Proceedings of the
44th IEEE Conference on Decision and Control, and
the European Control Conference 2005
Seville, Spain, December 12-15, 2005

A repetitive based controller for a shunt active filter to compensate for reactive power and harmonic distortion

G. Escobar, R. Ortega , A. Astolfi , M. F. Martínez and J. Leyva-Ramos

Abstract— **This paper presents a repetitive-based controller for a single phase active filter to compensate for reactive power and current harmonic distortion, i.e., to guarantee a power factor close to unity. The topology selected for the active filter consists of a single phase full-bridge Voltage Source Inverter (VSI). The controller is based on a frequency domain description of the periodic disturbances. Adaptation is then introduced to cope with uncertainties in the disturbance signals and system parameters. A first observation is that the adaptations are reduced to a bank of resonant filters by means of suitable rotations, which are tuned at the frequencies of the harmonics under compensation. The second observation is that there exists an equivalent expression for such a bank of resonators in terms of a single delay line. Moreover, we show that this new expression for the controller, which belongs to the family of repetitive schemes, has a relatively simple implementation. The proposed control scheme has been experimentally tested in a 1.5 KVA prototype and the results are presented here.**

I. INTRODUCTION

In the last years, the use of electronic equipment drawing highly distorted currents has increased considerably. As a consequence different problems have appeared, among these, low power factor, low efficiency, interference by the EMI, overheat of passive elements. These issues have received special attention in the power electronics and power systems applications where the disturbances to cancel are composed of specific higher harmonics of the fundamental frequency of the power supply. An interesting solution to alleviate such problems, caused by a distorted load current, is the use of active filters based on voltage-sourced inverters (VSI). These filters are connected in parallel with non-linear loads as shown in Fig. 1. In this case. the active filter is used to inject the higher harmonic components demanded by the distorted load and thus eliminating their effect on the source side. Usually, a decoupled technique comprising two loops, an inner (or current) and an outer (or voltage) loop, is one of the most appealing techniques to control the active filter.

It has been reported that conventional techniques [1], [2] used for active power filters, seem to have a limited achievable performance mainly due to the limited control bandwidth, and the delay of the digital implementation. These techniques may even exhibit stability problems due to the high gain techniques used in the inner loop. To overcome these limitations, we present a repetitive-based controller for a single phase active filter to compensate for reactive power and harmonic distortion in the source current. Repetitive control was proposed in the beginning as a practical solution to the tracking or rejection of periodic signals and is based on the well-known internal model principle [3]. First works on repetitive control were presented in [4] and [5]. Interesting theoretical development of repetitive control can be found in [6] and [7], and the numerous references within where the discrete-time formulation has also been treated. See [8], [9], [10] and the references therein for applications of repetitive control on power electronic systems such as rectifiers, inverters and active filters. In most of these works, the authors use a positive feedback scheme to implement the repetitive controller. Some of them place the delay line in the direct path and others in the feedback path. It is important to notice that a positive feedback structure has the disadvantage of compensating for every single harmonic either odd or even, including the dc component. Moreover, depending on the position of the delay line in the controller structure, it may even modify the phase shift, which explains the need of some extra filters to alleviate this problem.

In contrast, the repetitive scheme proposed here involves a negative feedback structure and is aimed to compensate for odd harmonics only, thus, reducing the possibility of reinjecting unnecessary distortion into the system. And is expected that this scheme will generate cleaner responses than traditional positive feedback based repetitive schemes in applications of power electronic systems containing mainly odd harmonics.

II. PROBLEM FORMULATION

Consider the single phase active filter shown in Fig. 1. The system dynamics are described by (1)-(2)

$$L\frac{di_S}{dt} = L\frac{d}{dt}i_0 - uv_C + v_S \quad (1)$$

$$C\frac{dv_C}{dt} = u(i_S - i_0) - \frac{v_C}{R} \quad (2)$$

where parameters L and C are the input inductance and dc side capacitance of the active filter, respectively, i_S is the current in the source, v_C the capacitor voltage; $i_0(t)$ represents the current load and $v_S(t)$ the line voltage; $i(t)$ is the injected current; $u(t)$, which takes values in the discrete set $\{-1, 1\}$, denotes the switch position function and acts as the control input, that is, for $u = 1$ transistors Q_1 and Q_4 are turned on, while transistors Q_2 and Q_3 are turned off, and the inverse for $u = -1$. Moreover, for $u = 0$, either, Q_1 and Q_2 are both on, or Q_3 and Q_4 are both on. Switching

R. Ortega is with the LSS-SUPELEC, CNRS, Gif-sur-Yvette, France. Email: rortega@lss.supelec.fr

A. Astolfi is with the Imperial College, UK. Email a.astolfi@imperial.ac.uk

G. Escobar, M. F. Martínez and J. Leyva-Ramos are with the IPICyT, San Luis Potosí, México. Email: [gescobar,fmartinez,jleyva]@ipicyt.edu.mx

0-7803-9568-9/05/$20.00 ©2005 IEEE

and other losses are collected and modelled as an unknown constant resistive element R.

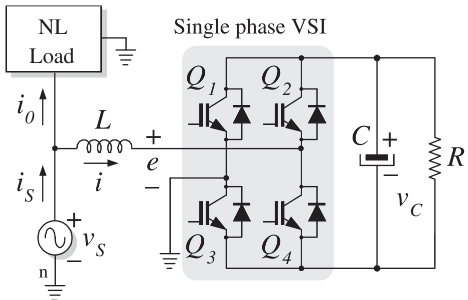

Fig. 1. Single phase full-bridge shunt active filter.

The *control objective* consists of the following two points: *(i) Tracking of the source current* towards a reference current signal proportional to the line voltage, so that, the *same* apparent resistance is observed by the voltage source at all frequencies, that is, $i_S \to i_S^* = \eta v_S$, where i_S^* represents the current reference, with η a scalar representing the apparent conductance observed by the source[1].
(ii) Regulation of the capacitor voltage towards a desired constant reference V_d, that is, $v_C \to V_d$. This guarantees that enough energy has been stored in the capacitor for the correct fulfillment of the previous objective.
Main assumptions:

A1. For controller design purposes the *averaged model* is considered, i.e., the control input u is considered as a continuous signal taking values in the range $[-1, 1]$. This is supported by the fact, for the real implementation, a pulse width modulation (PWM) technique with a relative high switching frequency is used to generate the switching sequences for each branch.[2].

A2. It is assumed that the inductor current dynamics are faster than the capacitor voltage dynamics. Thus, based on the time scale separation principle, the control design is split in two parts, an inner current control loop and an outer voltage control loop. Since both dynamics can be treated separately, this assumption is referred also as the *decoupling assumption*.

A3. The load current i_0 and source voltage v_S are *periodic signals (disturbances)* that contain higher *odd harmonics*[3] of the fundamental frequency $w_0 = 2\pi f_0$. They can be described in Fourier series as follows

$$v_S = \sum_{k\in H} \rho_k^\top V_{S,k} \ , \quad i_0 = \sum_{k\in H} \rho_k^\top I_{0,k} \tag{3}$$

where

$$\rho_k = \begin{bmatrix} \cos(kw_0t) \\ \sin(kw_0t) \end{bmatrix} \ , \ V_{S,k} = \begin{bmatrix} V_{S,k}^r \\ V_{S,k}^i \end{bmatrix} \ , \ I_{0,k} = \begin{bmatrix} I_{0,k}^r \\ I_{0,k}^i \end{bmatrix}$$

numbers $I_{0,k}^r$, $I_{0,k}^i$, $V_{S,k}^r$ and $V_{S,k}^i \in \mathbb{R}$ are the k^{th} harmonic coefficients of the Fourier series description (also referred as the phasors) of the load current and source voltage, respectively, and w_0 represents the fundamental frequency considered as a known constant. The harmonic coefficients are assumed unknown constants (or slowly varying) and $H = \{1, 3, 5, 7, ...\}$ is the set of indexes of the considered harmonic components. Superscripts $(\cdot)^r$ and $(\cdot)^i$ are used to distinguish the coefficients associated to $\cos(kw_0t)$ and $\sin(kw_0t)$, respectively.

A4. The system parameters L, C and R are assumed to be unknown positive constants.

III. Controller design

The control design starts by proposing a controller similar to the conventional one, to which adaptive laws are included to reject the periodic disturbances. The idea behind these adaptations is to estimate the frequency domain quantities of the periodic disturbances so they can be cancelled. These adaptations are then reduced, by suitable transformations to a bank of resonant filters. This observation is supported by the well-known internal model principle [3] which states that the controlled output can track a class of reference commands without a steady error if the generator (or the model) of the reference is included in the stable closed-loop system. It is well-known that the generator of a sinusoidal signal, i.e., containing only one harmonic component, is a harmonic oscillator, that is, a resonant filter. Roughly speaking, for each harmonic to compensate there should be included a resonant filter, and thus if an infinite number of harmonics are considered, then an infinite number of resonant filters should be included. The contribution here is the observation that, there is an equivalent and simpler expression of such an infinite bank of resonant filters in terms of a single *delay line* (also referred as *transport delay*). In other words, the exhausting implementation infinite bank of resonant filters can be replaced by a reasonably easy implementation of a particular feedback/feedforward array of a simple delay line, belonging to the family of repetitive schemes.

A. Inner control loop

In this subsection we design a controller which guarantees tracking of i_S towards its desired reference i_S^*. For this purpose, it is more convenient to rewrite the model (1)-(2) using the following coordinate transformations

$$x_1 = i_S \ , \quad x_2 = \frac{v_C^2}{2} \ , \quad e = uv_C$$

Thus, the model can be rewritten as

$$L\dot{x}_1 \quad = \quad L\frac{d}{dt}i_0 - e + v_S \tag{4}$$

$$C\dot{x}_2 \quad = \quad e(i_S - i_0) - \frac{2x_2}{R} \tag{5}$$

where e, the injected voltage, represents the actual control input. Notice that to accomplish the twofold control objective x_1 should track the reference $x_1^* = \eta v_S$, while x_2 should be driven towards $V_d^2/2$.

[1]This is equivalent to seek for an operation with power factor close to unity.

[2]A PWM unipolar modulation has been used, which consists in designing two duty ratios, one given by $(1+u)/2$ for the branch formed by $Q_1 - Q_3$ and another duty ratio for $Q_2 - Q_4$ given by $(1-u)/2$.

[3]In power electronics systems is common to find applications where the disturbances are composed mainly by odd harmonics.

It is straightforward to show that the following controller guaranties a stable tracking of subsystem (4)

$$e = L\frac{d}{dt}(i_0 - x_1^*) + v_S + k_1\tilde{x}_1 \tag{6}$$

where $\tilde{x}_1 = x_1 - x_1^*$ and $k_1 > 0$ is a design parameter.

Notice that, both the time derivative of i_0 and the parameter L are required to implement the controller above. In what follows we will show how this term can be estimated by means of adaptation. For this purpose, we appeal to the description of the disturbance in Fourier series (that is, in its harmonics components) to simplify this computation.

Using (3) the term $L\frac{d}{dt}(i_0 - x_1^*)$ can be expressed as follows

$$L\frac{d}{dt}(i_0 - x_1^*) = \sum_{k\in H} \rho_k^\top \Phi_k \tag{7}$$

where $\Phi_k \triangleq L\left(\dot{\eta} - kw\eta\mathcal{J}\right)V_{S,k} + Lkw\mathcal{J}I_{0,k}$ $(k \in H)$ are defined, and the following fact has been used

$$\dot{v}_S = -\sum_{k\in H} kw\rho_k^\top \mathcal{J}V_{S,k}\ , \quad \frac{di_0}{dt} = -\sum_{k\in H} kw\rho_k^\top \mathcal{J}I_{0,k}$$

$$\mathcal{J} = -\mathcal{J}^\top = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

According to the assumption **A3**, the vector Φ_k $(k \in H)$ practically converges towards an unknown constant[4]. Therefore, it is proposed to use an estimate $\hat{\Phi}_k$ in the control expression (6) instead of the term Φ_k. This yields the controller

$$e = v_S + k_1\tilde{x}_1 + \sum_{k\in\mathcal{H}} \hat{\Phi}_k \tag{8}$$

Subsystem (4) in closed-loop with controller (8) yields the following error dynamics:

$$L\dot{\tilde{x}}_1 = \sum_{k\in H} \rho_k^\top \tilde{\Phi}_k - k_1\tilde{x}_1 \tag{9}$$

where $\tilde{\Phi}_k \triangleq \hat{\Phi}_k - \Phi_k$ for every $k \in \mathcal{H}$.

To deal with the terms associated with the error signals $\tilde{\Phi}_k$, the following energy storage function is proposed

$$W = \frac{L}{2}\tilde{x}_1^2 + \sum_{k\in H}\frac{1}{2\gamma_k}\left[\left(\tilde{\Phi}_k^r\right)^2 + \left(\tilde{\Phi}_k^i\right)^2\right]$$

whose time derivative along the trajectories of (9) is given by

$$\dot{W} = -k_1\tilde{x}_1^2 + \tilde{x}_1\sum_{k\in H}\rho_k^\top\tilde{\Phi}_k + \sum_{k\in H}\frac{\dot{\tilde{\Phi}}_k^\top\tilde{\Phi}_k}{\gamma_k}$$

which is forced to be negative semidefinite if the error on the estimates is constructed according to the following adaptive laws

$$\begin{aligned} \dot{\hat{\Phi}}_k^r &= -\gamma_k\tilde{x}_1\cos(kwt)\ , \quad k \in H \\ \dot{\hat{\Phi}}_k^i &= -\gamma_k\tilde{x}_1\sin(kwt)\ , \quad k \in H \end{aligned}$$

[4]Normally η is forced to vary very slowly, taking a constant value in the steady state.

or in a more compact form

$$\dot{\hat{\Phi}}_k = -\gamma_k\tilde{x}_1\rho_k\ , \quad k \in H \tag{10}$$

where $\gamma_k > 0$, $k \in H$ are design parameters.

Following LaSalle's theorem arguments it is easy to see that $\tilde{x}_1 \to 0$ as $t \to \infty$ as long as e is well defined for all t. Moreover, $\tilde{\Phi} \to 0$ as $t \to \infty$ as long as $\tilde{x}_1 \to 0$. Notice that we have used the fact that $\dot{\tilde{\Phi}}k^p = \dot{\hat{\Phi}}k^p$ and $\dot{\tilde{\Phi}}k^n = \dot{\hat{\Phi}}k^n$.

The controller (8) with adaptive laws (10) can be further simplified using the following transformations

$$\begin{aligned} \Psi_k^r &= -\rho_k^\top\hat{\Phi}_k\ , \quad k \in H \\ \Psi_k^i &= -\rho_k^\top\mathcal{J}\hat{\Phi}_k\ , \quad k \in H \end{aligned}$$

The controller (8) is reduced to

$$e = \sum_{k\in H}\Psi_k^r + v_S + k_1\tilde{x}_1 \tag{11}$$

and the adaptive laws can be rewritten as

$$\begin{aligned} \dot{\Psi}_k^r &= \gamma_k\tilde{x}_1 - kw\Psi_k^i\ , \quad k \in H \\ \dot{\Psi}_k^i &= kw\Psi_k^r\ , \quad k \in H \end{aligned}$$

Expressing the dynamic extension (the adaptations) in the form of a transfer function $\tilde{x}_1 \mapsto \Psi_k^r$ $(k \in H)$ yields

$$\Psi_k^r = \frac{\gamma_k s}{s^2 + k^2w_0^2}\tilde{x}_1\ , \quad k \in H \tag{12}$$

The expression for the current or inner controller is given by

$$e = -k_1\tilde{x}_1 + v_S + \sum_{k\in H}\frac{2\gamma_k s}{s^2 + k^2w_0^2}\tilde{x}_1 \tag{13}$$

B. Outer control loop

As pointed out before, we consider that the dynamics of the subsystem (1) are much faster than the dynamics of subsystem (2), and moreover, that the controller e is bounded, which is true if all terms $\hat{\Phi}_k$ $(\forall k \in H)$ are bounded. Thus, in a relatively short time, practically $\tilde{x}_1 = 0$ and $\hat{\Phi} = \Phi$. Hence, subsystem (5) can be written in terms of the increments of x_2 as

$$\begin{aligned} C\dot{\tilde{x}}_2 &= \left(L\frac{d}{dt}(i_0 - x_1^*) + v_S\right)(x_1^* - i_0) - \frac{2x_2}{R} = \\ &= \eta v_S^2 - v_S i_0 - L\frac{d}{dt}(i_0 - x_1^*)(x_1^* - i_0) - \frac{2x_2}{R} \end{aligned}$$

where $\tilde{x}_2 \triangleq (x_2 - \frac{V_d^2}{2})$.

Considering only the average, or dc component[5], of $\tilde{x}_2$ dynamics

$$C\dot{\tilde{x}}_{20} = \eta v_{S,RMS}^2 - p_0 - \frac{2x_{20}}{R}$$

where $\tilde{x}_{20} = \langle\tilde{x}\rangle_0$ represents the dc part of the state $\tilde{x}_2$ and $p_0 = \langle v_S i_0\rangle_0$ represents the average output power, considered an unknown constant, while $v_{S,RMS}^2 = \langle v_S^2\rangle_0$ is the square of the RMS value of v_S which is also a constant.

[5]The extraction of the the dc component of a scalar x is defined at time t by the following averaging operation $\langle x\rangle_0(t) = \frac{1}{T}\int_{t-T}^{t} x(\tau)d\tau$.

It is common practice in power converter applications to obtain x_1^* as follows

$$x_1^* = \delta\left(\frac{v_S}{v_{S,RMS}^2}\right) \tag{14}$$

this is equivalent to make the following transformation in our developments

$$\delta = \eta v_{S,RMS}^2$$

This simple useful transformation keeps the values of most variables on the same order and thus reduces the risk of numerical errors since the value of η is usually very small. Here and in what follows δ is referred as the scaled equivalent conductance.

Finally, the error model can be written as

$$C\dot{\tilde{x}}_{20} = \delta - p_0 - \frac{2x_{20}}{R} \tag{15}$$

A controller that guarantees $\tilde{x}_{20} \to 0$ is

$$\begin{aligned} \delta &= -k_i\xi - k_p\zeta && (16)\\ \dot{\xi} &= \tilde{x}_{20} && (17)\\ \tau\dot{\zeta} &= \tilde{x}_{20} - \zeta && (18)\end{aligned}$$

where k_p, k_i are the proportional and integral gains of a proportional-integral (PI) controller, and τ the time constant of a low-pass filter (LPF).

Relying in the filtering capability of such a PI controller, we propose to use $\tilde{x}_2$ instead of $\tilde{x}_{20}$ for the control implementation. The controller (16)-(18) can thus be rewritten in the form of a transfer function as follows

$$\frac{\delta(s)}{\tilde{x}_2(s)} = -\frac{k_i}{s} - \frac{k_p}{\tau s + 1} \tag{19}$$

where s represents the complex Laplace variable.

Summarizing, the expression for the preliminary proposed controller is given by

$$\begin{aligned} e &= -k_1\tilde{x}_1 + v_S + \sum_{k\in H}\frac{2\gamma_k s}{s^2 + k^2 w_0^2}\tilde{x}_1 && (20)\\ \delta &= -\left(\frac{k_i}{s} + \frac{k_p}{\tau s + 1}\right)\tilde{x}_2 && (21)\\ x_1^* &= \delta\frac{v_S}{v_{S,RMS}^2} && (22)\end{aligned}$$

Clearly, the interesting part of this controller is the bank of resonators tuned at odd frequencies ($H = \{1, 3, 5, 7, ...\}$) of the fundamental w_0, each with an associated gain γ_k, $k \in H$.

In what follows, it is shown that this bank of resonators is equivalent to a hyperbolic tangent, and therefore, it can be implemented as a special feedback arrange of a delay line, i.e., as a repetitive scheme. In what follows it is shown that this bank of resonant filters has an equivalent expression that coincides with the repetitive scheme proposed in [12] which was intended for the compensation of odd harmonics only. Some conclusions from [12] are extracted and included here for the sake of completeness.

C. Proposed repetitive based controller

Let us manipulate the sum of resonant filters as follows

$$\begin{aligned} G(s) &= \sum_{\ell=1}^{\infty}\frac{2\gamma_{(2\ell-1)}s}{s^2 + (2\ell-1)^2 w_0^2} = \\ &= \frac{2k_r w_0}{\pi}\sum_{\ell=1}^{\infty}\frac{2s}{s^2 + (2\ell-1)^2 w_0^2} = \\ &= k_r \tanh\left(\frac{s\pi}{2w_0}\right) = k_r\left(\frac{e^{\frac{s\pi}{2w_0}} - e^{-\frac{s\pi}{2w_0}}}{e^{\frac{s\pi}{2w_0}} + e^{-\frac{s\pi}{2w_0}}}\right) = \\ &= k_r\left(\frac{1 - e^{-\frac{s\pi}{w_0}}}{1 + e^{-\frac{s\pi}{w_0}}}\right) \end{aligned} \tag{23}$$

where we have fixed $\gamma_{(2\ell-1)} = 2k_r w_0/\pi$, $\ell \in \{1, 2, ..., \infty\}$ with k_r a positive design parameter. Notice that we have used $(2\ell - 1)$ instead of $k \in \{1, 3, 5, ...\}$ to stress the fact that the relationships above are valid only for odd harmonic components (see [11] for the hyperbolic tangent equivalences used).

It is important to remark that the expression inside the parenthesis in the very last row of (23) can be easily implemented as a negative feedback of a single delay line plus a feedforward path as shown in the block diagram in Fig. 2. This particular scheme has been presented in [12] where is referred as the negative feedback plus feedforward repetitive scheme.

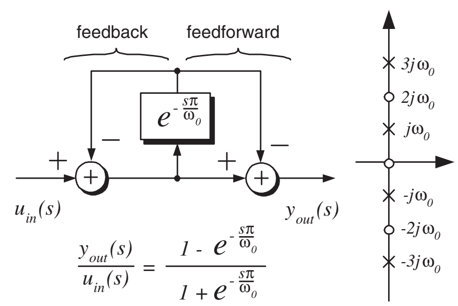

Fig. 2. Block diagram and poles location of the proposed negative feedback repetitive compensator with feedforward path.

The transfer function of the repetitive compensator can also be written as

$$G(s) = \frac{\sinh(\frac{s\pi}{2w_0})}{\cosh(\frac{s\pi}{2w_0})} = \frac{\frac{s\pi}{2w_0}\prod_{\ell=1}^{\infty}\left(\frac{s^2}{(2\ell)^2 w_0^2} + 1\right)}{\prod_{\ell=1}^{\infty}\left(\frac{s^2}{(2\ell-1)^2 w_0^2} + 1\right)} \tag{24}$$

Notice that the negative feedback scheme introduces an infinite number of poles in odd multiples of the fundamental frequency w_0, and thus it is able to compensate for odd harmonics (see Fig. 2). On the other hand the introduction of the feedforward path creates an infinite number of zeros on the imaginary axis located in between two consecutive poles, i.e., in the even multiples of the fundamental including a zero in the origin, thus improving the selective nature of the whole controller which will in principle allow bigger gains and better performance.

Remark III.1 Additionally, the poles can also be found from $e^{-\frac{s\pi}{w_0}}|_{s=jw} = -1$ which holds for every $w = (2\ell - 1)w_0$ ($\ell = 1, 2, 3, ...$), and the zeros from $e^{-\frac{s\pi}{w_0}}|_{s=jw} = 1$ which holds for every $w = (2\ell - 2)w_0$ ($\ell = 1, 2, 3, ...$). $\square$

In [12] it is proposed to multiply the delay line by a gain $0<K<1$ with the aim of limiting the resonant peaks gain. Indeed, it is shown that this gain K adds damping to all the poles by slightly shifting them to the left of the imaginary axis. To prove this, define $K = e^{-\frac{a\pi}{w_0}}$ with an $a > 0$, and multiply the pure delay by this term as follows

$$\frac{1 - Ke^{-\frac{s\pi}{w_0}}}{1 + Ke^{-\frac{s\pi}{w_0}}} = \frac{1 - e^{-\frac{a\pi}{w_0}}e^{-\frac{s\pi}{w_0}}}{1 + e^{-\frac{a\pi}{w_0}}e^{-\frac{s\pi}{w_0}}} = \frac{1 - e^{-\frac{(s+a)\pi}{w_0}}}{1 + e^{-\frac{(s+a)\pi}{w_0}}} \tag{25}$$

It is then clear that, if a gain $K>1$ is proposed then the poles move to the right, but if $0<K<1$ then they move to the left. Moreover, it is easy to show that the peaks, originally of infinite magnitude, get a magnitude of $(1 + K)/(1 - K)$ and the notches get a magnitude of $(1 - K)/(1 + K)$.

Summarizing the final expressions of the proposed controller including the repetitive scheme are

$$e = -k_1\tilde{x}_1 + v_S + k_r\left(\frac{1 + Ke^{-\frac{s\pi}{w_0}}}{1 - Ke^{-\frac{s\pi}{w_0}}}\right)\tilde{x}_1 \tag{26}$$

$$\delta = -\left(\frac{k_i}{s} + \frac{k_p}{\tau s + 1}\right)\tilde{x}_2 \tag{27}$$

$$x_1^* = \delta\left(\frac{v_S}{v_{S,RMS}^2}\right) \tag{28}$$

Notice that, the bank of resonant filters has been replaced by the proposed repetitive scheme.

The block diagram of the overall proposed repetitive-based controller is shown in Fig. 3. Notice that gain k_r allows additional control over the gain produced in the peaks.

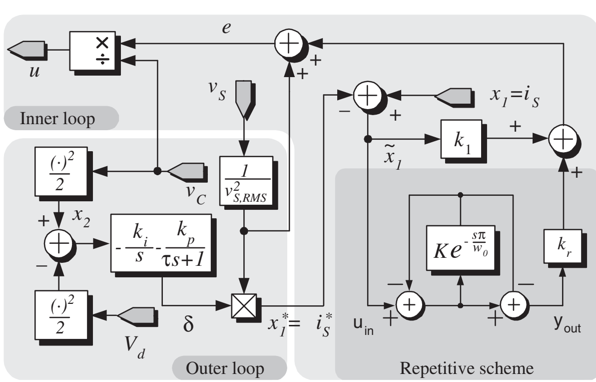

Fig. 3. Block diagram of the proposed repetitive-based controller.

IV. Experimental results

For the experiments a voltage source of 127 V_{rms}, f_0=60 Hz (w_0=377 rad/s) is considered. A diode bridge rectifier with a resistive load of either, 75 Ω or 150 Ω, and a bulky capacitor of 330μF is considered as the nonlinear load. This load produces a distorted current containing all odd harmonics of the fundamental frequency (60Hz). The active filter has been designed with parameters $L = 4$ mH, $C = 6800\,\mu$F, $R = 22$ KΩ. A 1104 dSPACE card has been used to implement the controller, at a sampling frequency of 12 kHz. The switching frequency for the switching devices is set to 12 kHz, therefore, the observed switching frequency is 24 kHz. The control parameters are fixed to $k_p = 0.018$, $k_i = 0.01$, $k_1 = 10$, $K = 0.944$ and $k_r = 0.75$.

Figure 4 shows (from top to bottom) the steady state responses of the compensated current, the load current and injected current. This figure shows that the compensated source current $i_S(t)$ (top plot) is an almost sinusoidal signal in phase with the source voltage $v_s(t)$, despite of the highly distorted load current $i_0(t)$ (medium plot). In the bottom plot the injected current $i(t)$ is presented. Although not shown here, the control algorithm is also able to deal with harmonic distortion in the voltage source which in this case has a total harmonic distortion (THD) of 4%.

Figure 5 shows (from top to bottom) the source voltage v_S, the source current i_S with harmonic compensation (including the repetitive scheme), and the source current i_S without harmonic compensation. Notice that there is a considerable improvement in the current shape when the repetitive scheme is introduced.

Figure 6 shows the capacitor voltage transient response when the load in the diode bridge rectifier is changed from 150 Ω to 75 Ω and back to 150Ω. It is shown that after a relatively small transient, the capacitor voltage $v_C(t)$ converges towards its reference 250V_{DC}, while the scaled apparent conductance $\delta(t) = v_{S,RMS}^2\eta(t)$ reaches a constant value which depends on the load characteristics.

Figure 7 shows the spectrum of the compensated current i_S in comparison with the load current i_0. It is shown that the compensated current is mainly composed by a fundamental harmonic, reaching a 12% of Total Harmonic Distortion (THD), despite of the highly distorted load current.

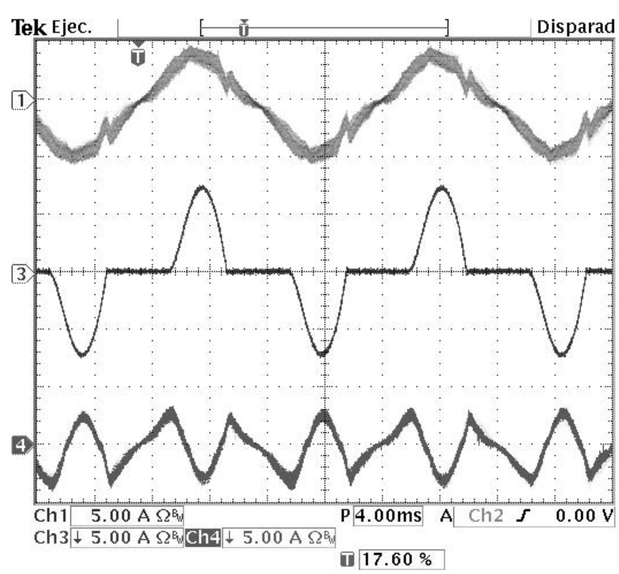

Fig. 4. **(from top to bottom)** compensated current $i_S(t)$, current load $i_0(t)$, and injected current $i(t)$.

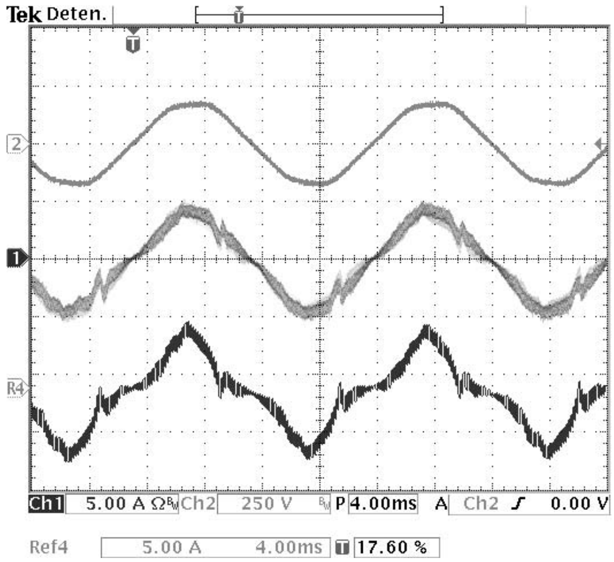

Fig. 5. **(from top to bottom)** Source voltage $v_S(t)$, source current $i_S(t)$ with harmonic compensation, and source current $i_S(t)$ without harmonic compensation.

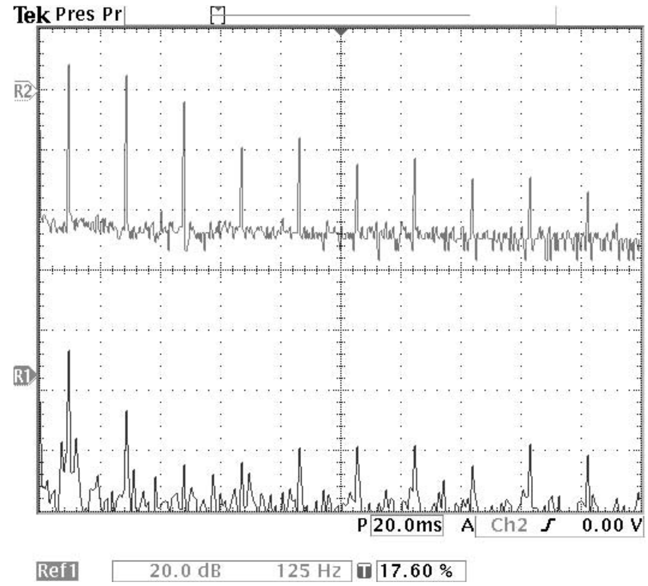

Fig. 7. Spectra of **(top)** the load current i_0, and **(bottom)** the compensated current i_S.

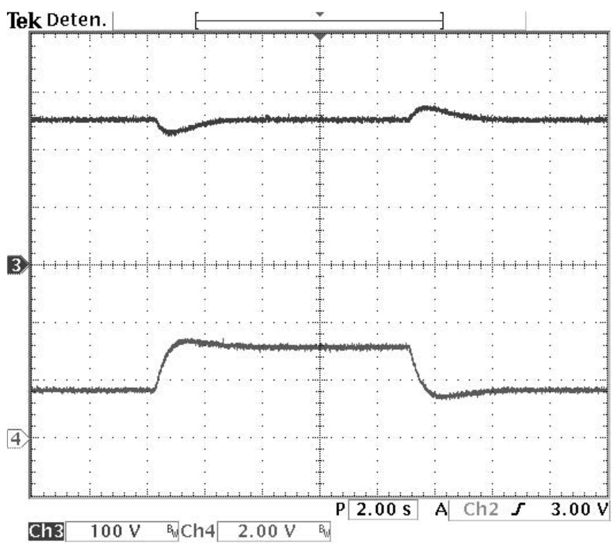

Fig. 6. **(top)** Capacitor voltage $v_C(t)$, and **(bottom)** scaled apparent conductance $\delta(t) = v_{S,RMS}^2 \eta(t)$ observed by the source.

V. Concluding remarks

This work presented a controller integrated by the cascade interconnection of inner and outer loops. The latter was designed as a low pass filter plus an integral term, while the former was composed by a repetitive scheme plus the usual proportional term. The idea behind the repetitive scheme was to compensate for the odd harmonic components of the periodic distortion in highly distorted load. Two remarks of interest have been highlighted here: First, the adaptations proposed to compensate the periodic disturbances can be transformed, by suitable rotations, into a bank of resonant filters, which is in agreement with the internal model principle. Second, the bank of resonant filters is equivalent to a repetitive scheme, and thus they were replaced by this simpler scheme thus facilitating the implementation. Finally, experimental results were obtained to show the effectiveness of the proposed control law.

References

[1] F. Pottker and I. Barbi. Power factor correction of non-linear loads employing a single phase active power filter: control strategy, design methodology and experimentation In *Proc. 28th Annual IEEE Power Electronics Specialists Conference PESC*, Vol. 1, 22-27 June 1997, pp. 412 - 417.

[2] H. Akagi and A. Nabae. “Control Strategy of Active Power Filters Using Multiple Voltage Source PWM Converters,” *IEEE Trans. on Ind. App.*, Vol. IA-22, No. 3, May/June 1986, pp.460-465.

[3] B. Francis and W. Wonham. “The internal Model Principle for Linear Multivariable Regulators,” *Applied Mathematics and Optimization*, Vol. 2, pp. 170-194, 1975.

[4] S. Hara, T. Omata and M. Nakano. “Synthesis of repetitive control systems and its applications,” in *Proc. 24th Conf. Decision and Control*, Vol. 3, pp. 1387-1392, 1985.

[5] T. Omata, S. Hara and M. Nakano. “Repetitive control for linear periodic systems,” *Elect. Eng. Jpn.*, Vol. 105, pp. 131-138, 1985.

[6] S. Hara, Y. Yamamoto, T. Omata and M. Nakano. “Repetitive control systems: A new type servo systems and its applications,” *IEEE Trans. Automat. Contr.*, Vol. 33, No. 7, pp. 659-667, 1988.

[7] J. Ghosh and J. Paden. “Nonlinear repetitive control,” *IEEE Trans. Automat. Contr.*, Vol. 45, No. 5, pp. 949-954, 2000.

[8] P. Mattavelli and F.P. Marafao, “Selective active filters using repetitive control techniques,” *IEEE Transactions on Industrial Electronics*, Vol. 51(5), October 2004, pp. 1018-1024.

[9] Ying-Yu Tzou, Shih-Liang Jung and Hsin-Chung Yeh, “Adaptive repetitive control of PWM inverters for very low THD AC-voltage regulation with unknown loads,” *IEEE Transactions on Power Electronics*, Vol. 14(5), pp. 973-981, Sept. 1999.

[10] Kai Zhang, Yong Kang, Jian Xiong and Jian Chen, “Direct repetitive control of SPWM inverter for UPS purpose,” *IEEE Transactions on Power Electronics*, Vol. 18(3), pp. 784-792, May 2003.

[11] I. S. Gradshteyn and I. M. Ryzhik. *Table of integrals, series and products*. Academic Press, 6th Ed., 2000.

[12] G. Escobar, J. Leyva-Ramos and P. R. Martínez, “Repetitive controllers with a feedforward path for harmonic compensation,” *IEEE Trans. on Ind. Electr.* (In press).