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The Behavioral LQ-problem for linear nD systems

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Abstract— **This paper deals with systems described by constant coefficient linear partial differential equations (n-D Systems) from a behavioral point of view. The performance functional is the integral of a quadratic differential form. In this case an appropriate tool to express quadratic functionals are 2-n variable polynomial matrices. We look for a characterization of the set of stationary trajectories and for the set of local minimal trajectories with respect to compact support variations, turning out that they are equal if the system is dissipative. Finally we implement these trajectories of the given plant as a (regular) interconnection of the plant and a controller.**

I. INTRODUCTION

Given the plant and a quadratic differential form (in the following abbreviated with QDF), the LQ-problem treated in this paper, is to characterize the trajectories of the plant that are stationary or optimal with respect to the integral of the QDF.

The linear quadratic (LQ) control was initially developed for input/state/output system with a performance functional given by an integral of the input and the state with one independent variable (usually time). However, often the systems do not have a clear input/state/output structure, may contain high order derivatives, or the cost may have derivatives in the control variables. Moreover, many if not most of the models of physical systems involve both time and space variables. The purpose of this paper is to approach the LQ-problem considering systems described by partial differential equations without assuming any special representation (behavioral approach) and considering the cost as an integral of an arbitrary quadratic expression in the system variables and their derivatives.

With this approach we can deal with systems that are not in state space form, and their dynamics depend on both time and space. The main result of the first part of this paper can be summarized as follows. We find an explicit representation of the behavior of the stationary trajectories and we prove that the set of local minimum trajectories are actually the same set if the system is dissipative or empty otherwise.

In the second part the so called *synthesis problem* is addressed i.e. find an n-D system called the controller that constrains the plant behavior through a distinguished set of variables, namely, the control variables, in order to implement the optimal trajectories. A representation of such a controller is found.

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II. MULTIDIMENSIONAL SYSTEMS

In behavioral system theory, the *behavior* is a subset of the space $\mathbb{W}^{\mathbb{T}}$ consisting of all trajectories from $\mathbb{T}$, the *indexing set*, to $\mathbb{W}$, the *signal space*. In this paper we consider systems with $\mathbb{T} = \mathbb{R}^n$ (from which the terminology "nD-system" derives) and $\mathbb{W} = \mathbb{R}^{\mathtt{w}}$. We call $\mathfrak{B}$ a *linear differential* nD *behavior* if it is the solution set of a system of linear, constant-coefficient partial differential equations; more precisely, if $\mathfrak{B}$ is the subset of $\mathfrak{C}^\infty(\mathbb{R}^n, \mathbb{R}^{\mathtt{w}})$ consisting of all solutions to

$$R(\frac{d}{d\mathbf{x}})w = 0 \tag{1}$$

where R is a polynomial matrix in n indeterminates ξ_i, $i = 1, \ldots, n$, and $\frac{d}{d\mathbf{x}} = (\frac{\partial}{\partial x_1}, \ldots, \frac{\partial}{\partial x_n})$. We call (1) a *kernel representation* of $\mathfrak{B}$ and write $\mathfrak{B}$=ker(R). We denote the set consisting of all linear differential nD-systems with $\mathtt{w}$ external variables by $\mathcal{L}_n^{\mathtt{w}}$. However, there are many other ways to represent an n-D system. One is using some auxiliary variables, called *latent variables*, that appear in order to express basic physical laws. Let us mention a few: *internal voltages* and *currents* in electrical circuits in order to express the external port behavior; *momentum* in Hamiltonian mechanics in order to describe the evolution of the position; *prices* in economics in order to explain the production and exchange of economic goods, etc... Hence, $\mathfrak{B}$ is the subset of $\mathfrak{C}^\infty(\mathbb{R}^n, \mathbb{R}^{\mathtt{w}})$ consisting of all functions w for which there exits $\ell \in \mathfrak{C}^\infty(\mathbb{R}^n, \mathbb{R}^{\mathtt{w}})$ such that

$$R(\frac{d}{d\mathbf{x}})w = M(\frac{d}{d\mathbf{x}})\ell. \tag{2}$$

Here R and M are polynomial matrices in n indeterminates ξ_i, $i = 1, \ldots, n$, and again $\frac{d}{d\mathbf{x}} = (\frac{\partial}{\partial x_1}, \ldots, \frac{\partial}{\partial x_n})$. We call (2) a *latent variable representation* of $\mathfrak{B}$ and the variable ℓ is called the *latent variable*.

III. QUADRATIC DIFFERENTIAL FORMS

A quadratic differential form (QDF) is a quadratic form in the components of a function $w \in \mathfrak{C}^\infty(\mathbb{R}^n, \mathbb{R}^{\mathtt{w}})$ and its derivatives. An appropriate tool to express quadratic functionals are 2n-variable polynomial matrices.

In order to simplify the notation, we denote the vector $\mathbf{x} := (x_1, \ldots, x_n)$, the multi-indices $\mathbf{k} := (k_1, \ldots, k_n)$ and $\mathbf{l} := (l_1, \ldots, l_n)$, and use the notation $\zeta := (\zeta_1, \ldots, \zeta_n), \xi := (\xi_1, \ldots, \xi_n)$ and $\eta := (\eta_1, \ldots, \eta_n)$. Let $\mathbb{R}^{\mathtt{w}_1 \times \mathtt{w}_2}[\zeta, \eta]$ denote the set of real polynomial $\mathtt{w}_1 \times \mathtt{w}_2$ matrices in the $2n$ indeterminates ζ and η; that is, an element of $\mathbb{R}^{\mathtt{w}_1 \times \mathtt{w}_2}[\zeta, \eta]$ is of the form

$$\Phi(\zeta, \eta) = \sum_{\mathbf{k},\mathbf{l}} \Phi_{\mathbf{k},\mathbf{l}} \zeta^{\mathbf{k}} \eta^{\mathbf{l}}$$

where $\Phi_{\mathbf{k},\mathbf{l}} \in \mathbb{R}^{\mathtt{w}_1\times\mathtt{w}_2}$; the sum ranges over the nonnegative multi-indices $\mathbf{k}$ and $\mathbf{l}$, and is assumed to be finite. Such 2n-variable polynomial matrix induces a *bilinear differential form* L_Φ

$$L_\Phi : \mathfrak{C}^\infty(\mathbb{R}^n,\mathbb{R}^{\mathtt{w}_1}) \times \mathfrak{C}^\infty(\mathbb{R}^n,\mathbb{R}^{\mathtt{w}_2}) \longrightarrow \mathfrak{C}^\infty(\mathbb{R}^n,\mathbb{R})$$
$$L_\Phi(v,w) := \sum_{\mathbf{k},\mathbf{l}} (\frac{d^{\mathbf{k}}v}{d\mathbf{x}^{\mathbf{k}}})^T \Phi_{\mathbf{k},\mathbf{l}} \frac{d^{\mathbf{l}}w}{d\mathbf{x}^{\mathbf{l}}}$$

where the $\mathbf{k}$-th derivative operator $\frac{d^{\mathbf{k}}}{d\mathbf{x}^{\mathbf{k}}}$ is defined as $\frac{d^{\mathbf{k}}}{d\mathbf{x}^{\mathbf{k}}} := \frac{\partial^{k_1}}{\partial x_1^{k_1}} \cdots \frac{\partial^{k_n}}{\partial x_n^{k_n}}$ (similarly for $\frac{d^{\mathbf{l}}}{d\mathbf{x}^{\mathbf{l}}}$). Note that ζ corresponds to differentiation of terms to the left and η refers to the terms to the right.

The $2n$-variable polynomial matrix $\Phi(\zeta,\eta)$ is called *symmetric* if $w_1 = w_2 =: w$ and $\Phi(\zeta,\eta) = \Phi(\eta,\zeta)^T$. Note that the former condition is equivalent to $L_\Phi(w_1,w_2) = L_\Phi(w_2,w_1)$ for all w_1,w_2. We denote the symmetric elements of $\mathbb{R}^{\mathtt{w}\times\mathtt{w}}[\zeta,\eta]$ as $\mathbb{R}_S^{\mathtt{w}\times\mathtt{w}}[\zeta,\eta]$. If Φ is symmetric then it induces also a quadratic functional

$$Q_\Phi : \mathfrak{C}^\infty(\mathbb{R}^n,\mathbb{R}^{\mathtt{w}}) \longrightarrow \mathfrak{C}^\infty(\mathbb{R}^n,\mathbb{R})$$
$$Q_\Phi(w) := L_\Phi(w,w)$$

We will call Q_Φ the *quadratic differential form* associated with Φ.

Definition 1: Given $\Phi \in \mathbb{R}_S^{\mathtt{w}\times\mathtt{w}}[\zeta,\eta]$. The plant $\mathfrak{B}$ is said to be Q_Φ-dissipative if

$$\int_{\mathbb{R}^n} Q_\Phi(w)\mathrm{d}\mathbf{x} \geq 0 \qquad \forall w \in \mathfrak{B} \text{ with compact support} \quad (3)$$

Dissipativity states that the system absorbs energy (in space and time) during any history in $\mathfrak{B}$ that starts and ends with the system at rest.

Definition 2: Let $\Phi' \in \mathbb{R}_S^{\ell\times\ell}[\zeta,\eta]$. $Q_{\Phi'}$ is said to be average non-negative if

$$\int_{\mathbb{R}^n} Q_{\Phi'}(\ell)\mathrm{d}\mathbf{x} \geq 0 \quad (4)$$
for all $\ell \in \mathfrak{C}^\infty(\mathbb{R}^n,\mathbb{R}^\ell)$ of compact support.

IV. STATIONARY AND LOCAL MINIMA

Definition 3: Let $\Phi \in \mathbb{R}_S^{\mathtt{w}\times\mathtt{w}}[\zeta,\eta]$. The trajectory $w \in \mathfrak{C}^\infty(\mathbb{R}^n,\mathbb{R}^{\mathtt{w}})$ is called stationary with respect to $\int_{\mathbb{R}^n} Q_\Phi(w)d\mathbf{x}$ if for all $\Delta \in \mathfrak{C}^\infty(\mathbb{R}^n,\mathbb{R}^{\mathtt{w}})$ of compact support, we have

$$\int_{\mathbb{R}^n} L_\Phi(\Delta,w)d\mathbf{x} = 0. \quad (5)$$

Oviously, $w \in \mathfrak{C}^\infty(\mathbb{R}^n,\mathbb{R}^{\mathtt{w}})$ is stationary if and only if for all $\Delta \in \mathfrak{C}^\infty(\mathbb{R}^n,\mathbb{R}^{\mathtt{w}})$ of compact support we have

$$\int_{\mathbb{R}^n} Q_\Phi(w+\Delta) - Q_\Phi(w)d\mathbf{x} = \int_{\mathbb{R}^n} Q_\Phi(\Delta)d\mathbf{x}. \quad (6)$$

Theorem 4: Let $\Phi \in \mathbb{R}_S^{\mathtt{w}\times\mathtt{w}}[\zeta,\eta]$. The set of stationary trajectories with respect to $\int_{\mathbb{R}^n} Q_\Phi(w)d\mathbf{x}$ consists of all $w \in \mathfrak{C}^\infty(\mathbb{R}^n,\mathbb{R}^{\mathtt{w}})$ that satisfy

$$\Phi(-\frac{d}{d\mathbf{x}},\frac{d}{d\mathbf{x}})w = 0 \quad (7)$$

Proof: If w is stationary then $\int_{\mathbb{R}^n} L_\Phi(\Delta,w)d\mathbf{x} = 0$ for all $\Delta \in \mathfrak{C}^\infty(\mathbb{R}^n,\mathbb{R}^{\mathtt{w}})$ with compact support. Then integrating by parts we obtain

$$\int_{\mathbb{R}^n} L_\Phi(\Delta,w)d\mathbf{x} = \int_{\mathbb{R}^n} \Delta^T\Phi(-\frac{d}{d\mathbf{x}},\frac{d}{d\mathbf{x}})wd\mathbf{x}, \quad (8)$$

Since this holds for all $\Delta \in \mathfrak{C}^\infty(\mathbb{R}^n,\mathbb{R}^{\mathtt{w}})$ with compact support, w is stationary if and only if (7) holds. ■

Now we look at the local minimum trajectories and their relation with stationary ones.

Definition 5: Let $\Phi \in \mathbb{R}_S^{\mathtt{w}\times\mathtt{w}}[\zeta,\eta]$. The trajectory w is called a local minimum for $\int_{\mathbb{R}^n} Q_\Phi(w)d\mathbf{x}$ with respect to compact support variations if

$$\int_{\mathbb{R}^n} Q_\Phi(w+\Delta) - Q_\Phi(w)d\mathbf{x} \geq 0, \quad (9)$$

for all $\Delta \in \mathfrak{C}^\infty(\mathbb{R}^n,\mathbb{R}^{\mathtt{w}})$ of compact support.

Theorem 6: Let $\Phi \in \mathbb{R}_S^{\mathtt{w}\times\mathtt{w}}[\zeta,\eta]$. If Q_Φ is average non-negative then the set of local minimum trajectories is equal to the set of stationary trajectories. If Q_Φ is not average non-negative, then the set of local minimum trajectories is empty.

Proof: It is easy to see that

$$\begin{aligned}&\int_{\mathbb{R}^n} Q_\Phi(w+\Delta) - Q_\Phi(w)d\mathbf{x}\\ &= 2\int_{\mathbb{R}^n} L_\Phi(w,\Delta)d\mathbf{x} + \int_{\mathbb{R}^n} Q_\Phi(\Delta)d\mathbf{x}\end{aligned} \quad (10)$$

Suppose Q_Φ is average non-negative. If $\int_{\mathbb{R}^n} L_\Phi(w,\Delta)d\mathbf{x}$ is not zero for all Δ of compact support, then one can always choose a Δ in such a way that $\int_{\mathbb{R}^n} L_\Phi(w,\Delta)d\mathbf{x} + \int_{\mathbb{R}^n} Q_\Phi(\Delta)d\mathbf{x}$ is negative since L_Φ is linear in Δ and Q_Φ is bilinear in Δ. Once we have $\int_{\mathbb{R}^n} L_\Phi(w,\Delta)d\mathbf{x} = 0$ for all Δ, then we clearly have that w is local minimum. Now suppose Q_Φ is not average non-negative. Using the same arguments as before we have that $\int_{\mathbb{R}^n} L_\Phi(w,\Delta)d\mathbf{x}$ must be zero as well. Then the set of local minimum is empty. ■

V. INTERCONNECTION

We now discuss the issue of control as interconnection. Since a plant behavior $\mathfrak{B} \in \mathcal{L}_{\mathrm{n}}^{\mathtt{w}}$ consists of all trajectories satisfying a set of differential equations, one would like to restrict this space of trajectories to a desired subsystem, $\mathcal{K} \subset \mathfrak{B}$. This restriction can be effected by increasing the number of equations that the variables of the plant have to satisfy. These additional laws themselves define a new system, called the controller (denoted by $\mathcal{C}$). The interconnection of the two systems (the plant and the controller) results in the controlled behavior $\mathcal{K}$. After interconnection, the variables have to satisfy the laws of *both* $\mathfrak{B}$ and $\mathcal{C}$. In this section we will look at two types of interconnections, full and partial.

The *full interconnection* of $\mathfrak{B}$ and $\mathcal{C}$ is defined as the system with behavior $\mathfrak{B}\cap\mathcal{C}$. Note that $\mathfrak{B}\cap\mathcal{C}$ is again an element of $\mathcal{L}_n^{\mathtt{w}}$. A given behavior $\mathcal{K} \in \mathcal{L}_n^{\mathtt{w}}$ is called *implementable with respect to* $\mathfrak{B}$ *by full interconnection* if there exists a $\mathcal{C} \in \mathcal{L}_n^{\mathtt{w}}$ such that $\mathcal{K} = \mathfrak{B}\cap\mathcal{C}$. The full interconnection of $\mathfrak{B}$ and $\mathcal{C}$ is called *regular*, if

$$\mathtt{p}(\mathfrak{B}\cap\mathcal{C}) = \mathtt{p}(\mathfrak{B}) + \mathtt{p}(\mathcal{C}).$$

Where p is equal to the rank of the polynomial matrix in any kernel representation of $\mathfrak{B}$. Let $Rw = 0$ and $Cw = 0$ be kernel representations of $\mathfrak{B}$ and $\mathcal{C}$ respectively. We know that the full interconnection of $\mathfrak{B}$ and $\mathcal{C}$ is regular if and only if $\mathfrak{B}$ + $\mathcal{C}$ = $\mathfrak{C}^\infty(\mathbb{R}^n, \mathbb{R}^w)$, (see [6]) . Regular interconnection expresses the idea of "restricting what is not restricted". In a regular interconnection, the controller imposes new restrictions on the plant; it does not reimpose restrictitions that are already present. In this sense, the controller in a regular interconnection has no redundancy. We call optimal controller, the controller that implements local minimum trayectories. We would want the controlled behavior to have non-zero trayectories (i.e. non-trivial), and so we look for nontrivial controllers.

Theorem 7: Given Q_Φ average non-negative and $\mathfrak{B}$ = ker R, there exit nontrivial optimal controllers if

$$\begin{bmatrix} R \\ \partial\Phi \end{bmatrix}$$

is no ZRP (zero right prime), that is, there does not exits a polynomial matrix S such that im (R($\frac{d}{d\mathbf{x}}$))= ker (S). If so, $\mathcal{C}$ = Ker $\partial\Phi$ is one controller. If it is ZRP only the trivial controller is optimal.

Where $\partial\Phi(\xi) = \Phi(\xi, \xi)$.

Proof: See [5]. ■

VI. Controllability and Observability

One of the properties of behaviors which is very convenient, in particular for LQ problems, is controllability.

Definition 8: A system $\mathfrak{B} \in \mathcal{L}_n^{\mathtt{w}}$ is said to be controllable if for all w_1 , $w_2 \in \mathfrak{B}$ and all sets U_1 ,$U_2 \subset \mathbb{R}^{\mathrm{n}}$ with disjoint closure, there exist a $w \in \mathfrak{B}$ such that $w\mid_{U_1} = w_1 \mid_{U_1}$ and $w\mid_{U_2} = w_2 \mid_{U_2}$.

There are a number of characterizations of controllability but the one useful for our purposes is the equivalence of controllability with the existence of an image representation. Consider the following special latent variable representation:

$$w = M(\frac{d}{d\mathbf{x}})\ell$$

with $M \in \mathbb{R}^{\mathtt{w}\times\ell}[\xi]$. Obviously, by the elimination theorem, see [3], its manifest behavior $\mathfrak{B}$ is a linear differential n-D system again, i.e. $\mathfrak{B} \in \mathcal{L}_n^{\mathtt{w}}$. Such special latent variable representations often appear in physics, where the latent variables in a such representation are called potentials. Clearly, $\mathfrak{B} = \mathrm{im}(M(\frac{d}{d\mathbf{x}}))$. For this reason this representation is called an *image representation* of its manifest behavior.

Theorem 9: (See [3]), $\mathfrak{B} \in \mathcal{L}_n^{\mathtt{w}}$ admits an image representation if and only if it is controllable.

Now, we will consider a useful property of n-D systems, observability. For this property one needs to split the variables of the system in two sets; the first set of variables are interpreted as the observed variables an the other is the set of 'to be deduced' variables.

Definition 10: Let $\mathfrak{B} \in \mathcal{L}_n^{\mathtt{w}}$ with manifest variable w, $w = (w_1, w_2)$ be a partition of w$\in \mathcal{L}_n^{\mathtt{w}}$ .Then w_2 it is said to be observable from w_1 in $\mathfrak{B}$ if given any two trajectories $(w_1{}', w_2{}'), (w_1{}'', w_2{}'') \in \mathfrak{B}$ we have that $w_1{}' = w_1{}''$ implies $w_2{}' = w_2{}''$.

So, observability only becomes an intrinsict property of the behavior after a partition of the manifest variable w is given. Although we can divide the set of variables in many ways, one natural way to do it, is when one is looking at a latent variable representation of the behavior is to ask whether the latent variables are observable from the manifest variables. If this is the case we call the latent variable representation *observable*. For controllable 1-D systems it can be shown that there always exists an observable image representation. This is not true for n-D systems. ¿From now on, we will assume that the plant is controllable and we will assume it has an observable image representation $\mathfrak{B} := \mathrm{Im}M(\frac{d}{d\mathbf{x}})$. If $w = M(\frac{d}{d\mathbf{x}})\ell$ is an observable image representation of $\mathfrak{B}$, then $w \in \mathfrak{B}$ has compact support if and only if the corresponding $\ell \in \mathfrak{C}^\infty(\mathbb{R}^n, \mathbb{R}^\ell)$ has compact support.

Given a 2n-variable polynomial matrix $\Phi(\zeta, \eta)$, suppose $w = M(\frac{d}{d\mathbf{x}})\ell$ is an observable image representation of $\mathfrak{B}$, we can then consider the induced 2n variable polynomial matrix Φ' defined by

$$\Phi'(\zeta, \eta) := M^T(\zeta)\Phi(\zeta, \eta)M(\eta) \quad (11)$$

Remark 1: Note that $\mathfrak{B}$ is Q_Φ-dissipative if and only if $Q_{\Phi'}$ is average non-negative.

We replace $Q_\Phi(w)$ by $Q_{\Phi'}(\ell)$ in the performance functional (or equivalently replace $\Phi(\zeta, \eta)$ by $M^T(\zeta)\Phi(\zeta, \eta)M(\eta)$), and obtain an LQ problem in which the dynamic variable w is replaced by the unconstrained variable ℓ.

Remark 2: Given a 2n-variable polynomial matrix $\Phi(\zeta, \eta)$, suppose $w = M(\frac{d}{d\mathbf{x}})\ell$ is an observable image representation of $\mathfrak{B}$. Then it can be shown that there exist a left inverse $M^\dagger(\xi) \in \mathbb{R}^{\mathtt{w}\times\bullet}[\xi]$ of M. Therefore from $w = M(\frac{d}{d\mathbf{x}})\ell$,

$$M^\dagger(\frac{d}{d\mathbf{x}})w{=}\ell. \quad (12)$$

For more details see [1].

Theorem 11: Let $\Phi \in \mathbb{R}_S^{\mathtt{w}\times\mathtt{w}}[\zeta, \eta]$ and a behavior $\mathfrak{B}$ = ker R and suppose $\mathfrak{B}$ has also $w = M(\frac{d}{d\mathbf{x}})\ell$ as an observable image representation. Assume that the polynomial matrix $\Phi'(\zeta, \eta)$, satisfy $det(\Phi'(-\xi, \xi)) \neq 0$. The set of stationary trajectories of $\mathcal{K}$ with respect to $\int_{\mathbb{R}^n} Q_\Phi(w)d\mathbf{x}$, is regularly implementable with respect to $\mathfrak{B}$ by full interconnection, and a controller $\mathcal{C}$ = Ker C that regularly implements $\mathcal{K}$ is represented by:

$$M^T(\frac{d}{d\mathbf{x}})\Phi(-\frac{d}{d\mathbf{x}}, \frac{d}{d\mathbf{x}})w = 0 \quad (13)$$

Proof: The proof that C is the controller which implement the stationary trayectories follows by sustitution. To see that the interconnection is regular we need to check that $\mathfrak{B}$ + $\mathcal{C}$ = $\mathfrak{C}^\infty(\mathbb{R}^n, \mathbb{R}^w)$. Now, a kernel representation of $\mathfrak{B}$ + $\mathcal{C}$ is obtain as folows: Consider $\begin{bmatrix} R \\ C \end{bmatrix}$ and let $\begin{bmatrix} N & L \end{bmatrix}$ be such that $ker \begin{bmatrix} N & L \end{bmatrix} = \mathrm{im} \begin{bmatrix} R \\ C \end{bmatrix}$.

Then obviously $NR = -LC$ and according to ([6], lemma 2.14), $\mathfrak{B}$ + $\mathcal{C}$ = $ker(NR)$. In our case we have $C(\xi) = M^T(\xi)\Phi(-\xi,\xi)$ so we get $N(\xi)R(\xi) + L(\xi)M^T(\xi)\Phi(-\xi,\xi) = 0$. Hence for every $\ell \in \mathfrak{C}^\infty(\mathbb{R}^n,\mathbb{R}^\ell)$ we get $L(\frac{d}{d\mathbf{x}})M^T(\frac{d}{d\mathbf{x}})\Phi(-\frac{d}{d\mathbf{x}},\frac{d}{d\mathbf{x}})M(\frac{d}{d\mathbf{x}})\ell = 0$. Recall that $\Phi^{'}(-\xi,\xi) = M^T(-\xi)\Phi(-\xi,\xi)M(\xi)$ which is assumed to be nonsingular. Hence $L(\xi) = 0$. Thus $NR = 0$ and $\mathfrak{B} + \mathcal{C} = \mathfrak{C}^\infty(\mathbb{R}^n,\mathbb{R}^w)$. This proofs that the interconnection is regular. ∎

Remark 3: It can be shown that $M^\dagger(\frac{d}{d\mathbf{x}})\Phi^{'}(\frac{d}{d\mathbf{x}},\frac{d}{d\mathbf{x}}) = w$ represents also a controller that regularly implements $\mathcal{K}$.

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