satisfying requirements related to disturbance rejections. The Bode plots for complementary sensitivity function, given by transfer function  $\omega_g/\omega_{sp}$ , shows that  $H_2$  controller attenuates measurements noise above 0.7 rad/s, assuring good robustness against uncertainties above this frequency. Considering speed difference  $\Delta\omega$ , the excitation of torsional modes is difficult since  $H_2$  control system provides an adequate attenuation for reference

variations and/or operational disturbances. Although power fluctuations on electric load are attenuated, system response to variations on electric torque  $Q_g$ , one of control input, will be very slow.

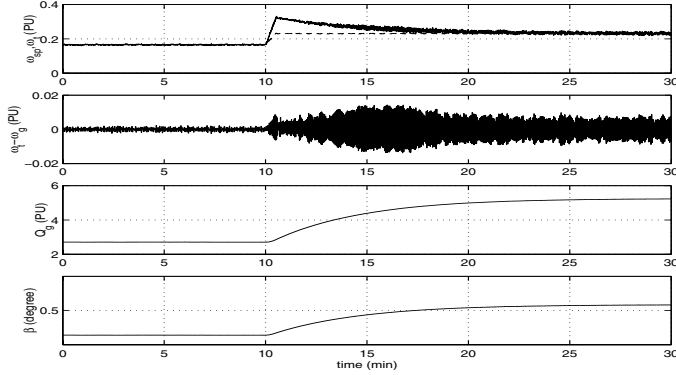


Fig. 7. Normalized dynamical behavior of close-loop WECS with  $H_2$  controller from wind step considering variable-speed operation

Admitting variable speed operation, simulation results of dynamic behavior of closed-loop WECS with  $H_2$  controller submitted to wind step variation of 7.5 m/s to 9.5m/s are shown in fig. 7. In this case, the speed reference is adjusted to:

$$\omega_{sp} = \frac{\lambda_{opt}}{R} V \quad (22)$$

With the occurrence of wind step,  $\omega_t$  follows speed reference  $\omega_{sp}$ , reaching zero error after 10 minutes. Aiming to adjust  $\omega_t$ , generator torque  $Q_g$  is practically duplicated, increasing the energy delivered to electric load. Considering that  $\beta$  has a detrimental effect on energy conversion efficiency, the relatively small contribution of this control input on speed regulation is positive to variable speed WECS. The control system provides a good attenuation for oscillations caused by wind fluctuations and operational phenomenon over shaft torque, represented through the difference  $\omega_t - \omega_g$ , reducing the stress on shaft. Using the same  $H_2$  controller, another simulation was realized admitting fixed speed operation, maintaining a fixed speed reference while a wind step is applied on WECS. Under these conditions, the close-loop system presented a unstable behavior due to effects of nonlinearities, since WECS does not operate near to linearization point  $\lambda_{opt}$  of nominal model.

#### IV. $H_\infty$ CONTROLLER DESIGN

Feedback control design can be also formalized in terms of  $H_\infty$  norm optimization. The sub-optimal  $H_\infty$  control problem is to find all admissible compensator  $\mathbf{K}_\infty(s)$  which stabilizes internally the generalized system  $\mathbf{G}(s)$  and minimizes the norm [11]:

$$\|\mathbf{H}_{zw}\|_\infty = \sup_{\omega} \bar{\sigma}[\mathbf{H}_{zw}] \quad (23)$$

such that  $\|\mathbf{H}_{zw}\|_\infty < \epsilon$ . Considering  $\mathbf{D}_{11} = \mathbf{0}$  and  $\mathbf{D}_{22} = \mathbf{0}$ , the solution of this problem can be given by:

$$\mathbf{K}_\infty(s) = \begin{cases} \dot{\hat{\mathbf{x}}} = (\mathbf{A}_\infty - \mathbf{B}_2\mathbf{B}'_2\mathbf{X}_\infty - \mathbf{L}_\infty\mathbf{C}_2) \hat{\mathbf{x}} + \mathbf{L}_\infty\mathbf{y} \\ \mathbf{u} = -\mathbf{B}'_2\mathbf{X}_\infty\hat{\mathbf{x}} \end{cases} \quad (24)$$

where

$$\mathbf{A}_\infty = \mathbf{A} + \epsilon^{-2}\mathbf{B}_1\mathbf{B}'_1\mathbf{X}_\infty \quad (25)$$

$$\mathbf{L}_\infty = (\mathbf{I} - \epsilon^{-2}\mathbf{Y}_\infty\mathbf{X}_\infty)^{-1}\mathbf{Y}_\infty\mathbf{C}'_2 \quad (26)$$

and  $\mathbf{X}_\infty$  and  $\mathbf{Y}_\infty$  are the solutions of two Riccati equations:

$$\mathbf{A}'\mathbf{X}_\infty + \mathbf{X}_\infty\mathbf{A} - \mathbf{X}_\infty\mathbf{B}_\infty\mathbf{B}'_\infty\mathbf{X}_\infty + \mathbf{C}'_1\mathbf{C}_1 = \mathbf{0} \quad (27)$$

$$\mathbf{Y}_\infty\mathbf{A} + \mathbf{A}'\mathbf{Y}_\infty - \mathbf{Y}_\infty\mathbf{C}'_\infty\mathbf{C}_\infty\mathbf{Y}_\infty + \mathbf{B}_1\mathbf{B}'_1 = \mathbf{0} \quad (28)$$

where

$$\mathbf{B}_\infty\mathbf{B}'_\infty = \epsilon^{-2}\mathbf{B}_1\mathbf{B}'_1 - \mathbf{B}_2\mathbf{B}'_2 \quad (29)$$

$$\mathbf{C}_\infty\mathbf{C}'_\infty = \epsilon^{-2}\mathbf{C}_1\mathbf{B}'_1 - \mathbf{C}_2\mathbf{C}'_2 \quad (30)$$

The conditions to assure the existence of a solution for  $H_\infty$  control problem are  $\mathbf{X}_\infty \geq \mathbf{0}$ ,  $\mathbf{Y}_\infty \geq \mathbf{0}$  and the eigenvalues  $\rho(\mathbf{X}_\infty\mathbf{Y}_\infty) \leq \epsilon^2$ . The best solution for sub-optimal optimal  $H_\infty$  controller can be computed using the loop-shifting two-Riccati formulae [12]. Considering the WECS model developed in section II and the weighting functions presented in the table I, the optimal  $H_\infty$  controller for WECS, obtained with  $\epsilon = 0.0674$ , is given by:

$$\mathbf{K}_\infty(s) = \left[ \begin{array}{c} \frac{2.297s^4 - 2.253s^3 - 8.93s^2 - 2.685s - 0.3474}{s^5 + 16.25s^4 + 52.89s^3 + 92.42s^2 + 51.75s + 5.18 \times 10^{-5}} \\ \frac{-1.138s^4 - 3.18s^3 - 6.059s^2 - 2.58s - 0.22}{s^5 + 16.25s^4 + 52.89s^3 + 92.42s^2 + 51.75s + 5.18 \times 10^{-5}} \end{array} \right]$$

The frequency response of closed-loop WECS with  $H_\infty$  controller is shown in fig. 8, where is noted that high frequency wind fluctuations are strongly attenuated. Below 0.7 rad/s, sensitivity function decays rapidly while above this frequency complementary sensitivity function is attenuated, which assures requirements related to disturbance rejections and robustness against uncertainties. Such as  $H_2$ ,  $H_\infty$  control system provides an adequate attenuation for reference variations and/or operational disturbances. If compared to  $H_2$ ,  $H_\infty$  controller provides a greatest attenuation for power fluctuations on electric load are attenuated, which become its response extremely slow for variations on electric torque  $Q_g$ .

Admitting variable speed operation, simulation results of dynamic behavior of closed-loop WECS with  $H_\infty$  controller submitted to wind step variation of 7.5 m/s to 9.5m/s are shown in fig. 9. It is noted that the performance of  $\beta$  on speed adjust is improved. Wind fluctuations and operational phenomena (such as wind shears) cause oscillations on difference  $\omega_t - \omega_g$  which are attenuated by control system, reducing mechanical stress in shaft. Although  $\omega_t$  will reach the reference  $\omega_{sp}$ , this adjustment is extremely slow. However, the simulation of fixed speed operation does not present an unstable behavior.

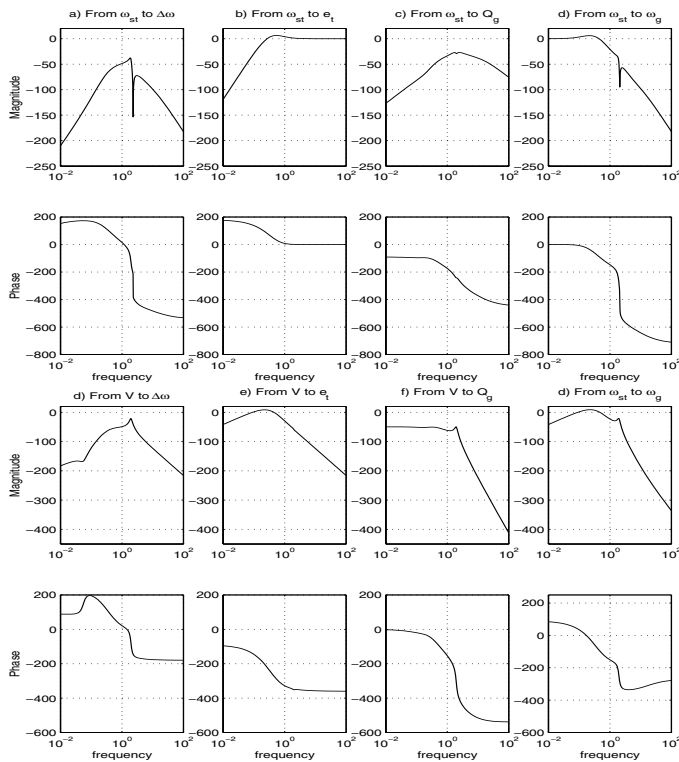


Fig. 8. Bode plots of close-loop with  $H_\infty$  controller

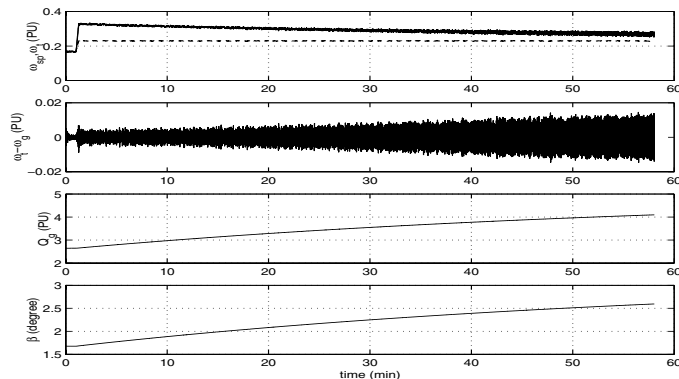


Fig. 9. Normalized dynamical behavior of close-loop WECS with  $H_\infty$  controller from wind step considering variable-speed operation

## V. CONCLUSION

This work focuses the application of  $H_2$  and  $H_\infty$  methodologies to design a multivariable controller for WECS. The controller design is based on a simple WECS model, linearized around  $\lambda_{opt}$  and augmented to weight control objectives. Using a complex nonlinear WT model, the dynamic response of close-loop system is simulated and the performance of  $H_2$  and  $H_\infty$  controllers are analyzed and compared. Similarly to several comparisons presented in the literature, there are issues that could cause the result of this work to be questionable.

Considering the problem presented in this work, it is observed that  $H_\infty$  methodology produces a more conservative (and consequently more robust) controller than a similar

$H_2$  design, although both approximations present several similarities and  $H_\infty$  solution admits sub-optimal controllers. However, the dynamic response of  $H_\infty$  controller is very slow if compared to a similar  $H_2$  controller. In this context,  $H_\infty$  controller presents characteristics more adequate for applications involving regulation problems such as fixed-speed WECS, where the rejection of disturbances is fundamental. In counterparts, the characteristics of  $H_2$  controller is more adequate for tracking problems of variable-speed WECS, where is required a fast response of the controller to follow a reference.

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## REFERENCES

- [1] L. L. Freris, ed., *Wind Energy Conversion Systems*. United King: Prentice Hall Inc., 1990.
- [2] P. Novak, T. Ekelund, I. Jovik, and B. Schmidtbauer, "Modeling and control of variable-speed wind-turbine drive-system dynamics," *IEEE Control Systems*, vol. 15, pp. 28–38, August 1995.
- [3] J. F. Manwell, J. G. McGowan, and B. H. Bailey, "Electrical / mechanical options for variable speed wind turbines," *Solar Energy*, vol. 46, no. 1, pp. 41–51, 1991.
- [4] O. Wasynczuk, D. T. Man, and J. P. Sullivan, "Dynamic behaviour of a class of wind turbine generators during random wind fluctuations," *IEEE Trans. on Power Apparatus and Systems*, vol. 100, pp. 2837–2845, June 1981.
- [5] W. E. Leithead, S. de la Salle, and D. Reardon, "Role and objectives of control for wind turbines," *IEE proceedings-C*, vol. 138, pp. 135–148, March 1991.
- [6] L. Dessaint, H. Nakra, and D. Mukhedkar, "Propagation and elimination of torque ripple in a wind energy conversion system," *IEEE Trans. on Energy Conversion*, vol. 1, pp. 104–112, June 1986.
- [7] S. Lefevbre and B. Dubé, "Control system analysis and design for an aerogenerator with eigenvalue methods," *IEEE Trans. on Power Systems*, vol. 3, pp. 1600–1608, November 1988.
- [8] J. M. Maciejowski, *Multivariable Feedback Design*. Wokingham - England: Addison-Wesley Publishing Company Inc., September 1989.
- [9] R. Rocha and L. de S. Martins Filho, "A multivariable  $H_\infty$  control for wind energy conversion system," in *IEEE Conference on Control Applications*, (Istanbul - Turkey), IEEE, June 2003.
- [10] A. Skogestad and I. Postlethwaite, *Multivariable Feedback Control - Analysis and Design*. John Wiley and Sons, 2.a ed., 2001.
- [11] J. C. Doyle, K. Glover, P. P. Khargonekar, and B. A. Francis, "State-space solutions to standart  $H_2$  and  $H_\infty$  control problems," *IEEE Trans. on Automatic Control*, vol. 34, pp. 831–846, August 1989.
- [12] R. Y. Chang and M. G. Safonov, *Robust control toolbox for use with matlab*. Mathworks Inc., USA, March 1996.