

# Adaptive Vehicle Following Control System with Variable Time Headways

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**Abstract**—In this paper, we design a vehicle following control system with variable time headways, using adaptive control design methodology. It is shown that the designed vehicle following control system guarantees closed-loop system stability, and that it regulates the speed and separation errors towards zero when the lead vehicle is at a constant speed. Simulation results are presented to demonstrate the performance of the proposed vehicle following control system when applied to a validated nonlinear vehicle model.

## I. INTRODUCTION

**D**URING last decade, considerable progress has been made in the area of Automated Highway Systems (AHS). The design of Adaptive Cruise Control (ACC) system serves as a preliminary step towards AHS, which allows one vehicle to cruise at a constant speed in the absence of obstacles in the longitudinal direction or to follow a preceding vehicle in the same lane automatically while maintaining a desired intervehicle spacing. Many efforts have been made to design ACC systems for both passenger vehicles and commercial trucks [1]-[3], and to study their impacts on highway traffic [3]-[9].

The intervehicle spacing policy between traveling vehicles in the same lane, or equivalently the time headway used by ACC systems, is a critical parameter of an AHS system. It should be chosen as small as possible to increase the highway capacity but never to violate the safety constraint. Several vehicle following controllers have been proposed and tested on a real vehicles with a constant time headway in [1]. Some studies indicate using variable time headways in the ACC systems may lead to better impacts on highway traffic [2], [6]-[9]. In [6] and [7], the vehicle following controller with variable time headways is designed using feedback linearization methodology based on a simplified second-order vehicle model. Though the design procedure is straightforward, this controller may generate high control effort, which is not desired due to the constraints for driving comfort. Furthermore, it is not applicable for certain time headways such as the one given in [2], as we will show in section II-D. In this paper, we design vehicle following controllers that can provide desired

stability properties for all of the variable time headways that have been considered in [2], [6]-[9] as well as the constant time headway. Simulation results are presented to demonstrate the validation of the proposed controllers.

## II. VEHICLE FOLLOWING CONTROL DESIGN

### A. Simplified Vehicle Model

The vehicle model used for simulation is taken from [1], which is built based on physical laws and has been experimentally validated. This model can be characterized by a set of differential equations, algebraic relations and look-up tables. Though the full nonlinear model is complicated, it can be simplified as a first-order system for control design purpose as [1], [2]

$$\dot{v} = -a(v - v_d) + b(u - u_d) + d \quad (1)$$

where  $v$  is the longitudinal speed,  $u$  is the throttle/brake command,  $v_d$  is the desired steady state speed,  $u_d$  is the corresponding steady state fuel command,  $d$  is the modeling uncertainty, and  $a$  and  $b$  are positive constant parameters that depend on the operating point. For a given vehicle, the relation between  $v_d$  and  $u_d$  can be described by a look-up table, or by a 1-1 mapping continuous function

$$v_d = f_u(u_d) \quad (2)$$

In our analysis, we assume  $f_u$  is differentiable and the derivative is bounded. In the vehicle following mode, the desired speed for the following vehicle is  $v_f$ , the speed of the preceding vehicle. Hence, the simplified vehicle model used for vehicle following control design is described by (1) and (2), with  $v_d$  replaced by  $v_f$ . It is reasonable to assume that  $d$ ,  $v_f$  and their derivatives are all bounded.

### B. Control Objective and Constraints

In the vehicle following mode, The control objective is to regulate the vehicle speed  $v$  to track the speed of the preceding vehicle  $v_f$  while maintaining the intervehicle spacing  $x_r$  as close to the desired spacing  $s_d$  as possible, as shown in Fig. 1.

With the time headway policy, the desired intervehicle spacing is given by

$$s_d = s_0 + hv \quad (3)$$

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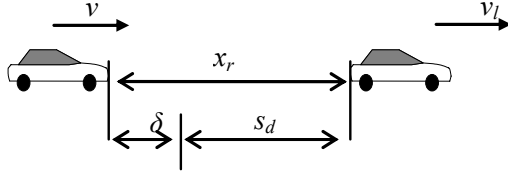


Fig. 1. Diagram of the vehicle following mode

where  $s_0$  is a fixed safety intervehicle spacing to avoid vehicle contact at low or zero speeds,  $v$  is the speed of the following vehicle, and  $h$  is the time headway. The control objective in the vehicle following mode can be expressed as

$$v_r \rightarrow 0, \delta \rightarrow 0 \text{ as } t \rightarrow \infty \quad (4)$$

where  $v_r = v_l - v$  is the speed error and  $\delta = x_r - s_d$  is the separation error. The following two constraints should also be satisfied:

- C1.  $a_{\min} \leq \dot{v} \leq a_{\max}$  where  $a_{\min}$  and  $a_{\max}$  are specified.
- C2. The absolute value of jerk defined as  $|\ddot{v}|$  should be small.

The above constraints are the results of driving comfort concerns and are established using human factor considerations [1].

### C. Variable Time Headways

Most of the previous studies for vehicle following control consider the constant spacing rule ( $h=0$ ) [10] and the constant time headway spacing rule ( $h = \text{nonzero constant}$ ) [1], [5]. Some studies indicate using variable time headways in the ACC systems may lead to better impacts on highway traffic. In [8], the spacing policy is chosen as

$$s_d = s_0 + h_1 v + h_2 v^2 \quad (5)$$

where  $h_1$  and  $h_2$  are two positive constants. The time headway incorporated in (5) can be expressed as

$$h = h_1 + h_2 v \quad (6)$$

This time headway increases with  $v$ . In practice, however, vehicle speed cannot exceed certain limit  $v_{\max}$ . Hence the time headway in (6) in fact is the same as

$$h = \begin{cases} h_1 + h_2 v, & \text{if } v < v_{\max} \\ h_1 + h_2 v_{\max}, & \text{otherwise} \end{cases} \quad (7)$$

In [6] and [7], the time headway is chosen based on the hypothesis proposed by Greenshields [11], and it can be written as

$$h = \frac{1}{k_{jam}(v_{free} - v)} \quad (8)$$

where  $k_{jam}$  is the traffic density corresponding to the jam conditions and  $v_{free}$  is the free speed when the traffic density is low. The time headway in (8) is expressed differently from that in [7] since the spacing considered in [7] incorporates the vehicle length. In [2], the time headway  $h$  proposed for tightly vehicle following control is given as

$$h = \text{sat}(h_0 - c_h v_r) \quad (9)$$

where  $h_0$  and  $c_h$  are positive constants to be designed, the saturation function  $\text{sat}(\bullet)$  has an upper bound 1 and a lower bound 0. Though  $\text{sat}(\bullet)$  is not analytical when  $v_r$  is equal to  $h_0/c_h$  or  $(h_0-1)/c_h$ , slight modifications will change  $h$  to a smooth function of  $v$  and  $v_l$ .

In this paper, we consider a general time headway as a smooth function of  $v$  and  $v_l$ , which has bounded partial derivatives. Let us define

$$H \triangleq \frac{\partial}{\partial v} s_d(v, v_l) \quad (10-a)$$

$$H_l \triangleq \frac{\partial}{\partial v_l} s_d(v, v_l) \quad (10-b)$$

This general time headway has the properties that  $H \geq 0$  and  $H$  and  $H_l$  are bounded. With the practical consideration as given in (7), and the proper modification for the smoothness of  $h$  in (9), we can see the general time headway includes all the time headways mentioned above. In particular,  $H$  and  $H_l$  are zeros for the constant spacing policy. For the constant time headway spacing rule,  $H$  is equal to the time headway  $h$  and  $H_l$  is zero.

### D. Control Design

The simplified longitudinal model described by (1) and (2) is used for the vehicle following control design. In [6] and [7], a vehicle following controller using the variable time headway in (8) was proposed based on feedback linearization. The desired closed-loop system is described by

$$\dot{\delta} = -k\delta \quad (11)$$

where  $k$  is a positive constant. If all the parameters in the simplified model are known, the vehicle following controller

$$u = \frac{1}{bH}(v_r + k\delta) - \frac{a}{b}v_r + u_d - \frac{d}{b} - \frac{H_l}{bH}\dot{v}_l \quad (12)$$

can be used to achieve the desired closed-loop system in (11). The design procedure is straightforward, but the controller in (12) has some obvious flaws. The control gains in (12) are high when  $H$  is small, which may not be desired

due to the control constraints. Furthermore, this controller cannot be implemented when  $H$  is equal to zero. Obviously  $H$  is zero in the constant spacing rule and may also be zero when the nonlinear time headway given in (9) is employed. The vehicle following controllers proposed in this paper guarantee global stability for any variable time headway that has bounded  $H$  and  $H_l$  with  $H \geq 0$ , and the control parameters can be properly chosen so that high control gains can be avoided. We first consider the simple situation in which the parameters  $a$ ,  $b$  and  $d$  in (1) are known and design a fixed-gain controller to solve the vehicle following problem. In the practical situation, where the parameters are unknown and may vary with time, we propose adaptive vehicle following controllers to accomplish the task.

**Lemma 1:** Consider the system in (1) and (2), with the following controller

$$u = k_1^* v_r + k_2^* \Delta(\delta, t) + k_3^* \quad (13)$$

where  $k_1^* = (a_m - a)/b$ ,  $k_2^* = a_m/b$ ,  $k_3^* = u_d - d/b$ ,  $\Delta$  is a design time varying function of  $\delta$  satisfying

$$k_l \delta \leq \Delta(\delta, t) \leq k_u \delta \quad (14)$$

and  $a_m$ ,  $k_l$  and  $k_u$  are positive design constants. All the signals in the closed-loop system are bounded if  $a_m$ ,  $k_l$  and  $k_u$  are designed such that there exists a positive constant  $p_1$  satisfying

$$a_m p_1 > 1 \quad (15a)$$

$$a_m p_1^2 (k_u - k_l)^2 - 4k_u (a_m p_1 - 1) < 0 \quad (15b)$$

$$a_m + a_m k_l p_1 - k_u \geq 0 \quad (15c)$$

$$\frac{4p_1 k_l}{a_m + a_m k_l p_1 - k_l} > \sup H \quad (15d)$$

where  $\sup H$  is the supremum of  $H$ . Furthermore, if  $v_l$  is a constant, then the control objective in (4) is achieved, i.e.  $v_r, \delta \rightarrow 0$  as  $t \rightarrow \infty$ .

**Proof:** Let  $\Delta(\delta, t) = k\delta$ , where  $k$  is a time varying function of  $\delta$  and satisfies  $k_l \leq k \leq k_u$ . Using (13) in (1) and (2), the closed-loop system is

$$\dot{v} = a_m (v_r + k\delta) \quad (17)$$

Denote  $x_1 = v_r$  and  $x_2 = \delta$ , then

$$\begin{cases} \dot{x}_1 = -a_m (x_1 + kx_2) + u_1 \\ \dot{x}_2 = (1 - a_m H)x_1 - a_m k H x_2 - H_l u_1 \end{cases} \quad (18)$$

where  $u_1 = \dot{v}_l$ ,  $H$  and  $H_l$  are bounded with  $H \geq 0$ . Consider

the following candidate Lyapunov function

$$V_a = \frac{1}{2} x^T P x \quad (19)$$

where  $P = \begin{bmatrix} p_1 & 1 \\ 1 & p_2 \end{bmatrix}$  a positive definite matrix, and  $p_1$  and  $p_2$  are positive constants. Hence,

$$\begin{aligned} \dot{V}_a &= p_1 x_1 \dot{x}_1 + p_2 x_2 \dot{x}_2 + x_1 \dot{x}_2 + \dot{x}_1 x_2 \\ &= p_1 x_1 [-a_m (x_1 + kx_2) + u_1] \\ &\quad + p_2 x_2 [(1 - a_m H)x_1 - a_m k H x_2 - H_l u_1] \\ &\quad + x_1 [(1 - a_m H)x_1 - a_m k H x_2 - H_l u_1] \\ &\quad + x_2 [-a_m (x_1 + kx_2) + u_1] \\ &= -x_1^2 (a_m p_1 - 1 + a_m H) - x_2^2 (a_m k + a_m k H p_2) \\ &\quad - x_1 x_2 (a_m k p_1 + a_m - p_2 + a_m H p_2 + a_m k H) \\ &\quad + u_1 (p_1 x_1 - H_l x_1 + x_2 - H_l p_2 x_2) \end{aligned} \quad (20)$$

We choose

$$p_2 = a_m k_l p_1 + a_m \quad (21)$$

When (15a) holds, the coefficient of  $x_1^2$  in (20) is negative and (21) guarantees that  $P$  is positive definite. With (21), (20) can be rewritten as

$$\begin{aligned} \dot{V}_a &= -x_1^2 (a_m p_1 - 1 + a_m H) - x_2^2 [a_m k + a_m^2 k H (k_l p_1 + 1)] \\ &\quad - x_1 x_2 (a_m k p_1 - a_m k_l p_1 + a_m^2 H + a_m^2 k_l H p_1 + a_m k H) \\ &\quad + u_1 (w_1 x_1 + w_2 x_2) \end{aligned} \quad (22)$$

where  $w_1 = p_1 - H_l$  and  $w_2 = 1 - H_l (a_m k_l p_1 + a_m)$ . Suppose  $u_1$  is zero. Then  $\dot{V}_a$  is negative definite if

$$\begin{aligned} &(a_m k p_1 - a_m k_l p_1 + a_m^2 H + a_m^2 k_l H p_1 + a_m k H)^2 \\ &< 4(a_m p_1 - 1 + a_m H) [a_m k + a_m^2 k H (k_l p_1 + 1)] \end{aligned} \quad (23)$$

always holds. (23) can be rewritten as

$$\begin{aligned} &a_m (a_m + a_m k_l p_1 - k)^2 H^2 \\ &- 2a_m p_1 (k + k_l) (a_m + a_m k_l p_1 - k) H \\ &+ a_m p_1^2 (k - k_l)^2 - 4k (a_m p_1 - 1) < 0 \end{aligned} \quad (24)$$

One necessary condition for (24) to be true for all  $H \geq 0$  is that

$$a_m p_1^2 (k - k_l)^2 - 4k (a_m p_1 - 1) < 0 \quad (25)$$

for all  $k \in [k_l, k_u]$ . This condition is equivalent to that (15b) is true. When (15a) and (15b) hold, a sufficient condition for (24) to hold is that (15c) and (15d) hold. Now we have

shown that if (15a) - (15d) hold and  $u_1$  is zero then  $\dot{V}_a$  is negative definite. Since  $w_1$  and  $w_2$  in (22) are bounded, it is easy to show that  $V_a$  is bounded, and then all the signals in the closed-loop system are bounded [12]. Furthermore, if  $v_l$  is a constant, i.e.  $u_1$  is zero, it can be verified that  $x_1, x_2 \in L_2 \cap L_\infty$ ,  $\dot{x}_1, \dot{x}_2 \in L_\infty$ . It follows from Barbalat's Lemma [12] that  $x_1, x_2 \rightarrow 0$  as  $t \rightarrow \infty$ , i.e., the control objective in (4) is achieved.  $\square$

The controller in (13) cannot be implemented in practice because  $a$ ,  $b$  and  $d$  are unknown parameters which may change with vehicle speed and other conditions. However, we can estimate  $k_i^*$  ( $i=1,2,3$ ) on-line and use their estimate  $k_i$  in the control law. In the next Lemma, we show that with proper update laws for  $k_i$ , the control law (13) where the  $k_i^*$  ( $i=1,2,3$ ) are replaced with their on-line estimates stabilizes the closed-loop system and meets the control objective when  $v_l$  and  $d$  are constants.

**Lemma 2:** Consider the system in (1) and (2), with the control law

$$u = k_1 v_r + k_2 \Delta(\delta, t) + k_3 \quad (26)$$

where  $k_i$  is the estimate of  $k_i^*$  (defined in Lemma 1) with initial condition  $k_{i0}$  ( $i=1,2,3$ ), generated by the adaptive laws

$$\begin{cases} \dot{k}_1 = \text{Proj}\{\gamma_1 x_1 [(p_1 x_1 + x_2) + (x_1 + a_m k_l p_1 x_2 + a_m x_2) H]\} \\ \dot{k}_2 = \text{Proj}\{\gamma_2 k x_2 [(p_1 x_1 + x_2) + (x_1 + a_m k_l p_1 x_2 + a_m x_2) H]\} \\ \dot{k}_3 = \text{Proj}\{\gamma_3 [(p_1 x_1 + x_2) + (x_1 + a_m k_l p_1 x_2 + a_m x_2) H]\} \end{cases} \quad (27)$$

where  $a_m, p_1, \gamma_1, \gamma_2$ , and  $\gamma_3$  are positive design parameters,  $\text{Proj}\{\bullet\}$  is the projection function keeping  $k_i$  within the intervals  $[k_{il}, k_{iu}]$  ( $i=1,2,3$ ).  $k_{il}$  and  $k_{iu}$  are chosen such that  $k_i^* \in [k_{il}, k_{iu}]$ . If we choose the parameters  $a_m, k_l, k_u$  and  $p_1$  such that (15a) - (15d) hold, then all the signals in the closed-loop system are bounded. Furthermore, if  $v_l$  and  $d$  are constants, then the control objective in (4) is achieved, i.e.  $v_r, \delta \rightarrow 0$  as  $t \rightarrow \infty$ .

**Proof:** With the proposed control law, the closed-loop system becomes

$$\dot{v} = a_m (v_r + k\delta) + b\tilde{k}_1 v_r + b\tilde{k}_2 \Delta(\delta, t) + b\tilde{k}_3 \quad (28)$$

where  $\tilde{k}_i = k_i - k_i^*$  ( $i=1,2,3$ ). We rewrite  $\Delta(\delta, t)$  as  $k\delta$  and denote  $x_1 = v_r$  and  $x_2 = \delta$ . Now we have

$$\begin{cases} \dot{x}_1 = -a_m (x_1 + kx_2) - b\tilde{k}_1 x_1 - b\tilde{k}_2 x_2 - b\tilde{k}_3 + u_1 \\ \dot{x}_2 = (1 - a_m H)x_1 - a_m k H x_2 - bH\tilde{k}_1 x_1 \\ \quad - bH\tilde{k}_2 x_2 - bH\tilde{k}_3 - H_1 u_1 \end{cases} \quad (29)$$

where  $u_1 = \dot{v}_l$ . Consider the following Lyapunov function

$$V = V_a + \sum_{i=1}^3 \frac{b}{2\gamma_i} \tilde{k}_i^2 \quad (30)$$

where  $V_a$  is the same as in (19). It is easy to verify by using the adaptive laws (27) and the knowledge of  $k_i^* \in [k_{il}, k_{iu}]$  that

$$\dot{V} \leq \dot{V}_a - \frac{b}{\gamma_3} \tilde{k}_3 \dot{k}_3^* \quad (31)$$

where  $\dot{V}_a$  is given in (22) and

$$\dot{k}_3^* = \dot{u}_d - \frac{\dot{d}}{b} \quad (32)$$

Since  $\dot{d}$ ,  $\dot{v}_l$  and the derivative of the function in (2) are bounded, it follows that  $\dot{k}_3^*$  is bounded. It is easy to show that if all the conditions in (15a) - (15d) are satisfied then  $\dot{V}$  is negative when  $x_1$  or  $x_2$  or both are large. This implies that  $V$  is bounded, and therefore all the signals in the closed-loop system are bounded [12].

When  $v_l$  and  $d$  are constants,  $\dot{V} < 0$  when either  $x_1$  or  $x_2$  is nonzero. It is true that  $x_1, x_2 \in L_2 \cap L_\infty$  and  $\dot{x}_1, \dot{x}_2 \in L_\infty$ . It follows from Barbalat's Lemma that  $x_1, x_2 \rightarrow 0$  as  $t \rightarrow \infty$ , i.e., the control objective in (4) is achieved.  $\square$

In [2], an adaptive controller was proposed with the nonlinear time headway in (9), and the gain  $k$  was chosen as

$$k = c_k + (k_0 - c_k) e^{-\sigma^2} \quad (33)$$

where  $k_0, c_k$ , and  $\sigma$  are positive constants to be designed (with  $c_k < k_0$ ). Even though it was shown in [2] that such choices for  $h$  and  $k$  could lead to good platoon performance, the system stability was not established. Since  $k$  in (33) is bounded by  $c_k$  and  $k_0$  all the time, Lemma 2 points out that if we choose the parameters properly, the adaptive controller in (26) with the update law in (27) makes the closed-loop system stable with  $h$  and  $k$  as chosen in [2]. Simulation results in Section III will demonstrate that the proposed controller can achieve the desired performance.

Since we have the flexibility to choose  $\Delta(\delta, t)$ , we can set  $\Delta(\delta, t) = k\delta$  where  $k$  is a positive constant. Hence we have the following lemma, which is a special case of Lemma 2 with the fact that  $k_l = k = k_u$ . The proof follows the same

steps as the proof for Lemma 3, and is omitted.

**Lemma 3:** Consider the system in (1) and (2), with the control law

$$u = k_1 v_r + k_2 \delta + k_3 \quad (34)$$

where  $k_i$  (with proper initial condition  $k_{i0}$  ( $i=1,2,3$ )) is generated according to the adaptive laws:

$$\begin{cases} \dot{k}_1 = \text{Proj}\{\gamma_1 x_1 [(p_1 x_1 + x_2) + (x_1 + a_m k p_1 x_2 + a_m x_2) H]\} \\ \dot{k}_2 = \text{Proj}\{\gamma_2 x_2 [(p_1 x_1 + x_2) + (x_1 + a_m k p_1 x_2 + a_m x_2) H]\} \\ \dot{k}_3 = \text{Proj}\{\gamma_3 [(p_1 x_1 + x_2) + (x_1 + a_m k p_1 x_2 + a_m x_2) H]\} \end{cases} \quad (35)$$

where  $a_m, p_1, \gamma_1, \gamma_2$ , and  $\gamma_3$  are positive design parameters, and  $\text{Proj}\{\bullet\}$  is defined in Lemma 2. All the signals in the closed-loop system are bounded if the design parameters are chosen such that

$$a_m p_1 > 1 \quad (36a)$$

$$\frac{4p_1 k}{a_m + a_m k p_1 - k} > \sup H \quad (36b)$$

Furthermore, if  $v_l$  and  $d$  are constants, then the control objective in (4) is achieved, i.e.  $v_r, \delta \rightarrow 0$  as  $t \rightarrow \infty$ .  $\square$

To avoid unnecessary switching between the brake and fuel systems, the following switching rules are incorporated in the vehicle following mode:

**S1.** If the separation distance  $x_r$  is larger than  $x_{\max}$  ( $x_{\max} > 0$  is a design constant), then the fuel system is on.

**S2.** If the separation distance  $x_r$  is smaller than  $x_{\min}$  ( $x_{\min} > 0$  is a design constant), then the brake system is on.

**S3.** If  $x_{\max} \leq x_r \leq x_{\min}$ , then the fuel system is on when  $u > 0$ , while the brake system is on when  $u < -u_0$  ( $u_0 > 0$  is a design constant). When  $-u_0 \leq u \leq 0$ , the brake system is inactive and the fuel system is operating as in idle speed.

There are several other practical issues associated with the application of the controller (26) or (34). To guarantee that the constraints **C1**, **C2** are not violated, we should avoid the generation of high or fast varying control signals. Though the controller in (26) or (34) is proposed without high gains, such high or fast varying control signals can still be generated if the lead vehicle accelerates rapidly or changes lanes creating a large spacing error or the ACC vehicle switches to a new target with large initial spacing error. To eliminate the adverse effect of large separation error, the control parameter  $k$  shown in (33) is proposed in [2]. When a constant  $k$  is to be used, the function  $\text{sat}(\delta)$  defined as

$$\text{sat}(\delta) = \begin{cases} e_{\max}, & \text{if } \delta > e_{\max} \\ e_{\min}, & \text{if } \delta < e_{\min} \\ \delta, & \text{otherwise} \end{cases} \quad (37)$$

should be used instead of  $\delta$  in order to eliminate the adverse effects of large separation error [1]. To eliminate the adverse effect of fast varying  $v_l$ , the nonlinear filter shown in Fig. 2 is used in [1] to smooth the speed trajectory of the lead vehicle. The filtered speed trajectory  $\hat{v}_l$  is then used by the controller. This modification is adopted in our vehicle following control system and.

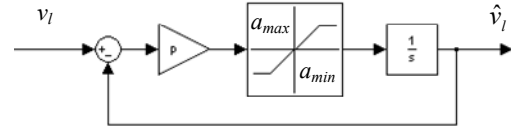


Fig. 2. Nonlinear filter used to smooth  $v_l$

### III. SIMULATION RESULTS

In this section, we present the simulation results that demonstrate the performance of the vehicle following controller given by (26) when applied to the validated nonlinear vehicle model used in [1]. In the simulation, the controller given by (26) with the update laws in (27) is tested using the  $h$  in (9) and  $k$  in (33). The control parameters are chosen as

$$s_0 = 4.5\text{m}, h_0 = 0.5, c_h = 0.1, k_0 = 0.5, c_k = 0.1,$$

$$a_m = 2, p_1 = 20,$$

$$a_{\max} = 1.0\text{m/s}^2, a_{\min} = -2.0\text{m/s}^2, p = 10$$

$$k_{10} = 6, k_{1u} = 12, k_{1l} = 4, \gamma_1 = 0.1,$$

$$k_{20} = 2, k_{2u} = 3, k_{2l} = 0.5, \gamma_2 = 0.05,$$

$$k_{30} = 0, k_{3u} = 30, k_{3l} = -30, \gamma_3 = 0.02$$

It can be verified that the control parameters are chosen such that (15a)-(15d) are satisfied.

Two vehicles are used in the simulation, and the following vehicle is equipped with the controller in (26). At time zero, the two vehicles have zero speed and are separated with a distance of  $s_0$ . From  $t = 0$ s to  $t = 20$ s, the lead vehicle increases its speed with a constant acceleration  $0.8\text{m/s}^2$ , and then cruises at  $16\text{m/s}$ . From  $t = 50$ s to  $t = 53$ s, the lead vehicle increases its speed with a constant acceleration  $2.0\text{m/s}^2$ , and then cruises at  $22\text{m/s}$ . From  $t = 90$ s to  $t = 100$ s, the lead vehicle increases its speed with a constant acceleration  $0.6\text{m/s}^2$ , and then cruises at  $28\text{m/s}$ . From  $t = 140$ s to  $t = 144$ s, the lead vehicle decreases its speed deceleration  $-2.0\text{m/s}^2$ , and then cruises at  $20\text{m/s}$ . The speed of the lead vehicle is presented using the dotted line in Fig. 3(a). The speed, acceleration, speed error, separation error, throttle angle and brake pressure responses of the following vehicle are presented in Fig. 3(a)-(f), respectively. As we can see, when the acceleration of the lead vehicle is

not too large ( $0.8$  or  $0.6\text{m/s}^2$ ), the throttle controller regulates the fuel system smoothly and the ACC vehicle follows the lead vehicle with small speed and separation errors. These errors are regulated towards zero when the lead vehicle reaches a constant speed. When the acceleration of the lead vehicle is large ( $2.0\text{m/s}^2$ ) for a short time, the following vehicle increases its speed in a smooth and comfortable way. The transient speed and separation errors are large due to the high acceleration of the lead vehicle. However, the errors are regulated towards zero as soon as the lead vehicle reaches a constant speed. When the lead vehicle decreases its speed rapidly, the brake system on the following vehicle is active to reduce the speed. From the acceleration and separation error responses, we can see that the vehicle following control system regulates the vehicle speed in a comfortable and safe way.

#### IV. CONCLUSION

In this paper, we design an adaptive vehicle following control system using a general time headway, which guarantees system stability and achievement of control objective when the speed of the lead vehicle is a constant. The analysis can be extended to any time headway with bounded partial derivatives with respect to  $v$  and  $v_l$ . The simulation results have demonstrated the performance of the proposed vehicle following control system when applied to the validated nonlinear vehicle model.

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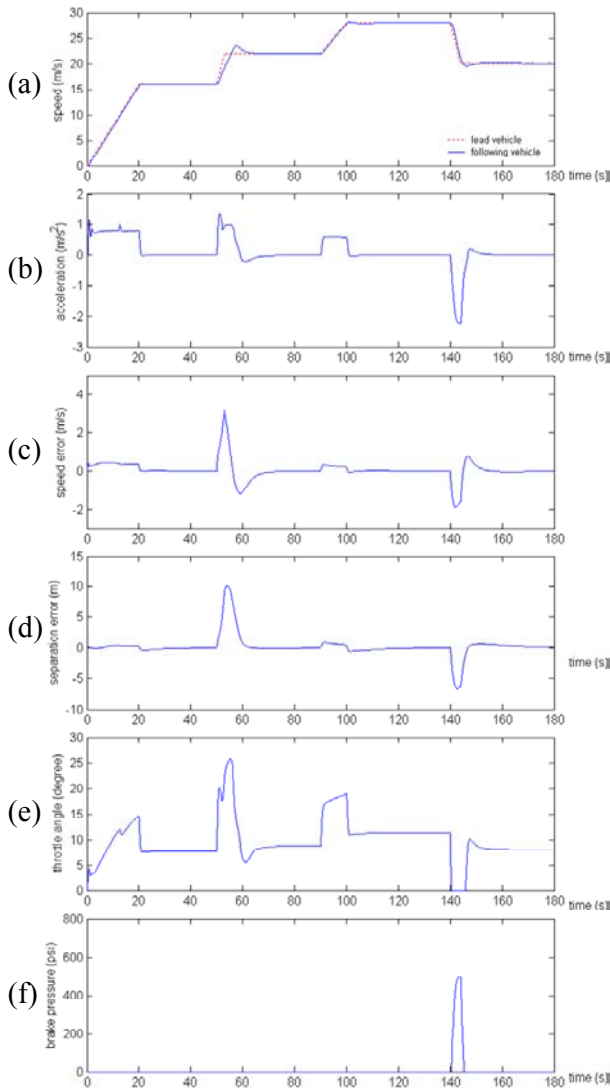


Fig. 3. Responses of the following vehicle: (a) speed, (b) acceleration, (c) speed error, (d) separation error, (e) throttle angle and (f) brake pressure in the simulation.