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Two Fuzzy Multiple Reference Model Tracking Control Designs with an Application to Vehicle Lateral Dynamics Control

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***Abstract*— This paper presents two control design methods for the general class of Takagi-Sugeno (T-S) fuzzy models. The main goal is to stabilize the system as well as to force the output vector to track a desired trajectory. These two control methods belong to the multi-controller approach by considering the dynamic output feedback case. But in the first approach, we derive analytical expressions of the dynamic output feedback controller matrices whereas in the second one, the controller is described by a convex interpolation of linear control laws, each of them is calculated by a linear dynamic output feedback controller. We give results on the stability analysis as well as the tracking performance of the two approaches by using H_∞ theory and Linear Matrix Inequalities (LMI). Note that the desired trajectory is calculated by a convex interpolation of multiple linear reference models. Finally, a simulation example for the vehicle lateral dynamics control is given to illustrate and compare the two control algorithms.**

I. Introduction

Takagi-Sugeno fuzzy model-based control has emerged as an attractive control technique during the past decade. It has been proved that T-S fuzzy models can effectively approximate complex nonlinear systems by a set of local linear dynamics with their linguistic description. The dynamic behavior of the nonlinear system is obtained by a convex interpolation of the linear dynamic equations. This class belongs to the more general class of Linear Parameter Varying (LPV) systems. Classical control techniques have been applied to T-S fuzzy models [9], [10], [11], [13], [14], [3], [7]. But note that all these research works concern stabilization. Nevertheless, classical control objectives are stabilization as well as trajectory tracking. There are few publications about tracking control design for T-S fuzzy models [11] without adaptive techniques.

What we want to propose in this paper is to generalize the approach from [5] where a set of reference model output feedback controllers are designed for a set of minimum phase SISO linear models. By considering MIMO T-S fuzzy models, we wish to design a set of linear dynamic output feedback controllers in order to guarantee the closed-loop stability and to force the system output vector to track a desired trajectory vector. We first give analytical expressions of the controller matrices which will be tuned on-line and therefore, this method is called fuzzy gain-scheduling approach because only one controller is considered and the idea differs from the second method, where the control signal which will be applied to the system is a convex interpolation of the local control signals. Thus, we keep the same philosophy of the multi-controller approach proposed in [5]. For the two methods and by using H_∞ theory, we give results on stability analysis as well as results on tracking performance and the two tracking control design problems are parameterized in terms of LMI problems. At last, the desired trajectory is the output vector of a nonlinear reference model which is also a convex interpolation of linear reference models. The proposed nonlinear reference model is suitable for practical applications. A unique reference model is usually not convenient for a nonlinear plant since slow dynamics may be necessary for some region state spaces whereas fast dynamics may be imposed for some other regions.

The paper is organized as follows: in section *2*, the problem formulation is presented. In section *3*, the Fuzzy Gain Scheduling (FGS) approach is presented and in section *4*, the multiple reference-model tracking Multi-Controller (MC) design method is explained. Moreover, we give conditions for which the two system control design methods are equivalent. Simulation results are given in section *5* to compare the tracking performance of the two control algorithms.

II. Problem Formulation

The studied class of nonlinear MIMO systems corresponds to the T-S fuzzy models which may approximate a large number of nonlinear systems. This continuous fuzzy dynamic model is described by fuzzy IF-THEN rules. Each of them represents linear input-output relations of nonlinear system. Let us consider a nonlinear system that can be described by the following T-S fuzzy model:

$$0 \le \mu_i(z(t)) \le 1; \quad (1)$$

$$\sum_{i=1}^{r} \mu_i(z(t)) = 1 \quad (2)$$

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$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(z(t)) A_i x(t) \\ \quad + \sum_{i=1}^{r} \mu_i(z(t)) B_{1_i} u(t) + \sum_{i=1}^{r} \mu_i(z(t)) B_{2_i} r(t) \\ y(t) = Cx(t) \end{cases} \tag{3}$$

Where $x \in \Re^n$, $u \in \Re^{k_1}$, $y \in \Re^{k_2}$ and $r \in \Re$ is a bounded exogenous input. Let $\mathrm{P} \in \Re^r$ be the set of membership functions satisfying (1) and (2). Let $p = [\mu_1 \quad \mu_2 \quad \dots \quad \mu_r] \in \mathrm{P}$, so, the dynamic fuzzy model (1) is equivalently defined by:

$$\begin{cases} \dot{x} = A(p)x + B_1(p)u + B_2(p)r \\ y = Cx \end{cases} \tag{4}$$

We consider the following nonlinear reference model:

$$\begin{cases} \dot{x}_m = \sum_{j=1}^{r} \mu_j(z(t)) A_{m_j} x_m + \sum_{j=1}^{r} \mu_j(z(t)) B_{m_j} r \\ y_m = Cx_m \end{cases} \tag{5}$$

Or, equivalently:

$$\begin{cases} \dot{x}_m = A_m(p)x_m + B_m(p)r \\ y_m = Cx_m \end{cases} \tag{6}$$

Where r is the reference input defined previously in (3) as a bounded exogenous input, A_{m_j}, B_{m_j}, C are known matrices of appropriate dimensions and A_{m_j} is Hurwitz. It is assumed that the system: $\dot{x}_m = \sum_{j=1}^{r} \mu_j(z(t)) A_{m_j} x_m$ is globally asymptotically stable.

Finally, we define: $e_c = y - y_m$ as the tracking error.

Our problem consists of finding a dynamic output-feedback controller with *two specific structures*:

$$K \begin{cases} \dot{\zeta} = \sum_{j=1}^{r} \mu_j(z(t))(A_{K_j}\zeta + B_{K_j} y_p) \\ u = \sum_{j=1}^{r} \mu_j(z(t))(C_{K_j}\zeta + D_{K_j} y_p + E_{K_j} r) \end{cases} \tag{7}$$

Or:

$$K \begin{cases} \dot{\zeta} = A_K(p)\zeta + B_K(p) y_p \\ u = C_K(p)\zeta + D_K(p) y_p + E_K(p) r \end{cases} \tag{8}$$

Such that [6]:

1. The closed loop with (7) or (8) and (3) is globally asymptotically stable.
2. The controller K dynamics (7) or (8) are stable.
3. The tracking error e_c is as small as possible. That means that we want to minimize the scalar $\gamma > 0$ where $\|T_{re_c}\|_\infty \le \gamma$ and T_{re_c} denotes the transfer function from r to e_c. Thus, we want to minimize the tracking error performance defined by the L_2 gain γ performance of the system (7) or (8) with (3).

The first method will concern the structure (7) whereas the second method will concern the structure (8). (7) implies (8) but inversely, the question is: is it possible to find the set of matrices A_{K_j}, B_{K_j}, C_{K_j}, D_{K_j}, E_{K_j} from the expressions $A_K(p)$, $B_K(p)\dots$ with $A_K(p) = \sum_{i=1}^{r} h_i(z(t)) A_{K_i} \dots$? The matrix functions $A_K(p)$, $B_K(p)$, $C_K(p)$, $D_K(p)$, $E_K(p)$ have to be determined in the first method which is called multiple reference model fuzzy tracking gain-scheduling approach. They are tuned on-line and they are function of membership functions whereas in the second method, we want to find the set of matrices A_{K_j}, B_{K_j}, C_{K_j}, D_{K_j}, E_{K_j} $\forall j$. Moreover, in comparison with [6], we want to remove the restrictive equality constraints and to find the set of variables E_{K_j}.

III. MULTIPLE REFERENCE MODEL FUZZY TRACKING GAIN SCHEDULING APPROACH

Before further analysis, the following lemma is needed for the design procedure.

Lemma 1: [2] For any matrices X and Y with appropriate dimensions, we have:

$$X^t Y + Y^t X \le X^t R^{-1} X + Y^t R Y \tag{9}$$

where R is any positive-definite symmetric matrix.

Let us recall that we want to find a controller whose structure is defined by (8), satisfying the stability and tracking performance conditions given in the previous section.

From (4), (6) and (8), we obtain:

$$\begin{pmatrix} \dot{\underline{x}} \\ \dot{\zeta} \end{pmatrix} = A_{cl}(p) \begin{pmatrix} \underline{x} \\ \zeta \end{pmatrix} + B_{cl}(p) r;\ z = C_{cl} \begin{pmatrix} \underline{x} \\ \zeta \end{pmatrix} \tag{10}$$

Where $A_{cl}(p) = \begin{pmatrix} \mathbf{A}(p) + \mathbf{B}(p) D_K(p) \mathbf{C} & \mathbf{B}(p) C_K(p) \\ B_K(p) \mathbf{C} & A_K(p) \end{pmatrix}$,

$B_{cl}(p) = \begin{pmatrix} \mathbf{B}(p) E_K(p) + \mathbf{B_r}(p) \\ 0 \end{pmatrix}$, $C_{cl} = \begin{pmatrix} \mathbf{D_z} & 0 \end{pmatrix}$

With $\underline{x} = (x \quad x_m)^t$,

$\mathbf{A}(p) = \begin{pmatrix} A(p) & 0 \\ 0 & A_m(p) \end{pmatrix} = \sum_{i=1}^{r} h_i(z(t)) \begin{pmatrix} A_i & 0 \\ 0 & A_{m_i} \end{pmatrix}$,

$\mathbf{B}(p) = \begin{pmatrix} B_1(p) \\ 0 \end{pmatrix}$, $\mathbf{B_r}(p) = \begin{pmatrix} B_2(p) \\ B_m(p) \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} C & 0 \end{pmatrix}$,

$\mathbf{D_z} = \begin{pmatrix} C & -C \end{pmatrix}$.

0 is the zero matrix of appropriate dimension and the transpose of a matrix A is denoted A^t.

By using the real bounded lemma [2], the H_∞ tracking performance is guaranteed for a prescribed value $\gamma > 0$ if and only if there exists a matrix $X = X^t > 0$ which is the common solution of the following LMI:

$$\begin{pmatrix} A_{cl}(p)X + XA_{cl}^t(p) & B_{cl}(p) & XC_{cl}^t(p) \\ B_{cl}^t(p) & -\gamma I & 0 \\ C_{cl}(p)X & 0 & -\gamma I \end{pmatrix} < 0 \qquad (11)$$

Where I is the identity matrix of appropriate dimension.

(11) is of course not linear but by using the classical variable change [10], [6] with $X = \begin{pmatrix} R & M \\ M^t & U \end{pmatrix}$, $X^{-1} = \begin{pmatrix} S & N \\ N^t & V \end{pmatrix}$ and by applying first lemma *1* then the Schur's lemma [2] because of the nonlinear term $S\mathbf{B}(p)E_K(p)$, we obtain after some manipulations the following optimization problem[1]:

$$\min \; trace(\varepsilon_1 \varepsilon_2) \qquad (12)$$

under the LMI system:

$$\begin{pmatrix} R & I \\ I & S \end{pmatrix} \geq 0, \; \begin{pmatrix} \varepsilon_1 & I \\ I & \varepsilon_2 \end{pmatrix} \geq 0 \qquad (13)$$

$$\psi_{ii} < 0 \;\; \forall i = 1 \ldots r \qquad (14)$$

$$\psi_{ij} + \psi_{ji} < 0 \;\; \forall i, j = 1 \ldots r \;\; i < j \qquad (15)$$

Where

$$\psi_{ij} = \begin{pmatrix} & & & & 0 & 0 \\ & \xi_{ij} & & & S & 0 \\ & & & & 0 & E_{K_j}{}^t \mathbf{B}_i^t \\ & & & & 0 & 0 \\ 0 & S & 0 & 0 & -\varepsilon_2 & 0 \\ 0 & 0 & \mathbf{B}_i E_{K_j} & 0 & 0 & -\varepsilon_1 \end{pmatrix} \qquad (16)$$

$$\xi_{ij} = \begin{pmatrix} sym(\mathbf{A}_i R + \mathbf{B}_i \mathbf{C}_{\mathbf{K}_j}) & * & * & * \\ \mathbf{A}_{\mathbf{K}_{ij}} + \mathbf{A}_i^t + \mathbf{C}^t D_{K_j}^t \mathbf{B}_i^t & sym(S\mathbf{A}_i + \mathbf{B}_{\mathbf{K}_{ij}} \mathbf{C}) & * & * \\ \mathbf{B}_{\mathbf{r}_i}{}^t + E_{K_j}^t \mathbf{B}_i^t & \mathbf{B}_{\mathbf{r}_i}{}^t S & -\gamma I & * \\ \mathbf{D_z} R & \mathbf{D_z} & 0 & -\gamma I \end{pmatrix}$$

The symmetric part of a square matrix A is denoted by *sym(A)*, i.e., $sym(A)=A^t+A$, * associated with the element a_{ij} is defined by the symmetric of a_{ji}.

The following relations are used:

$$\mathbf{C_K}(p) = \sum_{j=1}^{r} h_j(z(t))\mathbf{C}_{\mathbf{K}_j} = \sum_{j=1}^{r} h_j(z(t))(D_{K_j}\mathbf{C}R + C_{K_j}M^t) \qquad (17)$$

$$\mathbf{B_K}(p) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))\mathbf{B}_{\mathbf{K}_{ij}} = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))(NB_{K_j} + S\mathbf{B}_i D_{K_j}) \qquad (18)$$

$$\mathbf{A_K}(p) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))\mathbf{A}_{\mathbf{K}_{ij}} = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))(S(\mathbf{A}_i + \mathbf{B}_i D_{K_j}\mathbf{C})R + S\mathbf{B}_i C_{K_j}M^t + NA_{K_j}M^t + NB_{K_j}\mathbf{C}R) \qquad (19)$$

Where $I - RS = MN^t$.

For the tracking fuzzy gain-scheduling approach and by taking the controller order equal to $n + n_m$, we are ready to propose the following theorem to find the controller K (8).

Theorem 1: *If there exist* $R = R^t > 0$, $S = S^t > 0$, $\varepsilon_1 = \varepsilon_1{}^t > 0$, $\varepsilon_2 = \varepsilon_2{}^t > 0$, *a scalar* $\gamma > 0$ *and a set of matrices* D_{K_j}, E_{K_j}, $\mathbf{A}_{\mathbf{K}_{ij}}$, $\mathbf{B}_{\mathbf{K}_{ij}}$, $\mathbf{C}_{\mathbf{K}_j}$ *such that the optimization problem (12) under the LMI constraints (13), (14), (15) is solvable then, there exists a stable controller K (8) satisfying the closed-loop stability and the L_2 gain γ performance.*

The K controller matrices (8) deduced from (17), (18) and (19) for the fuzzy gain-scheduling approach are equal to:

$$E_K(p) = \sum_{j=1}^{r} h_j(z(t))E_{K_j}, \; D_K(p) = \sum_{j=1}^{r} h_j(z(t))D_{K_j} \qquad (20)$$

$$C_K(p) = \mathbf{C_K}(p)(M^t)^{-1} - D_K(p)\mathbf{C}R(M^t)^{-1} \qquad (21)$$

$$B_K(p) = N^{-1}\mathbf{B_K}(p) - N^{-1}S\mathbf{B}(p)D_K(p) \qquad (22)$$

$$A_K(p) = N^{-1}\mathbf{A_K}(p)(M^t)^{-1} - N^{-1}S(\mathbf{A}(p) + \mathbf{B}(p)D_K(p)\mathbf{C})R(M^t)^{-1} - N^{-1}S\mathbf{B}(p)C_K(p) - B_K(p)\mathbf{C}R(M^t)^{-1} \qquad (23)$$

IV. MULTIPLE REFERENCE MODEL TRACKING MULTI-CONTROLLER (MC) APPROACH

The problem is from now to determine the set of matrices A_{K_j}, B_{K_j}, C_{K_j} from (17), (18), (19) or from (21), (22), (23). Although this problem is obviously solved for C_{K_j}, we have from (18) and (19):

$$B_{K_j} = N^{-1}\mathbf{B}_{\mathbf{K}_{ij}} - N^{-1}S\mathbf{B}_i D_{K_j} \;\; \forall i = 1 \ldots r \qquad (24)$$

[1] The function $trace(\varepsilon_1 \varepsilon_2)$ is not convex but by using a linear approximation of $trace(\varepsilon_1 \varepsilon_2)$, we obtain the cone complementarity problem [2], [3]: min $trace(\varepsilon_{1_0} \varepsilon_2 + \varepsilon_{2_0} \varepsilon_1)$ subject to the LMI constraint (13), (14), (15).

$$A_{K_j} = N^{-1}\mathbf{A}_{\mathbf{K}_{ij}}(M^t)^{-1} - N^{-1}S\mathbf{A}_iR(M^t)^{-1}$$
$$- N^{-1}\mathbf{B}_{\mathbf{K}_{ij}}\mathbf{C}R(M^t)^{-1}\forall i = 1\ldots r \tag{25}$$

An immediate and obvious remark is that constraints (24) and (25) specify an over-determined system[2], since they involve r matrix equations in one matrix unknown. It is well known that a solution x to an over-determined system $Ax = b$ exists and is unique if $rank(A) = rank([A \quad b])$. That is why (24) admits one solution B_{K_j} if $rank(Y) = rank([Y \quad Z_j])$ where:

$Z_j = [\mathbf{B}_{\mathbf{K}_{1j}} - S\mathbf{B}_1D_{K_j};\ldots;\mathbf{B}_{\mathbf{K}_{rj}} - S\mathbf{B}_rD_{K_j}]$ and $Y = [N;N;\ldots;N]$. Since Y has full rank, the solution is unique. We have the same conclusion for A_{K_j}.

Thus, we propose the following theorem which gives sufficient conditions to find K (8) and (7):

Theorem 2: *If there exist $R = R^t > 0$, $S = S^t > 0$, $\varepsilon_1 = \varepsilon_1{}^t > 0$, $\varepsilon_2 = \varepsilon_2{}^t > 0$, a scalar $\gamma > 0$ and a set of matrices D_{K_j}, E_{K_j}, $\mathbf{A}_{\mathbf{K}_{ij}}$, $\mathbf{B}_{\mathbf{K}_{ij}}$, $\mathbf{C}_{\mathbf{K}_j}$ such that the optimization problem (12) under the LMI constraints (13), (14), (15) is solvable.*

Then, there exists a stable controller K (8) satisfying the closed-loop stability and the L_2 gain γ performance. The K controller matrices (8) are given by (20), (21), (22) and (23).

Moreover,

$$\text{if } rank(Y) = rank([Y \quad Z_j]) = rank([Y \quad W_j]) \tag{26}$$

Where $Y = [N;N;\ldots;N]$,

$$Z_j = [\mathbf{B}_{\mathbf{K}_{1j}} - S\mathbf{B}_1D_{K_j};\ldots;\mathbf{B}_{\mathbf{K}_{rj}} - S\mathbf{B}_rD_{K_j}],$$

$$W_j = \begin{bmatrix} \mathbf{A}_{\mathbf{K}_{1j}}(M^t)^{-1} - S\mathbf{A}_1R(M^t)^{-1} - \mathbf{B}_{\mathbf{K}_{1j}}\mathbf{C}R(M^t)^{-1} \\ \ldots \\ \mathbf{A}_{\mathbf{K}_{rj}}(M^t)^{-1} - S\mathbf{A}_rR(M^t)^{-1} - \mathbf{B}_{\mathbf{K}_{rj}}\mathbf{C}R(M^t)^{-1} \end{bmatrix}.$$

The K controller matrices (7) are given by:

$$\begin{cases} C_{K_j} = \mathbf{C}_{\mathbf{K}_j}(M^t)^{-1} - D_{K_j}\mathbf{C}R(M^t)^{-1} \\ B_{K_j} = (Y^tY)^{-1}Y^tZ_j;\ A_{K_j} = (Y^tY)^{-1}Y^tW_j \end{cases} \tag{27}$$

Note that (7) and (8) are equivalent with condition (26).

[2] In [10], the authors have proposed the dynamic parallel dynamic compensation (DPDC) which only concerns stabilization. But the problem does not appear since from (7), they have: $u = C_K\xi + D_Ky$. Let us assume in our approach that C_K and D_K are unique, then it becomes obvious to calculate A_{K_j}, B_{K_j} from (18) and (19).

The last approach step is to determine the controller (7) matrices in the case where the LMI problem (13), (14), (15) is feasible but (26) is not satisfied.

From (24) and (25), we can easily deduce the following relations for a fixed value $i^* \in \{1;\ldots;r\}$:

$$\begin{cases} \mathbf{B}_{\mathbf{K}_{ij}} = \mathbf{B}_{\mathbf{K}_{i^*j}} + S(\mathbf{B}_i - \mathbf{B}_{i^*})D_{K_j} \quad \forall i, j = 1\ldots r \\ \mathbf{A}_{\mathbf{K}_{ij}} = \mathbf{A}_{\mathbf{K}_{i^*j}} + S(\mathbf{A}_i - \mathbf{A}_{i^*})R + S(\mathbf{B}_i - \mathbf{B}_{i^*})D_{K_j}\mathbf{C}R \end{cases} \tag{28}$$

By using (28) with (14), (15), (16) and by using lemma *1* and the Schur's lemma, we also obtain the optimization problem (12) under the LMI system (13) and

$$\chi_{ii}^* < 0 \ \forall i = 1\ldots r \tag{29}$$

$$\chi_{ij}^* + \chi_{ji}^* < 0 \ \forall i, j = 1\ldots r \ i < j \tag{30}$$

Where

$$\xi_{ij}^* = \begin{pmatrix} sym(\mathbf{A}_iR + \mathbf{B}_i\mathbf{C}_{\mathbf{K}_j}) & * & * & * \\ \mathbf{A}_{\mathbf{K}_{i^*j}} + \mathbf{A}_i^t + \mathbf{C}^tD_{K_j}^t\mathbf{B}_i^t & sym(S\mathbf{A}_i + \mathbf{B}_{\mathbf{K}_{i^*j}}\mathbf{C}) & * & * \\ \mathbf{B}_{\mathbf{r}_i}{}^t + E_{K_j}^t\mathbf{B}_i^t & \mathbf{B}_{\mathbf{r}_i}{}^tS & -\gamma I & * \\ \mathbf{D}_\mathbf{z}R & \mathbf{D}_\mathbf{z} & 0 & -\gamma I \end{pmatrix}$$

$$\chi_{ij}^* = \begin{pmatrix} & & & & 0 & R(\mathbf{A}_i^t - \mathbf{A}_{i^*}^t) & R & 0 \\ & \xi_{ij}^* & & & S & \mathbf{C}^tD_{K_j}^t(\mathbf{B}_i^t - \mathbf{B}_{i^*}^t) & 0 & 0 \\ & & & & 0 & E_{K_j}^t\mathbf{B}_i^t & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon_2 & 0 & 0 & 0 \\ * & * & * & * & * & -\varepsilon_1 & 0 & (\mathbf{B}_i - \mathbf{B}_{i^*})D_{K_j}\mathbf{C} \\ * & * & * & * & * & * & -\varepsilon_2 & 0 \\ * & * & * & * & * & * & * & -\varepsilon_1 \end{pmatrix} \tag{31}$$

So, we propose the following theorem:

Theorem 3: *If there exist $R = R^t > 0$, $S = S^t > 0$, $\varepsilon_1 = \varepsilon_1{}^t > 0$, $\varepsilon_2 = \varepsilon_2{}^t > 0$, a strictly positive scalar γ and a set of matrices D_{K_j}, E_{K_j}, $\mathbf{A}_{\mathbf{K}_{i^*j}}$, $\mathbf{B}_{\mathbf{K}_{i^*j}}$, $\mathbf{C}_{\mathbf{K}_j}$ for a fixed value $i^* \in \{1;\ldots;r\}$ such that the optimization problem (12) under the LMI constraints (13), (29), (30) is solvable.*

Then, there exists a stable controller K (7) satisfying the closed-loop stability and the L_2 gain γ performance. The K controller matrices (7) are the unique solution of the linear system:

$$\begin{cases} \mathbf{B}_{\mathbf{K}_{i*j}} = NB_{K_j} + S\mathbf{B}_{i*}D_{K_j} \\ \mathbf{C}_{\mathbf{K}_j} = D_{K_j}\mathbf{C}R + C_{K_j}M^t \\ \mathbf{A}_{\mathbf{K}_{i*j}} = S(\mathbf{A}_{i*} + \mathbf{B}_{i*}D_{K_j}\mathbf{C})R + S\mathbf{B}_{i*}D_{K_j}M^t \\ \quad + NA_{K_j}M^t + NB_{K_j}\mathbf{C}R \;\forall j = 1\ldots r \end{cases} \tag{32}$$

V. APPLICATION TO THE VEHICLE LATERAL DYNAMICS CONTROL AND SIMULATION RESULTS

A. Vehicle lateral dynamics T-S model

An illustrative example which concerns the vehicle lateral dynamics is given to demonstrate the validity of the two control methods. The lateral dynamics are characterized by the following nonlinear LPV model:

$$\dot{x} = \begin{pmatrix} \dot{\beta} \\ \dot{r} \end{pmatrix} = \begin{pmatrix} \frac{-2(C_{fb}+C_{rb})}{mv} & -1-\frac{2(a_f C_{fb} - a_r C_{rb})}{mv^2} \\ \frac{-2(a_f C_{fb} - a_r C_{rb})}{I_z} & \frac{-2(a_f^2 C_{fb} + a_r^2 C_{rb})}{I_z v} \end{pmatrix} \begin{pmatrix} \beta \\ r \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{1}{I_z} \end{pmatrix} u + \begin{pmatrix} \frac{2C_{fb}}{mv} \\ \frac{2a_f C_{fb}}{I_z} \end{pmatrix} \delta_f \tag{33}$$

Where β denotes the side slip angle, r is the yaw velocity, v is the time-varying longitudinal vehicle speed which is supposed to be independent of the lateral dynamics, I_z is the yaw moment of inertia, m is the vehicle mass, u is the direct yaw moment control, δ_f is the front steering angle. For further details, see [12], [4], [6]. Note here that:

$a_f = 1.104m$, $a_r = 1.421m$, $I_z = 2630kg.m^2$, $m = 1526kg$,

$B_f = 6.7651$, $B_r = 9.0051$, $D_f = \text{-}6436.8$, $D_r = \text{-}5430$

$C_f = C_r = 1.3kg.rad^{-1}$, $C_{fb} = -D_f B_f C_f$, $C_{rb} = -D_r B_r C_r$

To obtain the T-S fuzzy model, unlike [1], we consider two speed zones: a small speed zone $(v_{\min} = 8m/s)$ and a high speed zone $(v_{\max} = 40m/s)$. For each zone, we obtain a linear model from (33) and the following rule base system:

If v is *V_Small* then $\dot{x} = A_1 x + B_{1_1}\delta_f + B_{2_1}u;\; y = Cx$ (34)

If v is *V_Big* then $\dot{x} = A_2 x + B_{1_2}\delta_f + B_{2_2}u;\; y = Cx$

C is the identity matrix. The membership functions associated with the two symbols *V_Small* and *V_Big* and denoted by $\mu_{V\_Small}(v)$ and $\mu_{V\_Big}(v)$, are solutions of:

$$\begin{cases} \frac{1}{v^2} = \frac{1}{v_{\min}^2}\mu_{V\_Small}(v) + \frac{1}{v_{\max}^2}\mu_{V\_Big}(v) \\ \mu_{V\_Small}(v) + \mu_{V\_Big}(v) = 1 \end{cases} \tag{35}$$

The T-S fuzzy model (34) is a good approximation over the set: $v = [v_{\min} \quad v_{\max}]$. Note that the TS fuzzy model is exact by using four models and by decomposing the two nonlinearities $\frac{1}{v^2}$ and $\frac{1}{v}$ as (35).

The next step is to choose a reference model. From [8], we consider the following two local reference models:

$\dot{x}_m = A_{m_i}x_m + B_{m_i}\delta_f$; $y_m = Cx_m$

Where $A_{m_i} = \begin{pmatrix} \varepsilon_i & 0 \\ 0 & -\frac{1}{\tau_{a_i}} \end{pmatrix}$; $B_{m_i} = \begin{pmatrix} 0 \\ \frac{\kappa_{a_i}}{\tau_{a_i}} \end{pmatrix}$; $C = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

And $\kappa_{a_1} = 4.0819$; $\tau_{a_1} = 0.1300$; $\varepsilon_1 = -1$ for $v_{\min} = 8m/s$

$\kappa_{a_2} = 2.5776$; $\tau_{a_2} = 0.1951$; $\varepsilon_2 = -1$ for $v_{\max} = 40m/s$.

The global reference model is defined by:

$$\begin{aligned} \dot{x}_m &= \mu_{V\_Small}(v)(A_{m_1}x_m + B_{m_1}\delta_f) \\ &+ \mu_{V\_Big}(v)(A_{m_2}x_m + B_{m_2}\delta_f);\; y_m = Cx_m \end{aligned} \tag{36}$$

B. Simulation results

We consider a speed profile and a front steering angle profile both given by the driver and we want to find a controller K (7) and (8) (the control signal is the moment control) such that the output vector of (33) (here $y = (\beta \quad r)^t$) "tracks" the output vector of the nonlinear reference model (36) (here $y_m = (0 \quad r_m)^t$).

Computer simulations have been carried out to show first the performances of the multiple reference model fuzzy tracking gain scheduling approach then the performances of the multi-controller approach.

We have chosen: $x_0 = x_{m_0} = [0.1 \quad 0.5]$.

For the fuzzy tracking gain scheduling approach, we have found a solution for (13), (14) and (15) with $\gamma = 0.46$ but conditions (26) are not satisfied. However, for the multi-controller approach and thanks to theorem *3*, we have found a solution for (13), (29) and (30) with $\gamma = 1.5$.

The figure *1* shows the vehicle speed profile as well as the front steering angle evolution which corresponds to a slalom maneuver.

For the FGS approach, the figure *2* (left figure) shows the evolution of the vehicle yaw velocity in comparison with the yaw velocity from the reference model (for the MC approach, see the right figure). In the two approaches, the controlled system demonstrates good tracking performances with respect to the desired response even under large front steering angles. However, by comparing the side slip angle responses (see figure *3*: for the FGS approach, see the left figure, for the MC approach, see the right figure), we find out that the two control systems can not guarantee good tracking performance. This fact may imply that there exists a trade-off relationship between the side slip angle and the yaw velocity in such control systems.

The Figure *4* shows the evolution of the yaw moment as control input for the two approaches (the left figure for the FGS approach and the right figure for the MC approach).

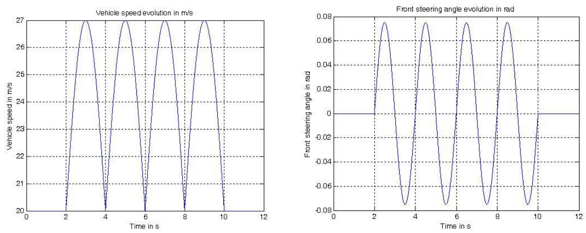

Fig. 1. Vehicle speed profile in *m/s* (left figure) and front steering angle evolution in *radians* (right figure).

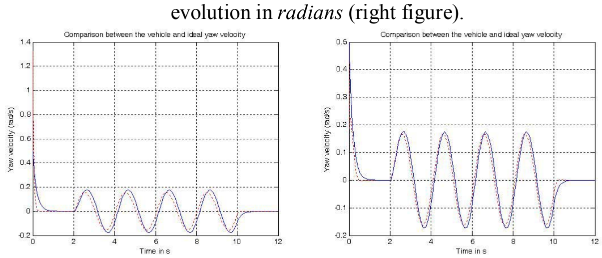

Fig. 2. Evolution of the vehicle yaw velocity in dotted line with the evolution of the yaw velocity from the reference model in solid line.

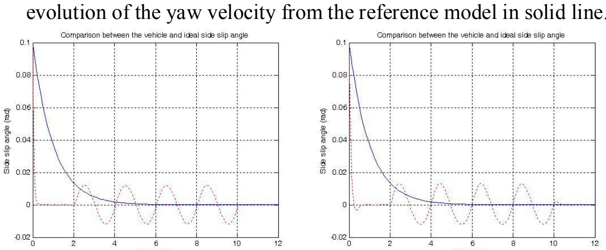

Fig 3. Evolution of the vehicle side slip angle in dotted line with the evolution of the side slip angle from the reference model in solid line.

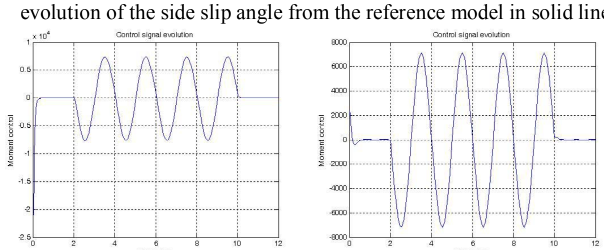

Fig. 4. Control signal evolution in $kg.m^2.rad.s^{-2}$.

VI. CONCLUSION

In this paper, we have presented two control designs for the T-S fuzzy models. The goal is to solve two problems: the stabilization problem as well as the tracking problem by minimizing a tracking H_∞ performance. For the first approach which is called fuzzy tracking gain scheduling approach, only one controller is designed and the controller matrices are tuned on-line. For the second approach which is called multi-controller approach, the control law is a convex interpolation of dynamic output feedback controllers. Note at last in order to compare the two methods that the first approach is less conservative than the second one.

The application concerns the vehicle lateral dynamics control and we have proposed a control system to make the yaw velocity and the side slip angle track desired trajectories provided by a nonlinear reference model using the direct yaw moment as the control input. Note that the reference model is a convex interpolation of two linear reference models. Computer simulations show the effectiveness of the two approaches with a better H_∞ performance for the fuzzy tracking gain scheduling approach.

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