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Optimal Control of the Boost dc-dc Converter

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Abstract— This paper extends the recently introduced approach for modelling and solving the optimal control problem of fixed frequency switch-mode dc-dc converters using hybrid system methodologies to the boost circuit topology, including parasitic elements. The concept of the ν-resolution model is employed to capture the hybrid nature of these circuits. As the resulting equations are nonlinear, two models are formulated, one featuring additional piecewise affine approximations of the nonlinearities and another nonlinear model that retains the nonlinearities in the related system description. An optimal control problem is formulated and solved online for both cases. Simulation results are provided to compare the outcomes of these approaches.

I. INTRODUCTION

Dc-dc converters are a class of electronic power circuits that is used extensively in regulated dc power supplies and dc motor drive applications due to its advantageous features in terms of size, weight and reliable performance.

The main difficulty in controlling dc-dc converters stems from their hybrid nature as their switched circuit topology entails different modes of operation, each with its own associated linear continuous-time dynamics. Hard constraints are also present on the input variable (duty cycle), and additional constraints may be imposed as safety measures, such as current limiting.

This paper analyzes the modelling and controller synthesis of the fixed-frequency boost dc-dc converter, where the semiconductor switch is operated by a pulse sequence with constant switching frequency f_s (resp. period T_s). It is then possible to regulate the dc component of the output voltage via the duty cycle $d = \frac{t_{on}}{T_s}$, where t_{on} denotes the interval within the switching period during which the switch is in conduction.

By employing this operation principle, the main control objective of the converter is to drive the semiconductor switch with a duty cycle such that the dc component of the output voltage is equal to its reference. As the name suggests, for the boost converter this reference value is higher than that of the input voltage. This regulation needs to be maintained despite variations in the load or the input voltage. The different control techniques that are used in practice have in common the employment of PI-type controllers that are tuned on the basis of linearized averaged models [1], [2] and that employ two control loops for the inductor current and output voltage. Different strategies involving nonlinear and feedforward control methods have been proposed for improved controller design [3], [4], [5]; these approaches

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however all employ simplified models that do not capture the hybrid dynamics of dc-dc converters. Furthermore, none of the proposed controllers allow one to explicitly incorporate constraints in the design procedure.

Recently there has been an increased interest and research in the direction of alternative control techniques, also in order so as to compensate for such drawbacks; in particular passivity-based control design for switched-mode power converters has gained significant attention [6], [7], [8], [9] and has also been considered for the boost circuit in [10], in a way such that the physical structure (e.g., energy dissipation, and interconnection) is explicitly incorporated in the model and control scheme. These approaches however do not take into account variations neither on the input voltage nor on the output load. A hybrid approach is given in [11], where a hybrid automaton is synthesized to opportunely switch a boost converter among operating modes. All of the foregoing methods in any case consider simplified boost converter models that neglect the presence of parasitic elements in the circuit parameters. Such an approach may prove to be difficult to implement in industrial practice, as the steady state characteristics of the circuit change considerably when the parasitics are taken into account.

Motivated by these issues, this paper extends the recently introduced ν-resolution model approach [12] for the analysis and controller synthesis problem of step-down dc-dc converters to the boost converter topology featuring parasitic elements. As the corresponding equations are nonlinear two models are formulated to deal with this aspect, that is i) a piecewise polynomial (PWP) model that directly includes the resulting nonlinearities and ii) a Mixed Logic Dynamic (MLD) [16] framework model with a piecewise affine (PWA) description of the relevant nonlinear expressions.

Both formulations are valid for the whole operating regime and capture the evolution of the state variables within the switching period. For each version a finite time optimal control problem is formulated and solved online, for the case where the state update is considered to be ideal and variations on the load are assumed to be known. Future research work will address these assumptions with the intent of implementing an adequate estimation setup, as well as working towards the derivation of an offline representation for the solution [13].

This paper is organized as follows. In Section II, the two aforementioned models for the boost converter are derived by employing the notion of the ν-resolution model. In Section III, an optimal control problem incorporating the appropriate control objectives is formulated. Section IV contains simulation results illustrating the control schemes'

performance in comparison to the behaviour of a traditional industry-standard PI type controller tuned on the basis of a locally linearized averaged model [14]. Finally, conclusions and further research directions are outlined in Section V.

II. MODELLING THE BOOST CONVERTER

A. Continuous-time Model

The circuit topology of the boost converter is shown in Fig.1; only the continuous conduction mode will be considered, that is operating points for which the inductor current remains positive.

Using normalized quantities, r_o denotes the output load resistance, r_c the equivalent series resistance (ESR) of the capacitor, r_ℓ is the internal resistance of the inductor and x_ℓ and x_c represent the inductance and capacitance of the remaining circuit parameters. The boost converter features two operation modes with two different affine dynamics. The controller selects the control input, the duty cycle $d(k)$, for each period k, determining when the switch from the first mode to the second takes place. During the time interval $kT_s \leq t < (k + d(k))T_s$ the switch S is in the s_1 position and the inductor is charged. At the end of this interval S is switched to s_2 and power is transferred to the load. Subsequently, at the end of the period, the switched is set back to the s_1 position.

By taking $x(t) = [i_\ell(t)\ v_c(t)]^T$ as the state vector, where $i_\ell(t)$ is the inductor current and $v_c(t)$ the capacitor voltage, the system is described by the following pair of affine continuous time state-space equations. The following equations hold while S is in s_1

$$\dot{x}(t) = A_1 x(t) + B v_s, \quad kT_s \leq t < (k + d(k))T_s \quad \text{(1a)}$$

$$v_o = C_1 x(t) \quad \text{(1b)}$$

whereas while S is in the s_2 position one has

$$\dot{x}(t) = A_2 x(t) + B v_s, \quad (k + d(k))T_s \leq t < (k + 1)T_s \quad \text{(2a)}$$

$$v_o = C_2 x(t) \quad \text{(2b)}$$

where the matrices A_1, A_2, B, C_1 and C_2 are given by

$$A_1 = \begin{bmatrix} -\frac{r_L}{x_L} & 0 \\ 0 & -\frac{1}{x_c(r_o+r_c)} \end{bmatrix}, \quad \text{(3a)}$$

$$A_2 = \begin{bmatrix} -\frac{r_o r_L + r_c r_L}{x_L(r_o+r_c)} & -\frac{r_o}{x_L(r_o+r_c)} \\ -\frac{2r_o}{x_c(r_o+r_c)} & -\frac{1}{x_c(r_o+r_c)} \end{bmatrix}, \quad B = \begin{bmatrix} \frac{1}{x_L} \\ 0 \end{bmatrix} \quad \text{(3b)}$$

$$C_1 = \begin{bmatrix} 0 & \frac{r_o}{r_o+r_c} \end{bmatrix}, \quad C_2 = \begin{bmatrix} \frac{r_o r_c}{r_o+r_c} & \frac{r_o}{r_o+r_c} \end{bmatrix} \quad \text{(4)}$$

As the control problem is to regulate the output voltage across the load r_o during each switching period the difference between the output voltage and its reference over this time interval should be computed; however, as will be motivated in Section III, the control problem is reformulated into a current reference scheme, so that the difference between the inductor current and its reference is computed

$$i_{\ell,err}(k) = \int_{kTs}^{(k+1)Ts} (i_\ell(t) - i_{\ell,ref})\, dt, \quad \text{(5)}$$

where $i_{\ell,ref}$ denotes the inductor current reference.

Variations in the input voltage v_s are considered to be measurable in accordance with common practice; furthermore, as a simplifying assumption for the present work, the load r_o is also taken to be measurable at each instant. Therefore, since the inductor current $i_\ell(k)$ and the output voltage $v_o(k)$ can be directly measured, from these two measurements the second state $v_c(k)$ can be directly evaluated.

B. ν-Resolution Discrete-Time Hybrid Model

The formulation of an adequate model for the boost converter is of fundamental importance for the subsequent derivation and implementation of the optimal control problem. Such a model must capture the typically hybrid nature of the switched circuit topology and should contain certain other characteristics namely, i) given the discrete time variation of the input variable, it should be formulated in the discrete time domain ii) it ought to be able to monitor the behaviour of the states within a single switching period and iii) it must include an expression approximating the average inductor current error, the reason for which is further detailed in Section III.
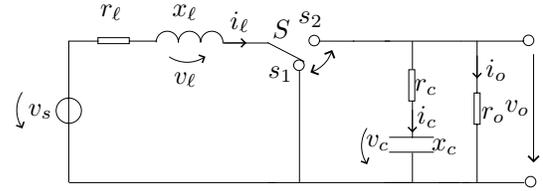

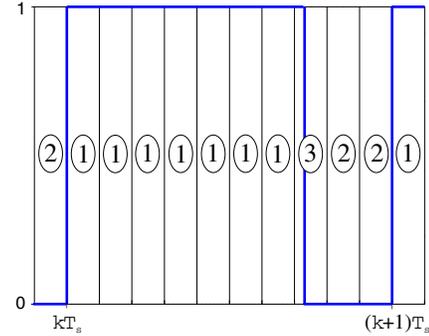Fig. 1.   Topology of the boost converter

Fig. 2.   Position of the switch S and the number of the mode which is active in the respective subperiod, for the case $\nu =10$.

On the basis of the previous considerations the ν-resolution model is introduced similarly as in [12]. The period of length T_s is divided in subperiods of duration $\tau_s = \frac{T_s}{\nu}$ with $\nu \in \mathbb{N}$, $\nu \geq 2$. This concept is illustrated in Fig.2. The states sampled within a subperiod are denoted by $\xi(n)$ and the discrete-time instants of the subperiod are referred to with $n \in \{0, 1, \ldots, \nu - 1\}$. Furthermore, $\xi(0) = x(k)$ and $x(k+1) = \xi(\nu - 1)$ hold by definition. Subsequently, ν binary variables, defined as

$$\sigma_n = \text{true} \iff d(k) \geq \frac{n}{\nu}, \quad n = 0, \ldots, \nu - 1 \quad \text{(6)}$$

are introduced. These represent the sampled switch position of S at time instants $n\tau_s$. The two previously discussed modes of evolution (switch on or off), plus a third mode that captures the transition from the first to the second mode are then associated to each subperiod. More specifically, the modes are (i) the switch S is in the s_1 position for the whole subperiod, (ii) the switch S is in the s_2 position for the whole subperiod and (iii) S is switched from s_1 to s_2 during the subperiod. Hence, for the ν-th subperiod the state update equations are

$$\xi(n+1) = \begin{cases} \Phi_1\xi(n) + \Psi_1, & \text{if } \sigma_n \,\wedge\, \sigma_{n+1}, \\ \Phi_2\xi(n) + \Psi_2, & \text{if } \bar{\sigma}_n, \\ \Xi_3(\xi(n)), & \text{if } \sigma_n \,\wedge\, \bar{\sigma}_{n+1}, \end{cases} \quad (7)$$

with

$$\Xi_3(\xi(n)) := (\Phi_1(Nd(k) - n) + \Phi_2(1 - Nd(k) + n))\xi(n) +$$
$$\Psi_1(Nd(k) - n) + \Psi_2(1 - Nd(k) + n) \quad (8)$$

where Φ_1, Φ_2, Ψ_1 and Ψ_2 are the discrete time representations (with sampling time τ_s) of A_1, A_2 and B (with S in the positions s_1 and s_2 respectively).

The third (auxiliary) mode refers to the mode transition where S switches from s_1 to s_2 within a subperiod. Note that in the third mode, i.e. when $\sigma_n \,\wedge\, \bar{\sigma}_{n+1}$ holds, $Nd(k) - n$ is bounded by zero and one. Thus, the third mode constitutes a weighted average of modes one and two. Furthermore, notice that differently from the step-down converter [12], the existence of two distinct matrices A_1, A_2 and correspondingly of two discrete-time equivalents Φ_1, Φ_2 produces the term $d(k)\xi(n)$ in the third mode, that is the multiplication of the state times the input.

In the present work this nonlinearity is dealt with by means of two different approaches: in the first method it is simply retained and featured in the model equations to yield a PWP model, wherein the polynomial elements correspond to the multiplicative terms in consideration and the integer variables and components capture the discrete features of the ν step model. The second approach consists of approximating the nonlinearities by means of piece wise affine (PWA) functions, so that a complete description of the hybrid dynamics of the model in terms of linear equalities and inequalities featuring continuous and binary variables is available. The two approaches are further detailed in the two following subsections.

Finally, the current error integral for the k-th period can be approximated by

$$i_{\ell,err}(k) = \sum_{n=0}^{\nu-2} \frac{i_\ell(n) + i_\ell(n+1)}{2(\nu-1)} - i_{\ell,ref} \quad (9)$$

C. PWP Framework

Equations (7) describe a system whose update equation differs on the basis of which duty cycle is selected, yielding a total of ν possible cases. Specifically, the state update

equation of such a system may be formulated as

$$x(k+1) = F_j x(k) + G_j d(k) + H_j d(k)x(k) + K_j \quad (10a)$$
$$\text{if } d(k) \in [\frac{j-1}{\nu}, \frac{j}{\nu}] \quad j = 1, \dots, \nu \quad (10b)$$

where the F_j, G_j, H_j, and K_j, are matrices of appropriate dimension resulting from the consecutive application of (7) for all possible choices of the input. As the update equation features the product $d(k)x(k)$, the resulting model is then piecewise polynomial.

D. MLD Framework

The second approach towards modelling the system employs an MLD formulation as it captures the associated hybrid features and allows the definition of the optimal control problem in a convenient way. Furthermore, efficient software (HYSDEL, HYbrid System Description Language) is available to easily translate a high level system description into the chosen formalism [15], and PWA elements can also be treated directly and conveniently cast in an MLD representation. The general form of MLD hybrid systems as introduced in [16] is

$$x(k+1) = Ax(k) + B_1u(k) + B_2\delta(k) + B_3z(k) \quad (11a)$$
$$y(k) = Cx(k) + D_1u(k) + D_2\delta(k) + D_3z(k) \quad (11b)$$
$$E_2\delta(k) + E_3z(k) \leq E_1u(k) + E_4x(k) + E_5, \quad (11c)$$

where $k \in \mathbb{N}$ denotes the discrete time-instant, and $x \in \mathbb{R}^{n_c} \times \{0,1\}^{n_\ell}$ denotes the states, $u \in \mathbb{R}^{m_c} \times \{0,1\}^{m_\ell}$ the inputs and $y \in \mathbb{R}^{p_c} \times \{0,1\}^{p_\ell}$ the outputs, with both real and binary components. Furthermore, $\delta \in \{0,1\}^{r_\ell}$ and $z \in \mathbb{R}^{r_c}$ represent binary and auxiliary continuous variables, respectively. These variables are introduced when translating propositional logic or PWA functions into linear inequalities. Only MLD systems that are *completely well-posed* are considered, i.e. systems for which for given $x(k)$ and $u(k)$, the values of $\delta(k)$ and $z(k)$ are uniquely determined by the inequality (11c). This assumption is not restrictive and is always satisfied when real plants are considered.

III. THE CONTROL PROBLEM

A. Control Issues And Objectives

The main control objective for the boost dc-dc converter is to regulate the dc component of the output voltage v_o to its reference $v_{o,ref}$. This regulation has to be achieved in the presence of the hard constraints on the manipulated variable (the duty cycle) which is bounded between 0 and 1, and needs to be maintained despite the changes in the load r_o and the input voltage v_s. Moreover, the controller must render a steady state operation under a constant duty cycle, thus avoiding the occurrence of fast-scale instabilities (subharmonic oscillations).

However, before proceeding with the design of a control scheme that aims at fulfilling these objectives, there are some peculiarities of the physical system that need to be considered. More specifically, one needs to account for the fact that the output voltage exhibits a non-minimum

phase behavior with respect to the duty cycle [17], and for the existence of multiple steady-state equilibria due to the existence of the parasitic elements r_c and r_ℓ [18], the inclusion of which in the model is of crucial importance, as depicted in Fig. 3. Indeed, neglecting them leads to a model that yields an arbitrarily high output voltage for duty cycles close to unity, whereas it can be clearly seen that a realistic setup will yield significantly different results. For an assigned voltage ratio $\frac{v_o}{v_s} > 1$ there will always be two possible stationary values for the duty cycle for a typical boost converter configuration.

These two problems are bypassed by formulating the control problem of the boost dc-dc converter as a current (rather than a voltage) regulation problem, aiming at steering the average value of the inductor current i_ℓ to a reference $i_{\ell,ref}$. This approach is the common industrial practice [17], and yields satisfactory results in terms of the closed-loop performance for the following reasons: (i) the inductor current has a minimum phase behavior with respect to the duty cycle [17], and (ii) the steady state characteristic of the average value of i_ℓ is monotonically increasing as a function of the duty cycle. This implies that the region of high duty cycles is to be avoided for the steady state operation of the converter, since due to the high currents the efficiency of the converter drops significantly [18]. Therefore, despite the fact that there will always be two possible stationary values for the duty cycle for a given output voltage reference $v_{o,ref}$, the current corresponding to the lower duty cycle is selected and taken as the reference $i_{\ell,ref}$. In this paper, we will consider that $i_{\ell,ref}$ can be exactly calculated[1]; this represents a simplifying assumption that implies full knowledge of the circuit parameters and will be addressed in our future research work.

B. Model Predictive Control

Model Predictive Control (MPC) has been traditionally and successfully employed in the process industry and recently also for hybrid systems. The control action is obtained by minimizing an objective function at each time step over a finite horizon subject to the equations and constraints of the model. Depending on the model and on the length of the prediction horizon used in the objective function, this minimization problem varies considerably in complexity [16].

The major advantage of MPC is its straight-forward design procedure. Given a model of the system, one only needs to set up an objective function that incorporates the control objectives. Hard constraints can be easily dealt with by modelling them directly as inequality constraints, whereas soft constraints can be accounted for in the objective function by using penalties. Further details about MPC can be found in [19].

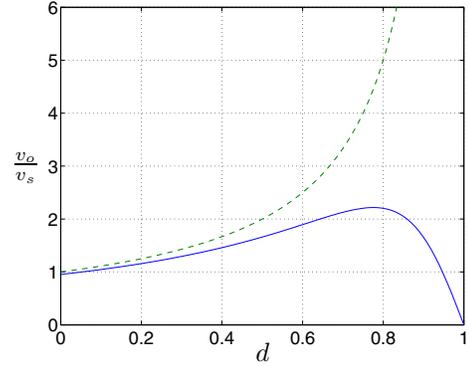[1]In practice, an outer voltage control loop with a PI controller is used to produce $i_{\ell,ref}$.

Fig. 3. Steady state characteristic of the boost converter with (continuous) and without (dashed) parasitics

C. Constrained Finite Time Optimal Control Scheme

The control objectives are to regulate the average output voltage to its reference as fast and with as little overshoot as possible, or equivalently, to minimize the inductor current error $i_{\ell,err}(k)$, despite changes in the input voltage v_s or changes in the load resistance r_o, and to respect the given constraint on the maximal inductor current denoted by $i_{\ell,max}$. Let

$$\Delta d(k) = |d(k) - d(k-1)| \qquad (12)$$

indicate the absolute value of the difference between two consecutive duty cycles. This term is introduced in order to reduce the presence of unwanted chattering in the input when the system has almost reached stationary conditions.

Define the penalty matrix $Q = \mathrm{diag}(q_1, q_2)$ with $q_1, q_2 \in \mathbb{R}^+$ and the vector $\varepsilon(k) = [i_{\ell,err}(k), \Delta d(k)]^T$, with $i_{\ell,err}(k)$ as defined in (9). Consider the objective function

$$J(D(k), x(k), d(k-1)) = \sum_{\ell=0}^{L-1} \|Q \; \varepsilon(k+\ell|k)\|_1 \qquad (13)$$

which penalizes the predicted evolution of $\varepsilon(k+\ell|k)$ from time-instant k on over the finite horizon L using the 1-norm. The control law at time-instant k is then obtained by minimizing the objective function (13) over the sequence of control moves $D(k) = [d(k), \ldots, d(k+L-1)]^T$ subject to the related system equations and constraints for the MLD and PWP models
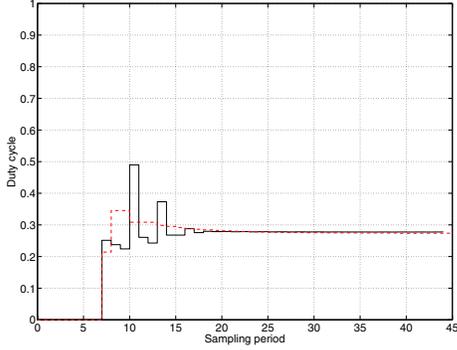
IV. SIMULATION RESULTS

In this section, simulation results demonstrating the potential advantages of the proposed control methodology are presented. The circuit parameters expressed in the per unit system are given by $x_c = 70\,\mathrm{p.u.}$, $x_\ell = 3\,\mathrm{p.u.}$, $r_c = 0.005\,\mathrm{p.u.}$ and $r_\ell = 0.05\,\mathrm{p.u.}$ If not otherwise stated the output resistance is given by $r_o = 1\,\mathrm{p.u.}$ and the output voltage reference is $v_{o,ref} = 1$.

The cases included here represent different scenarios that are of interest in practical applications. In all cases, the current limit for the converter has been set to $i_{\ell,max} = 5\,\mathrm{p.u.}$ The penalty matrix is chosen to be $Q = \mathrm{diag}(5,4)$. The
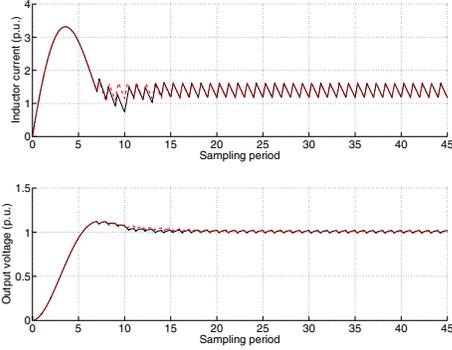
(a) Inductor current (above) and output voltage(below) for the MLD (continuous) and PWP (dashed) control schemes

(b) Duty cycle for the MLD (continuous) and PWP (dashed) control schemes during startup

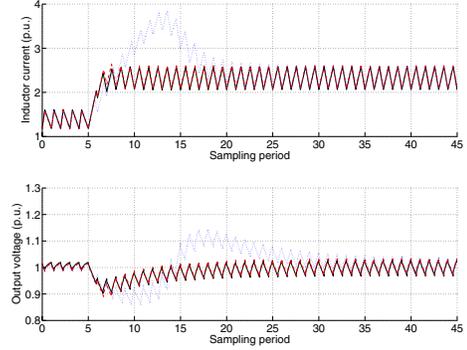Fig. 4.   Simulation results for the startup scenario

(a) Inductor current (above) and output voltage(below) for the MLD (continuous), PWP (dashed) and averaged (dotted) control schemes

(b) Duty cycle for the MLD (continuous), PWP (dashed) and averaged (dotted) control schemes

Fig. 5.   Simulation results for the scenario featuring the step-down of v_s from 0.8 p.u. to 0.55 p.u.

prediction horizon in all cases is $L = 2$ and $\nu = 3$ subperiods are chosen for the ν-resolution model. For the MLD model, both PWA representations feature three regions and cover values of the state $x(t) = [i_\ell(t), v_c(t)]^T$ in $[-10, 10] \times [0, 3]$.

All results presented in the following figures are normalized, including the time scale where one time unit is equal to one switching period.
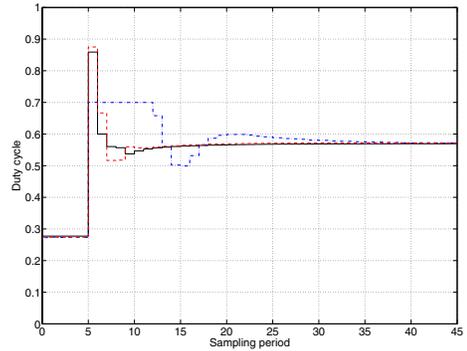
The first case to be analyzed is that of the transient behaviour during startup. Fig.4(a) and Fig.4(b) depict the step responses of the different schemes during start-up. The initial state is given by $x(0) = [0, 0]^T$, the input voltage is $v_s = 0.8$ p.u.. Both proposed optimal control schemes yield a current transient that respects the current constraint at all times and an output voltage that reaches its stationary conditions with a limited overshoot and within 15 switching periods.

For the remaining cases the performance of the proposed MLD and PWP control schemes was compared to that of a traditional industry-standard PI-type controller tuned on the basis of a linearized averaged model (*averaged control scheme*) [14]. The averaged control scheme has not been taken as a term of comparison for the startup scenario since it is linearized around a chosen operating point and as such not specifically designed for this sort of transient.

For the second case results stemming from a step variation in the input voltage v_s during steady state operation from 0.8

p.u. to 0.55 p.u. are shown in Fig.5(a) -5(b); the new value of the input voltage is assumed to be measurable, consistently with actual practice [17]. The currents and voltages of the MLD and PWP models are substantially identical and yield improved transient behaviour and overshoot values if compared to the averaged scheme.

The third and final case concerns a step variation in the load resistance r_o during steady state operation from 1 p.u. to 1.5 p.u.; results are displayed in Fig.6(a)-6(b). The new value of the load is taken to be known, as an initial simplifying assumption; further research work will deal with providing an adequate estimation for implementing a more realistic setup. As can be seen, the optimal control methods give better transient performance; the MLD scheme however produces a small offset in the steady state value of the voltage obtained. This can be ascribed to the PWA expressions featured in the former and can be reduced by augmenting the number of regions employed to approximate the nonlinear terms.

V. CONCLUSIONS AND OUTLOOK

An extension of the ν-resolution model for switch mode dc-dc converters has been formulated for the boost converter topology, inclusive of parasitic elements, by including a PWA
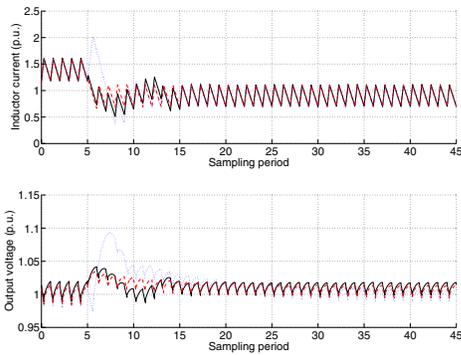
(a) Inductor current (above) and output voltage(below) for the MLD (continuous), PWP (dashed) and averaged (dotted) control schemes
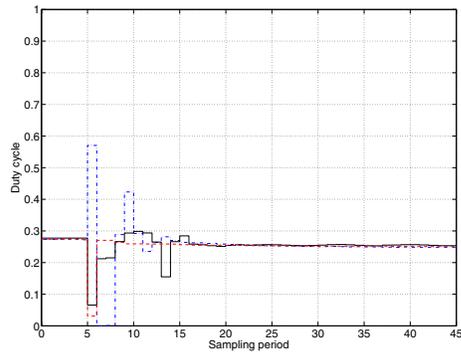
(b) Duty cycle for the MLD (continuous), PWP (dashed) and averaged (dotted) control schemes

Fig. 6. Simulation results for the scenario featuring the step-up of r_o from 1 p.u. to 1.5 p.u.

representation to account for the non-linear dynamics of the model. On the basis of this an optimal control problem has been formulated and solved online. The use of MPC allows to explicitly account for the hard physical constraints on the duty cycle, which must be between zero and one, and the selected form and tuning of the cost function yields an inductor current behaviour that, with no further provisions, respects the bounds imposed by the safety constraints of current limiting; this is achieved by appropriately taking as reference operating point one of the two possible steady state values resulting from the non-ideal topology considered. Simulation results show that this leads to a system with favorable closed loop properties.

The present work considers the case where the state update is considered to be ideal and information concerning the system, such as variations on the load, is assumed to be known. Future research work will have to address such issues to provide an adequate estimation scheme. Furthermore, the online solution of the optimal control problem necessitates computation times that are well above the sampling times used in real-life applications. Therefore, the experimental validation of the proposed approach also needs the computation of the optimal state-feedback control law parameterized

over the state space. This operation reduces the online optimization involving MILPs to a simple search in a look-up table requiring only matrix multiplications.

VI. ACKNOWLEDGEMENTS

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