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Final Glide-back Envelope Computation for Reusable Launch Vehicle Using Reachability

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Abstract— **The main limitation of existing computational tools for continuous system reachability problems is that, due to the exponential growth of the computation with the dimension of the continuous state space, the tools can be applied effectively to relatively low dimensional problems (typically 1-4 dimensions). In this paper we adopt a two time scale approach to extend the use of continuous system reachability tools to six dimensions, thus making them applicable to a number of interesting case studies in the area of aeronautics. To prove the effectiveness of our approach, we apply it in the final glideback envelope computation for safe landing of a small reusable launch vehicle (RLV) in a non-steady atmosphere. The mathematical model of the RLV that is used is a three-degree of freedom (six state) nonlinear point mass model, modified to contain ambient winds and several state constraints for the final approach phase. The results show that it is feasible to do exacting computations with nonlinear continuous dynamics in higher dimensions, if one can exploit additional structure in the model, in our case, the separation into slow and fast dynamics.**

I. Introduction

Because of their importance in applications ranging from engineering to biology and economics, questions of reachability, viability and invariance have been studied extensively in the dynamics and control literature. Most recently, the study of these concepts has received renewed attention through the study of safety problems in hybrid systems. Reachability computations have been used in this context to address problems in the safety of ground transportation systems [1], [2], air traffic management systems [3]-[5], flight control [6] etc.

Direct characterisation of reachability concepts is one of the topics addressed by viability theory [7]. The development of computational tools to support the numerous viability theory concepts is an on going effort [8]. An alternative, indirect approach to reachability is using optimal control. In this case, the reachable, viable, or invariant sets are characterised as level sets of the value function of an appropriate optimal control problem. Using dynamic programming the value function is subsequently characterised by a partial differential equation. It can be shown that reachability questions can indeed be encoded as optimal control problems where the cost is the minimum of a function of the state over a given horizon [4], [[9], [20]. The objective of the controller is either to maximise this quantity (SUPMIN problem), or to minimise it (INFMIN problem). It can be shown [9] that these two problems are equivalent to viability and invariance computations respectively. The value functions of these two problems can be characterised as viscosity solutions to first order partial differential equations, which are variants of the standard Hamilton-Jacobi equation. Therefore, efficient algorithms developed for this class of equations [10]-[13] whose properties have been extensively tested in theory and in applications, can be directly applied to reachability problems.

Both viability theory and viscosity solution type algorithms for performing reachability computations suffer from what is known as the "curse of dimensionality". Since both classes of algorithms are numerical and rely on gridding the state space, the memory and time necessary for the computation grows exponentially in the dimension of the state. This limits the applicability of the tools to relatively low dimensional problems; to the best of our knowledge the highest dimensional system addressed by these methods was of dimension five [14]; even at this relatively low dimension the computation had to be rather coarse to remain manageable.

In recent work we have been investigating to what extent it is possible to alleviate this limitation by exploiting the structure of the system. In particular, for systems that exhibit a partition between fast and slow dynamics, we propose to approximate the reachability computation by a sequence of two computations, one for the fast dynamics, followed by one for the slow dynamics. Even though not all high dimensional problems can be decomposed into fast and slow sub-problems, many can. This is typically the case, for example, for problems in flight dynamics.

In this paper we demonstrate the feasibility of this approach by applying it to the glideback flight envelope computation for safe landing of a small Reusable Launch Vehicle [15] (RLV) An added source of difficulty in this

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problem is the presence of the wind, which enters as an unpredictable and uncontrolled disturbance. This disturbance input effectively turns the reachability computation into a pursuit evasion game problem between the control inputs (angle of attack and roll angle) and the wind velocity, whose value function can be characterized using appropriate variants of the Hamilton-Jacobi-Isaacs partial differential equation [20].

The mathematical model of the RLV that is used, is a three degree of freedom (six state) nonlinear point mass model having a lookup table of subsonic aerodynamic coefficients, ambient winds as a function of altitude and several state constraints for the final approach phase.

The paper is organized into six sections. Section II deals with the psychical description and the dynamics of the RLV, Section III provides some background material on the reachability problem and Section IV discusses how the landing problem for the RLV can be formulated in this context. In Section V, we present the two time scale approximation applied to reachability computations. Finally, in the Section VI, we provide some concluding remarks and directions for further research.

II. SPACE VEHICLE DYNAMICS

A. *Physical Description and Dynamics*

As stated in [15] "A need exists for new small RLVs for the deployment of military, civil and commercial satellites either singly or in satellite constellations and architecture". These vehicles must be highly responsive in providing rapid deployment of small payloads at significantly lower launch costs, improved reliability and maintainability. To address this issue the Vehicles Analysis branch of NASA's Langley Research Center with [15],[16] proposed a new in-house small RLV. The schematic diagram of the vehicle is shown in Fig. 1.

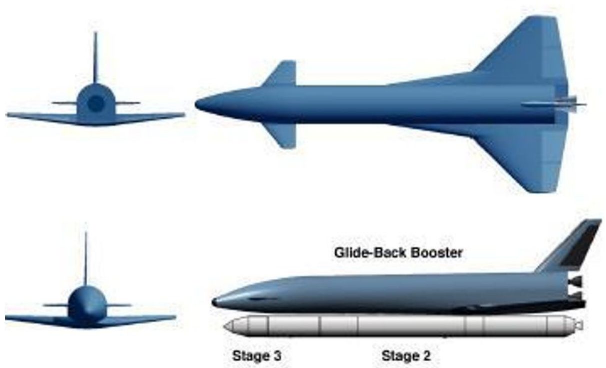

Fig. 1. Small Reusable Launch Vehicle (from [15]).

The booster glideback aerodynamic database covers Mach range from 0.2 to 4.5, angle of attack from -4 to 48deg, elevon deflections form -30 to 20deg, canards from -30 to 20deg and speedbrake at 90deg. In our approach we will use simplified aerodynamics of the RLV consisting of lift and drag coefficients (C_L, C_D) as a function of Mach number and angle of attack. These coefficients correspond to longitudinal aerodynamic coefficients [15],[16] for the baseline vehicle (control surfaces undeflected) and are plotted in Fig 2.

A fine-tuned study of optimum glideback performance would also have to include an optimization problem [18] to obtain trim C_L and C_D as a function of Mach number and angle of attack to minimize the total drag coefficient using elevon and canard deflections. However, even without this step, the baseline aerodynamic coefficients provide a good insight to our approach and will be used here.

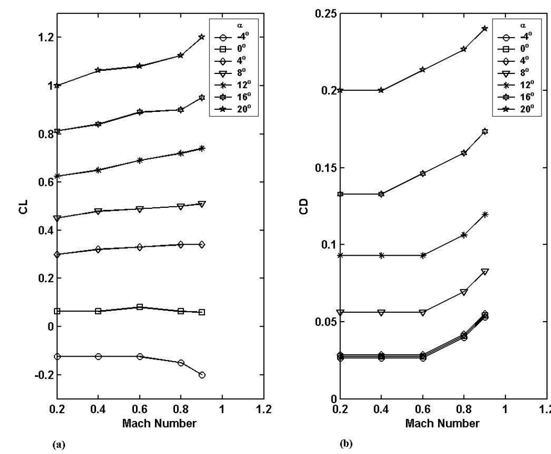

Fig. 2. (a) Lift coefficient
(b) Drag coefficient

A six state point mass zero-thrust model of the RLV for 3-dimensional Flat Earth navigation is governed by the following equations of motion [17]:

$$\dot{h} = V \sin\gamma + W_h \tag{1}$$

$$\dot{x} = V \cos\gamma \cos\psi + W_x \tag{2}$$

$$\dot{y} = V \cos\gamma \sin\psi + W_y \tag{3}$$

$$\dot{V} = -\frac{0.5\rho S_{ref} V^2 C_D(\alpha, M)}{m} - \frac{g}{V}\sin\gamma \tag{4}$$

$$\dot{\gamma} = \frac{0.5\rho S_{ref} V^2 C_L(\alpha, M)}{mV}\cos\phi - V\cos\gamma \tag{5}$$

$$\dot{\psi} = \frac{0.5\rho S_{ref} V^2 C_L(\alpha, M)}{mV\cos\gamma}\sin\phi \tag{6}$$

The states are the altitude h, Earth Cartesian coordinates x and y, velocity V, flight path angle γ and heading ψ.

The control inputs $u, (u \in U \subseteq \mathbb{R}^2)$ for equations (1)-(6)

are angle of attack α and bank angle ϕ. In equations (1)-(3) the inertial movement of the RLV is affected by the wind speed $W = (W_x, W_y, W_h) \in \mathbb{R}^3$ which acts like a statistically controlled disturbance with mean and standard deviation adopted from [19] and presented in Fig 3

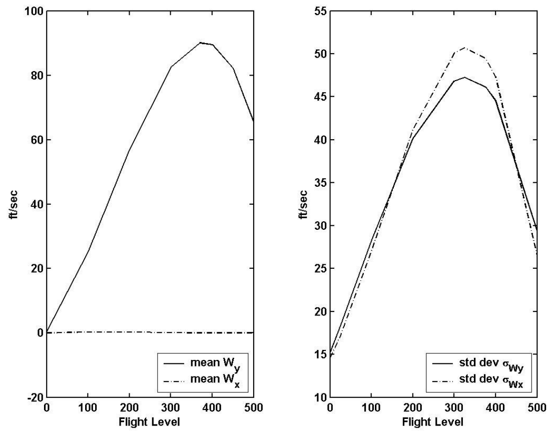

Fig. 3. Wind mean and standard deviation (from [19]).

Using the data from Fig. 3 we used the following wind bounds in our calculations:

$$W_h(h) = 0 \tag{7}$$

$$\bar{W}_x(h) - 2\sigma_{W_x}(h) \le W_x(h) \le \bar{W}_x(h) + 2\sigma_{W_x}(h) \tag{8}$$

$$\bar{W}_y(h) - 2\sigma_{W_y}(h) \le W_y(h) \le \bar{W}_y(h) + 2\sigma_{W_y}(h) \tag{9}$$

The equations for the air density ρ, the speed of sound a and the gravitational acceleration g are [18]:

$$\rho = \rho_0 e^{-\beta h} \tag{10}$$

$$a = a_0 + a_1 h + a_2 h^2 + a_3 h^3 \tag{11}$$

$$g = g_0 \left(R_E / (R_E + h) \right)^2 \tag{12}$$

The coefficients are obtained from [15], [18]

III. Reachability problem: Background

Consider a dynamical system

$$\dot{x} = f(x, u, d) \tag{13}$$

$x \in \mathbb{R}^n, u \in U \subseteq \mathbb{R}^m, d \in D \subseteq \mathbb{R}^k, f(\cdot,\cdot): \mathbb{R}^n \times U \to \mathbb{R}^n$.

Assume that f is Lipschitz in x and continuous in u and d and that $u(\cdot)$ in $U_{[t,t']}$ and $d(\cdot)$ in $D_{[t,t']}$ (the sets of Lebesgue measurable functions from the interval $[t,t']$ to $U \subseteq \mathbb{R}^m$ and $D \subseteq \mathbb{R}^k$ respectively); in this case (13) has a unique solution over $[t,t']$ starting at every $x(t) = x$.

We formulate a pursuit evasion game over the horizon $T \ge 0$ with target set $K \subseteq \mathbb{R}^n$. We assume that u is trying to keep the state in the set K, whereas d is trying to drive it out of K. To ensure the game is well-posed, we restrict u to play non-anticipative strategies. Recall that a function $g(\cdot): D_{[0,T]} \to U_{[0,T]}$ is called non-anticipative [21] if for all $d(\cdot), d'(\cdot) \in D$ and all $t \in [0,T]$, if $d(s) = d'(s)$ for almost all s in $[0,t]$, then $g(d)(s) = g(d')(s)$ for almost all s in $[0,t]$.

For the types of safety problems considered here, we would like to establish the set of initial states for which u can win the game, in other words the set

$$\begin{aligned} \mathrm{Disc}(t,K) = \{ & x \in \mathbb{R}^n \mid \exists \textit{ nonanticipative } g(\cdot), \\ & \forall d \in D_{[t,T]}, \forall \tau \in [t,T], x(\tau) \in K \} \end{aligned} \tag{14}$$

The characterization of this set (known as the *discriminating kernel* [22]) can be done either directly [22], or by characterizing it as a SUPMIN problem [20]. For the latter characterization one needs to introduce a bounded, uniformly continuous function $l: \mathbb{R}^n \to \mathbb{R}$ such that

$$K = \{ x \in \mathbb{R}^n \mid l(x) \ge 0 \} \tag{15}$$

If one defines a value function

$$V(x,t) = \sup_{\textit{nonanticipative } u(\cdot)} \inf_{d(\cdot) \in \mathcal{D}_{[t,T]}} \min_{\tau \in [t,T]} l(x(\tau)) \tag{16}$$

then it is easy to show that

$$Disc(t,K) = \{ x \mid V(x,t) \ge 0 \} \tag{17}$$

The following characterization of V was derived in [20].

Theorem 1: V is the unique bounded and uniformly continuous viscosity solution of the terminal value problem

$$\frac{\partial V}{\partial t}(x,t) + \min\left\{ 0, \sup_{u \in \mathcal{U}} \inf_{d \in \mathcal{D}} \frac{\partial V}{\partial x}(x,t) f(x,u,d) \right\} = 0 \tag{18}$$

over $(x,t) \in \mathbb{R}^n \times [0,T]$, with terminal boundary condition $V(x,T) = l(x)$.

IV. Reachability approach to RLV landing problem

The computation of the aerodynamic envelope for RLV safe landing is a reachability problem: *We need to characterize the initial states for which despite the action of the wind, the controls can ensure that the vehicle will land safely at a certain landing strip while several constraints on flight path, landing point and velocity are satisfied*. In this section we show how to approximate this problem by a discriminating kernel computation so that it can be solved by algorithms developed for solving the PDE of Theorem 1.

If we set the origin of the Cartesian coordinates at the beginning of the landing strip, and align the runway with the x axis, the trajectory constraints that will be used in conjunction with the equations (1)-(9) are:

1) *Normal velocity at touchdown:* $V_z \le 20\,ft/\sec$

2) *Total landing velocity*: $150knts \le V \le 220knts$

3) *Touchdown point*: $-100 \le \mathrm{x} \le 100ft$,

$$-100 \le \mathrm{y} \le 100ft,\ -10^\circ \le \psi \le 10^\circ$$

Based on the previous definitions and equations (1)-(6), we encode an admissible flight domain

$$F_D := \left[\underline{h},\overline{h}\right]\times\left[\underline{\mathrm{x}},\overline{\mathrm{x}}\right]\times\left[\underline{\mathrm{y}},\overline{\mathrm{y}}\right]\times\left[\underline{V},\overline{V}\right]\times\left[\underline{\gamma},\overline{\gamma}\right]\times\left[\underline{\psi},\overline{\psi}\right] \tag{19}$$

and trajectory constraints as a cost function $l(\cdot):\mathbb{R}^6 \to \mathbb{R}$ in a SUPMIN problem :

$$\begin{aligned} l(x) = \min\{ & h-\underline{h},\overline{h}-h,\mathrm{x}\text{-}\underline{\mathrm{x}},\overline{\mathrm{x}}\text{-}\mathrm{x},\mathrm{y}\text{-}\underline{\mathrm{y}},\overline{\mathrm{y}}\text{-}\mathrm{y},V-\underline{V},\overline{V}-V, \\ & \gamma-\underline{\gamma},\overline{\gamma}-\gamma,\psi-\underline{\psi},\overline{\psi}-\psi,h-\tan\varphi\left(V\sin\gamma-V_{z_c}\right), \\ & h-\tan\varphi\left(x-V_{x_c}\right),h-\tan\varphi\left(x-\overline{\mathrm{x}}_{\mathrm{c}}\right),h+\tan\varphi\left(x-\underline{\mathrm{x}}_{\mathrm{c}}\right), \\ & h-\tan\varphi\left(y-\overline{\mathrm{y}}_{\mathrm{c}}\right),h+\tan\varphi\left(y-\underline{\mathrm{y}}_{\mathrm{c}}\right), \\ & h-\tan\varphi\left(\psi-\overline{\psi}_c\right),h+\tan\varphi\left(\psi-\underline{\psi}_c\right)\} \end{aligned} \tag{20}$$

with

$\underline{h} = 0\,ft, \overline{h} = 3.5\times10^4\,ft, \underline{x} = -10^4\,ft, \overline{x} = 10^4\,ft,$

$\underline{y} = -10^4\,ft, \overline{y} = 10^4\,ft, \underline{V} = 150knts, \overline{V} = 300knts$,

$\underline{\gamma} = -60^\circ, \overline{\gamma} = 60^\circ, \underline{\psi} = -180^\circ, \overline{\psi} = 180^\circ, \varphi = 20^\circ$ and

$V_{z_c} = 20\,ft/\sec, V_c = 220knts, \underline{\mathrm{x}}_{\mathrm{c}} = -100\,ft, \overline{\mathrm{x}}_{\mathrm{c}} = 100\,ft$

$\underline{\mathrm{y}}_{\mathrm{c}} = -100\,ft, \overline{\mathrm{y}}_{\mathrm{c}} = 100\,ft, \underline{\psi}_c = -180^\circ, \overline{\psi}_c = 180^\circ.$

Notice that the landing constrains, which really apply only to states with *h=0*, have been coded as linear functions of altitude. These functions are selected to meet the constraints at touchdown and extend them to the total envelope. For example the horizontal distance x from the origin, at the touchdown point, must be inside the box $[-100,100]\,ft$. A linear function with appropriate slope ($\tan\varphi$), to encode this constraint is:

$$h-\tan\varphi\left(x-\overline{x}_c\right)\ge 0, \quad h+\tan\varphi\left(x-\underline{x}_c\right)\ge 0 \tag{21}$$

Equations like these give rise to the terms in the second to fifth lines of equation (22).

Clearly, $l(x)\ge 0, \forall x\in K$, $l(x)\le 0, \forall x\notin K$ and *l(x)* is Lipschitz continuous. To keep l bounded (and since we are only interested in the behavior in and around K) we "saturate" the function l outside the set

$$\begin{aligned} & \left[\underline{h}-\delta h,\overline{h}+\delta h\right]\times\left[\underline{\mathrm{x}}-\delta\mathrm{x},\overline{\mathrm{x}}+\delta\mathrm{x}\right]\times\left[\underline{\mathrm{y}}-\delta\mathrm{y},\overline{\mathrm{y}}+\delta\mathrm{y}\right]\times \\ & \left[\underline{V}-\delta V,\overline{V}+\delta V\right]\times\left[\underline{\gamma}-\delta\gamma,\overline{\gamma}+\delta\lambda\right]\times\left[\underline{\psi}-\delta\psi,\overline{\psi}+\delta\psi\right] \end{aligned} \tag{22}$$

for some $\delta h, \delta\mathrm{x}, \delta\mathrm{y}, \delta V, \delta\gamma, \delta\psi > 0$ small.

The controls α and ϕ are assumed to be varied through the following regions:

$$0 \le \alpha \le 20^\circ, -60^\circ \le \phi \le 60^\circ \tag{23}$$

With this formulation, we can solve the Hamiltonian of the Theorem 1 to obtain optimal controls using reachability computational tools.

V. REACHABILITY COMPUTATIONS

The main limitation of existing computational tools for solving reachability problems for systems with continuous states is that, due to the exponential growth of the computation with the dimension of the continuous state space, the tools can be applied effectively to relatively low dimensional problems (typically 1-4 dimensions). The problem considered here is 6 dimensional and for this reason we approach the solution by decomposing it into fast and slow dynamics.

A. *Theoretical formulation*

A noteworthy point in the equations (1)-(6), is that the Earth Cartesian coordinates x,y ,h and the heading ψ are functions of velocity V, flight path angle γ, drag coefficient C_D and lift coefficient C_L. Having in mind that $\dot{V},\dot{\gamma}$ due to flight dynamics are faster than $\dot{\mathrm{x}},\dot{\mathrm{y}},\dot{\psi},\dot{h}$ we can evolve effectively the $\dot{\mathrm{x}},\dot{\mathrm{y}},\dot{\psi},\dot{h}$ equations if we use pseudo-controls $V(h),\gamma(h)$ combined with actual controls α and ϕ. This suggests the following decomposition of the reachability problem.

1) Fast dynamics problem: To produce the pseudo-controls $V(h),\gamma(h)$ we define the reachability problem with states *V*, *γ* and *h*, and apply the results from the previous section. The Hamiltonian of Theorem 1 becomes:

$$H_1(x,p) = \min\left\{0,\sup_{u\in\mathcal{U}} p^T f(x,u,d)\right\} \tag{24}$$

Here $f(x,u,d)$ represents equations (1),(4),(5) with optimal controls $\hat{\alpha},\hat{\phi}$ computed over a finite grid of the control set $\left[\underline{\alpha},\overline{\alpha}\right]\times\left[\underline{\phi},\overline{\phi}\right]$; notice that since $W_h=0$, the disturbance does not enter the computation yet. The function used to encode the cost is

$$\begin{aligned} l_1(x) = \min\{ & h-\underline{h},\overline{h}-h,V-\underline{V},\overline{V}-V,\gamma-\underline{\gamma},\overline{\gamma}-\gamma, \\ & h-\tan\varphi\left(V\sin\gamma-V_{z_c}\right),h-\tan\varphi\left(V-V_c\right)\} \end{aligned} \tag{25}$$

The solution of the problem gives us a range $\left[\underline{V}(h),\overline{V}(h)\right]\times\left[\underline{\gamma}(h),\overline{\gamma}(h)\right]$ of *V* and *γ* that can be used at each altitude *h*.

2) Slow dynamics problem: After obtaining ranges for the pseudo-controls $V(h),\gamma(h)$ from the fast dynamics, we form the Hamiltonian:

$$H_2(x,p) = \min\left\{0, \sup_{u\in\mathcal{U}} \inf_{d\in\mathcal{D}} p^T f(x,u,d)\right\} \qquad (26)$$

Here $f(x,u,d)$ now represents equations (1)-(3),(6) with optimal controls $\hat{V}(h), \hat{\gamma}(h), \hat{\alpha}, \hat{\phi}$ computed over a finite grid of the control set $[\underline{V}, \bar{V}] \times [\underline{\gamma}, \bar{\gamma}] \times [\underline{\alpha}, \bar{\alpha}] \times [\underline{\phi}, \bar{\phi}], \forall h$. Here we set:

$$\begin{aligned} l_2(x) = \min\{&h-\underline{h}, \bar{h}-h, \mathrm{x}-\underline{\mathrm{x}}, \bar{\mathrm{x}}-\mathrm{x}, \mathrm{y}-\underline{\mathrm{y}}, \bar{\mathrm{y}}-\mathrm{y}, \\ &\psi-\underline{\psi}, \bar{\psi}-\psi, h-\tan\varphi(\mathrm{x}-\bar{\mathrm{x}}_c), h+\tan\varphi(\mathrm{x}-\underline{\mathrm{x}}_c), \\ &h-\tan\varphi(\mathrm{y}-\bar{\mathrm{y}}_c), h+\tan\varphi(\mathrm{y}-\underline{\mathrm{y}}_c), \\ &h-\tan\varphi(\psi-\bar{\psi}_c), h+\tan\varphi(\psi-\underline{\psi}_c)\} \end{aligned} \qquad (27)$$

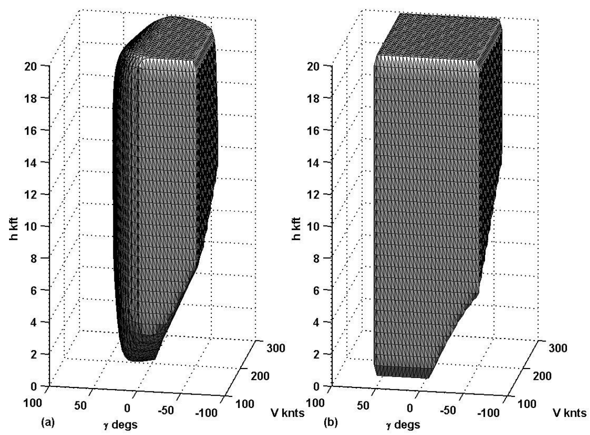

Fig. 4. Level sets of (a) $V_1(x,0)$ and (b) $V_1(x,T)$ for T=6s

B. Implementation and results

The two Hamiltonians were coded in a numerical tool developed at Stanford University [10], [11] for computing viscosity solutions to Hamilton-Jacobi equations using the algorithm of [12], [13].

1) Pseudo-control computation: The computation was performed on a 30×30×30 grid for h, V, γ, with a 5×5 grid for controls $\alpha \times \phi$ required a 640 seconds on a Pentium 4, 1.8GHz processor running windows XP. Fig. 4 depicts the initial sets $\mathrm{Viab}(0,K) = \{x \in \mathbb{R}^3 \mid V_1(x,0) \geq 0\}$ for initial level set envelope with a missing part due to implementation of the cost function (25) and the level set after a horizon $T = 6\,\mathrm{sec}$. As expected the level set "shrinks" as T increases but for $T \approx 6s$ the shrinking stops.

If we compare Fig 4(a) and Fig 4(b) we see that in Fig. 4(a) there is a missing part for high positive flight path angle and low velocity for any altitude, this part corresponds to a stall region. The general shape of the level sets suggests that if the RLV ever gets to an unsafe state (unshaded area) then, whatever the flight management system does from them on, it will sooner or later violate the flight envelope requirements. If the initial condition is inside the level set, however, unsafe states can be avoided by applying the optimal controls determined by the Hamiltonian whenever the state trajectory hits the boundary of the level set.

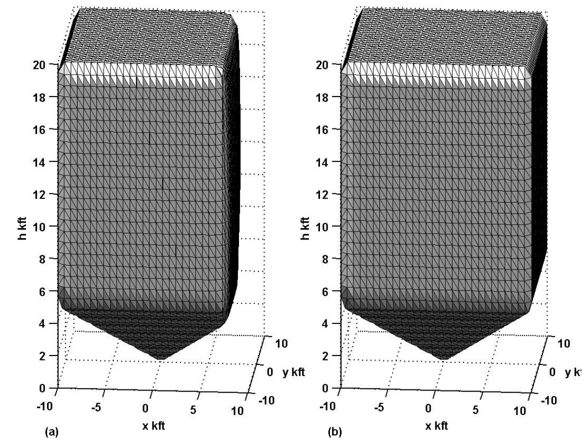

Fig. 5. Level sets of (a) $V_2(x,0)$ and (b) $V_2(x,T)$ for T=20s with ψ=0° and zero wind

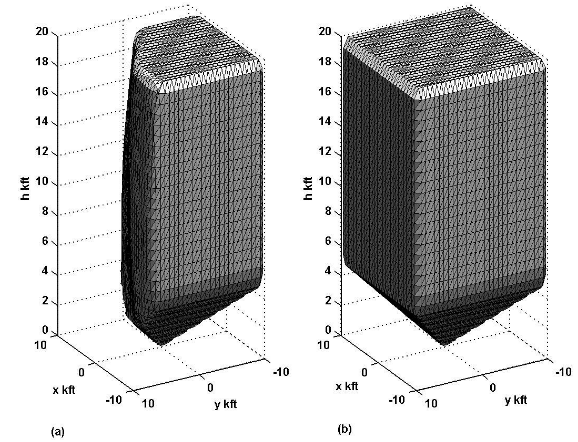

Fig. 6. Level sets of (a) $V_2(x,0)$ and (b) $V_2(x,T)$ for T=20s with ψ=0° and winds per equations (7)-(9)

2) Safe envelope computation: To obtain the controls for the computation of the slow dynamics problem we combine safe envelope grid points (pseudo-controls) and actual control inputs with the following algorithm:

For every h, combine every safe γ with safe $\underline{V}$ and safe $\bar{V}$, and 5×5 grid of controls α, ϕ.

We must mention that on the boundary of the pseudo control envelope (Fig 4a) we use only optimal α, ϕ [obtained by solving (24)] combined with $V(h), \gamma(h)$.

The computation was performed on a 30×30×30×30 grid for h, x, y, ψ and a 1500 safe controls for each h, required a 84600 seconds on a Pentium 4, 1.8GHz processor running Windows XP.

As expected the level set "shrinks" as T increases but for $T \approx 20s$ the shrinking stops. The lower part of initial level set in Fig 4(b) and Fig 5(b) is shaped like a pyramid because of cost function (27). Fig 5(a) represents all the safe h,x,y states when the $\psi = 0^{\circ}$ for zero winds. In addition in Fig 6(a) we have an extra missing part due to the existence of ambient winds during the final landing phase. The two figures are representative 3D slices from the total 4D aerodynamic envelope.

VI. CONCLUDING REMARKS

The contribution of this paper was to apply a few tricks, first to decompose a 6-dimensions aeronautical problem into fast and slow dynamics and second to incorporate specifications for the landing point, the velocity and the heading into a viability computation. These tricks broaden the capabilities of existing computational reachability tools to new challenging areas.

The numerical results show that it is feasible to do exacting computations for systems nonlinear continuous dynamics in higher dimensions, if one can exploit additional structure in the model (in our case, the separation into slow and fast dynamics).

In further work, we plan to obtain the total aerodynamic database for the RLV and demonstrate the numerical convergence of our computation by analyzing the results as the continuous state space grid is made finer, a standard method of validation of scientific computing codes

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