Proceedings of the
44th IEEE Conference on Decision and Control, and
the European Control Conference 2005
Seville, Spain, December 12-15, 2005

An Experiment-Based Methodology for Robust Design of Optimal Residual Generators

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***Abstract*—A novel and pragmatic methodology is presented for the design of fault detection and isolation systems by using parity relations in a fully stochastic setting. The approach is based on data obtained, either from the real process or an analytical model, through a statistically designed experiment. The inspiring concept is that design optimality can only be attained if detection/isolation performance and robustness are simultaneously addressed in an integrated framework. Advantages with respect to the state of the art are pointed out, both in terms of general applicability of the method and optimality of the design solution. An illustrative case-study concludes the paper.**

I. Introduction

It has been recognized for a long time that proper handling of robustness issues is fundamental for successful implementation of model-based fault detection and isolation (FDI) systems. The purpose of this paper is to present a data-driven methodology for the design of linear parity functions, as defined in [1], that optimizes specific criteria for detection and isolation of faults while accounting for linearization error, parametric modelling uncertainties and noise effects.

Previous work on robust residual generation based on linear parity relations can be found in [1-6]. Handling noise effect has been considered in [7-9], where parity relations are suitably combined with statistical approaches for residual evaluation. However, modelling uncertainties are not introduced in [7-9].

There are several shortcomings in the methods described in [1-9]. One should first notice that, in all cases, the design relies on a linear dynamic model of the supervised process, or on a set of such models. Yet, most processes are nonlinear, and modelling errors are due to both linearization errors and uncertainties in model parameters. Besides, parity relations based on linear models use measurements that correspond to deviations with respect to a nominal set point. Slight variations of this set point and/or uncertainties on the nominal values of the measurements associated to this set point may affect significantly the performance of a FDI system. Such changes may possibly be tracked by averaging signals over a long time period to determine their nominal value, but this causes cancellation of the effect of slow drifts and of step like faults in the long term [10]. To avoid such problems, the nonlinear plant model will be used directly for the design of linear parity relations in this paper.

Another issue is the choice of the objective function to be optimized in the design of the parity relations. As residuals are typically processed through statistical tests or change detection algorithms, criteria that measure detection and isolation potential in a statistical sense are relevant. This means that both the mean and the variance of the residual must be explicitly considered in the objective function and motivates the use of the Kullback divergence as a measure of residual decoupling.

To be able to introduce such features in the design of parity relations, the methodologies of statistical design of experiments and robust design will be exploited.

The paper is organized as follows. The considered framework for residual generation is presented in section II. The proposed design methodology is described in section III and its application to the case study of a fluid mixer is shown in section IV. Final remarks conclude the paper.

II. Problem Statement

The following class of dynamic non linear systems is considered:

$$\begin{aligned} x(k+1) &= f\left(x(k),u(k),e(k),\gamma,\theta\right) \\ y(k) &= g\left(x(k),u(k),e(k),\gamma,\theta\right)+v(k) \end{aligned} \qquad (1)$$

where $x(k)$, $u(k)$, $e(k)$, $y(k)$, $v(k)$ are real-valued vectors with dimensions n, q, p, m, m respectively denoting the state, the known input, the unknown input, the measured output and the measurement noise. $\gamma \in R^{N_p}$ is a vector of uncertain parameters, and $\theta \in R^{N_f}$ is the fault vector. The latter is zero in the absence of faults, and its components take a non zero value upon occurrence of one out of N_f possible faults. For the work reported here faults are assumed to induce changes on the process output at steady-state. The input vectors $u(k)$ and $e(k)$ take the following form:

$$u(k) = u_s + u_d(k); \qquad e(k) = e_s + e_d(k) \qquad (2)$$

where u_s, e_s are stochastic vectors with mean μ_u, μ_e and variance Σ_u, Σ_e characterizing the range of inputs around which the fault detection system should work properly.

Manuscript received March 7, 2005. This work was supported by the Brussels Region, Ministry of Economy Employment and Scientific Research, under Grant "Research in Brussels-2004".

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Process noise, $\{u_d(k)\}_k$ and $\{e_d(k)\}_k$, and measurement noise $\{v(k)\}_k$ are mutually uncorrelated white noise sequences with variance Q_u, Q_e and Q_v respectively. They are also uncorrelated with u_s and e_s. γ is a random vector with mean μ_γ and variance Σ_γ supposed to be uncorrelated with u_s, e_s, $\{u_d(k)\}_k$, $\{e_d(k)\}_k$ and $\{v(k)\}_k$. The initial state vector is also assumed to be described by a random vector $x(0)$ with known mean and variance, uncorrelated with all previously defined random vectors and processes.

Our aim is to design a set of parity relations of the affine form

$$r(k) = \alpha^T \begin{bmatrix} y(k-s+1) \\ \vdots \\ y(k) \end{bmatrix} + \beta^T \begin{bmatrix} u(k-s+1) \\ \vdots \\ u(k) \end{bmatrix} + \delta \qquad (3)$$

where s is a fixed integer, δ is a scalar parameter, α and β are $m{\cdot}s$- and $q{\cdot}s$- dimensional vectors of parameters. The objective is to determine δ, α and β so that each residual achieves certain optimality properties that make the whole set of residuals suitable for fault detection and isolation.

III. Methodology

A. Conceptual Background

The distinctive feature of the methodology is its reliance on process data, no matter whether they come from a physical set-up or a system model. In the latter case the model is in fact mainly used as a data generator. Implications are quite profound. On one hand closed form solutions are no longer possible. On the other hand, several limitations of the existing approaches drop, thus allowing for more general applicability. This is certainly attractive for practitioners.

The methodology is inspired by Robust Design, a well known technique of industrial statistics applied to engineering design [11-13]. It is aimed at designing products and processes (i.e., finding optimal setting of design parameters) whose performance is on a desired target and random variability around this target is minimised. Random variability is caused by all sources of uncontrolled variation occurring when a product is being used or a process is running (*in-process* conditions). In the language of Robust Design, design parameters are referred to as control factors (usually deterministic) whereas uncontrolled variables as noise factors (random); the performance of interest is called the response. Technically the two goals, response on target with minimum variability, are obtained by optimising an objective function involving in some way the first two moments of the response. A key feature of the approach is that expected value and variance of the response are estimated using the information drawn from statistically designed experiments conducted on the product or the process. From this regard, Robust Design largely borrows from another fundamental discipline of industrial statistics, Design of Experiments (DoE) [14], [15]. Statistically designed experiments are a form of stratified sampling where samples of the response are observed for different settings of the factors.

While DoE traditionally deals with the mean performance, Robust Design addresses also the random variation about the mean. Opposed to DoE, Robust Design assumes that the size of the variation, which is directly related to the scatter of the noise factors, may also depend on the control factors. Thus an appropriate setting of the control factors may accommodate both the mean (close to the target) and the variance (sufficiently small).

Application of Robust Design to the design of an FDI system is fully justified but not straightforward. Both adaptations and extensions are needed. First we define all factors involved in the case at hand, then the proposed methodology is described in a step-by-step fashion.

B. Classification of FDI Variables

Control factors

Control factors are those that the analyst has to set at optimal values. Here they are mainly vectors α and β in the affine form (3) defining the residual. The scalar parameter δ is not included as it has the only function to adjust the mean of the residual to zero in absence of fault. This will be apparent later in the section. An additional control factor is the length of the observation window of the residual, s.

Noise factors

Noise factors are out of the analyst control in *in-process* conditions, where they are considered random variables. Nevertheless noise factors may be, and often are, controlled in a lab environment during the experimental phase. This has the twofold advantage of reducing the experiment size and giving the analyst more insight on how noise factors affect the response as they can explicitly appear in the response model. Of course controlling noise factors, even in the lab, can be difficult, costly or unfeasible in some instances. However, this is not the case when experiments are not conducted on a physical set-up but on a simulation model of the product or the process. This kind of experiments is known in the literature as *Computer Experiments* [16]. Such experiments turn out very useful when functions f and g in model (1) are known.

The main set of noise factors are the random vectors of known input u_s, unknown input e_s and parameter γ. Another set includes process noise and sensor noise, as represented by all (infinite) random variables in the processes $\{u_d(k)\}_k$, $\{e_d(k)\}_k$ and $\{v(k)\}_k$. However, there are differences between the two sets. Random variables in the first set generally account for the major contribution of uncertainty, as due to an unknown (and slow) change of the set point or of the system dynamics whereas process and sensor noise can be regarded as a smaller disturbance acting on a shorter time scale without a definite connection with single assignable causes. The effect of this disturbance on the response is very much alike to what is called "experimental

error" in DoE, namely the unexplained difference between values of the response measured in repeated experimental runs, i.e. where factors setting is unchanged. The practical consequence is that process and sensor noise are totally uncontrollable in the experiments while system inputs and parameters are not.

The random vector of the initial state should also be considered a noise factor. However we make the assumption that the process is running around a stable set point and that, at steady-state, the effect of the initial state on the output has vanished.

Fault factors

The fault vector θ forms a set of special factors which are uncontrollable during process operation but must be necessarily controlled in the computer (and possibly physical) experiment since the primary objective of the analysis is to characterize how the system reacts to different faults.

C. Design Procedure

Step 1. It involves an experiment on the process. The experiment can be either physical or simulated, depending on whether we work on the real process or on a computer code implementing a known process model (1). Since no assumption of linearity was claimed for that model, any computable form of it is acceptable in principle. Interestingly only fault factors and noise factors appear in the experiment, see Fig. 1. In fact, the control factors come into play only at the residual generation stage via Equation (3). This makes the method a sequential one. Fault factors have fixed levels in the experiment and the same levels will be carried forward in the subsequent steps. Let N_θ be the number of the values of θ investigated in the experiment. If the case of simultaneous faults is not of interest N_θ equals $(l_f - 1)\cdot N_f + 1$ where l_f is the number of levels for the fault factors including the zero level. The selected range for the component θ_i should match the desired specification range within which fault i is to be detected.

Noise factors, u_s , e_s and γ may have either random or fixed levels. Let N_n be the number of the different experimental settings for the ensemble of u_s, e_s and γ. For the reasons provided earlier in the section, process noise and sensor noise are necessarily treated as random variables from stochastic processes. Let h be the number of different realizations of the random processes considered in the experiment for each setting of the other factors.

By fully crossing the levels of fault and noise factors a total of $N_\theta \cdot N_n \cdot h$ experimental runs results. Given the interpretation of the effect of the random sequences as an "experimental error", in the jargon of DoE one would say that the experiment is made up of h replications of a $N_\theta \cdot N_n$-run design. Output y and known input u form the response of the experiment and are observed in a sufficiently long time horizon (t_{K+1} , t_{K+T}) after reaching stationary condition. Let us denote by $\mathrm{Y}_{zwb}^{(c)}(k)$ the c-th component (c=1,$m+q$) of the response measured at time t_k for the experimental run identified by the b-th setting for θ, the z-th for u_s , e_s and γ, the w-th for process and sensor noise (b=1,N_θ; z=1,N_n; w=1,h).

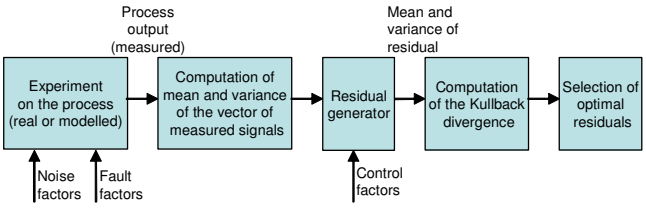

Fig. 1. Overall scheme of the proposed methodology.

Step 2. Here the expected value and the variance, conditional on the fault vector θ, of the random vector $Y=[y(k-s_{\max}+1)\ \dots\ y(k)\ u(k-s_{\max}+1)\ \dots\ u(k)]^T$ at steady-state are estimated. $s_{\max}$ is the maximum number of signals that the analyst will allow for in Equation (3). Since under the hypothesis of process stationarity the mean, the variance and the autocorrelation structure do not change over time, signal variation over time around the steady-state level can be used as a substitute of variation among different realizations at fixed time. Thus estimates of the mean and variance of Y/θ at the N_θ settings analysed at Step 1 are obtained by averaging the h sample mean and variance computed over the noise factors and the whole observation window. Notice that also h=1 (no replication in the experiment) is viable with a considerable saving of experimental burden. In a component-wise fashion the estimates write as:

$$\hat{\mathrm{E}}(Y_i/\theta) = \frac{1}{N_n \cdot h \cdot T}\sum_{z=1}^{N_n}\sum_{w=1}^{h}\sum_{k=1}^{T}\mathrm{Y}_{zwb_\theta}^{(i)}(K+k) \qquad i=1,(m+q)\cdot s_{\max}$$

$$\hat{Cov}(Y_i,Y_j/\theta) = \frac{1}{h}\sum_{w=1}^{h}\frac{1}{[N_n\cdot(T-l)-1]}\cdot \sum_{z=1}^{N_n}\sum_{k=1}^{T-l}\left(\mathrm{Y}_{zwb_\theta}^{(c_i)}(K+k)-\overline{\mathrm{Y}}_{\bullet wb_\theta}^{(c_i)}\right)\cdot \left(\mathrm{Y}_{zwb_\theta}^{(c_j)}(K+k+l)-\overline{\mathrm{Y}}_{\bullet wb_\theta}^{(c_j)}\right) \qquad i,j=1,(m+q)\cdot s_{\max} \tag{4}$$

with

$$\overline{\mathrm{Y}}_{\bullet wb_\theta}^{(c_i)} = \frac{1}{N_n(T-l)}\sum_{z=1}^{N_n}\sum_{k=1}^{T-l}\left(\mathrm{Y}_{zwb_\theta}^{(c_i)}(K+k)\right)$$

$$\overline{\mathrm{Y}}_{\bullet wb_\theta}^{(c_j)} = \frac{1}{N_n(T-l)}\sum_{z=1}^{N_n}\sum_{k=1}^{T-l}\left(\mathrm{Y}_{zwb_\theta}^{(c_j)}(K+k+l)\right)$$

where l denotes the time lag ($0\le l\le s_{\max}$) between Y_i and Y_j, b_θ is the number of the fault setting θ, c_i and c_j the number of the components of the response Y associated with the components i and j of Y. Note that at this stage we might

prefer to estimate the moments conditional also on u_s, e_s and γ. This would give information on how they affect the moments. In this case, the unconditional moments will be eventually estimated by using the fundamental identities:

$$\begin{aligned} \mathrm{E}(\bullet/\theta) &= \mathrm{E}_{u_s,e_s,\gamma}(\mathrm{E}(\bullet/\theta;u_s,e_s,\gamma)) \\ Var(\bullet/\theta) &= \mathrm{E}_{u_s,e_s,\gamma}(Var(\bullet/\theta;u_s,e_s,\gamma)) + \\ &\quad Var_{u_s,e_s,\gamma}(\mathrm{E}(\bullet/\theta;u_s,e_s,\gamma)) \end{aligned} \tag{5}$$

Mean and variance of Y conditional on θ are valuable information; a simple screening may give the analyst a practical understanding of how the process responds to the faults and what are the critical requirements of the FDI system. Light can be shed on basic questions: which faults are difficult to detect or to isolate, which components of Y are most sensitive to each fault, what is the size of random variability characterizing the process and how it is affected by the faulty conditions.

Step 3. Expected value and variance of the residual, conditional on θ, can now be estimated via (3):

$$\begin{aligned} \hat{\mathrm{E}}(r/\theta) &= \left[\alpha^T \ \beta^T\right]\cdot\hat{\mathrm{E}}(Y/\theta)+\delta \\ \hat{\mathrm{Var}}(r/\theta) &= \left[\alpha^T \ \beta^T\right]\cdot\hat{\Xi}_Y\cdot\left[\alpha^T \ \beta^T\right]^T \end{aligned} \tag{6}$$

where $\hat{\Xi}_Y$ is the covariance matrix of the signal vector at steady-state whose elements are calculated as in (4). Some considerations are in order on the (seemingly) free parameters α, β and δ. The scalar parameter δ, being independent from θ, is clearly irrelevant in the context of detection and isolation. It could be set to zero without any loss in performance. However, after optimal value α and β have been found, it can be used to adjust the residual mean so that it is zero in absence of fault, i.e.:

$$\delta = -\left[\alpha^T \ \beta^T\right]\cdot\hat{\mathrm{E}}(Y/\theta=0) \tag{7}$$

As for α and β, we note that the SN ratio $(\mathrm{E}(r/\theta)-\delta)^2/\mathrm{Var}(r/\theta)$ is invariant with respect to the multiplication of vector $\left[\alpha^T \ \beta^T\right]$ by any real constant. Recalling that δ can be safely set to zero, we conclude that a gain in sensitivity to a fault due to an increase in the residual mean is cancelled by a loss in robustness due to a proportional increase of the standard deviation. Thus it is reasonable to impose the unit norm constraint

$$\left[\alpha^T \ \beta^T\right]\cdot\left[\alpha^T \ \beta^T\right]^T = 1 \tag{8}$$

Step 4. The knowledge of the first two moments of the residual conditional on θ is extremely useful for tackling the problem of the design of a robust FDI system. Basically, detection and isolation rely on residual decoupling. What is needed, then, is a consistent metric for residual decoupling in a stochastic setting. In fact, the two moments can be used as building blocks for measuring the "distance" between a pair of probability distributions of the residual, each distribution being associated with a different value of the fault vector. Two such distances are of interest, one for detection and the second for isolation:

$$\mathrm{dist}(r/\theta_i, r/\theta=0)\ ;\mathrm{dist}(r/\theta_i, r/\theta_j), \quad i,j=1,N_f;\ j\neq i \tag{9}$$

An appropriate metric is the Kullback divergence J, an operator derived from the Kullback information K [17]:

$$\begin{aligned} &J(\phi_1,\phi_2) = K(\phi_1,\phi_2)+K(\phi_2,\phi_1) \\ &\text{with} \quad K(\phi_1,\phi_2) = \int \ln\frac{f_{\phi_1}(y)}{f_{\phi_2}(y)} f_{\phi_1}(y)\mathrm{d}y \end{aligned} \tag{10}$$

where ϕ_1 and ϕ_2 are two settings of the parameters of the density function f. An important property of the Kullback information is that the power of a statistical test for discriminating the two distributions is not decreasing for increasing values of K. As J is symmetric in the arguments whereas K is not, it qualifies better for being a distance measure. We note that the residual derives from the sum of several random variables. Therefore the Central Limit theorem gives assurance that the normality assumption for the residual can be reasonably made. In the case of a Normal random variable where a change affects both the mean and the variance, the Kullback divergence is:

$$J(\phi_1,\phi_2) = \frac{1}{2}\left[(\mu_2-\mu_1)^2\cdot\left(\frac{1}{\sigma_1^2}+\frac{1}{\sigma_2^2}\right)+\left(\frac{\sigma_1^2}{\sigma_2^2}+\frac{\sigma_2^2}{\sigma_1^2}-2\right)\right] \tag{11}$$

where $\phi_1 = (\mu_1\ \sigma_1^2)^T$ and $\phi_2 = (\mu_2\ \sigma_2^2)^T$ are the vectors of the distribution parameters before and after the change respectively. For equal variances J reduces to the SN ratio $(\mu_2-\mu_1)^2/\sigma^2\cdot$ and, for a constant difference of the means, it increases with the difference of the variances. When the Kullback divergence is applied to a pair of residuals these parameter vectors can be estimated by using Eqs. (6).

Step 5. Once an appropriate metric for residual decoupling is available we address the problem of designing a set of residuals with optimal detection and isolation capabilities. This means finding a set of optimal vectors $\left[\alpha_i^T \ \beta_i^T\right]$. A modular strategy can envisage to find a set for detection first, then another set for isolation. The detection set includes the residuals which, for each fault, are most distant from the fault-free condition (first line of (9)), i.e. the ones that maximizes the appropriate Kullback divergences. Therefore the detection set will contain as many residuals as faults at most. A similar criterion is adopted for isolation and consists in including in the set the residual which, for each fault, is most distant from all the other faults. A reasonable implementation of the criterion is to maximize the minimum

Kullback divergence between the considered fault and each of the remaining ones. Also the isolation set will contain a maximum number of N_f residuals.

Implementation of the above criteria requires stating two sets of N_f optimization problems, one for detection and one for isolation:

$$\begin{array}{ll} \textit{Detection} & \textit{Isolation} \\ i=1,N_f & i=1,N_f \\ \max\limits_{\alpha,\beta}\ \min\limits_{k=1,l_f-1} J(\phi_{ik},\phi_0) & \max\limits_{\alpha,\beta}\ \min\limits_{\substack{j\neq i \\ k,l=1,l_f-1}} J(\phi_{ik},\phi_{jl}) \\ \text{subject to} & \text{subject to} \\ \left[\alpha^T\ \beta^T\right]\cdot\left[\alpha^T\ \beta^T\right]^T=1 & \left[\alpha^T\ \beta^T\right]\cdot\left[\alpha^T\ \beta^T\right]^T=1 \end{array} \qquad (12)$$

where ϕ_{ik} is the vector of distribution parameters corresponding to a fault vector having all zero components but the *i*-th which is at level $\theta_{ik}\neq 0$, and ϕ_0 is the parameter vector for the fault-free condition $\theta=0$. Of course the fault levels considered are the $N_\theta-1$ non zero settings used in the experiment at Step 1. Finally, parameter δ is determined by Eq. (7).

IV. Case-Study

The methodology is applied to a fluid mixer. Two independent input flows, u_1 and u_2, containing dissolved material with concentrations e_1 and e_2 respectively, feed a tank of cross-section area γ_2. Concentration y_2 in the output flow F equals the concentration in the tank x_2 because the latter is continuously stirred. Conservation laws yields the following state space model for the process

$$\begin{aligned} \dot{x}_1 &= u_1+u_2-\gamma_1\cdot\gamma_2^{-\frac{1}{2}}\cdot x_1^{-\frac{1}{2}} \\ \dot{x}_2 &= x_1^{-1}\cdot\left((e_1-x_2)\cdot u_1+(e_2-x_2)\cdot u_2\right) \end{aligned} \qquad (13)$$

with γ_1 being an experimental constant. The process output y coincides with the state, namely the fluid volume in the tank and the concentration of the output flow. Faults are bias on the input and the output. Normality is assumed for all random variables. The 6-sigma interval around the mean input accounts for a ±10% change of the set-point. There is no uncertainty in the parameters γ. Process noise is considered only on the known input and its standard deviation is one third of that of the nominal set-point (see [18] for numerical values). The experiment is conducted on the numerical code implementing a finite difference integration of (13) in its discrete form. Three levels, including the zero central level, are assigned to each fault with the extreme levels symmetrical around zero (N_θ=9). Noise factors u_s and e_s are given 100 random levels drawn from their distributions (N_n=100). At each of the $N_\theta{\cdot}N_n$ experimental settings three realizations of y are computed (h=3) and observed at steady-state in the interval (5001, 10000) s (K=T=5000). Mean and variance of the signal vector, conditional on θ, are estimated up to $s_{\max}$=5. For s=1, Table I shows $\hat{\mathrm{E}}(Y/\theta)$ for the fault free condition and the high level of each fault ((u_{10}, u_{20}), (x_{10}, x_{20}) denote respectively the nominal input and state values), whereas Table II and III report the estimated variance in absence of fault and for the highest level of fault 1. One can see that fault 1 and fault 4 determine the largest change in the mean of y_1 and y_2 respectively whereas fault 2 determines the smallest change in both. As the standard deviation of y_1 and y_2 is comparable with the change in their mean, robustness is critical to FDI performance. Thus in the selection of optimal vectors α and β an important issue is the exploitation of the correlations among signals in order to minimize the residual variance. A final consideration is that the input faults affect also the variance of the signal vector with the obvious exception of the constant input variances. As an example, variance of y_1 exhibits a 31% increase from θ=0 to θ_1 at its highest level. This fact, known as heteroscedasticity, is a consequence of the process non linearity in the input. Interestingly it goes totally unseen in the commonly used linear model of the process with a clear detriment to the optimality of the FDI system. The optimization problem for detection is solved by using MATLAB routines for non-linear constrained optimization. In Fig. 2 the Kullback divergence is plotted for each fault with s= 1,2…,5. In fact, the extra degrees of freedom turn to be useful for detection as the Kullback divergence steadily increases with s. Results confirm that fault 2 is the most critical, as previously argued. Detection capability for fault 4 is inferior to the expectation and exhibits a minimal increase with s. A deeper scrutiny reveals that mean and variance of this residual are strongly correlated ($\sigma\approx 0.12\mu$) thus the Kullback divergence collapses to a constant ($J\approx 69$). To better interpret numbers in Fig. 2 we note that, in the simple case of equal variances, a 6-sigma separation between the means, which corresponds to a 0.27% overlapping probability, produces a Kullback divergence of 36. This threshold is overcome for fault 2 from s=2. Looking at the size of the mean and standard deviation of the optimal residuals for s=1 in Table IV one can discover that the solution corresponds to a condition with very low variability. Notice that the mean of the residual in absence of fault is not null. This bias is cancelled by choosing the constant δ as in (7). Finally, Table V shows the critical (minimum) Kullback divergence of the optimal residuals for isolation for s=2,3. Acceptable isolation capability is achieved already with s=2; the most critical separation is between fault 2 and fault 3.

TABLE I
Estimated Means of the Output Signals, Conditional on θ

signal	θ=0	θ_1=0.2· u_{10}	θ_2=0.2· u_{20}	θ_3=0.2· x_{10}	θ_4=0.2· x_{20}
y_1, m^3	1.0008	1.3234	1.1033	1.2008	1.0008
y_2, kg m^{-3}	1.2469	1.2143	1.2827	1.2469	1.4969

TABLE II
ESTIMATED VARIANCE OF THE SIGNAL VECTOR, CONDITIONAL ON θ=0

x10^{-6}	y_1	y_2	u_1	u_2
y_1	2200			
y_2	-54.84	1055		
u_1	18.67	-1.62	0.211	
u_2	2.35	1.050	-0.001	0.027

TABLE III
ESTIMATED VARIANCE OF THE SIGNAL VECTOR, CONDITIONAL ON THE HIGHEST LEVEL OF FAULT 1

x10^{-6}	y_1	y_2	u_1	u_2
y_1	2880			
y_2	-17.37	1026		
u_1	21.50	-1.12	0.211	
u_2	2.71	0.950	-0.001	0.027

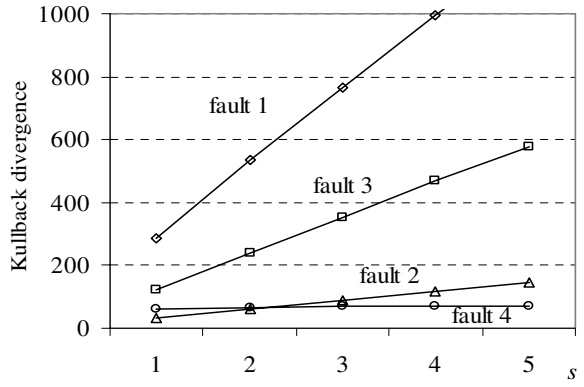

Fig. 2. Optimal values of the Kullback divergence in the detection set of residuals for different time horizons.

TABLE IV
ESTIMATED MEANS AND VARIANCE OF OPTIMAL DETECTION RESIDUALS, CONDITIONAL ON $\theta(s=1)$

	θ_1=0.2· u_{10}	θ_2=0.2· u_{20}	θ_3=0.2· x_{10}	θ_4=0.2· x_{20}
$\hat{E}(r/\theta)$	$46\cdot10^{-4}$	$-43\cdot10^{-4}$	$-46\cdot10^{-4}$	$350\cdot10^{-4}$
$\hat{V}ar(r/\theta)$	$2.33\cdot10^{-8}$	$2.09\cdot10^{-8}$	$2.02\cdot10^{-8}$	$62.2\cdot10^{-8}$
$\hat{E}(r/\theta=0)$	$70\cdot10^{-4}$	$-52\cdot10^{-4}$	$-62\cdot10^{-4}$	$288\cdot10^{-4}$
$\hat{V}ar(r/\theta=0)$	$1.87\cdot10^{-8}$	$1.95\cdot10^{-8}$	$2.02\cdot10^{-8}$	$62.2\cdot10^{-8}$

TABLE V
OPTIMAL KULLBACK DIVERGENCES FOR ISOLATION (s=2,3)

s	Fault 1	Fault 2	Fault 3	Fault 4
2	77.9	55.5	55.5	108.4
3	108.5	81.6	81.6	138.8

V. FINAL REMARKS

The intent of the proposed approach is to extend generality and optimality of the existing methods for fault diagnosis based on residuals obtained via parity relations. Increased generality stems from a more realistic consideration of the process which is not necessarily described by a linear model and can be affected by any source of random variability. The contribution to optimality relies on the concept that, in a stochastic framework, the simultaneous consideration of the mean and the variance of the residuals is a necessary condition for achieving optimal residual decoupling. This is particularly important when the process is non linear since residual variance may vary depending on the fault which is acting and on the fault level. The Kullback divergence, which involves the first two moments of the residuals, is proposed as a measure of residual decoupling. In the paper it is assumed that diagnosis is made in stationary process condition. However, in principle there is no limitation to the extension of the method in a dynamic condition.

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