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Fault tolerant monitoring of vehicle lateral dynamics stabilization systems

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Abstract— **In this paper, a model based fault tolerant monitoring (FTM) scheme for vehicle lateral dynamics stabilization systems is presented. The major focus is on the handling of model uncertainties. The developed FTM scheme has been tested using real driving data.**

I. INTRODUCTION

The wide integration of electronic control systems like ABS (Anti-lock Breaking System), ESP (Electronic Stability Program), TCS (Traction Control System) into cars marks an important technological progress in the automotive industry in the past years. A central functionality of these control systems is to improve the active safety by stabilizing the vehicle in extreme driving situations [1], [2]. Associated with this development, integration of model based fault detection and isolation (FDI) units into the vehicle stabilization control systems is receiving more and more attention [3]. Driven by the enhanced demands for high system reliability and availability, considerable research efforts have been devoted to the development of advanced model based FDI methods. One of the focuses of this development is on the monitoring of those sensors that are fully embedded in the control loops and play a key role in the vehicle dynamics stabilization.

The study reported here is a part of European project Intelligent Fault Tolerant Control in Integrated Systems (IFATIS, website: http://ifatis.uni-duisburg.de). One of the objectives of IFATIS is to develop model-based FTM-schemes for vehicle lateral dynamics control systems. This study is strongly motivated by the practical demands for such monitoring systems that a) deliver reliable and fault tolerant estimates of the vehicle lateral dynamics b) automatically and reliably detect and isolate the faults in sensors c) are modularly structured and d) realizable on the embedded electronic control unit (ECU). The objective of this paper is to present the results achieved during the design, construction and implementation of such an FTM system.

II. PROBLEM FORMULATION AND PRELIMINARIES

A. System models

There are a number of mathematical models for the description of vehicle lateral dynamics [4]. In this study, the well-known bicycle model is used:

$$\begin{bmatrix}\dot{\beta}\\ \dot{r}\end{bmatrix}=\begin{bmatrix}\frac{Y_\beta K_{\phi R}}{mv} & \frac{Y_r K_{\phi R}}{mv}-1\\ \frac{N_\beta}{I_z} & \frac{N_r}{I_z}\end{bmatrix}\begin{bmatrix}\beta\\ r\end{bmatrix}+\begin{bmatrix}\frac{c'_{\alpha V}K_{\phi R}}{mv}\\ \frac{l_V c'_{\alpha V}}{I_z}\end{bmatrix}\delta_L^*-\begin{bmatrix}\frac{g}{v}\\ 0\end{bmatrix}\sin\alpha_x$$

$$Y_\beta=-\left(c'_{\alpha V}+c_{\alpha H}\right),\ Y_r=\left(l_H c_{\alpha H}-l_V c'_{\alpha V}\right)/v \quad (1)$$

$$N_\beta=l_H c_{\alpha H}-l_V c'_{\alpha V},\ N_r=\left(l_V^2 c'_{\alpha V}-l_H^2 c_{\alpha H}\right)/v$$

	Unit	Explanation
v	$[m/s]$	longitudinal velocity
a_y	$[m/s^2]$	lateral acceleration
β	$[rad$	vehicle side slip angle
r	$[rad/s]$	yaw rate
δ_L^*	$[rad]$	vehicle steering angle
α_x	$[rad]$	unknon road bank angle
g	$[m/s^2]$	gravity constant
$c'_{\alpha V}, c_{\alpha H}$	$[N/rad]$	cornering stiffness (front, rear)
l_V, l_H	$[m]$	distance front/rear axle - center of gravity
I_z	$[kgm^2]$	moment of inertia about the z-axis
m	$[kg]$	mass

The decision for the use of bicycle model has been made based on a compromise between the needed on-line computation and sufficient description of system dynamics. As shown in (1), the bicycle model is a system of the second order and thus the associated on-line computation for the model based FTM system is acceptable. However, the validity conditions of model (1), for instance (lateral acceleration) $a_y \leqslant 4m/s^2$ or $\dot{v}\approx 0$, may lead to considerable model uncertainties in some driving situations. Moreover, tyre cornering stiffness $c'_{\alpha V},\ c_{\alpha H}$ may considerably vary during driving on road with low or varying friction values [4], [5], [6], [7].

Following the relationship [4]

$$K_{\phi R}a_y=v\left(\dot{\beta}+r\right)+g\sin\alpha_x$$

the (first) sensor model

$$\begin{bmatrix}a_y\\ r\end{bmatrix}=\begin{bmatrix}\frac{Y_\beta}{m} & \frac{Y_r}{m}\\ 0 & 1\end{bmatrix}\begin{bmatrix}\beta\\ r\end{bmatrix}+\begin{bmatrix}\frac{c'_{\alpha V}}{m}\\ 0\end{bmatrix}\delta_L^* \quad (2)$$

holds. Note that model (1) can also be re-written into

$$\begin{bmatrix}\dot{\beta}\\ \dot{r}\end{bmatrix}=\begin{bmatrix}0 & -1\\ \frac{c_{\alpha H}(l_H+l_V)}{I_z} & \frac{l_H c_{\alpha H}(l_H+l_V)}{-vI_z}\end{bmatrix}\begin{bmatrix}\beta\\ r\end{bmatrix}+\begin{bmatrix}\frac{K_{\phi R}}{v}\\ \frac{l_V m}{I_z}\end{bmatrix}a_y-\begin{bmatrix}\frac{g}{v}\\ 0\end{bmatrix}\sin\alpha_x \quad (3)$$

with a_y as input. Then, the associated sensor model is described by

$$\begin{bmatrix}\delta_L^*\\ r\end{bmatrix}=\begin{bmatrix}\frac{-Y_\beta}{c'_{\alpha V}} & \frac{-Y_r}{c'_{\alpha V}}\\ 0 & 1\end{bmatrix}\begin{bmatrix}\beta\\ r\end{bmatrix}+\begin{bmatrix}\frac{m}{c'_{\alpha V}}\\ 0\end{bmatrix}a_y \quad (4)$$

The faults in the lateral acceleration, yaw rate and steering wheel angle sensors will be denoted by $f_{a_y},\ f_r,\ f_{\delta_L^*}$ respectively. They are modelled, following the practical

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requirements, as an additive term in the associated sensor models. Both of the system models, (1) and (3), with their associated sensor models (2) and (4) will be used below.

B. Technical requirements on the FTM system

The main objective of developing an FTM system for the lateral stabilization systems is to integrate all functionalities related to the signals delivered by the sensors into a modularly structured platform. These functionalities include a) FDI-function: detection and isolation of faults in the yaw rate, lateral acceleration and steering wheel angle sensors b) estimation function: estimation of the major vehicle motion variable side slip angle β c) FT function: ensuring functionalities a) and b) with a degraded performance in case of a detected sensor fault. Aiming at meeting the demands of the control loops on the reliability of the sensor signals and the estimates of the vehicle motion, the following technical requirements are specified for the above-mentioned functionalities:

- Requirements on the FDI function
  Low false alarm rate. For the test purpose, a catalog with more than 600 driving maneuvers is defined.
- Requirements on the estimation function
  Each estimate should be specified by a confidential interval.

In addition, the modular structure should ensure that a system extension caused by the integration of an additional sensor will not lead to a total system re-design.

C. The central problem

The performance of a model based monitoring system depends decisively on the quality of the models used. Concerning the bicycle model, there are two major facts that may cause considerable model uncertainties. First, the bicycle model is derived on a large number of assumptions and most of these assumptions will temporarily not hold during some maneuvers. Indeed, a mostly complete vehicle dynamic model is nonlinear, of high order (more than 30) and consists of a number of sub-models [8]. Secondly, the road conditions and driving maneuvers may cause considerable changes in the model parameters. Some of these influences, for instance bank angle, can be modelled as unknown inputs which may temporarily strongly change the dynamics of the vehicle. On the other hand, statistical data show that more than 80% of driving situations can be classified as stationary one which is well described by the bicycle model. This fact delivers not only a convincing argument for the use of the bicycle model but also the motivation for an alternative design scheme which will be described in Section 3.

D. Preliminary work

Regarding to the FTM of vehicle dynamics control systems, there are only few published results. The most popular scheme is the so-called fail-safe strategy, in which a sensor signal will not be used if a fault in this sensor is detected. The core of such systems is an FDI unit [6]. Indeed, an overview of the published results in recent years shows that the most efforts have been made to apply advanced FDI methods to improve the performance of FDI systems, see for instance [3], [5], [6], [7], [8], [9], [10], only mention some of them.

III. DESIGN OF THE FAULT TOLERANT MONITORING SYSTEM

In this section, the design of the FTM system is presented. The major focus is on the system structure and on the so-called model uncertainty indicator. Some details, for instance the design of the FDI unit, will not be included in this paper. The readers are referred to the published report [9].

A. System structure

As shown in Fig.1, the FTM system consists of four blocks.

FDI functional block: In this block, faults in yaw rate, lateral acceleration and steering wheel angle sensors will be detected and isolated. The corresponding information is stored in bits $A_{a_y}, A_r, A_{\delta_L^*}$ with "high" indicating a fault and "low" fault-free. In addition, three signal blocks are available, each of them includes an estimate for a sensor signal, the side slip angle and model uncertainties respectively. It also delivers a residual signal.

Switch block: The switching logic,

$$
\begin{aligned}
&S_1 \text{ is on only if } A_r = high;\\
&S_2 \text{ is on only if } A_{a_y} = high;\\
&S_3 \text{ is on only if } A_{\delta_L^*} = high;\\
&S_4 \text{ is on only if } A_r \cup A_{a_y} \cup A_{\delta_L^*} = low
\end{aligned}
$$

decides the working mode. In the fault-free mode, the estimation functional block will run, otherwise one of the signal blocks, regarding to the fault situation, will be switched to the Backup block.

Estimation functional block: An observer and a model uncertainty indicator are integrated into this block, which deliver an estimate for the side slip angle and the confidential interval.

Backup block: This block will be activated if one of the sensors fails. Suppose that a fault in the yaw rate sensor has been detected. Signal block I will be switched on and deliver the estimates for the side slip angle and the yaw rate together with an estimate for the model uncertainty. Based on the latter, the confidential intervals for the estimates will be computed. In addition, the residual signal R_{1,a_y} together with a simple evaluation algorithm is used for the detection of possible faults in the other two sensors. Note that the estimates delivered in Signal block I are achieved based on the sensor signals a_y, δ_L^*. Thus, a degradation of the estimation and fault detection performance is expected.

B. Observer and model uncertainty indicator design

Remember that the major objective of this study is to develop an alternative FTM scheme for handling of model uncertainties. The basic idea behind the FTM scheme presented here is to indicate the changes in residual signals or in the estimates, which are caused by the model uncertainties, instead of making the monitoring system being robust against

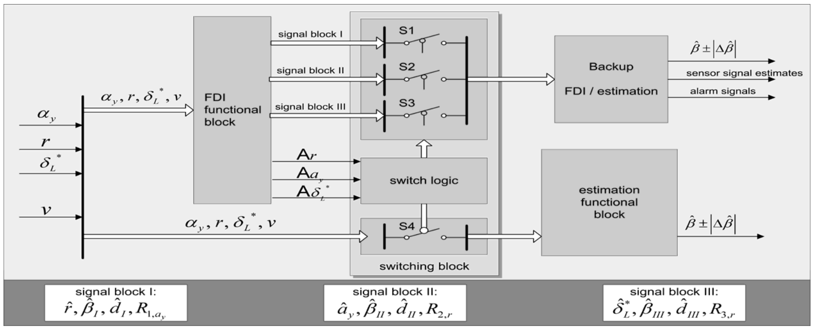

Fig. 1. Structure of the FTM system

the model uncertainties or identifying the parameter changes. An automatic fitting of the thresholds and the computation of the confidential interval for the estimates of the vehicle motion will then be realized based on the information delivered by the indicator.

Consider a system described by

$$\begin{aligned} x(k+1) &= (A+\Delta A)\,x(k)+(B+\Delta B)\,u(k)+(E+\Delta E)\,w(k) \\ y(k) &= (C+\Delta C)\,x(k)+(D+\Delta D)\,u(k)+(F+\Delta F)\,w(k) \end{aligned} \quad (5)$$

where $x \in R^n$, $u \in R^{k_u}$, $y \in R^m$ and $w \in R^{k_w}$ denote the state, input, output and unknown input vectors of the plant, respectively. A, B, C, D, E, F are known system matrices with appropriate dimensions and $\Delta A, \Delta B, \Delta C, \Delta D, \Delta E, \Delta F$ represent model uncertainties. For the FTM purpose, the observer of the form

$$\hat{x}(k+1) = A\hat{x}(k)+Bu(k)+L(y(k)-C\hat{x}(k)-Du(k)) \quad (6)$$

is used. Let $e(k) = x(k) - \hat{x}(k), r(k) = y(k) - \hat{y}(k)$. The estimation error dynamics is governed by

$$\begin{bmatrix} x(k+1) \\ e(k+1) \end{bmatrix} = \begin{bmatrix} A+\Delta A & O \\ \Delta A - L\Delta C & A-LC \end{bmatrix} \begin{bmatrix} x(k) \\ e(k) \end{bmatrix}$$
$$+ \begin{bmatrix} B+\Delta B \\ \Delta B - L\Delta D \end{bmatrix} u(k) + \begin{bmatrix} E+\Delta E \\ E+\Delta E - L(F+\Delta F) \end{bmatrix} w(k)$$
$$r(k) = Ce(k)+\Delta Cx(k)+\Delta Du(k)+(F+\Delta F)w(k)$$

Note that the overall system (the plant + the observer) is stable iff the plant is stable and $(A,\ C)$ is detectable, since the observer gain L has no influence on the system dynamics (5). The above equation can be further written as

$$e(k+1) = (A-LC)\,e(k)+\varphi(k)-Lv(k), r(k) = Ce(k)+\nu(k)$$
$$\varphi(k) = \Delta Ax(k) + \Delta Bu(k) + (E+\Delta E)\,w(k)$$
$$\nu(k) = \Delta Cx(k) + \Delta Du(k) + (F+\Delta F)\,w(k)$$

Suppose that input and output data in time interval $(k-s, k)$ are available. Then, after a straightforward calculation the above equation can be re-written into

$$r_s(k) = \Gamma d_s(k), r_s(k) = \begin{bmatrix} r(k-s) \\ \vdots \\ r(k) \end{bmatrix}, d_s(k) = \begin{bmatrix} e(k-s) \\ \varphi(k-s) \\ v(k-s) \\ \vdots \\ \varphi(k) \\ v(k) \end{bmatrix} \quad (7)$$

$$\Gamma = \begin{bmatrix} C & 0 & I & & & & \\ C\bar{A} & C & -CL & 0 & I & & \\ \vdots & \vdots & & \ddots & \ddots & & \\ C\bar{A}^s & C\bar{A}^{s-1} & -C\bar{A}^{s-1}L & \cdots & & 0 & I \end{bmatrix}$$

with $\bar{A} = A - LC$. Suppose that Γ has the full row rank and denote $\Gamma^+ = \Gamma^T \left(\Gamma\Gamma^T\right)^{-1}$. Then, it follows from (7) that

$$d_s(k) = \Gamma^+ r_s(k) + \left(I - \Gamma^+\Gamma\right) z \quad (8)$$

with some (unknown) vector z. Note that

$$\|d_s(k)\|_2 = \left\|\Gamma^+ r_s(k)\right\|_2 + \left\|\left(I - \Gamma^+\Gamma\right) z\right\|_2 \quad (9)$$

Thus, for a small $\|(I - \Gamma^+\Gamma)\, z\|_2$ signal $\|\Gamma^+ r_s(k)\|_2$ indicates the "size" of the model uncertainties over the evaluation window $(k-s, k)$. It is worth remarking that it is of primary interest for the below study that $\|d_s(k)\|_2$ can be directly estimated from the residual signal $r_s(k)$. In the following of this subsection, it will be demonstrated how to use (8) as well as (9) to build model uncertainty indicators. Suppose that observer (6) delivers an estimate $C_1\hat{x}(k)$ for $C_1 x(k)$. The estimation error is then described by

$$r_1(k) = C_1 e(k) \Rightarrow r_{s,1}(k) = \Gamma_1 d_s(k), r_{s,1}(k) = \begin{bmatrix} r_1(k-s) \\ \vdots \\ r_1(k) \end{bmatrix}$$

$$\Gamma_1 = \begin{bmatrix} C_1 & 0 & 0 & & & & \\ C_1\bar{A} & C_1 & -C_1 L & 0 & 0 & & \\ \vdots & \vdots & & \ddots & \ddots & & \\ C_1\bar{A}^s & C_1\bar{A}^{s-1} & -C_1\bar{A}^{s-1}L & \cdots & & 0 & 0 \end{bmatrix} \quad (10)$$

Substituting (8) into (10) leads to

$$r_{s,1}(k) = \Gamma_1 \Gamma^+ r_s(k) + \Gamma_1 \left(I - \Gamma^+\Gamma\right) z \quad (11)$$

It is evident that the influence of z on $r_{s,1}$ should be minimized in order to achieve a satisfactory estimate of $r_{s,1}$ in terms of r_s. To this end, the following optimization problem is formulated

$$\min_L \left[\left(I - \Gamma^+\Gamma\right)^T \Gamma_1^T \Gamma_1 \left(I - \Gamma^+\Gamma\right)\right] \quad (12)$$

In [12], it has been proven that for any observer gain L, Γ

can be written into

$$\Gamma = Q_L \Gamma_0, Q_L = \begin{bmatrix} I & 0 & & \cdots & 0 \\ -CL & I & 0 & & \vdots \\ & -CL & I & \ddots & \\ \vdots & \vdots & \ddots & \ddots & 0 \\ -C\bar{A}^{s-1}L & -C\bar{A}^{s-2}L & \cdots & -CL & I \end{bmatrix}$$

$$\Gamma_0 = \begin{bmatrix} C & 0 & I & 0 & \cdots & 0 \\ CA & C & 0 & 0 & I & \vdots \\ \vdots & \vdots & & \ddots & \ddots & \\ CA^s & C\bar{A}^{s-1} & 0 & \cdots & 0 & I \end{bmatrix}$$

It yields

$$\Gamma^+\Gamma = \Gamma_0^T Q_L^T \left(Q_L \Gamma_0 \Gamma_0^T Q_L^T\right)^{-1} Q_L \Gamma_0 = \Gamma_0^T \left(\Gamma_0 \Gamma_0^T\right)^{-1} \Gamma_0 \quad (13)$$

That means $\Gamma^+\Gamma$ is independent of the observer gain. As a result, optimization problem (12) can be equivalently reformulated as

$$\min_L \left[\left(I - \Gamma_0^T (\Gamma_0 \Gamma_0^T)^{-1} \Gamma_0\right)^T \Gamma_1^T \Gamma_1 \left(I - \Gamma_0^T (\Gamma_0 \Gamma_0^T)^{-1} \Gamma_0\right) \right] \quad (14)$$

It is worth remarking that the solution of optimization problem (14) will also lead to a sub-minimization of $\Gamma_1^T \Gamma_1$ which means, noting (10), a reduction of the influence of the model uncertainties on the estimation. Considering that (14) is a nonlinear optimisation problem and moreover the selected observer gain should guarantee a suitable dynamic behavior, it will be solved iteratively by solving the following nonlinear matrix inequalities (NMI)

$$\min \alpha \quad (15)$$

$$\begin{bmatrix} \alpha I & \left(I - \Gamma_0^T (\Gamma_0 \Gamma_0^T)^{-1} \Gamma_0\right)^T \Gamma_1^T \\ \Gamma_1 \left(I - \Gamma_0^T (\Gamma_0 \Gamma_0^T)^{-1} \Gamma_0\right) & I \end{bmatrix} > 0$$

$$\beta^{-2} \bar{A}^T P \bar{A} - P < 0$$

Note that the latter inequality ensures that the poles of the observer are smaller than β $(0 < \beta < 1)$. The (sub-)optimal observer gain is finally given by

$$L^* = \arg\{\min \alpha\} \quad (16)$$

Having set the observer gain optimally (in the sense of (15)), the model uncertain indicator can be defined either as

$$I = \frac{\|r_{s,1}(k)\|_2}{\sqrt{s+1}} = \frac{\|\Gamma_1 \Gamma^+ r_s(k)\|_2}{\sqrt{s+1}} \quad (17)$$

which delivers an average value over the evaluation interval $(k-s, k)$ or more conservatively as

$$I = \max_i \{\|r_1(k-i)\|_2, i = 0, \cdots s\} \quad (18)$$

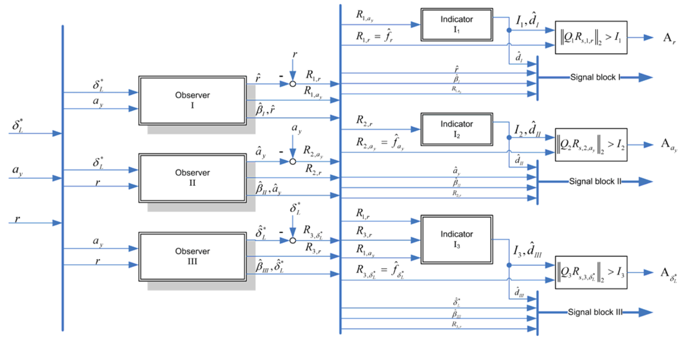

Fig. 2. Structure of the FDI block

C. Design of FDI functional block

The structure of the FDI functional block is shown in Fig.2. It consists of three observers and three model uncertainty indicators. The signals delivered by these units are used both for the FDI and fault tolerant purposes. Since each observer is driven by only two of the three available sensor signals, the following essential relations hold:

$$\begin{array}{ll} \hat{\beta}_I, \hat{r}, R_{1,a_y} & \text{are independent of sensor signal } r \\ \hat{\beta}_{II}, \hat{a}_y, R_{2,r} & \text{are independent of sensor signal } a_y \\ \hat{\beta}_{III}, \hat{\delta}_L^*, R_{3,r} & \text{are independent of sensor signal } \delta_L^* \end{array} \quad (19)$$

The FDI scheme developed in this study follows a strategy which is different from the well-established observer based FDI methods [11], [13], [14], [15]. The residual signals $R_{1,a_y}, R_{2,r}, R_{3,r}$ are used for the estimation of the model uncertainties and so the construction of model uncertainty indicators, while

$$R_{1,r} = r - \hat{r}, R_{2,a_y} = a_y - \hat{a}_y, R_{3,\delta_L^*} = \delta_L^* - \hat{\delta}_L^*$$

build the so-called detection residual signals. It follows from relations (19) that on the assumptions of no simultaneous fault and no model uncertainties the detection and isolation logic could be

$R_{1,r}$ is large and R_{1,a_y} is low $\Rightarrow$ fault in r - sensor
R_{2,a_y} is large and $R_{2,r}$ is low $\Rightarrow$ fault in a_y - sensor
R_{3,δ_L^*} is large and $R_{3,r}$ is low $\Rightarrow$ fault in δ_L^* - sensor

Thus, $R_{1,a_y}, R_{2,r}, R_{3,r}$ can be interpreted as thresholds.

Remember that the handling of the model uncertainties is the major task of the FDI scheme. To this end, model uncertainty indicators are constructed based on $R_{1,a_y}, R_{2,r}, R_{3,r}$ and the detection and isolation logic is modified as follows:

$$\begin{array}{l} \|Q_1 R_{s,1,r}\|_2 > I_1 \Rightarrow A_r = high \\ \|Q_2 R_{s,2,a_y}\|_2 > I_2 \Rightarrow A_{a_y} = high \\ \|Q_3 R_{s,3,\delta_L^*}\|_2 > I_3 \Rightarrow A_{\delta_L^*} = high \end{array} \quad (20)$$

where I_1, I_2, I_3 denote the outputs of the model uncertainty indicators, Q_1, Q_2, Q_3 the weighting matrices and $R_{s,1,r}, R_{s,2,a_y}, R_{s,3,\delta_L^*}$ the vectors constructed by $R_{1,r}, R_{2,a_y}, R_{3,\delta_L^*}$ over the time interval $(k-s, k)$. The basic idea behind the FDI logic (20) is that a sensor fault would be detected and isolated if the model uncertainties are moderate (i.e. $I_i, i = 1, 2, 3$ is small) or the size of the fault is very large. Note that in case of a sensor fault one of the three observers will work independent of the fault and delivers estimates for β and for the sensor which fails. Moreover, the residual signal can be used for the purpose of detecting the possible faults in the other two sensors. These signals will be integrated into the corresponding signal block and switched on to the backup block. In this way, a fault tolerant monitoring is achieved.

The first step to the FDI system design is a re-modelling which includes a state transformation and system discretization. The state transformation serves for the purpose of making the dynamics of the observers independent of the velocity v.

Discrete-time model I

$$\begin{aligned} z_I(k+1) &= A_1 z_I(k) + B_1 \begin{bmatrix} a_y(k) \\ \delta_L^*(k) \end{bmatrix} + E_{1,\,w} w_I(k) \\ &\quad + E_{1,\,\alpha_x} \sin \alpha_x(k) \qquad (21) \\ a_y(k) &= c_1 z_I(k) + v_I(k) + d_1 \delta_L^*(k), r(k) = c_2 z_I(k) \end{aligned}$$

Discrete-time model II

$$\begin{aligned} z_{II}(k+1) &= A_2 z_{II}(k) + B_2 \begin{bmatrix} r(k) \\ \delta_L^*(k) \end{bmatrix} + E_{2,\,w} w_I(k) \\ &\quad + E_{2,\,\alpha_x} \sin \alpha_x, \qquad (22) \\ a_y(k) &= c_4 z_{II}(k) + v_I(k) + d_1 \delta_L^*(k), r(k) = c_3 z_{II}(k) \end{aligned}$$

Discrete-time model III

$$\begin{aligned} x(k+1) &= A_3 x(k) + B_3 \begin{bmatrix} r(k) \\ a_y(k) \end{bmatrix} + E_{3,\,w} w_{II}(k) \\ &\quad + E_{3,\,\alpha_x} \sin \alpha_x(k) \qquad (23) \\ \delta_L^*(k) &= c_6 x(k) + v_{II}(k) + d_2 a_y(k), r(k) = c_5 x(k) \end{aligned}$$

In the above models, $w_I, w_{II} \in R^2, v_I, v_{II} \in R$ are unknown and represent the model uncertainties. Based on models (21)-(23), three observers are constructed as follows:

Observer I:

$$\begin{aligned} \hat{z}_I(k+1) &= A_1 \hat{z}_I(k) + B_1 \begin{bmatrix} a_y(k) \\ \delta_L^*(k) \end{bmatrix} + L_1 R_{1,\,a_y}(k) \\ \hat{a}_y(k) &= c_1 \hat{z}_I(k) + d_1 \delta_L^*(k),\ \hat{r}(k) = c_2 \hat{z}_I(k) \qquad (24) \\ R_{1,\,a_y}(k) &= a_y(k) - \hat{a}_y(k),\ R_{1,\,r}(k) = r(k) - \hat{r}(k) \end{aligned}$$

whose dynamics is, for $e_I(k) = z_I(k) - \hat{z}_I(k)$

$$\begin{aligned} e_I(k+1) &= (A_1 - L_1 c)\, e_I(k) + \bar{w}_I(k) - L_1 v_I(k) \\ \bar{w}_I(k) &= E_{1,w} w_I(k) + E_{1,\alpha_x} \sin \alpha_x(k) \qquad (25) \\ \begin{bmatrix} R_{1,a_y}(k) \\ R_{1,r}(k) \end{bmatrix} &= \begin{bmatrix} c_1 e_I(k) + v_I(k) \\ c_2 e_I(k) + f_r \end{bmatrix} \end{aligned}$$

Observer II:

$$\begin{aligned} \hat{z}_{II}(k+1) &= A_2 \hat{z}_{II}(k) + B_2 \begin{bmatrix} r(k) \\ \delta_L^*(k) \end{bmatrix} + L_2 R_{2,r}(k) \\ \hat{r}(k) &= c_3 \hat{z}_{II}(k),\ \hat{a}_y(k) = c_4 \hat{z}_{II}(k) + v_I(k) + d_1 \delta_L^* \qquad (26) \\ R_{2,r}(k) &= r(k) - \hat{r}(k),\ R_{2,a_y}(k) = a_y(k) - \hat{a}_y(k) \end{aligned}$$

whose dynamics is, for $e_{II}(k) = z_{II}(k) - \hat{z}_{II}(k)$

$$\begin{aligned} e_{II}(k+1) &= (A_1 - L_2 c_2)\, e_{II}(k) + \bar{w}_{II}(k) - L_2 f_r \\ \begin{bmatrix} R_{2,\,r}(k) \\ R_{2,\,a_y}(k) \end{bmatrix} &= \begin{bmatrix} c_3 e_{II}(k) + f_r \\ c_4 e_{II}(k) + \bar{v}_I(k) + f_{a_y} \end{bmatrix} \qquad (27) \\ \bar{w}_{II}(k) &= E_{2,w} w_I(k) + E_{2,\alpha_x} \sin \alpha_x, \bar{v}_I(k) = v_I(k) - f_{a_y} \end{aligned}$$

Observer III:

$$\begin{aligned} \hat{x}(k+1) &= A_3 \hat{x}(k) + B_3 \begin{bmatrix} r(k) \\ a_y(k) \end{bmatrix} + L_3 R_{3,\,r}(k) \\ \hat{\delta}_L^*(k) &= c_6 \hat{x}(k) + d_2 a_y(k), \hat{r}(k) = c_5 \hat{x}(k) \qquad (28) \\ R_{3,\,r}(k) &= r(k) - \hat{r}(k), R_{3,\,\delta_L^*}(k) = \delta_L^*(k) - \hat{\delta}_L^*(k) \end{aligned}$$

whose dynamics is, for $e_{III}(k) = x(k) - \hat{x}(k)$

$$\begin{aligned} e_{III}(k+1) &= (A_3 - L_3 c_5) e_{III}(k) + \bar{w}_{II}(k) - L_3 f_r \\ \begin{bmatrix} R_{3,r} \\ R_{3,\delta_L^*} \end{bmatrix} &= \begin{bmatrix} c_5 e_{III}(k) + f_r \\ c_6 e_{III}(k) + \bar{v}_{II}(k) + f_{\delta_L^*} \end{bmatrix} \qquad (29) \\ \bar{w}_{II}(k) &= E_{3,w} w_{II}(k) + E_{3,\alpha_x} \sin \alpha_x, \bar{v}_{II}(k) = v_{II} - f_{\delta_L^*} \end{aligned}$$

Using the design scheme for the model uncertainty indicators given in the last subsection, three indicators are designed. They are driven by $R_{1,\,a_y}, R_{2,\,r}, R_{3,\,r}$ respectively.

Indicator I:

$$I_1 = \left\| Q_1 \Gamma_{r,1} \Gamma_{a_y,1}^+ R_{s,1,a_y}(k) \right\| + \sigma_1^{1/2} \left\| f_{r,\,\min} \right\|_2 \qquad (30)$$

$$R_{s,1,a_y}(k) = \begin{bmatrix} R_{1,\,a_y}(k-3) \\ \vdots \\ R_{1,\,a_y}(k) \end{bmatrix}, \Gamma_{a_y,1}^+ = \Gamma_{a_y,1}^T \left(\Gamma_{a_y,1} \Gamma_{a_y,1}^T \right)^{-1}$$

$$\Gamma_{a_y,1} = \begin{bmatrix} c_1 & & 0 & 1 & & & \\ c_1 \bar{A}_1 & & c_1 & -c_1 L_1 & 0 & 1 & \\ \vdots & & \vdots & & \ddots & \ddots & \\ c_1 \bar{A}_1^3 & c_1 \bar{A}_1^2 & -c_1 \bar{A}_1^2 L_1 & \cdots & & 0 & 1 \end{bmatrix}$$

$$\Gamma_{r,1} = \begin{bmatrix} c_2 & & 0 & & & \\ c_2 \bar{A}_1 & c_2 & -c_2 L_1 & & 0 & \\ \vdots & & \vdots & & \ddots & \ddots \\ c_2 \bar{A}_1^3 & c_2 \bar{A}_1^2 & -c_2 \bar{A}_1^2 L_1 & \cdots & & 0 \end{bmatrix}$$

Indicator II:

$$I_2 = \left\| Q_2 \Gamma_{a_y,2} \Gamma_{r,2}^+ R_{s,2,r}(k) \right\|_2 + \sigma_2^{1/2} \left\| f_{a_y,\,\min} \right\|_2 \qquad (31)$$

$$R_{s,2,r}(k) = \begin{bmatrix} R_{2,r}(k-3) \\ \vdots \\ R_{2,r}(k) \end{bmatrix}, \Gamma_{r,2}^+ = \Gamma_{r,2}^T \left(\Gamma_{r,2} \Gamma_{r,2}^T \right)^{-1}$$

$$\Gamma_{r,2} = \begin{bmatrix} c_3 & & 0 & 1 & & & \\ c_3 \bar{A}_2 & & c_3 & -c_3 L_2 & 0 & 1 & \\ \vdots & & \vdots & & \ddots & \ddots & \\ c_3 \bar{A}_2^3 & c_3 \bar{A}_2^2 & -c_3 \bar{A}_2^2 L_2 & \cdots & & 0 & 1 \end{bmatrix}$$

$$\Gamma_{a_y,2} = \begin{bmatrix} c_4 & & 0 & & & \\ c_4 \bar{A}_1 & c_4 & -c_4 L_2 & & 0 & \\ \vdots & & \vdots & & \ddots & \ddots \\ c_4 \bar{A}_2^3 & c_4 \bar{A}_2^2 & -c_4 \bar{A}_2^2 L_2 & \cdots & & 0 \end{bmatrix}$$

Indicator III:

$$I_3 = \left\| Q_3 \Gamma_{\delta_L^*,\,3} \Gamma_{r,\,3}^+ R_{s,\,3,\,r}(k) \right\|_2 + \sigma_3^{1/2} \left\| f_{\delta_L^*,\,\min} \right\|_2 \qquad (32)$$

$$R_{s,3,r}(k) = \begin{bmatrix} R_{3,r}(k-3) \\ \vdots \\ R_{3,r}(k) \end{bmatrix}, \Gamma_{r,3}^+ = \Gamma_{r,3}^T \left(\Gamma_{r,3} \Gamma_{r,3}^T \right)^{-1}$$

$$\Gamma_{r,3} = \begin{bmatrix} c_5 & & 0 & 1 & & & \\ c_5\bar{A}_3 & c_5 & -c_5L_3 & & 0 & 1 & \\ \vdots & & \vdots & & & \ddots & \ddots \\ c_5\bar{A}_3^3 & c_5\bar{A}_3^2 & -c_5\bar{A}_3^2L_3 & \cdots & & 0 & 1 \end{bmatrix}$$

$$\Gamma_{\delta_L^*,3} = \begin{bmatrix} c_6 & & 0 & & \\ c_6\bar{A}_3 & c_6 & -c_6L_3 & & 0 \\ \vdots & & \vdots & \ddots & \ddots \\ c_6\bar{A}_3^3 & c_6\bar{A}_3^2 & -c_6\bar{A}_3^2L_3 & \cdots & 0 \end{bmatrix}$$

Note that in indicators (30), (31) and (32) additional terms are introduced, where $f_{r,\min}, f_{a_y,\min}, f_{\delta_L^*,\min}$ denote the minimum detectable faults and $\sigma_1, \sigma_2, \sigma_3$the maximum singular values of matrices $Q_1\Gamma_{r,1}, Q_2\Gamma_{a_y,2}, Q_3\Gamma_{\delta_L^*,3}$ respectively. These are added to take into account the influence of the unknown vector z and the existing disturbance in the output equations of $R_{2,\,a_y}, R_{3,\,\delta_L^*}$.

D. Design of backup block

The backup block will be activated if a fault is detected. Corresponding to the detected fault, a signal block will be switched to the backup block. To illustrate the working principle, suppose that the a_y sensor fails. Having detected and isolated the fault, $\hat{\beta}_{II}, R_{2,r}, \hat{d}_{II}$ will be delivered to the backup block. In the backup block, $\Delta\beta$ will then be computed based on $\hat{d}_{II}$. For the purpose of fault detection, the detection logic:

$$\|R_{s,2,r}\|_2 > J_{th} \Rightarrow \text{ fault in } r \text{ or } \delta_L^* \text{ sensor} \quad (33)$$

is used, where J_{th} is a constant threshold. Note that in this case only a large-sized fault can be detected. Also, no fault isolation is possible.

E. Design of estimation functional block

After all sensor signals have been tested and no fault has been detected, an observer driven by all three sensor signals can be implemented. For this purpose, the following discrete time model is derived:

$$x(k+1) = A_4x(k) + B_4 \begin{bmatrix} a_y(k) \\ r(k) \\ \delta_L^*(k) \end{bmatrix} + E_{4,\,w}w_{III}(k) + E_{4,\,\alpha_x}\sin\alpha_x(k)$$

$$r(k) = c_8x(k),\ a_y(k) = c_7x(k) + v_{III}(k) + d_8\delta_L^*(k) + d_9r(k)$$

where matrix A_4 is independent of the velocity. Correspondingly, the observer is constructed as follows:

$$\hat{x}(k+1) = A_4\hat{x}(k) + B_4 \begin{bmatrix} a_y(k) \\ r(k) \\ \delta_L^*(k) \end{bmatrix} + L_4 \begin{bmatrix} R_{4,\,a_y}(k) \\ R_{4,\,r}(k) \end{bmatrix}$$

$$\hat{r}(k) = c_8\hat{x}(k),\ \hat{a}_y(k) = c_7\hat{x}(k) + d_8\delta_L^*(k) + d_9r(k)$$

$$R_{4,\,a_y}(k) = a_y(k) - \hat{a}_y(k),\ R_{4,\,r}(k) = r(k) - \hat{r}(k) \quad (34)$$

The estimate for the side slip angle is delivered by

$$\hat{\beta}(k) = \hat{x}_1(k) := c_9\hat{x}(k)$$

Using the method introduced in Subsection III.B, one can then establish the confidential interval for the above estimation. As a result,

$$|\Delta\beta| = \frac{\left\|\Gamma_\beta\Gamma_4^+R_{s,4}(k)\right\|_2}{2},\ R_{s,4}(k) = \begin{bmatrix} R_4(k-3) \\ \vdots \\ R_4(k) \end{bmatrix}$$

$$\Gamma_4 = \begin{bmatrix} C_4 & & 0 & I & & & \\ C_4\bar{A}_4 & C_4 & -C_4L_4 & & 0 & I & \\ \vdots & & \vdots & & & \ddots & \ddots \\ C_4\bar{A}_4^s & C_4\bar{A}_4^{s-1} & -C_4\bar{A}_4^{s-1}L_4 & \cdots & & 0 & I \end{bmatrix}$$

$$\Gamma_\beta = \begin{bmatrix} c_9 & & 0 & 0 & & & \\ c_9\bar{A}_4 & c_9 & -c_9L_4 & & 0 & 0 & \\ \vdots & & \vdots & & & \ddots & \ddots \\ c_9\bar{A}_4^3 & c_9\bar{A}_4^2 & -c_9\bar{A}_4^2L_4 & \cdots & & 0 & 0 \end{bmatrix}$$

IV. AN EXAMPLE OF THE TEST RESULTS

To test the performance of the developed FTM system, real data collected during driving tests. The test car undertook more than 600 driving tests on different roads and with different driving maneuvers. Most of these driving maneuvers are the critical ones. Due to the space limitation, the successful results can not be presented in this paper. An extended version of this paper can be downloaded under http://aks.uni-duisburg.de (staff section).

V. CONCLUSION

In this paper, a model based FTM system for the sensors integrated in the lateral dynamics stabilization systems has been presented. The first tests using the real driving data show promising results.

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