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Combined a priori knowledge and structural model closed-loop identification of high temperature industrial furnaces

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Abstract— **The paper describes a systematic approach which combines model identification techniques that are appropriately tailored to accommodate *a priori* knowledge of plant behaviour. The plant under consideration is a high temperature gas-fired multi-zone industrial furnace. The aim is to obtain a model which replicates the nonlinear temperature control loop to be used for improved control system design. The approach is demonstrated using recorded measurements of closed-loop operational data. Generic features of the developed approach are highlighted.**

Index Terms— **Bilinear system, Closed-loop identification, Industrial furnace, Modelling, Nonlinear systems, Optimisation, Relay, Regularisation techniques.**

I. Introduction

With attention given to the nonlinear temperature control loop corresponding to a single-zone within a multi-zone industrial furnace, the paper describes the development of a systematic procedure to obtain a model for the purpose of enhanced closed-loop control system design. Whilst the application is specific, the modelling problem has much in common with many other industrial processes, where demands for enhanced control leads to the need for more sophisticated models and, hence, increasingly complex solution methodologies. The key to solving such complex modelling problems lies in the ability to accommodate available *a priori* knowledge of the process, whilst at the same time being able to obtain an appropriate balance, or trade-off, between the use of *a priori* knowledge and a black box modelling approach based on measured data only. The proposed procedure consists of three logical stages, namely: initialisation of parameters in accordance with available *a priori* knowledge, followed by sequential optimisation of the model parameters in a decoupled manner, and finally sequential optimisation in a closed-loop configuration. At each stage, the search in the parameter space is 'guided' using application specific functionals within a regularisation framework. This provides a sensible compromise, effectively apportioning the importance, or confidence, between the available *a priori* knowledge and the measured data.

The specific application in the paper is an example of a high temperature gas-fired industrial furnace, which has been a subject of interest for over a decade (see for example [1], [2], [3] and [4]). Due mainly to the disruptive nature of the regenerative burners employed, the temperature control problem is considered to be a non-trivial task for enhanced modelling and model-based control solutions.

Previous research on high temperature industrial furnaces has led to the use of bilinear models to replicate the dominant nonlinear heat transfer characteristic. Additionally, since the use of three-term PID control is already well established for such industrial processes, an aim was to extend the control strategy to encompass the bilinear behaviour, thus leading to a four-term bilinear PID (BPID) scheme.

Recent work [4] has focused on the identification of the fourth term of the BPID controller, i.e. the value of the tuning parameter within a bilinear compensator which is cascaded with a standard PID scheme. The value of the bilinear tuning parameter, together with knowledge of the effect of the regenerative burner firing cycle on closed-loop behaviour, form the basis of *a priori* knowledge used in this paper. It is further acknowledged that the disruptive nature of the firing cycle is severely exacerbated due to a hardware limitation whereby use is made of a single temperature sensor positioned in each zone. The latter observation, coupled with the effect of the regenerative burners, prompted the concept of an unmeasured 'global temperature' as well as the measured 'local temperature'. This forms a key working hypothesis adopted in this paper.

The paper is organised as follows: the industrial furnace is presented in Section II, together with a consideration of closed-loop system identification, with emphasis on control system design and model structures, whilst recognising the need to incorporate *a priori* knowledge. A systematic three-stage approach to the parametrisation of a structured model is described in Section III. The overall results indicate good agreement with closed-loop data recorded from the system. General conclusions are presented in Section IV.

II. Industrial furnace modelling and control

A. Industrial furnace control problem

The industrial system under consideration consists of three furnaces in series (labelled Furnace 1, 2 and 3; see Fig. 1) based at Outokumpu Stainless Limited, Sheffield, UK. To simplify the modelling procedure, each zone within each furnace is treated independently. However, it is considered that model predictions, hence closed-loop control, could be improved by taking into account temperature interdependencies between zones/furnaces, as shown in [5].

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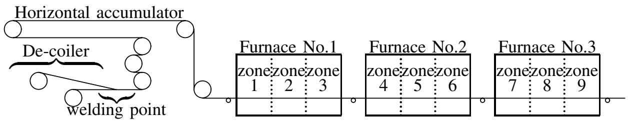

Fig. 1. Schematic illustration of furnace system

Each furnace is subdivided into three zones. A schematic illustration of Furnace No.2 is given in Fig. 2.

To maintain the required temperature, each of the Zones 1-7 use three pairs of regenerative burners, where each pair switches over a ninety second firing cycle, i.e. one (burner) fires whilst the second (regenerator) exhausts waste combustion gases, and stores the wasted heat in a ceramic chamber. Zones 8 and 9 make use of recuperative burners to reduce the effect of temperature variation. Each zone has its own temperature set point and control loop. Temperatures are measured by thermocouples positioned closest to the central burner pair within each zone (e.g. Pair 6 in Zone 4). A summary of the furnace operation can be found in [6].

Although regenerative burner technology is efficient, it does give rise to an undesirable oscillation in measured zone temperature. This inherently 'uncontrollable' part of the system arises due to an externally triggered cycle which drives the regenerative burners. The burners are fired sequentially. For example, in Furnace No. 2, the firing cycle of Pair 1, Pair 2, …Pair 9 is interchanged at a ten-second interval giving rise to a 180 second firing cycle for a given pair. Another oscillatory mode is present due to the firing of adjacent burner pairs (i.e. Pair 3 and Pair 9 in Zone 4), as the switch-over of the pairs of burners appears every 30 second in each zone. The firing cycle gives rise to an almost anti-phase closed-loop reaction between the controller output and the measured temperature; constituting *a priori* knowledge on closed-loop behaviour.

B. A priori knowledge and model structure

The model developed in this paper stems from a conceptual idea, originally described in [4], which advocates the existence of an unmeasurable 'global temperature', dependent on the amount of gas combusted in the system. A bilinear relationship is assumed between these two quantities. To obtain the 'global temperature', which represents an aggregate volumetric temperature value for each zone, one would ideally require a measurement of the mean value of the zone temperature. However, the sensor is known to be influenced mainly by the closest burner pair so that the locally measured temperature exhibits an oscillatory characteristic due to the firing cycle. Consequently, the measured

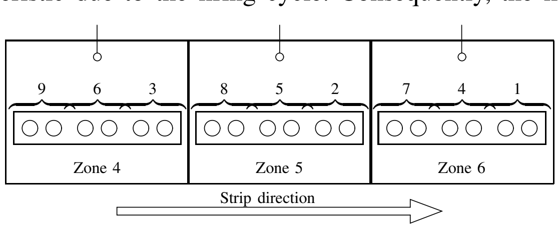

Fig. 2. Illustrating arrangement of burners in Furnace 2

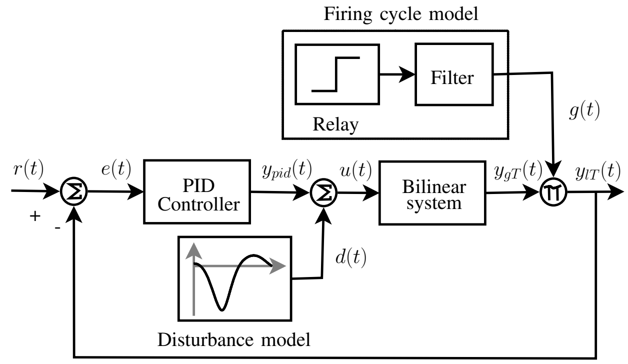

Fig. 3. Conceptual model of a single furnace zone

temperature possesses information about both the firing cycle and the notional 'global temperature'; with the hypothesis being that the latter 'exhibits' a lower variance.

A relay is used in the model to attempt to replicate the effect of the externally triggered firing cycle. A filter is included within the firing cycle model, to smooth the 'generated' edges at the switched relay output. The resulting instantaneous output of the combined relay and filter model is utilised as a cyclic static gain term, and is implemented as a multiplier. As no energy is 'induced' by the firing cycle, the overall gain is normalised such that it has a mean value of unity over a given period.

An additional externally driven event, which arises as a consequence of a butt-weld connecting two adjacent coils, results in a perturbation having a periodicity of around 1200 seconds. This is modelled as a disturbance. Its value is forced to be zero, when its effects on the observations are negligible. This typically results in allowing the disturbance to be non-zero for around 350 seconds, every period.

These concepts are illustrated schematically in the model configuration shown in Fig. 3, where $e(t)$ is the error signal between the reference signal $r(t)$ and the measured output signal or local temperature $y_{lT}(t)$, $y_{pid}(t)$ is the output of the controller, $d(t)$ is a disturbance signal representing the almost periodic perturbation, $u(t)$ is the plant input defined such that $u(t) := y_{pid}(t) + d(t)$, y_{gT} is the assumed 'global temperature' signal. The signal $g(t)$ representing the firing cycle allows the relationship between the global and local temperature to be expressed such that $y_{lT}(t) = g(t)y_{gT}(t)$.

C. Closed-loop system identification

The problem of system identification from closed-loop data has received much interest in the literature (see [7] and the references therein). The subject has witnessed renewed interest, due probably to the widespread need to improve closed-loop system performance, coupled with the fact that some plants, for various reasons (see for example [8]), are unable to operate in open-loop. It is shown, also in [8], which compares estimators based on instrumental variables for closed-loop system identification, that an ideal minimum variance biased estimator requires the noise model to be known exactly. The desire to have compatible criteria for control and identification was one of the motivations for introducing minimum variance control and maximum likelihood estimation [9]. The issue of identification and control system design is also discussed in [10]. The study in [11] shows that for model-based control design, closed-loop identification gives better performance. In this paper, the interest in closed-loop system identification is motivated by the desire to improve the existing closed-loop control system for an industrial furnace. The problem faced here has similarities to [9], [10] and [11] in that the aim of the resulting model is to improve the tuning of the controller, hence improve system performance. However, the major difference is that there is a need for a model to have a higher degree of sophistication than those previously cited: the application is an actual production plant; it is not a practical option to iteratively trial and test on the plant; and the plant is nonlinear. Although the excitation of the reference signal is insufficient, the presence of an externally triggered disturbance improves the identifiability of the closed-loop system [7].

III. Model parameter estimation via combined output error and a priori knowledge

This section describes the systematic approach to parameterising the model structure described in Section II-B. The closed-loop system represented by the model in Fig. 3 is estimated in order to replicate the measured system output, whilst accommodating *a priori* knowledge concerning the firing cycle and system bilinearity as outlined earlier. Model parameters are identified using an output error (OE) minimisation technique [12] combined with a regularisation approach (see [13], [14] and [15] and the references therein) in order to incorporate and quantify the available *a priori* knowledge in a structured manner. The general cost function, minimised using an optimisation approach [16], is given by:

$$\hat{\Theta} = \arg\min_{\Theta} \left\{ \|y - \hat{y}(\Theta, x_0)\|_2^2 + \alpha \mathcal{F}(\Theta, \beta) \right\} \qquad (1)$$

where $\mathcal{F}$ is a column vector of functionals which are dependent on the hyperparameter vector β. The hyperparameter row vector α defines the balance, or weighting, between the relative importance, or confidence, in the available *a priori* knowledge of the process with respect to the data. The hyperparameters are defined here as tuning parameters for the estimation procedure, but do not parametrise the models themselves. They can either be user defined, chosen in accordance with expectation of results (the case here), or, alternatively, estimated in a possibly more rigorous manner using Bayesian inferences [15].

The choice of an initial set of parameters is required, upon which the optimisation takes place. It is important for this initial set of parameters to be close to the global optimum in order to avoid becoming trapped in a local minima. It is normal to obtain an initial set of parameters using prediction error minimisation techniques which reflect the *a priori* knowledge of the process; i.e. resulting in a 'grey box' approach. However, depending on the particular application, inappropriate choice of initial parameter values can lead to instabilities causing divergence of the optimisation. To overcome this problem, a systematic approach to optimisation is performed in three logical steps: *i)* fit/select initial parameters in accordance with *a priori* knowledge; *ii)* optimisation of parameters in a decoupled manner (e.g. by performing an OE optimisation on the open-loop configuration of the model, while using closed-loop observations); and *iii)* refinements and final optimisation of parameters in a closed-loop configuration. At each stage, the search in the parameter space is 'guided' using functionals which are utilised to incorporate *a priori* knowledge of the process.

The advantage of adopting the above procedure is to confine the search in the parameter space to a limited region, thereby preventing erroneous parameter values to inappropriately satisfy the cost function, leading to the corruption of other parameters and ultimate divergence.

A. A priori knowledge and functionals

This sub-section describes how certain functionals, within the regularisation framework, are used to realise the *a priori* knowledge relating to the furnace system. With reference to Fig. 3, model parameters are to be obtained for: the PID controller, the bilinear system and the combined relay and filter. The model adopted for the bilinear system is:

$$y_{gT}(t+1) = -a y_{gT}(t) + b y_{pid}(t) + \eta y_{gT}(t) y_{pid}(t) + \rho_T \quad (2)$$

where η is the coefficient of the bilinear term, and ρ_T is known as the local ambient offset temperature, which represents a lower operating value for modelling purposes. Experience suggests that there exists a 'balance' between the parameters a and ρ_T in (2). For stability of the model, it is required that $a < 0$ and $|a| < 1$ (i.e. the linear part of the system (2) with $\eta = 0$ is assumed to be stable). Such knowledge can be incorporated, by making use of any suitable functional, e.g.

$$\mathcal{F}_{fd}(x) = -\frac{\log(x)}{x} - \frac{\log(1-x)}{1-x} \quad \forall \quad x \in]0,1[\quad (3)$$

which forces the parameter x to be within the interval $]0,1[$. It is to be noted here, that $\mathcal{F}_{fd}(x)$ is not defined for $x \notin]0,1[$, but this should not be a problem if the optimisation is initiated with $x_0 \in]0,1[$. A situation where a step within the optimisation procedure could lead to $x \notin]0,1[$ can be accommodated by defining $\mathcal{F}_{fd}(x)$ to be equal to an arbitrarily high value. The functional $\mathcal{F}_a$ defined as $\mathcal{F}_a := \mathcal{F}_{fd}(a)$ is included within the cost function to incorporate this *a priori* knowledge (i.e. $a \in]-1,0[$).

The discrete PID controller parameters are initially identified using measured signals, while ensuring realistic estimated values. It is usual in practice to specify the ratio of the derivative and integral time constants denoted by $T_d : T_i$, where both terms are positive, and typically chosen to satisfy $0 < \frac{T_d}{T_i} < 0.25$ (e.g. see [17]). Therefore to ensure the ratio of $T_d : T_i$ lies in the interval $]0, 0.25[$, use is made of the functional $F_{pid} := \mathcal{F}_{fd}(4\frac{T_d}{T_i})$.

Previous research [4] on the furnace application has led to knowledge of the sign and approximate magnitude of the bilinear tuning parameter, denoted by K_b, and expressed as the ratio of the bilinear model parameters, i.e. $K_b = \frac{\eta}{b}$. The use of the 'crossed entropy', otherwise known as 'Kullback length', allows a positivity constraint to be imposed on K_b:

$$\mathcal{F}_{K_b}(K_b, m_{K_b}) = K_b \log \frac{K_b}{m_{K_b}} - K_b + m_{K_b} \qquad (4)$$

where m_{K_b} is a hyperparameter which can be chosen as an approximation of the expected value of K_b.

One of the key hypotheses proposed in [4] is that whilst the mean values of the local and global temperatures y_{lT} and y_{gT}, respectively, are necessarily equal, the variance of y_{gT} is lower than that of y_{lT}. However, while performing an OE optimisation to identify the parameters of the overall model, it is necessary to take into consideration the presence of the disturbance $d(t)$, as it will typically degrade the performance of the model, even under 'normal' relatively undisturbed conditions. This is because the effects of the unmodelled disturbance interval will be 'distributed' over the full period. Additionally, it will be beneficial to estimate the unmeasured disturbance signal $d(t)$ prior to performing the final closed-loop optimisation. To facilitate both the estimation of the disturbance signal and to remove the corrupted periods from the analysis of mean and variance, it is useful to define the vector of hyperparameters $T \in \mathbb{R}^{3p}$ as follows:

$$T = \left[\begin{array}{ccccccc} T_{1,1} & T_{1,2} & T_{1,3} & \ldots & T_{p,1} & T_{p,2} & T_{p,3} \end{array} \right]^T \qquad (5)$$

where p indicates the number of periods of the disturbance which occurs in the observations, and where the effect of the i^{th} period on the time interval $T_{i,1}$ to $T_{i,3}$, cannot be neglected. To alleviate the effect of the disturbance signal, of non-zero mean value, on both the mean and the variance of the global temperature, the following functional is utilised:

$$\mathcal{F}_{gT}(T, \beta_1) = \left[\begin{array}{c} \left(\overline{\tilde{y}_{gT}} - \overline{\tilde{y}_{lT}}\right)^2 \\ -\log\left(var(\tilde{y}_{gT}) - \beta_1 \times var(\tilde{y}_{lT})\right) \end{array} \right] \qquad (6)$$

where the bar notation $(\bar{.})$ signifies the mean of the variable, $\tilde{y}_x$ is defined such that $\tilde{y}_x := \Lambda(T) y_x$, $\Lambda(T) \in \mathbb{R}^{M \times N}$ is a matrix formed by subtracting p blocks of $(T_{i,3} - T_{i,1})/T_s$ rows from the identity matrix $\mathbf{I}_N$ (i.e. $M = N - \sum_{i=1}^{p}(T_{i,3} - T_{i,1})/T_s$), where T_s is the sampling interval (taken here to be unity) and N is the number of samples. Here, β_1 is a hyperparameter satisfying $0 < \beta_1 < 1$ which is a ratio between the variance of the global temperature and that of the local temperature, and is assumed to be known *a priori*. The first element of the functional (6) enforces the mean values of the global and local temperatures to be almost equal and the second element fulfils the hypothesis regarding their relative variances.

In this work, it is proposed to force the unknown disturbance signal to be null when its effect is negligible. For the time interval when the disturbance is non-negligible, an estimation of this signal may be carried out in two different ways: by reconstructing each sample value of the disturbance signal, or alternatively by estimating a model of the disturbance which is simple and sufficiently generalisable to fit the disturbance intervals of interest.

The first method iteratively increments/decrements the disturbance value for each discrete time intermediate sample

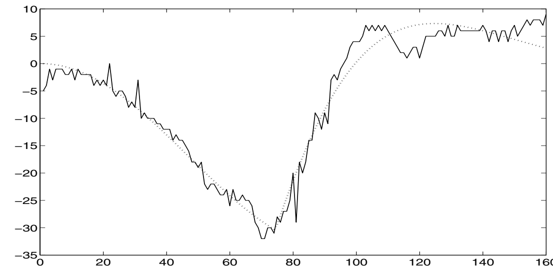

Fig. 4. Estimated (solid line) and modelled (dotted line) disturbance.

interval $t \in [T_{i,1}, T_{i,3}]$ until a minimum value of the cost function of interest is achieved (e.g. (1)). The second method (adopted in this work) exploits the fact that the disturbance can be divided into two parts. The first part is the arrival of the butt-weld within the furnace zone, which provokes a rapid decrease of the temperature, and is modelled using a cosine function. The second part, starting at $T_{i,2}$, corresponding to the time instant $T_{i,2} \in [T_{i,1}, T_{i,3}]$, where the value of $d(t)$ is a minimum, is modelled using an exponentially decaying sine function. The nonlinear model for a period of the disturbance, can therefore be expressed as:

$$d(t, \theta_6, T) = \delta_{i,1}(t,T) A_{i,1} \left(\cos(\omega_{i,1}(t - T_{i,1}) - \phi_{i,1}) - 1\right)$$
$$- \delta_{i,2}(t,T) A_{i,2} \sin(\omega_{i,2}(t - T_{i,1}) - \phi_{i,2}) e^{-a_i(t - T_{i,1})}$$

where $\theta_6 = \left[\begin{array}{ccc} \theta_{6,1} & \ldots & \theta_{6,p} \end{array} \right] \in \mathbb{R}^{7p}$ in which $\theta_{6,i} = \left[\begin{array}{ccccccc} A_{i,1} & \omega_{i,1} & \phi_{i,1} & A_{i,2} & \omega_{i,2} & \phi_{i,2} & a_i \end{array} \right]$ with $i = 1, \ldots, p$ for p periods and $\delta_{i,j}$ is defined as:

$$\delta_{i,j}(t, T) := \begin{cases} 1 & \text{for } T_{i,j} < t < T_{i,j+1} \\ 0 & \text{for } T_{i,j} \leq t, \text{ and } t > T_{i,j+1} \end{cases}$$

with $j = 1, 2$. Estimation of such a model is faster than evaluating each disturbance value on the interval $[T_{i,1}, T_{i,3}]$ (i.e. less parameters to be estimated). A comparison of results for the two methods is given in Fig. 4.

In practice, it has been found that some functionals are required to be quite heavily weighted in order to enforce certain properties of the solution (especially the functional in (6)). As the initial conditions of the models are estimated, the optimisation algorithm often tends to iteratively modify these initial conditions in order to minimise the cost function composed of the functionals, such as given in (6). Typically, it has been observed that, with a relatively large weight on (6), the minimisation of the cost function, e.g. (1), can give rise to an unsatisfactory set of initial conditions, while the cost attributed to being in agreement with the data may be only slightly affected due to the imbalance of the weighting. Such an observation is referred in to [15] as ensuring that the compound criterion corresponds to a 'straightforward convex bowl', to yield a unique solution. It is therefore of interest to introduce the following functional on the initial conditions

$$\mathcal{F}_{init}(x, t, \beta) = \sum_x \sum_t \mathcal{F}_{fd}(\tilde{y}_x(t, \beta)) \qquad (7)$$

in which the variables and respective initial time instants of interest are defined as $(x, t) \in$

$\{(pid, \{2\}), (gT, \{3\}), (lT, \{1, 2, 3\}\}$, and where $\tilde{y}_x(t, \beta)$ is defined as:

$$\tilde{y}_x(t, \beta) = ||y_x(t) - \hat{y}_x(t)| - \Delta_x| \quad (8)$$

where the individual hyperparameters $\Delta_x \in \beta$ represent the variance allowed between the estimated and measured variables. The hyperparameter vector β is defined by:

$$\beta = \left[\begin{array}{cccccc} \beta_1 & m_{K_b} & T & \Delta_{pid} & \Delta_{gT} & \Delta_{lT} \end{array} \right] \in \mathbb{R}^{2+3(p+1)}, \quad (9)$$

where T is specified in (5). As no measurement of the global temperature is available, it is proposed to use the mean value of the local temperature as the initial value, i.e. $y_{gT}(3) := \bar{y}_{lT}$.

The combined relay and filter constitute the largest uncertainty in terms of lack of *a priori* knowledge. The relay parameters are initialised while retaining the unity mean hypothesis given in Section II-B. They are separated into two sets, one called the 'temporal' set, which specifies the switching instants for the regenerative cycle for each pair of burners, and another called the 'amplitude' set which determines the 'relay height' for each burner.

For the 'temporal' parameter set, representing the switching instants of the relay, it is considered more appropriate and pragmatic to test if changing the switching instants by one sample period, reduces the cost function of interest, e.g. (1). By following such a procedure, the effects of adjusting the switching instants are iteratively evaluated until the cost function ceases to decrease.

In addition to the two sets, the relative weights between the three pairs of burners are required. Whilst two of these weights are estimated, the third is deduced to satisfy the unity mean hypothesis for the relay gain over a 180s period. For the 'amplitude' set, three of the four parameters required for a single pair of burners are estimated. The remaining value, for example a_{i3}, can be obtained as follows:

$$a_{i3} = \frac{180 - a_{i1}\Delta_{i_{14}} - a_{i2}\Delta_{i_{21}} - a_{i4}\Delta_{i_{43}}}{\Delta_{i_{32}}}$$

where the a_{ij} specify one segment of amplitude during the period $\Delta_{i_{lj}} = t_{il} - t_{ij}$, where t_{il} and t_{ij} denote the 'switching instants' of the i^{th} pair of burners with

$$j = 1, \ldots, 4, \quad l := \begin{cases} j+1, & j < 4; \\ 1, & j = 4. \end{cases}$$

For convenience, the filter adopted has a second order Butterworth structure, which is characterised by the two parameters (ω_0^2, Q), and defined by $f(\omega_0^2, Q) = \omega_0^2 / \left(s^2 + s\omega_0/Q + \omega_0^2\right)$. The filter representation is transformed to the z-domain using a Tustin approximation. Initial conditions of the filter within the combined relay-filter are required for any variation in the firing cycle parameters (e.g. the filter and relay parameters). This leads to ensuring that the initial conditions of the filter are chosen such that they do not generate undesirable transients, which would violate the hypothesis that the equality $\bar{g} = 1$ is maintained over consecutive periods of $180s$. This is solved by simulating the relay-filter block until it achieves steady-state, and the corresponding 'initial conditions' of interest retrieved.

B. Estimation procedure

The model parameter optimisation problem, even for this sub-system (pertaining to temperature control) of the overall complex furnace system has $39 + 7p$ model parameters to be optimised, including 7 initial conditions. This is a non-trivial task and, due to the non-uniqueness of the potential solutions, requires care to be taken in effectively 'guiding' the search in the parameter space. Define $\Theta = \left[\begin{array}{ccc} \theta_1^T & \ldots & \theta_6^T \end{array} \right]^T \in \mathbb{R}^{39+7p}$ where θ_i, $i = 1, \ldots, 6$, correspond to the vectors of parameters for: the PID controller, bilinear model for a single zone, relay switching times, relay heights and weights, filter terms, and the disturbance, respectively.

1) Decoupled optimisation: In this stage, the problem is 'decoupled' to optimise two sub-problems. Because these sub-problems are in fact inherently linked, the results of the decoupled optimisation step only yields a sub-optimal solution. However, these results serve as a good starting point to perform the global optimisation, i.e. the final closed-loop optimisation stage in the developed procedure. Initially, the PID parameters, denoted by θ_1, are estimated, based on a constrained optimisation [16], as follows:

$$\hat{\theta}_1, \hat{\alpha}_1 = \arg \min_{\theta_1 \in \mathbb{R}^3} \left\{ \|y_{pid} - \hat{y}_{pid}(\theta_1)\|_2^2 + \alpha_1 \mathcal{F}_{pid} \right\} \quad (10)$$

where α_1 is the first element in the vector $\alpha \in \mathbb{R}^6$. The resulting parameters are then used to iteratively optimise the combined relay and filter and the bilinear model $\theta_i (i = 2, \ldots, 5)$ according to the cost function:

$$\mathcal{J}_{ol}(\Theta, \alpha, \beta) = \left\| y_{lT} - \hat{y}_{lT}(\theta_i|_{\Theta_{j \neq i}}) \right\|_F^2 + \tilde{\alpha}\mathcal{F}(\Theta, \beta) \quad (11)$$

with $\Theta_{j \neq i} = \{\theta_j, j \neq i\}$ and the vector $\mathcal{F}$ of functionals is defined as:

$$\mathcal{F}(\Theta, \beta) = \left[\begin{array}{cccc} \mathcal{F}_a(a) & \mathcal{F}_{K_b}(K_b, \beta) & \mathcal{F}_{gT}(\beta) & \mathcal{F}_{init}(\beta) \end{array} \right]^T, \quad (12)$$

where $\tilde{\alpha}$ denotes the reduced vector α with its first element removed. The optimisations are performed iteratively until the decreasing sequence of errors converges within a certain user pre-specified threshold.

2) Closed-loop optimisation: With reference to Fig. 3, the feedback loop is closed, and a similar approach to that explained in *1)* above is repeated. There are, however, two important differences. Firstly, the closed-loop optimisation is formulated using a compound (single) cost function so that the resulting solution is 'more optimal' than that for the uncoupled case. Secondly, a refinement is introduced, whereby the effect of the disturbance at each period is taken into account. The approach adopted is that of estimating a model for each interval where the disturbance is non-negligible, as described in Section III-A. The cost function to be minimised is defined by:

$$\mathcal{J}_{cl}(\Theta, \alpha, \beta) = \left\| Y - \hat{Y}(\theta_i|_{\Theta_{j \neq i}}) \right\|_F^2 + \alpha\mathcal{F}(\Theta, \beta) \quad (13)$$

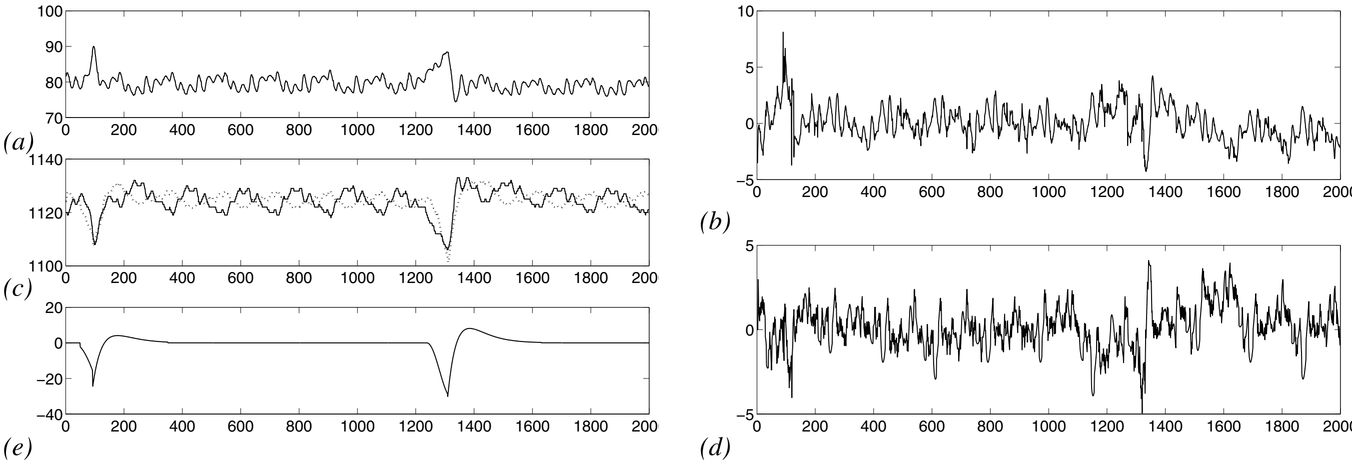

Fig. 5. Results of the final closed-loop optimisation stage, showing *(a)* actual control signal, *(b)* error between actual and reconstructed $y_{pid}(t)$ control signal, *(c)* measured local temperature (solid) with estimated global temperature $y_{gT}(t)$ anti-phased (dotted), *(d)* error between actual and reconstructed $y_{lT}(t)$ measured temperature, and *(e)* final disturbance $d(t)$ using the model based approach. The horizontal axes correspond to sample index in seconds, the vertical axes *(a)*, *(b)* and *(e)* correspond to control action in engineering units ($\pm 100\%$) and vertical axes *(c)* and *(d)* correspond to temperature $^\circ C$.

where $Y = \begin{bmatrix} y_{lT} & y_{pid} \end{bmatrix}^T$, and $\Theta_{j \neq i} = \{\theta_j, j \neq i\}$. The introduction of y_{pid} into the cost function allows a refinement of the PID parameters with respect to the optimised decoupled system representation, under the hypotheses previously cited. The signals introduced in Fig. 3 that are reconstructed following the minimisation of (13) are given in Fig. 5.

IV. Conclusion

The paper has described the development of a systematic approach to obtaining a sophisticated model of an order of complexity currently demanded for the purpose of enhanced closed-loop control system design. The developed approach is able to accommodate, and indeed regulate, the influence of available *a priori* knowledge, with respect to measured data. The resulting three-stage optimisation procedure combines *a priori* knowledge and structured data-based model identification methodologies within a regularisation framework. Whilst the approach is generic, the paper has shown how functionals having certain properties may be appropriately selected to incorporate specific *a priori* knowledge for an industrial furnace application.

In the case of the industrial furnace, assumptions were made on the relative variances of global and local temperatures. Whilst results obtained from the model are in good agreement with closed-loop operational data, further work is required to be carried out to investigate the effect of the relative variance of the global and local temperatures on the resulting closed-loop control system.

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