Proceedings of the
44th IEEE Conference on Decision and Control, and
the European Control Conference 2005
Seville, Spain, December 12-15, 2005

Uncertainty Identification for a Nominal LPV Vehicle Model Based on Experimental Data

Gábor Rödönyi and József Bokor

Abstract— **In this paper a practical method is presented for modelling uncertainty of a nominal linear parameter-varying (LPV) vehicle model. The aim with the uncertainty model is to bound the nominal model-error and satisfy robust stability and performance objectives during robust control design. Existing frequency-domain model-validation methods are applied to perform the first aim. The linear fractional uncertainty structure and the distribution of nominal model-error among the uncertainty blocks and disturbances are chosen to perform the second aim. The paper is motivated by the problem of steering a vehicle by alternately braking the front wheels in emergency situations. The identification is performed on real experiment data. The method and the results are demonstrated on a yaw-rate tracking problem and μ-controller design on constant scheduling variable of the LPV model. Using the proposed algorithm, on the supposition that nominal model error remains below the bound estimated from the validation data set, an unfalsified model is constructed for robust control guaranteeing robust performance against worst-case uncertainty and disturbance.**

I. INTRODUCTION

Model validation for control is a central research area of system identification. The product of identification, i.e. the model, should be a reliable base of controller design. The validation or invalidation approaches depend on the control strategy. Robust control typically requires strict bounds on the model-error. Usually this error is structured as disturbances originating from outer signals and a variety of model uncertainty types due to neglected dynamics.

Both time- and frequency-domain techniques have been developed for model validation problems for robust control. Concerning time-domain approaches Ljung maintains the concept of model error modelling based on statistical tools in many papers, e.g. in [1] and references therein. Another fundamental paper is by Poolla at al. [2]. Sizable research has been devoted to algorithmize the creation of validated, more exactly unfalsified models for a specific control approach. The validation step of the identification process and the control design step get interlacing. For example, in the field of adaptive control directly the controller is falsified before it is implemented [3]. Applications of unfalsification concept in adaptive control can also be found e.g. in [4] and [5].

Of the works most relevant to this paper Smith [6], [7] and Lim [8] studied model validation problems on the frequency-domain. Their concept will be applied in constructing unfalsified models.

This work was supported by the Hungarian National office for Research and Technology through the project "Advanced Vehicles and Vehicle Control Knowledge Center" (No. OMFB-01418/2004).

G. Rödönyi and J. Bokor is with Systems and Control Laboratory, Computer and Automation Research Institute of Hungarian Academy of Sciences. H-1518 Budapest P.O. Box 63, Hungary rodonyi@sztaki.hu

A. Problem setup

The nominal model is assumed to be linear time-invariant (LTI) or linear parameter-varying provided that the scheduling variable can be fixed constant while performing a single experiment. This is required because the measured data will be transformed into frequency-domain. The nominal model can be the result of any classical identification method, e.g. [9], or constructed by the help of physical considerations. All signals are assumed to be elements of $\mathcal{L}_2$ and the model perturbations are stable, causal and LTI, as usual in robust control [10]. The structure of the uncertainty is a designer choice. The only limitation on it that at least a part of the disturbance should be additive and the perturbation is structured in linear-fractional transformation (LFT) form. Once the structure is selected, the unfalsification means finding the minimum required sizes of the uncertainty blocks and disturbances along some defined trade-off between them. The unfalsified model describes all model-error in the validation data set.

In the robust control setup the control problem is defined by weighting functions. They reflect the frequency-contents of outer signals, like reference, noise, and disturbances and penalize signals that should be small. Some of these are objective: reference, noise, control energy and tracking are nature and requirements of the environment. But disturbance and model-perturbation are only modelling tools to capture the nominal-error. Once an unfalsified uncertainty structure is created it remains to test whether the specified, objective performance goals can be achieved. If not, the designer's freedom stands in the uncertainty structure and in the trade-off.

B. The application problem

In the most common vehicles, where no electronic steering system is available but the braking is controlled by onboard computers, the only way to automate or assist steering is the use of the electronic brake system, the application of individual or unilateral wheel brakes. There are many papers concerning different approaches that develop steering by the braking systems. In [11] active steering and individual wheel braking is compared in yaw and roll control point of view. A method for unilateral braking for rollover avoidance can be found in [12], [13], for avoiding unintended lane departure in e.g. [14], [15]. The model presented in this paper is designed for this latter application. All the details of the identification of the nominal vehicle model is described in [16]

The frequency-domain invalidation method of [6], [7] and [8] is summarized in section II. Then the concept of the unfalsification for achieving robust performance is detailed in section III. The nominal vehicle model is presented in section IV. The computation results for the vehicle and controller simulation results can be found in section V.

II. Model validation method - the basis of unfalsification

The following frequency-domain model validation method borrowed from Smith [6], [7] and Lim [8] is the basis of unfalsification and therefore summarized here.

Given a nominal discrete-time, linear time-invariant system in the frequency-domain with linear fractional transformation (LFT) type uncertainty structure and disturbance signal w in the form as follows

$$\begin{aligned} \begin{bmatrix} \eta \\ \hat{y} \end{bmatrix} &= \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \end{bmatrix} \begin{bmatrix} \xi \\ w \\ u \end{bmatrix}, \quad (1) \\ \xi &= \Delta\eta, \qquad \Delta \in \mathcal{D} \quad (2) \\ \mathcal{D} &:= \{\Delta \in \mathcal{C}^{m\times n} : \Delta = diag\{\Delta_i, i = 1, \ldots, \tau\}\}, \end{aligned}$$

where the signals are given in the frequency-domain. Practically the control input $u \in \mathcal{C}^{n_u}$ and the measured output of the real system $y_n \in \mathcal{C}^{n_y}$ is computed via N-point discrete Fourier-transformation (DFT) on the frequencies $z_k = e^{j\omega_k}$, $\omega_k = \frac{2\pi}{T_s}\frac{k}{N}$, $k = 1, \ldots, \lfloor \frac{N}{2} \rfloor$, where T_s is the sampling time. Introduce real scalar bounds ν_i and γ_w so that

$$\|\Delta_i\|_\infty \le \nu_i, i = 1, \ldots, \tau, \qquad |w| \le \gamma_w, \quad (3)$$

then the model validation problem can be stated like this: if there exist $\Delta \in \mathcal{D}$ and $w \in \mathcal{C}^{n_w}$ such that $\hat{y} = y_n$ and (2) and (3) are satisfied, then the data does not falsify the model with the given bounds $\nu := [\nu_1 \cdots \nu_\tau]^T$ and γ_w.

The condition $\hat{y} = y_n$ means that the nominal model-error $e = y_n - P_{23}u$ can be written as

$$e = M \begin{bmatrix} \xi \\ w \end{bmatrix}, \qquad M := \begin{bmatrix} P_{21} & P_{22} \end{bmatrix} \quad (4)$$

If $e \in Im(M)$ does not hold, the validation fails. This can be avoided by appropriate selection of the uncertainty structure. If M is invertible, there is a trivial unique solution.

If the dimension of the space of e is less then that of $\begin{bmatrix} \xi \\ w \end{bmatrix}$, then we can extend the vector e by θ so that the augmented coefficient matrix will be invertible and we get

$$\begin{bmatrix} \xi \\ w \end{bmatrix} = \begin{bmatrix} M_\xi^+ & N_\xi \\ M_w^+ & N_w \end{bmatrix} \begin{bmatrix} e \\ \theta \end{bmatrix},$$

where $\begin{bmatrix} M_\xi^+ \\ M_w^+ \end{bmatrix}$ is the Moore-Penrose pseudo-inverse of M. We are looking for an uncertainty vector $\begin{bmatrix} \xi \\ w \end{bmatrix}$ so that (4) is satisfied with the constraint (2). Clearly we have $dim(N_\xi)$ freedom, i.e infinitely many solutions. The aim can be to find the minimum norm solution, i.e the smallest uncertainty and/or disturbance required.

Now consider the inequality constraint on w:

$$|w| = \left| \begin{bmatrix} M_w^+ & N_w \end{bmatrix} \begin{bmatrix} e \\ \theta \end{bmatrix} \right| < \gamma_w.$$

The space of θ can be divided according to the kernel space of N_w. Using singular value decomposition (SVD) (* denotes conjugate transpose)

$$\begin{aligned} N_w &= U_w S_w V_w^* = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} S_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_1^* \\ V_2^* \end{bmatrix} \\ \theta &= V_1\gamma + V_2\psi \\ w &= M_w^+ e + U_1 S_1 \gamma \end{aligned}$$

The norm of a vector remains unchanged when multiplied by a unitary matrix. A new variable ϕ is introduced:

$$\begin{aligned} \begin{bmatrix} U_1^* \\ U_2^* \end{bmatrix} w &= \begin{bmatrix} U_1^* M_w^+ e + S_1\gamma \\ U_2^* M_w^+ e \end{bmatrix} =: \begin{bmatrix} \phi \\ U_2^* M_w^+ e \end{bmatrix} \\ w &= U_1\phi + (I - U_1U_1^*)M_w^+ e \\ &\Rightarrow \quad \phi = U_1^* w. \end{aligned}$$

The ϕ is the free variable in w. The inequality constraint can be written as follows

$$|\phi|^2 + |U_2^* M_w^+ e|^2 \le \gamma_w^2 \quad (5)$$

Now consider again equality constraint (2)

$$\begin{aligned} \xi &= \begin{bmatrix} M_\xi^+ & N_\xi \end{bmatrix} \begin{bmatrix} e \\ \theta \end{bmatrix} \\ &= M_\xi^+ e + N_\xi(V_1\gamma + V_2\psi), \text{ where} \\ \gamma &= S_1^{-1}(\phi - U_1^* M_w^+ e), \text{ from the def. of } \phi \\ \xi &= (M_\xi^+ - N_\xi V_1 S_1^{-1} U_1^* M_w^+)e + N_\xi V_1 S_1^{-1}\phi + \\ &+ N_\xi V_2 \psi \quad (6) \\ \eta &= \begin{bmatrix} P_{11} & P_{12} \end{bmatrix} \begin{bmatrix} \xi \\ w \end{bmatrix} + P_{13}u \\ &= \begin{bmatrix} P_{11} & P_{12} \end{bmatrix} \left(\begin{bmatrix} M_\xi^+ - N_\xi V_1 S_1^{-1} U_1^* M_w^+ \\ (I - U_1U_1^*)M_w^+ \end{bmatrix} e + \right. \\ &+ \left. \begin{bmatrix} N_\xi V_1 S_1^{-1} \\ U_1 \end{bmatrix} \phi + \begin{bmatrix} N_\xi V_2 \\ 0 \end{bmatrix} \psi \right) + P_{13}u \quad (7) \end{aligned}$$

There are two free variables in ξ and η: ϕ and ψ. Once they are found, a Δ satisfying (2) can be constructed. A general optimization problem can be stated with the introduction of positive-semidefinite trade-off variables Λ_ν for the uncertainty blocks and Λ_w for the disturbance

$$min_{\phi,\psi,\nu} \nu^T \Lambda_\nu \nu + w^* \Lambda_w w \quad (8)$$

subject to the constraints

$$\nu_i \ge \frac{|\xi_i|}{|\eta_i|}, \text{ i=1..}\tau\text{, and (5),}$$

This optimization is performed on each frequency points ω_k, $k = 1, \ldots, \lfloor \frac{N}{2} \rfloor$. The validation method can handle also real or complex repeated scalar uncertainty. For details see [8].

In the next section a procedure is presented that applies this validation algorithm to create unfalsified models.

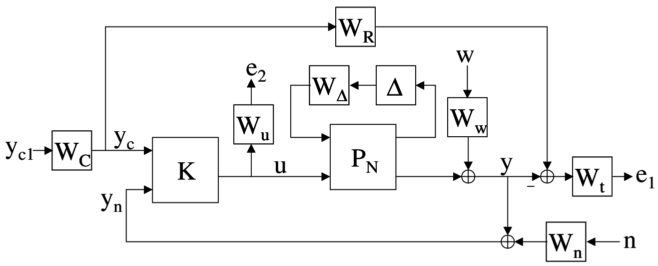

Fig. 1. A general robust control design structure.

III. THE CONCEPT OF IDENTIFYING AN UNCERTAINTY MODEL FOR ROBUST CONTROL

In the following part of the paper the disturbance is assumed to be additive on the outputs, i.e. $P_{12} = 0$ and $P_{22} = I_{n_y}$ in (1). This is the only restriction on the uncertainty model structure. The signals belong to the $\mathcal{L}_2$ space as usual in the robust control context [10]. The goal of identifying an uncertainty model for robust control can be comprise within the next sentence.

Select an uncertainty LFT structure and an additive disturbance bound as function of frequency so that an unfalsified model be created with minimum size of uncertainty and robust performance objectives of the control setup be satisfied.

An example for robust performance objectives can be seen on Fig. 1. The disturbance weight W_w and the uncertainty weight W_Δ are the results of the unfalsification process, while the other weighting functions represent the objective requirements of the closed-loop system and are considered to be given. They are defined in section V.

In order to perform the goal, the following steps are applied.

Step 1. Preparations. Transform the nominal model into discrete-time and perform DFT for input u and output y_n of each experiments in the validation data set.

Step 2. Minimum disturbance. Compute the minimum disturbance for each experiments on each frequency ($\phi = 0$ in (5)) and take a bound function B_1 defined as

$$B_1(\omega) \geq |U_2^*(\omega) M_w^+(\omega) e(\omega)| \tag{9}$$

for all ω, in all experiments as plotted on Fig. 2, where the points represent the right side of (9).

Step 3. Maximum disturbance. Define a real-valued bound function B_2 over the Euclidean norm of the nominal model-error, i.e. $B_2(\omega) \geq |e(\omega)|$ for all ω, in all experiments. On Fig. 2 B_2 is plotted. Would B_2 be defined for the components of $e(\omega)$ and were these bounds the magnitude of the disturbance weight functions, there were no need for uncertainty.

Step 4. Disturbance bound. Design a disturbance bound function $\gamma_w(\omega) \geq B_1(\omega)$ that guaranties the existence of the solution to optimization problem (8). With the relative distance of γ_w from the lower and upper bounds the trade-off between disturbance and uncertainty can be planned. On Fig. 2 the upper bound B_2 and γ_w can be seen. Also the DC data of the minimum disturbances are plotted with

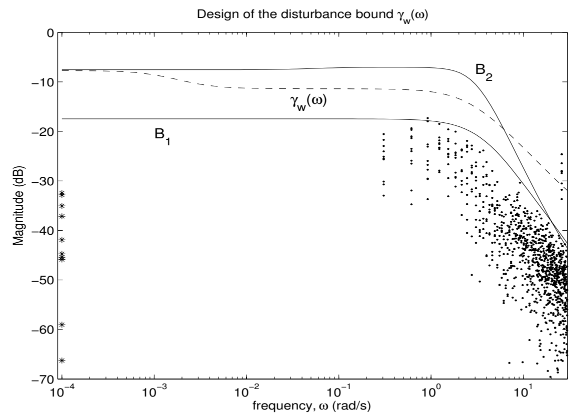

Fig. 2. Design of the disturbance bound $\gamma_w(\omega)$.

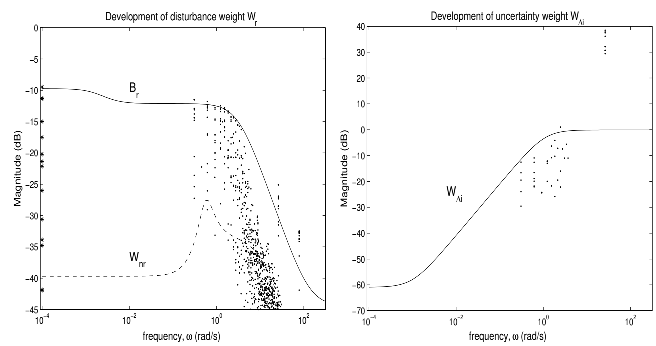

Fig. 3. Design of the yaw rate disturbance bound B_r (left) and the input-multiplicative uncertainty bound $W_{\Delta_i}(right)$. W_{nr} is the yaw rate sensor model (left).

stars, shifted from $\omega = 0$ to $\omega = 1e - 4$ in order to see them in the logarithmic scale. It's worth adverting loop shaping viewpoints in selecting the trade-off variable (see [10], chapter 5).

Step 5. Optimization. Select a trade-off parameter Λ_ν for the uncertainty blocks and perform the optimization problem (8) with $\Lambda_w = 0$, i.e minimize the uncertainty subject to the selected disturbance constraint.

Step 6. Bounds over the results. Define bounds $B_{w_i}(\omega)$ over the resulted disturbance points $|w_i(\omega)|$ $i = 1, \ldots, n_y$ and $W_{\Delta_i}(\omega)$ over $\nu_i(\omega)$ $i = 1, \ldots, \tau$. See Fig. 3 for the case of $n_y = 2$ and $\tau = 1$. In the example of the figures maximum disturbance was specified for the DC frequency which corresponds to zero uncertainty. Therefore in designing $W_{\Delta_i}(\omega)$ any small values can be placed to frequencies below ω_1. If information is required below ω_1, the number of DFT points N should be increased.

Step 7. Correction with sensor model If a sensor model is available, the magnitude of its weight function $|W_n(\omega)|$ can be subtracted from $B_w(\omega)$ since noise is also additive and was included in the unfalsification process. The real disturbance weight is defined accordingly as $|W_w(\omega)| = |B_w(\omega)| - |W_n(\omega)|$

As soon as the diagonal normalization weights W_w and W_Δ are computed, the following assumptions are applied on the disturbance and uncertainty

$$\|\Delta\|_\infty \leq 1, \qquad |w| \leq 1.$$

Step 8. Validation by controller design. Now we have an unfalsified model and all weighting functions in the robust control setup. It remained to check whether robust performance of the closed loop system can be guaranteed against worst-case disturbances and uncertainties. If not, there is a lots of freedom in the trade-off variables $\gamma_w(\omega)$ and $\Lambda_\nu(\omega)$, go back to steps 4 or 5.

In the next section the method is demonstrated on a linear velocity-scheduled vehicle model. It makes no difficulty about extending the proposed method for LTI systems to LPV systems of that kind. The vehicle driven on constant velocity defines an LTI model per experiments. In this paper LTI uncertainty model is identified, i.e. W_w and W_Δ are velocity-independent, while the nominal model changes from experiment to experiment due to the velocity.

IV. The nominal LPV model of the vehicle

The experimental vehicle is an MAN truck, with disk breaks at the front axle. During the data acquisition the vehicle is driven with constant velocity v. The control input is the brake pressure difference Δp. The measured outputs are the yaw-rate r and steering angle δ. The nominal model consists of the yaw dynamics of the vehicle body and the steering system. The details of the derivation and identification of the LPV model can be found in [16]. From the resulted model in [16], which is in predictor form, only the A, B, C, D state-space matrices are kept, the predictor gain is omitted for the robust control setup. So the nominal model is as follows

$$\begin{bmatrix} \dot{r} \\ \dot{\delta} \\ \ddot{\delta} \end{bmatrix} = \begin{bmatrix} \frac{-p_1 l}{v} + p_2 v & p_1 & 0 \\ 0 & 0 & 1 \\ \frac{p_3 l}{v} + p_4 v & -p_3 & p_5 \end{bmatrix} \begin{bmatrix} r \\ \delta \\ \dot{\delta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ p_6 \end{bmatrix} \Delta p$$

and $y = \begin{bmatrix} r & \delta \end{bmatrix}^T$ where the parameters given in table I.

TABLE I

PARAMETERS OF THE NOMINAL MODEL

p_1	p_2	p_3	p_4	p_5	p_6
14.54	0.06	20.60	-0.25	-4.96	-0.32

Let $P_v(s)$ denote the Laplace-transform of the nominal model on velocity v.

Fig. 4 shows the outputs of the real system and the model in the time-domain on a low (v=8 m/s) and a higher (v=17m/s) velocity.

The model-error is mainly due to disturbances of the lateral road-slope and the neglected roll dynamics, i.e. tire-load variation.

V. Uncertainty modelling for the LPV vehicle model

In this section three type of uncertainty structures are compared to demonstrate the procedure. The unfalsified uncertainty models are compared on the same objectives of the control setup, see Fig. 1.

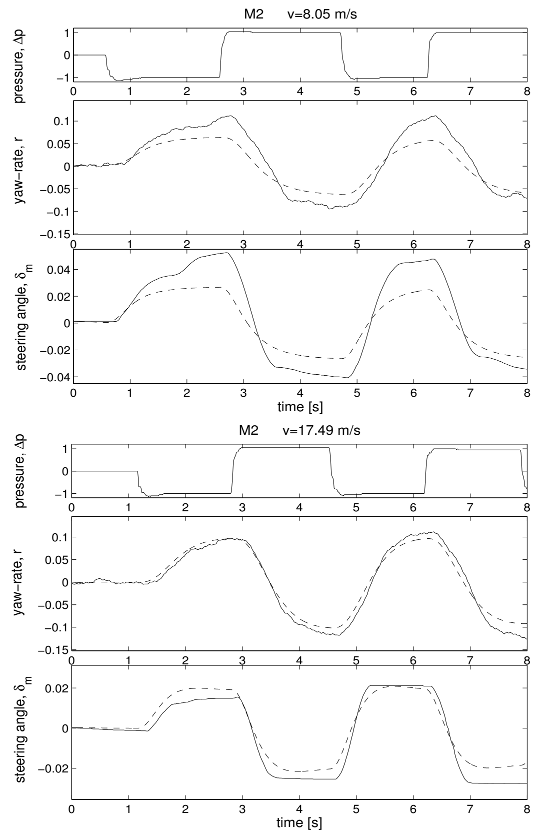

Fig. 4. The nominal model in time-domain. Measurements (solid), nominal model (dashed).

The reference command y_c is the yaw-rate command r_c. Its weight function W_C that normalizes $y_c = r_c$ to y_{c1} is computed by bounding the DFT of the measured yaw-rate. It is assumed that similar commands can occur during the identification experiments. See Fig. 5 for W_C and other performance weights.

The reference model W_R is a well-damped second-order LTI system.

The weight functions $W_n = \begin{bmatrix} W_{nr} \\ W_{n\delta} \end{bmatrix}$ for the noises are computed by bounding the DFT of the outputs of an experiment with zero control input ($\Delta p = 0$).

The penalty functions for the control energy W_u and tracking-error W_t are parameterized by the DC-gain in order to compare the three uncertainty model on different performance specifications.

$$\begin{aligned} W_t(s) &= A_t \frac{s/10+1}{s/0.02+1} \\ W_u(s) &= A_u \frac{(s/0.02+1)^2}{(s/50+1)^2} \end{aligned}$$

H_∞ **control design** (K_{H_∞}**).** In the first structure all of the model errors are considered as additive disturbance and noise. Steps 4 and 5 can be omitted, since the optimization

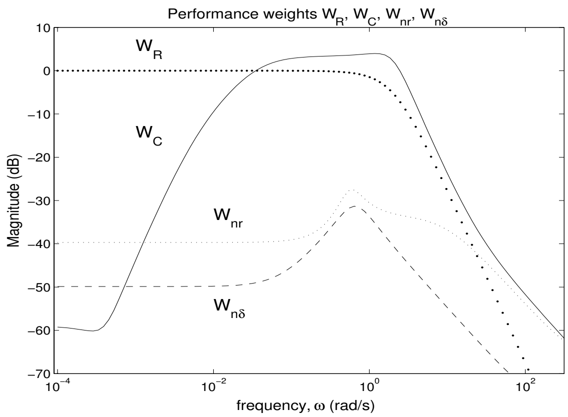

Fig. 5. Performance weight functions of the control setup in Fig. 1

would have trivial solution with the choice $\gamma_w = B_2$. The resulted disturbances w would be equal to e. Bounding the components of the nominal error vector $e(\omega)$ the procedure can be continued from step 7 with $W_\Delta = 0$. The controller is a simple H_∞ controller designed by γ-iteration with the *hinfsyn* command in the μ-Analysis and Synthesis Toolbox in MATLAB [17].

μ-control design with input-multiplicative uncertainty (K_i) . Figures 2 and 3 show the design steps of this type. The Laplace-transform of the output of the uncertain model can be written as

$$\begin{aligned} y(s) &= P_v(s)(1 + W_{\Delta_i}(s)\Delta_i(s))u(s) + W_w(s)w(s), \\ w &= \begin{bmatrix} w_r \\ w_\delta \end{bmatrix}, \\ \|\Delta_i\|_\infty &\leq 1, \\ \|w\|_2^2 &= \int_{-\infty}^{\infty} |w(t)|^2 dt \leq 1, \end{aligned}$$

where $P_v(s)$ is the nominal model on velocity v.

μ control design with input-multiplicative and state-space uncertainty (K_{is}). The state-space uncertainty is represented by a 3 by 3 full complex block Δ_s defined with the following open-loop plant model specification

$$\begin{aligned} \dot{x} &= Ax + B\tilde{u} + \xi_s \\ \xi_s &= W_{\Delta_s}\Delta_s\eta_s \\ \eta_s &= x, \\ \|\Delta_s\|_\infty &\leq 1, \end{aligned}$$

where the input is loaded with the input-multiplicative uncertainty block:

$$\tilde{u} = (1 + W_{\Delta_i}\Delta_i)u$$

The closed-loop robust performance is summarized in table II. The last three columns of the table corresponds to three different performance objectives in the penalty functions. The rows corresponds to the three type of uncertainty descriptions. The table shows the achieved performance values and controller order. The K_{H_∞} and K_{is} controllers achieved similar performance in each specification. The conclusion can be drawn that further improvement on the trade-off variables γ_w and Λ_ν are required. The second controller K_i could not satisfy robust performance by the first specification. Either control energy should be allowed to increase or tracking performance should be abated.

TABLE II
COMPARISON OF THE THREE UNCERTAINTY STRUCTURES.

	A_t	95	75	95
	A_u	1e-6	1e-6	1e-7
K_{H_∞}	γ	**0.984**	0.777	0.984
	order	**23**	23	23
K_i	μ	1.053	**0.999**	0.986
	order	30	**30**	24
K_{is}	μ	**0.986**	0.777	0.985
	order	**29**	29	29

For time-domain simulation the nominal models are feed back with worst-case uncertainty of size $\|\Delta_i\|_\infty = 1$. The reference command r_c is the filtered yaw-rate measurement of the experiment shown on the right of Fig. 4 and the disturbance is proportional to the nominal model-error

$$w(t) := e(t) \begin{bmatrix} 0.63 & 0 \\ 0 & 1 \end{bmatrix}$$

The yaw-rate error is decreased in order to keep the DC disturbance bound. The noise is simulated, band limited and white, of power 0.01.

On Fig. 6 the three solution denoted with boldface in table II are compared. It can be seen, that the worst tracking performance was produced by the second controller with input-multiplicative uncertainty, while K_{H_∞} and K_{is} presented similar results. Note, that in case of K_{is} the worst-case input-multiplicative and state-space uncertainty was simulated in addition to the same disturbances as at the K_{H_∞} controller, where the nominal model did not have uncertainty.

VI. CONCLUSIONS

In the general robust control setup the proposed method ensures a reliable uncertainty model of a nominal LTI or a special kind of LPV model. Assuming appropriate (informative) validation data set an unfalsified model can be guaranteed by some simple design steps. The selection from the candidate models was performed by μ-control design.

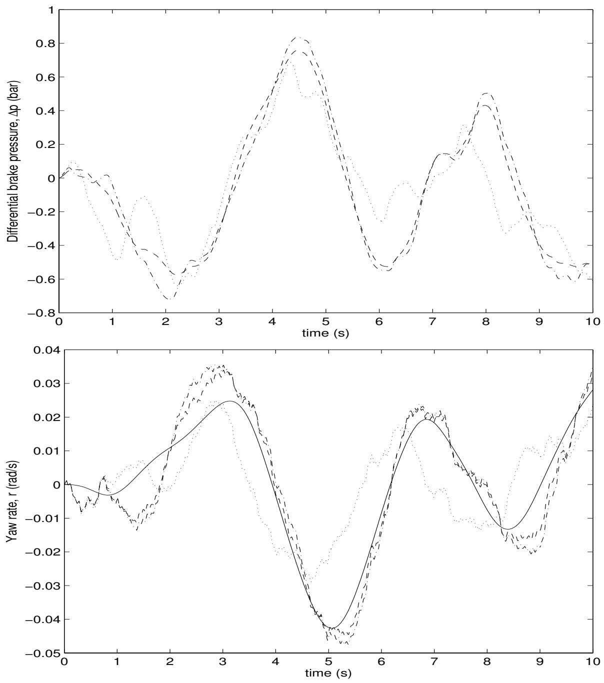

Fig. 6. Validation of the model in the time-domain by controller-simulation. Differential pressure (top), yaw-rate (down). (worst-case uncertainty, measured disturbances and reference). K_i dotted, reference solid, K_{is} dashed, K_{H_∞} dash-dot.

The procedure was demonstrated using real experimental data on a velocity-scheduled linear vehicle model. Some practical viewpoints were presented to the selection of the design parameters.

VII. ACKNOWLEDGEMENT

This work has been supported by the Hungarian National Science Foundation (OTKA) under grant number T043111, and by the the Hungarian National office for Research and Technology through the project "Advanced Vehicles and Vehicle Control Knowledge Center" (No. OMFB-01418/2004). The free run of the truck was given by Knorr-Bremse Fékrendszerek Kft, which is gratefully acknowledged by the authors.

REFERENCES

[1] L. Ljung, "Model validation and model error modeling," *The Astrom Symposium on Control*, pp. 15–42, 1999.

[2] K. Poolla, P. Khargonekar, A. Tikku, J. Krause, and K. Nagpal, "A time-domain approach to model validation," *IEEE Transactions on Automatic Control*, vol. 39, pp. 951–958, May 1994.

[3] M. Safonov and T. Tsao, "The unfalsified control concept and learning," *IEEE Transactions on Automatic Control*, vol. 42, No. 6, pp. 843–847, June 1997.

[4] W. Lee, B. Anderson, , I. Mareels, and R. Kosut, "On some key issues in windsurfer approach to robust adaptive control," *Automatica*, vol. 31, No. 11, pp. 1619–1636, 1995.

[5] R. Kosut, "An uncertainty model validation approach to adaptive robust linear control," *Proceedings on the CD-ROM of the IEEE Mediterranean Conference on Control and Systems, Cyprus*, 1997.

[6] R. S. R. Smith, "Model validation for uncertain systems," *PhD dissertation*, 1990.

[7] R. S. Smith and J. C. Doyle, "Model validation: a connection between robust control and identification," *IEEE Transactions on Automatic Control*, vol. 37, No 7, pp. 942–952, July 1992.

[8] K. B. Lim and D. P. Giesy, "Parameterization of model validating sets for uncertainty bound optimizations," *AIAA report*, vol. 98-4135, 1998.

[9] L. Ljung, *System identification: theory for the user*. Prentice Hall, Inc, 1987.

[10] K. Zhou, J. C. Doyle, and K. Glover, *Robust and optimal control*. Prentice-Hall Inc., Upper Saddle River, New Jersey, 1996.

[11] J. Ackermann, D. Odenthal, and T. Bünte, "Advantages of active steering for vehicle dynamics control," *In Proc. 32nd International Symposium on Automotive Technology and Automation, Vienna*, vol. ME, pp. 263–270, 1998.

[12] P. Gáspár, I. Szászi, and J. Bokor, "Fault-tolerant control structure to prevent the rollover of heavy vehicles," *Safeprocess, Washington*, 2003.

[13] ——, "Rollover avoidance for steer-by-wire vehicles by using linear parameter varying methods," *Mediterranean Conference on Control and Automation*, 2003.

[14] G. Kovács, J. Bokor, L. Palkovics, L. Gianone, A. Semsey, and P. Széll, "Lane-departure detection and control system for commercial vehicles," *IEEE Int. Conference on Intelligent Vehicles, Stuttgart*, pp. 46–50, 1998.

[15] T. Pilutti, G. Ulsoy, and D. Hrovat, "Vehicle steering intervention through differential braking," *American Control Conference, Washington*, pp. 1667–1671, 1995.

[16] G. Rödönyi and J. Bokor, "Identification of an lpv vehicle model based on experimental data," *Accepted in the 16th IFAC World Congress, Prague*, 2005.

[17] G. Balas, J. Doyle, K. Glover, A. Packard, and R. Smith, *μ-Analysis and Synthesis Toolbox, for Use with MATLAB*. The MathWorks Inc., 1993.