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Output robust stabilisation of uncertain Takagi-Sugeno model

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Abstract— **The output feedback control of uncertain fuzzy models is one of the most challenging problems in the fuzzy control field. In this paper an approach to design an observer-based robust fuzzy control of uncertain fuzzy models is proposed. Based on quadratic Lyapunov function and Linear Matrix Inequalities (LMI) formulation, sufficient conditions are derived for robust asymptotic output stabilization. An example is given to illustrate the proposed results.**

I. INTRODUCTION

THERE have been several recent studies concerning the stability and the synthesis of controllers and observers for nonlinear systems described by Takagi-Sugeno fuzzy (T-S) models [10]. Based on quadratic Lyapunov function some papers have discussed the feedback control and the state estimation for fuzzy systems [5][7][1][3][13][19]. For example in [7], stability conditions for fuzzy control systems are reported to relax the conservatism of the basic conditions. To less of conservatism another method to relaxed quadratic stability conditions are also proposed in [19]. More recently, in [2] new stability conditions are obtained by relaxing the stability conditions of the previous works and particularly these derived in [19] allowing the symmetrical off-diagonal matrix blocks to be non-symmetrical. However the majority of these papers dealt with a state feedback control design that requires all states of systems to be measured. In many cases, this requirement is too restrictive. To overcome this situation, some papers [4][6][7][9][11] have discussed the control based on observers for fuzzy systems. However, in the above paper, the output feedback controllers with parametric uncertainties are not considered. Consequently the robustness of the closed-loop fuzzy model is not guaranteed. In [6] an approach to design an observer-based robust fuzzy control of uncertain fuzzy models was proposed. Unfortunately, the synthesis conditions are very conservative.

In this paper news derived stability conditions for robust output feedback stabilization relaxing the conservatism of the conditions obtained by the previous works (such as [6] and references therein) are given. The proposed results are based on the work of [7][2] developed in the case of state feedback control.

Motivated by the aforementioned works, sufficient conditions are derived for robust asymptotic output stabilization using quadratic Lyapunov function. These stability conditions are solved using LMI optimization techniques. A T-S model of four-wheel drive vehicles is used to compare the proposed result and to show the effectiveness of the derived conditions.

Notation: symmetric definite positive matrix P is defined by $P>0$, the set $\{1,2,..,n\}$ is defined by $I_n=\{1,2,..,n\}$, the symbol * denotes the transpose elements in the symmetric positions and $\sum_{i<j}^{n} x_i x_j = \sum_{i=1}^{n}\sum_{i<j}^{n} x_i x_j$.

II. PRELIMINARY

The uncertain fuzzy model considered is described as:

$$\begin{aligned}\dot{x}(t) &= \sum_{i=1}^{q} \mu_i(z(t))\big((A_i+\Delta A_i)x(t)+(B_i+\Delta B_i)u(t)\big)\\ y(t) &= \sum_{i=1}^{q} \mu_i(z(t))C_i x(t)\end{aligned} \tag{1}$$

With the properties:

$$\sum_{i=1}^{q} \mu_i(z(t))=1,\ \mu_i(z(t))\geq 0 \quad \forall\, i\in I_q \tag{2}$$

Where q is the number of sub-models, $x(t)\in\mathbb{R}^n$ is the state vector, $u(t)\in\mathbb{R}^m$ is the control input vector, $y(t)\in\mathbb{R}^l$ is the output vector, $A_i\in\mathbb{R}^{n.n}$, $B_i\in\mathbb{R}^{n.m}$, $C_i\in\mathbb{R}^{l.n}$ are the i[th] state matrix, the i[th] input matrix and the i[th] output matrix respectively. Vector $z(t)$ is the premise variables depending on measurable variables. ΔA_i and ΔB_i are time-varying matrices representing parametric uncertainties in the plant model. These uncertainties are admissibly norm-bounded and structured.

Assumption 1: The considered parameter uncertainties are norm-bounded:

$$\begin{aligned} &\Delta A_i = D_i F_i(t) E_{Ai}, \quad \Delta B_i = D_i F_i(t) E_{Bi} \\ &F_i(t)^T F_i(t) < I \end{aligned} \tag{3}$$

where D_i, E_{Ai}, E_{Bi} are known real constant matrices of appropriate dimension, $F_i(t)$ is an unknown matrix function with Lebesgue-measurable elements and I is the identity matrix.

The following lemma, commonly used in several papers, is recalled.

Lemma 1[1]: Given constant matrices D and F, symmetric constant matrix S and unknown constant matrix F_i of appropriate dimension satisfying the constraint $F^T F < R$. The following two propositions are equivalent:

i) $S + DFE + E^T F^T D^T < 0$

ii) $S + \begin{pmatrix} E^T & D \end{pmatrix} \begin{pmatrix} \varepsilon^{-1} R & 0 \\ 0 & \varepsilon I \end{pmatrix} \begin{pmatrix} E \\ D^T \end{pmatrix} < 0$ for some $\varepsilon > 0$.

III. Quadratic Output Stabilisation

The fuzzy observer is defined as follows

$$\begin{aligned} &\dot{\hat{x}}(t) = \sum_{i=1}^{q} \mu_i(z(t)) \left(A_i x(t) + B_i u(t) + G_i (y(t) - \hat{y}(t)) \right) \\ &\hat{y}(t) = \sum_{i=1}^{q} \mu_i(z(t)) C_i \hat{x}(t) \end{aligned} \tag{4}$$

where $G_i \in \mathbb{R}^{n.l}$ are the observer gains to be determined. The estimation error is defined as

$$e(t) = x(t) - \hat{x}(t) \tag{5}$$

The objective is to design an observer-based controller of model (1), in the form:

$$u(t) = -\sum_{i=1}^{q} \mu_i(z(t)) K_i \hat{x}(t) \tag{6}$$

where $K_i \in \mathbb{R}^{m.n}$ are the feedback gains to be determined. Tacking into account (1), (5) and (6), we obtain:

$$\begin{aligned} \dot{x}(t) = &\sum_{i=1}^{q}\sum_{j=1}^{q} \mu_i(z(t)) \mu_j(z(t)) \left(A_i + \Delta A_i - (B_i + \Delta B_i) K_j \right) x(t) + \\ &\sum_{i=1}^{q}\sum_{j=1}^{q} \mu_i(z(t)) \mu_j(z(t)) (B_i + \Delta B_i) K_j e(t) \end{aligned} \tag{7}$$

$$\begin{aligned} \dot{\hat{x}}(t) = &\sum_{i=1}^{q}\sum_{j=1}^{q} \mu_i(z(t)) \mu_j(z(t)) \left(A_i - B_i K_j \right) \hat{x}(t) + \\ &\sum_{i=1}^{q}\sum_{j=1}^{q} \mu_i(z(t)) \mu_j(z(t)) L_i C_j e(t) \end{aligned} \tag{8}$$

$$\begin{aligned} \dot{e}(t) = &\sum_{i=1}^{q}\sum_{j=1}^{q} \mu_i(z(t)) \mu_j(z(t)) \left(A_i - L_i C_j - \Delta B_i K_j \right) e(t) + \\ &\sum_{i=1}^{q}\sum_{j=1}^{q} \mu_i(z(t)) \mu_j(z(t)) \left(\Delta A_i - \Delta B_i K_j \right) x(t) \end{aligned} \tag{9}$$

The following result, based on the works [1][6], proposes stabilisation conditions designing an observer-based robust fuzzy control of uncertain fuzzy models (1) and gives some relaxation by introducing supplementary semi-definite matrices and tacking account the numbers of sub-models simultaneously activated (r).

Theorem 1: suppose that there exist matrices $Q > 0$, $P > 0, \Upsilon \geq 0, \Gamma \geq 0$, M_i, N_i and scalars ε_{ij} satisfying the following conditions $\forall \left(i, j\right) \in I_q^2, i \leq j$:

$$\bar{\Phi}_{ii} = \begin{pmatrix} \Phi_{ii} & * & * \\ E_{Ai} Q - E_{Bi} M_i & -\left(\varepsilon_{ii}^{-1} + 1\right)^{-1} I & * \\ D_i^T & 0 & -\left(\varepsilon_{ii} + 1\right)^{-1} I \end{pmatrix} < 0 \tag{10a}$$

$$\bar{\Psi}_{ij} = \begin{pmatrix} \Psi_{ij} & * & * & * & * \\ E_{Ai} Q - E_{Bi} M_j & -\left(\varepsilon_{ij}^{-1} + 1\right)^{-1} I & * & * & * \\ E_{1j} Q - E_{Bj} M_i & 0 & -\left(\varepsilon_{ij}^{-1} + 1\right)^{-1} I & * & * \\ D_i^T & 0 & 0 & -\varepsilon_{ij}^{-1} & * \\ D_j^T & 0 & 0 & 0 & -\varepsilon_{ij}^{-1} \end{pmatrix} < 0 \tag{10b}$$

$$\bar{T}_{ii} = \begin{pmatrix} T_{ii} & * & * \\ E_{Bi} K_i & -\left(\varepsilon_{ii}^{-1} + 1\right)^{-1} I & * \\ D_i^T P & 0 & -\left(\varepsilon_{ii} + 1\right)^{-1} I \end{pmatrix} < 0 \tag{11a}$$

$$\bar{\Theta}_{ij}=\begin{pmatrix}\Theta_{ij} & * & * & * & * \\ E_{Bi}K_j & -\left(\varepsilon_{ij}^{-1}+1\right)^{-1}I & * & * & * \\ E_{Bj}K_i & 0 & -\left(\varepsilon_{ij}^{-1}+1\right)^{-1}I & * & * \\ (PD_{ii})^T & 0 & 0 & -\left(\varepsilon_{ij}+1\right)^{-1}I & * \\ (PD_j)^T & 0 & 0 & 0 & -\left(\varepsilon_{ij}+1\right)^{-1}I\end{pmatrix}<0 \tag{11b}$$

with

$$\Phi_{ii}=QA_i^T+A_iQ-M_i^TB_i^T-B_iM_i+(r-1)\Upsilon+I \tag{12a}$$

$$\begin{aligned}\Psi_{ij}=&QA_i^T+A_iQ+QA_j^T+A_jQ-M_j^TB_i^T-B_iM_j-\\&M_i^TB_j^T-B_jM_i+D_iD_i^T+D_jD_j^T-2\Upsilon+2I\end{aligned} \tag{12b}$$

$$T_{ii}=A_i^TP+PA_i-C_i^TN_i^T-N_iC_i+K_i^TB_i^TB_iK_i+(r-1)\Gamma \tag{12c}$$

$$\begin{aligned}\Theta_{ij}=&A_i^TP+PA_i+A_j^TP+PA_j-C_i^TN_j^T-N_iC_j-C_j^TN_i^T-\\&N_jC_i+K_i^TB_j^TB_jK_i+K_j^TB_i^TB_iK_j-2\Gamma\end{aligned} \tag{12d}$$

and r is the number of sub-models simultaneously activated.
Then, fuzzy model (1) is globally asymptotically stable via control law (6) based on estimated state (4) with

$$K_i=M_iQ^{-1} \tag{13a}$$

$$G_i=P^{-1}N_i \tag{13b}$$

Proof: Considering the Lyapunov function candidate:

$$V\left(x(t),e(t)\right)=V_1\left(x(t)\right)+V_2\left(e(t)\right) \tag{14}$$

with

$$\begin{aligned}&V_1\left(x(t)\right)=x(t)^TP_1x(t),\quad P_1=Q^{-1}>0;\\&V_2\left(e(t)\right)=e(t)^TPe(t),\quad P>0\end{aligned} \tag{15}$$

The time-derivative of $V_1\left(x(t)\right)$ along the trajectory of (7) is

$$\begin{aligned}\dot{V}_1\left(x(t)\right)=&\sum_{i=1}^{q}\mu_i\left(z(t)\right)^2x(t)^T\left(H_{ii}^TP_1+P_1H_{ii}\right)x(t)+\\&2\sum_{i<j}^{q}\mu_i\left(z(t)\right)\mu_j\left(z(t)\right)x(t)^T\left(\left(\frac{H_{ij}+H_{ji}}{2}\right)^TP_1+P_1\left(\frac{H_{ij}+H_{ji}}{2}\right)\right)x(t)\\&+2\sum_{i=1}^{q}\sum_{j=1}^{q}\mu_i\left(z(t)\right)\mu_j\left(z(t)\right)x(t)^TP_1\left(B_i+\Delta B_i\right)K_je(t)\end{aligned} \tag{16}$$

With

$$H_{ij}=A_i+\Delta A_i-\left(B_i+\Delta B_i\right)K_j \tag{17}$$

Using lemma 1 and assumption 1, the following inequality holds:

$$\begin{aligned}2x(t)^TP_1\left(B_i+\Delta B_i\right)K_je(t)\le\ & x(t)^T\left(P_1^2+P_1D_iD_i^TP_1\right)x(t)+\\&e(t)^TK_j^T\left(B_iB_i^T+E_{Bi}E_{Bi}^T\right)K_je(t)\end{aligned} \tag{18}$$

Thus, the first and the second sum of (18) are bounded respectively as follows:

$$\begin{aligned}&\sum_{i=1}^{q}\sum_{j=1}^{q}\mu_i\left(z(t)\right)\mu_j\left(z(t)\right)x(t)^T\left(P_1^2+P_1D_iD_i^TP_1\right)x(t)\le\\&\sum_{i=1}^{q}\mu_i\left(z(t)\right)^2x(t)^T\left(P_1^2+P_1D_iD_i^TP\right)x(t)+\\&2\sum_{i<j}^{q}\mu_i\left(z(t)\right)\mu_j\left(z(t)\right)x(t)^T\left(\frac{2P_1^2+P_1D_iD_i^TP_1+P_1D_jD_j^TP_1}{2}\right)x(t)\end{aligned} \tag{19}$$

$$\begin{aligned}&\sum_{i=1}^{q}\sum_{j=1}^{q}\mu_i\left(z(t)\right)\mu_j\left(z(t)\right)e(t)^TK_i^T\left(B_iB_i^T+E_{Bi}E_{Bi}^T\right)K_ie(t)\le\\&\sum_{i=1}^{q}\mu_i\left(z(t)\right)^2e(t)^TK_i^T\left(B_iB_i^T+E_{Bi}E_{Bi}^T\right)K_ie(t)+\\&2\sum_{i<j}^{q}\mu_i\left(z(t)\right)\mu_j\left(z(t)\right)e(t)^T\left(\frac{K_j^T\left(B_iB_i^T+E_{Bi}E_{Bi}^T\right)K_j+K_i^T\left(B_jB_j^T+E_{Bj}E_{Bj}^T\right)K_i}{2}\right)e(t)\end{aligned} \tag{20}$$

Taking account of (19) and (20) in (16), the time-derivative of $V_1\left(x(t)\right)$ is as follows:

$$\begin{aligned}\dot{V}_1\left(x(t)\right)\le&\sum_{i=1}^{q}\mu_i\left(z(t)\right)^2x(t)^T\left(H_{ii}^TP_1+P_1H_{ii}+P_1^2+P_1D_iD_i^TP_1\right)x(t)\\&+2\sum_{i<j}^{q}\mu_i\left(z(t)\right)\mu_j\left(z(t)\right)x(t)^T\begin{pmatrix}\left(\frac{H_{ij}+H_{ji}}{2}\right)^TP_1+P_1\left(\frac{H_{ij}+H_{ji}}{2}\right)\\+\frac{2P_1^2+P_1D_iD_i^TP_1+P_1D_jD_j^TP_1}{2}\end{pmatrix}x(t)\\&+\sum_{i=1}^{q}\mu_i\left(z(t)\right)^2e(t)^TK_i^T\left(B_iB_i^T+E_{Bi}E_{Bi}^T\right)K_ie(t)\\&+2\sum_{i<j}^{q}\mu_i\left(z(t)\right)\mu_j\left(z(t)\right)e(t)^T\begin{pmatrix}\frac{K_j^T\left(B_iB_i^T+E_{Bi}E_{Bi}^T\right)K_j}{2}+\\\frac{K_i^T\left(B_jB_j^T+E_{Bj}E_{Bj}^T\right)K_i}{2}\end{pmatrix}e(t)\end{aligned} \tag{21}$$

The time-derivative of $V_2\left(e(t)\right)$ along the trajectory of (9) is

$$\dot{V}_2(e(t))=\sum_{i=1}^{q}\mu_i(z(t))^2 e(t)^T\left(\Sigma_{ii}^T P+P\Sigma_{ii}\right)e(t)+$$
$$2\sum_{i<j}^{q}\mu_i(z(t))\mu_j(z(t))e(t)^T\left(\left(\frac{\Sigma_{ij}+\Sigma_{ji}}{2}\right)^T P+P\left(\frac{\Sigma_{ij}+\Sigma_{ji}}{2}\right)\right)e(t)$$
$$+2\sum_{i=1}^{q}\sum_{j=1}^{q}\mu_i(z(t))\mu_j(z(t))e(t)^T P\left(\Delta A_i-\Delta B_i K_j\right)x(t) \tag{22}$$

with

$$\Sigma_{ij}=A_i-G_iC_j+\Delta B_iK_j \tag{23}$$

Using lemma 1 and assumption 1, the following inequality holds:

$$2e(t)^T P\left(\Delta A_i-\Delta B_iK_j\right)x(t)\le e(t)^T\left(PD_iD_i^TP\right)e(t)+ x(t)^T\left(\left(E_{Ai}-E_{Bi}K_j\right)^T\left(E_{Ai}-E_{Bi}K_j\right)\right)x(t) \tag{24}$$

The same technique is used to bound (24) as the one used in (18). The derivative-time of $V_2(e(t))$ is then as follows:

$$\dot{V}_2(e(t))\le\sum_{i=1}^{q}\mu_i(z(t))^2 e(t)^T\left(\Sigma_{ii}^T P+P\Sigma_{ii}+PD_iD_i^TP\right)e(t)+$$
$$2\sum_{i<j}^{q}\mu_i(z(t))\mu_j(z(t))e(t)^T\left(\begin{array}{c}\left(\frac{\Sigma_{ij}+\Sigma_{ji}}{2}\right)^T P+P\left(\frac{\Sigma_{ij}+\Sigma_{ji}}{2}\right)\\+\frac{PD_iD_i^TP+PD_jD_j^TP}{2}\end{array}\right)e(t)$$
$$+\sum_{i=1}^{q}\mu_i(z(t))^2 x(t)^T L_{ii}x(t)+$$
$$2\sum_{i=1}^{n}\sum_{j=1}^{n}\mu_i(z(t))\mu_j(z(t))x(t)^T\left(\frac{L_{ij}+L_{ji}}{2}\right)x(t) \tag{25a}$$

With

$$L_{ij}=\left(E_{Ai}-E_{Bi}K_j\right)^T\left(E_{Ai}-E_{Bi}K_j\right) \tag{25b}$$

Finally, by combining (21) and (25), the time derivative of $V(x(t),e(t))$ can be obtained.

The negativity of $\dot{V}(x,e)$, $\forall\left(x(t)^T,e(t)^T\right)\neq 0$, is ensured by exploiting the basic result of [7] and lemma 1. ■

We note that the proposed result is less conservative than these proposed in [6] where the derived synthesis conditions are based on the negativity of each terms. Moreover the derived conditions introduce supplementary parameters (Γ,Υ,r) which lead to more relaxation.

In the objective of advantage of less conservatism, we propose in the following section to exploit the result of [2].

IV. Relaxed Quadratic Output Stabilisation

In this section a new stability conditions are obtained by relaxing the stability conditions derived in the above section. Theorem 2 is obtained by extending the result of [2] to the case of uncertain fuzzy models. The reduction of conservatism is obtained by introducing multiple matrices, instead of single matrix [7], where the off-diagonal matrices are allowed to be non-symmetric.

Theorem 2: suppose that there exist matrices $Q>0$, $P>0$, $M_i, N_i, \Upsilon_{ij}, \Gamma_{ij}$ and scalars ε_{ij} satisfying the following conditions $\forall (i,j)\in I_q^2, i\le j$:

$$\bar{\Phi}_{ii}<0 \tag{26a}$$
$$\bar{\Psi}_{ij}<0 \tag{26b}$$
$$\bar{T}_{ii}<0 \tag{27a}$$
$$\bar{\Theta}_{ij}<0 \tag{27b}$$

$$\begin{pmatrix}\Upsilon_{11} & \Upsilon_{12} & \cdots & \Upsilon_{1q}\\ \Upsilon_{12}^T & \Upsilon_{22} & \cdots & \Upsilon_{2q}\\ \vdots & \vdots & \ddots & \vdots\\ \Upsilon_{1q}^T & \Upsilon_{2q}^T & \cdots & \Upsilon_{qq}\end{pmatrix}>0 \tag{28}$$

$$\begin{pmatrix}\Gamma_{11} & \Gamma_{12} & \cdots & \Gamma_{1q}\\ \Gamma_{12}^T & \Gamma_{22} & \cdots & \Gamma_{2q}\\ \vdots & \vdots & \ddots & \vdots\\ \Gamma_{1q}^T & \Gamma_{2q}^T & \cdots & \Gamma_{qq}\end{pmatrix}>0 \tag{29}$$

where $\bar{\Phi}_{ii}$, $\bar{\Psi}_{ij}$, $\bar{T}_{ii}$, $\bar{\Theta}_{ij}$ are defined respectively in (10a), (10b), (11a) and (11b) with

$$\Phi_{ii}=QA_i^T+A_iQ-M_i^TB_i^T-B_iM_i+\Upsilon_{ii}+I \tag{30a}$$
$$\Psi_{ij}=QA_i^T+A_iQ+QA_j^T+A_jQ-M_j^TB_i^T-B_iM_j-M_i^TB_j^T- B_jM_i+\Upsilon_{ij}+\Upsilon_{ij}^T+D_iD_i^T+D_jD_j^T+2I \tag{30b}$$
$$T_{ii}=A_i^TP+PA_i-C_i^TN_i^T-N_iC_i+K_i^TB_i^TB_iK_i+\Gamma_{ii} \tag{30c}$$
$$\Theta_{ij}=A_i^TP+PA_i+A_j^TP+PA_j-C_i^TN_j^T-N_iC_j-C_j^TN_i^T- N_jC_i+K_i^TB_j^TB_jK_i+K_j^TB_i^TB_iK_j+\Gamma_{ij}+\Gamma_{ij}^T \tag{30d}$$

Then, fuzzy model (1) is globally asymptotically stable via control law (6) based on estimated state (4) with

$$K_i=M_iQ^{-1} \tag{31a}$$

$$G_i = P^{-1}N_i \tag{31b}$$

Proof: To ensure the negativity of $\dot{V}\left(x(t),e(t)\right), \forall\left(x(t)^T, e(t)^T\right) \neq 0$, we consider the result [2]. The detail of proof is omitted due to lack of space. ■

Remarks and discussion:

• Comparing theorem 1 and theorem 2, we note that second result allows introducing multiple matrices and non-symmetrical off-diagonal matrices instead of single symmetric semi-definite positive matrix. Hence there are more variables in the second result than in the first one which leads to more great freedom in guaranteeing the stability of the output robust control of fuzzy systems. The same remark can be stated comparing the proposed constraints with previous results ([1][6] and references therein).

• We note that the obtained conditions are in the BMI (bilinear matrix inequality) forms which are impossible to solve directly by LMI tool (LMI toolbox of Matlab software for example). However to be able to use the LMI tool, we propose to solve sequentially synthesis conditions (26)-(28):

-First, we solve the LMI (26) and (28) in the variables Q, Υ_{ij} and M_i,

-Once gains K_i have been calculated from (31a), conditions (27) and (29) become linear in P, Γ_{ij} and N_i, and can be easily resolved using the LMI tool to determine gains L_i from (31b).

V. SIMULATION

Considering the Takagi-Sugeno model of four-wheel drive vehicles represented by two sub-models [12][14]:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{2}\mu_i\left(z(t)\right)\left((A_i + \Delta A_i)x(t)\right) + \\ \quad \sum_{i=1}^{2}\mu_i\left(z(t)\right)\left((B_{fi} + \Delta B_{fi})\delta_f(t) + (B_{ri} + \Delta B_{ri})\delta_r(t)\right) \\ y(t) = Cx(t) \end{cases} \tag{32}$$

Where

$$A_1 = \begin{pmatrix} -4.7964 & -0.988 \\ 2.3982 & -3.7532 \end{pmatrix}, A_2 = \begin{pmatrix} -0.636 & -1.0054 \\ -1.0757 & -0.4907 \end{pmatrix} \tag{33a}$$

$$B_{f1} = \begin{pmatrix} 2.3982 \\ 28.7784 \end{pmatrix}, B_{f2} = \begin{pmatrix} 0.3738 \\ 4.4852 \end{pmatrix} \tag{33b}$$

$$B_{r1} = \begin{pmatrix} 2.3982 \\ -31.1766 \end{pmatrix}, B_{r2} = \begin{pmatrix} 0.2623 \\ -3.4095 \end{pmatrix} \tag{33c}$$

$$C = \begin{pmatrix} 0 & 1 \end{pmatrix} \tag{33d}$$

$$\mu_1(r) = 1 - \mu_2(r) = \frac{1}{\left(1 + abs\left(\frac{|r| - c_1}{a_1}\right)\right)^{2b_1}} \tag{34}$$

and $x(t) = \begin{pmatrix} \beta & r \end{pmatrix}^T$ denotes respectively the side slip angle (β) and the yaw velocity (r), δ_f is the front steer angle and δ_r is the rear steer angle.

The parameters uncertainties are defined as follows:

$$D_1 = D_2 = \alpha\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \alpha \in \mathbb{R}^+ \tag{35}$$

$$E_{A1} = E_{A2} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, E_{B1} = E_{B2} = \begin{pmatrix} 1.2 \\ 1.2 \end{pmatrix} \tag{36}$$

It is well known that the stability domain of vehicle is reduced when the speed and/or the steering angle increase, which is source of instability. So to improve the performances of the vehicle, active control systems must be developed and installed. For this, a control system operating on the rear wheel-axle of the vehicle to improve its stability is proposed.

Thus, the objective is to design a robust rear control ($\delta_r = -\sum_{i=1}^{2}\mu_i(z(t))K_i\hat{x}(t)$) based on observer of the closed-loop uncertain fuzzy models (32) when the front steering angle $\left(\delta_f\right)$ is as given in figure 1.

The conditions proposed in [6] fail to design the observer and the control law $\left(\delta_r\right)$ guaranteeing the stability of the closed loop fuzzy model (32) when $\alpha > 0.2$ whereas these derived in theorem 1 fails to give solution when $\alpha > 0.3$. However until $\alpha = 0.35$, the derived constraints of theorem 2 remain feasible. Thus the resolution of LMI conditions (26)-(28) allow to design the controller gains:

$$K_1 = \begin{pmatrix} 1.3359 & -3.4860 \end{pmatrix}, K_2 = \begin{pmatrix} 1.3367 & -2.7140 \end{pmatrix}$$
$$Q = \begin{pmatrix} 0.8325 & 0.2741 \\ 0.2471 & 0.2680 \end{pmatrix} \tag{37}$$

Then the resolution of the constraints (27)-(29) becoming linear in P, Γ_{ij} and N_i make it possible to design the observer gains:

$$L_1 = (\text{-}8.8395\ \ 163.7199)^T, L_2 = (\text{-}49.1678\ \ 165.6981)^T$$

$$P = 10^3 \begin{pmatrix} 0.2523 & 0.0539 \\ 0.0539 & 4.2730 \end{pmatrix} \quad (38)$$

The simulation shows that the designed controller based observer for the fuzzy model (32) is robust against norm-bounded parametric uncertainties (figure 1).

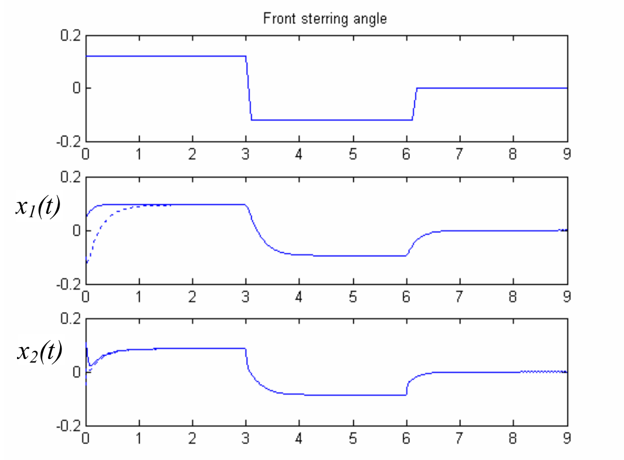

Figure 1. Closed-loop response with initial conditions $\left(x(0)^T, \hat{x}(0)^T\right) = (0.05; 0.1; \text{-}0.1; \text{-}0.05)$

VI. CONCLUSION

This paper presents new stability conditions for robust asymptotic output stabilization. The derived results are based on the use of the quadratic Lyapunov function and solved using LMI optimization techniques. The proposed stability conditions are proved to be less conservative than previous results. An example is given to illustrate the utility of the proposed results.

VII. REFERENCES

[1] H. J. Lee, J. B. Park, G. Chen, “Robust fuzzy control of nonlinear systems with parametric uncertainties”, IEEE Trans. on Fuzzy Systems, 9(2), pp. 369379, 2001.

[2] Liu Xiaodiong, Zhang Qingling, “New approach to Hinf controller designs based on observers for T-S fuzzy systems via LMI”, Automatica, 39, pp. 1571-1582, 2003.

[3] S.C Cao, N. W. Rees, G. Feng. “Hinf control of uncertain fuzzy continuous-time systems”, Fuzzy sets and systems, 115, pp. 171-190, 2000.

[4] X. J. Ma, Z. Q Sun “Analysis and design of fuzzy controller and fuzzy observer”, IEEE Trans. Fuzzy Systems, 6(1), pp. 41-51, 1998

[5] K. Kiriadis, “fuzzy model-based control of complex plants”, IEEE Trans. Fuzzy systems, 6(4), pp 517-529, 1998.

[6] M. Chadli, A. Elhajjai “comment on ‘S. Tong, H-H. Li, Observer-based robust fuzzy control of nonlinear systems with parametric uncertainties’, Fuzzy Sets and Systems, 131, pp 165-184, 2002”, Fuzzy sets and systems, *accepted*.

[7] K. Tanaka, T. Ikeda, O. Wang, “Fuzzy regulator and fuzzy observer : relaxed stability conditions and LMI based design”, IEEE Trans. on Fuzzy Systems, 6 (2), pp. 250-256, 1998.

[8] M. Chadli, D. Maquin, J. Ragot "Multiple observers for discrete-time multiple model", IFAC Congres, Safeprocess, Washington, June 2003.

[9] M. Chadli, D. Maquin, J. Ragot "An LMI formulation for output feedback stabilisation in multiple model approach ". IEEE 41th Conference on Decision Control, USA, 10-13 Dec. 2002

[10] M. Takagi and M. Sugeno, “Fuzzy identification of systems and its application to moddeling and control”, IEEE Trans. on Systems Man and Cybernetics-part C, 15 (1), pp. 116-132, 1985.

[11] M. Chadli, D. Maquin, J. Ragot, “Output stabilization in multiple model approach”, IEEE Control Conference Application CCA’02, Sep. 2002.

[12] A. Hajjaji, A. Ciocan, D. Hamad, “Four Wheel steering control by fuzzy approach”, Journal of intelligent and robotics systems Vol 41(2-3) pp.141-156, 2004.

[13] A. Akhenak, M. Chadli, D. Maquin and J. Ragot, “State estimation of uncertain multiple model with unknown inputs”, IEEE 43th Conference on Decision and Control, Bahamas, Dec. 14-17, 2004.

[14] M. Chadli, M. Oudghiri, A. Hajjaji “Commande robuste basée sur observateur de modèle flou incertain : application au véhicule à quatre roues ”, JDMACS-JNMACS, Lyon, France, 5-7 Sep. 2005.

[15] P. Apkarian., H. D. Tuan “Robust control via concave minimisation: local and global algorithms”. IEEE Trans. on Automatic Control, vol. 45, no 2, pp. 299-305, 2000.

[16] S. Boyd. et al. Linear matrix inequalities in systems and control theory. Philadelphia, PA: SIAM, 1994.

[17] E. Kim, H. Lee, “New approaches to relaxed quadratic stability condition of fuzzy control systems“ IEEE Trans. on Fuzzy Systems, vol. 8, no. 5, pp. 523-534, 2000.