Proceedings of the
44th IEEE Conference on Decision and Control, and
the European Control Conference 2005
Seville, Spain, December 12-15, 2005

Uncertain Nonlinear Receding Horizon Control Systems Subject to Non-Zero Computation Time

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Abstract— **In this paper Receding Horizon Control (RHC) of an uncertain nonlinear system is considered where the computation time is non-negligible. In a well-known method, the solution process of the optimal control problem is started one sampling period in advance by using the prediction of the initial conditions, thus giving the controller a reasonable deadline to complete the optimization process. The current work suggests the use of the theory of neighboring extremal paths to improve the performance of the existing method by adding a correction phase to the previous method and therefore recovering the exact solution in the presence of prediction errors. An immediate result would be that the properties of the RHC techniques involving zero computation time would be valid for practical systems in the actual implementation, where a zero computation time is unachievable. The new approach is applied for the control of a mobile robot system which demonstrates significant performance improvements over the existing method.**

I. Introduction

RECEDING Horizon Control (RHC), also known as Model Predictive Control (MPC), was first introduced in the process control community and attracted the attention of numerous researchers due to its ability to handle constraints on the states and inputs in multi-variable control problems [1]. The RHC method is basically a repeated on-line solution to a finite horizon open loop optimal control problem. Based on the current state values, an optimal control problem is solved for a period of time called the *prediction horizon*. The first part of the computed optimal input is applied to the plant in a period of time called the *execution horizon* until the next sampling of the states becomes available, where again the same procedure is repeated. The reader is referred to [2] for a comprehensive review of RHC literature.

In the past, the repeated on-line solution of an open loop optimal control problem limited the application of RHC mainly to process control problems. In process control problems found in chemical industries, the dynamics of the plant is slow enough to allow for the computation of the optimal control and therefore computation time is not an issue. However, recent advances in computing performance and distributed computation and the introduction of faster optimization algorithms, such as the one suggested in [3], have allowed the approach to be applied to mechanical systems with fast dynamics such as mobile robot and aerospace systems. In the method suggested in [4], the concept of differential flatness has been used to generate optimal trajectories, offering a shorter computation time. This was later successfully applied to a vector thrust flight experiment [5]. In [6] and [7], authors propose dividing the nonlinear optimal control system architecture into two parts: An outer loop which generates the reference optimal outputs to be followed by the plant and an inner loop stabilizing the states of the system around the generated reference trajectory using neighbouring extremal paths theory. The Legendre pseudospectral method is used to approximate the states, co-states and inputs in [6] instead of the B-Spline approximation proposed in [4]. In reference [6], the approximation method allows for rapid generation of optimal trajectories, which can potentially be useful for RHC problems.

In the literature, there are several studies on the stability of closed-loop systems obtained with a RHC strategy, albeit with a zero computation time assumption [6]. In practice, however, there is no guarantee of closed-loop stability as the zero computation time assumption is violated, especially for systems with fast dynamics. Milam *et. al.* address the issue of computation time in [5], by proposing a method involving prediction of the states at the next sampling time before hand, which gives the controller enough time to generate the optimal trajectories. The predicted states serve as initial conditions for the open loop optimal control problem, giving the controller a computation deadline equal to the sampling period to solve the optimization problem. At the next sampling time, same prediction and optimal trajectory generation procedure is repeated.

In this paper, we build upon the work presented in [5] by proposing the addition of a correction phase to the

Manuscript received March 7, 2005. This work was partially funded through the Defense Research and Development Canada (DRDC) Technology Investment Fund under the project entitled "Trusted Uninhabited Vehicle Autonomy Through Time-Constrained Decentralized Model Predictive Control" and the Natural Sciences and Engineering Research Council of Canada (NSERC).

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prediction and trajectory generation in the RHC of systems with non-zero computation time. In our proposed method, the control signal design for the RHC system is obtained in three stages, the first two being those proposed in [5]. The prediction phase estimates the values of the states of the system in the next RHC sampling time. An optimal trajectory is designed with the initial conditions of the optimal open loop problem being the predicted values calculated in the prediction phase. The generation of this trajectory is the most time consuming part of the computation of the RHC inputs. The actual values of the states available at the next sampling period can be used to modify this pre-calculated trajectory using neighbouring extremal paths theory [9]. This constitutes the third phase, *i.e.* the correction phase. The modification can be assumed to have zero computation time, even in systems with fast dynamics such as aerospace systems, as discussed later in the paper.

Using our proposed method, the solution to the open loop optimal control problem obtained by assuming zero computation time can be recovered as long as the conditions put forth in the paper are satisfied. The method proposed in this paper allows for use of RHC in practical systems, while the theoretical results assuming zero computation time for uncertain nonlinear systems still applicable to such practical systems; a property that does not hold for the method presented in [5] as the actual solution to the optimal control problem is not obtained due to the state prediction errors.

The outline of the paper is as follows: in Sections II theory of Receding horizon control of nonlinear systems is reviewed followed by a brief discussion on the theory of neighboring extremal paths in Section III. We will propose our method in Section IV and discuss about its validity in Section V. In Section VI the proposed method is applied to a mobile robot of unicycle type, where the simulations show significant improvement in the performance compared to the existing method found in [5].

II. Receding Horizon Control of Nonlinear Systems

In this section, we review the general RHC scheme briefly. The class of systems considered is described by the set of equations

$$\dot{\boldsymbol{x}} = \boldsymbol{f}\big(\boldsymbol{x}(t),\boldsymbol{u}(t)\big) \qquad \boldsymbol{x}(0)=\boldsymbol{x_0} \tag{1}$$

where $\boldsymbol{x}(t)\in\Re^n$ is the state of the system and $\boldsymbol{u}(t)\in\Re^m$ is the input vector satisfying the constraints

$$\boldsymbol{u}(t)\in U \qquad \forall t\geq 0 \tag{2}$$

U is the set of allowable inputs. Furthermore, we assume that assumptions (A1-A3) in [10] are also satisfied; that is, $\boldsymbol{f}$ is twice differentiable, U is compact and convex, and system (1) has a unique solution for a given initial condition. Receding horizon control is the repeated solution of the following problem.

Problem 1 Find

$$J_T^*\big(\boldsymbol{x}(t)\big) = \min_{\boldsymbol{u}(.)} J(\boldsymbol{x}(t),\boldsymbol{u}(.),T) \tag{3}$$

with

$$\begin{aligned} J(\boldsymbol{x}(t),\boldsymbol{u}(.),T) = \int_t^{t+T} \Big(\|\boldsymbol{x}(\tau;\boldsymbol{x}(t))\|_{\mathbf{Q}}^2 + \|\boldsymbol{u}(\tau)\|_{\mathbf{R}}^2\Big) d\tau \\ + \big\|\boldsymbol{x}\big(t+T;\boldsymbol{x}(t)\big)\big\|_{\boldsymbol{P}}^2 \end{aligned} \tag{4}$$

subject to

$$\left.\begin{aligned} \dot{\boldsymbol{x}}(s) &= \boldsymbol{f}\big(\boldsymbol{x}(s),\boldsymbol{u}(s)\big) \\ \boldsymbol{u}(s) &\in U \end{aligned}\right\} \qquad s\in[t,t+T] \tag{5}$$

$\mathbf{Q}\in\Re^{n\times n}$ and $\mathbf{R}\in\Re^{m\times m}$ denote positive-definite, symmetric weighting matrices, T is a finite prediction time and $\boldsymbol{x}(t;\boldsymbol{x_0})$ denotes the trajectory of the system (1) driven by $\boldsymbol{u}(t)$ starting from the initial condition $\boldsymbol{x_0}$. Furthermore, the weighted norms in (4) are defined as $\|\boldsymbol{x}\|_{\mathbf{Q}}^2 = \boldsymbol{x}^T\mathbf{Q}\boldsymbol{x}$.

Let h denote the receding horizon sampling period, where h lies in the $(0,T]$ interval. The closed-loop system is described by

$$\begin{aligned} &\dot{\boldsymbol{x}}(\tau) = \boldsymbol{f}\big(\boldsymbol{x}(\tau),\boldsymbol{u}^*(\tau)\big) \\ &\boldsymbol{u}^*(\tau) = \boldsymbol{u}^*(\tau;\boldsymbol{x}(t)) \qquad \tau\in[t,t+h), \quad 0<h\leq T \end{aligned} \tag{6}$$

where $\boldsymbol{u}^*(\tau;\boldsymbol{x}(t))$, $\tau\in[t,t+T]$, is the optimal control of the problem stated above with the initial condition $\boldsymbol{x}(t)$, t being the start time of the optimization process and the instant at which states are sampled.

As discussed in [2], numerous methods have been suggested to guarantee the stability of closed-loop system by requiring a terminal constraint at the end-time of the optimization horizon or a special way to select the terminal cost. Therefore, it is straightforward to adapt the RHC scheme to the specific method, one would like to implement. As an instance, [10] guarantees the stability of the closed-loop system provided that the following terminal inequality constraint is added to Problem 1

$$\begin{aligned} &\boldsymbol{x}\big(t+T;\boldsymbol{x}(t)\big)\in \Omega_\alpha \\ &\Omega_\alpha := \Big\{\boldsymbol{x}\in\Re^n \,\Big|\, \boldsymbol{x}^T\mathbf{P}\boldsymbol{x}\leq\alpha\Big\} \end{aligned} \tag{7}$$

where α is a positive constant and the matrix $\mathbf{P}$, the solution to the Lyapunov equation, is selected as described in [10].

III. Neighbouring Extremal Paths

In this section, we briefly review the perturbation analysis of the open loop optimal control problem presented in [9]. This will be used in Section IV, where we state our proposed solution for dealing with uncertainties in the presence of computational delay.

Assume that Problem 1 in Section II is solved for the given initial conditions $\boldsymbol{x}(t_0)$. Introduction of a small perturbation in the initial conditions, $\delta\boldsymbol{x_0}$, will cause a change in the optimal trajectories, *i.e.* $\delta\boldsymbol{x}$ and $\delta\boldsymbol{u}$. The solution to the perturbed problem can be retrieved by solving a linear optimal control problem. More specifically, this problem is composed of finding the optimal change in the input signal $\delta\boldsymbol{u}$, minimizing

$$\delta^2 J = \frac{1}{2}\left(\delta\boldsymbol{x}^T \Phi_{\boldsymbol{xx}} \delta\boldsymbol{x}\right)_{t=T} + \frac{1}{2}\int_t^{t+T} \begin{bmatrix} \delta\boldsymbol{x}^T & \delta\boldsymbol{u}^T \end{bmatrix} \begin{bmatrix} H_{\boldsymbol{xx}} & H_{\boldsymbol{xu}} \\ H_{\boldsymbol{ux}} & H_{\boldsymbol{uu}} \end{bmatrix} \begin{bmatrix} \delta\boldsymbol{x} \\ \delta\boldsymbol{u} \end{bmatrix} d\tau \tag{8}$$

subject to the following constraint

$$\begin{aligned} \delta\dot{\boldsymbol{x}} &= \boldsymbol{f_x}\delta\boldsymbol{x} + \boldsymbol{f_u}\delta\boldsymbol{u} \\ \delta\boldsymbol{x}(t_0) &= \delta\boldsymbol{x_0} \end{aligned} \tag{9}$$

with Hamiltonian described by

$$H = \left\|\boldsymbol{x}(\tau;\boldsymbol{x}(t))\right\|_{\mathbf{Q}}^2 + \left\|\boldsymbol{u}(\tau)\right\|_{\mathbf{R}}^2 + \lambda^T \boldsymbol{f} \tag{10}$$

and $\boldsymbol{\lambda}$ is the vector of co-state variables. As this problem is a linear optimal control problem, it is straightforward to show that the optimal input is given by

$$\delta\boldsymbol{u}(t) = -H_{\boldsymbol{uu}}^{-1}(H_{\boldsymbol{ux}}\delta\boldsymbol{x} + f_{\boldsymbol{u}}^T\delta\lambda) \tag{11}$$

where the perturbation in the states and co-states is given by

$$\begin{aligned} \delta\dot{\boldsymbol{x}} &= \mathbf{A}(t)\delta\boldsymbol{x} - \mathbf{B}(t)\delta\lambda \\ \delta\dot{\lambda} &= -\mathbf{C}(t)\delta\boldsymbol{x} - \mathbf{A}^T(t)\delta\lambda \end{aligned} \tag{12}$$

Matrices $\mathbf{A}(t)$, $\mathbf{B}(t)$ and $\mathbf{C}(t)$ are defined as follows

$$\begin{aligned} \mathbf{A}(t) &= \boldsymbol{f_x} - \boldsymbol{f_u}H_{\boldsymbol{uu}}^{-1}H_{\boldsymbol{ux}} \\ \mathbf{B}(t) &= \boldsymbol{f_u}H_{\boldsymbol{uu}}^{-1}\boldsymbol{f_u}^T \\ \mathbf{C}(t) &= H_{\boldsymbol{xx}} - H_{\boldsymbol{xu}}H_{\boldsymbol{uu}}^{-1}H_{\boldsymbol{ux}} \end{aligned} \tag{13}$$

Among the possible ways to solve the Two Point Boundary Value Problem (TPBVP) in (12), we choose the backward sweep method described in [9].

Remark 1. Although Problem 1 assumes a quadratic cost for the performance index J in (4), in the perturbation analysis, the theory of neighbouring extremal paths is general enough to be applied to optimal control problems with a nonlinear cost in the performance index. This allows its application to RHC schemes such as those described in [8], where the cost in the performance index J is not necessarily quadratic and can have a general nonlinear form.

IV. PROBLEM STATEMENT AND PROPOSED METHOD OF SOLUTION

Consider the RHC of system (1), as described in Problem 1 in Section II. The optimization problem has to be solved on-line implying that the process of finding the optimal value will require a certain computation time, not known *a priori*. As proposed in [5], the following computation algorithm can be used allowing the RHC scheme to be applied to practical systems: At time t, predict the state of the system at time $t+h$, using the current state values available from the sampling operation. Then, solve the optimal control problem using the predicted states as the initial conditions. This gives the system a computation deadline equal to h to compute the optimal input.

If there is no uncertainty present in the modelling of system (1), the predicted and the actual values of the states are the same. However, uncertainties in the model and exogenous disturbances cause a mismatch in the predicted and the actual values of the states. We propose to modify this pre-computed input before its application to the plant using the theory of neighbouring extremal paths [9] reviewed in Section III. The modification process is composed of two parts: (i) Solving a differential equation by the backward sweep method, a differential Riccati equation resulting from the TPBVP (12); this is essentially an initial value problem (ii) Solving an initial value differential equation to calculate the changes in the input profile using (11).

The process of modification can be regarded as a zero computation time task even in the case of fast dynamic mechanical systems, as it is composed of the solution of two initial value problems, noting that $t \in [0,T]$ in the first initial value problem and $t \in [0,h]$ in the second one. Note that the parameters of the differential Riccati equation are computed off-line (refer to Chapter 5 in [9]), therefore only the solution of such initial value problems has to be carried out on-line. The complete algorithm is summarized below.

Algorithm 1. (a) Assume a zero input for the first execution horizon, *i.e.* $\boldsymbol{u_0}=0$. Let $k=0$.
(b) Sample the states at times $t=t_k$
(c) At time t_k, predict the states of the system at time $t=t_{k+1}=t_k+h$ based on the current states and current input $\boldsymbol{u_k}$.
(d) Solve the open loop optimal control problem using the predicted states calculated in step (c). The solution of the optimization takes place in the time interval $[t_k,t_{k+1}]$ having a computation deadline of h.
(e) Sample the states $\boldsymbol{x_{actual}}$, at $t=t_{k+1}$.
(f) Calculate the difference between the predicted states, $\boldsymbol{x_{predict}}$ and the actual states $\boldsymbol{x_{actual}}$ at $t=t_{k+1}$.
(g) Solve for the change in the optimal input, $\delta\boldsymbol{u}$, and update the input $\boldsymbol{u_{k+1}}$.
(h) $k=k+1$. Goto step (c).

V. VALIDITY OF THE PROPOSED ALGORITHM

As the neighbouring extremal paths theory, used in Algorithm 1 in Section IV, is only valid in a sufficiently small neighbourhood of the original extremal trajectories, in this section we find a more rigorous description for the condition mentioned above. More specifically, satisfaction of a condition stated in Proposition 1 is sufficient for the validity of the proposed method for a general nonlinear system subject to bounded perturbations. We require the following assumptions to hold.

Assumption 1. In the open loop optimal control problem described in Problem 1, small perturbation in the initial conditions will result in perturbation in other variables, $\delta\boldsymbol{x}$, $\delta\boldsymbol{u}$ and $\delta\boldsymbol{\lambda}$, of the same order, *i.e.*

$$\mathrm{O}(\delta\boldsymbol{x}_0) = \mathrm{O}(\delta\boldsymbol{x}) = \mathrm{O}(\delta\boldsymbol{u}) = \mathrm{O}(\delta\boldsymbol{\lambda}) \tag{14}$$

Assumption 2. The RHC problem defined in Problem 1 has no constraints on the input, *i.e.* $U = \Re^m$, where m is the dimension of the input space. This is due to the fact that the perturbation analysis in Section III is based on the optimal control problem with no constraints on the input. This assumption can be removed, if the corresponding theory is modified accordingly.

Assumption 3. Among the possible RHC schemes discussed in [2] we choose those for which a terminal constraint is not used to guarantee stability of the closed loop. In order to remove this assumption, the perturbation analysis should be appropriately changed (refer to [9]).

Remark 2. Assumption 3 confines the use of the method described in [10] to linear unconstrained problems. A novel RHC technique to stabilize nonlinear systems has been recently introduced in [8]. The latter does not require a terminal constraint. Therefore, the RHC method described in [8] satisfies Assumption 3.

Assumption 4. The computation time of the parts (f) and (g) of Algorithm 1 are negligible compared to the dynamics of the closed-loop system.

Remark 3. Note that Assumption 4 is only necessary from the practical point of view and not for the validity of the correction phase in Algorithm 1. It is practically achievable, considering the fact that two initial value problems (with the parameters of the differential equations calculated off-line) have to be solved in a time span equal to the execution and prediction horizon, for the corresponding initial value problems. Use of the method described in [6] can even reduce the computation time further. TPBVP can be avoided using the pseudospectral approximation. Instead, a set of coupled algebraic equations should be solved on-line, reducing the computation burden significantly.

Proposition 1. Consider the system

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x},\boldsymbol{u}) \tag{15}$$

which is used as a nominal model for the RHC synthesis and

$$\boldsymbol{x}_{actucal} = \boldsymbol{f}(\boldsymbol{x}_{actual},\boldsymbol{u}) + \boldsymbol{g}(t,\boldsymbol{x}_{actual},\boldsymbol{u}) \tag{16}$$

which serves as the model of the real system. In (16), $\boldsymbol{g}(t,\boldsymbol{x}_{actual},\boldsymbol{u})$ accounts for disturbances, uncertainties and unmodelled dynamics. Algorithm 1 is valid provided that the following condition is satisfied

$$\left\|\boldsymbol{g}(t,\boldsymbol{x}_{actual},\boldsymbol{u})\right\| \le b \tag{17}$$

where b is a positive constant. In addition, $(bh)^2$ is negligible (h is the execution horizon) and Assumptions 1-3 hold.

Proof. See [11]. □

VI. EXAMPLE

In this section, we apply the proposed Algorithm 1 to the point stabilization of a differentially driven wheeled mobile robot, which is especially useful in formation stabilization, where it is desirable that each agent take a predefined position. The reader is referred to [11] for a discussion of cooperative control of mobile robots. A mobile robot of unicycle type is described by the following set of kinematic equations

$$\begin{aligned}\dot{x} &= v\cos\theta \\ \dot{y} &= v\sin\theta \\ \dot{\theta} &= \omega\end{aligned} \tag{18}$$

where x and y are the coordinates of a point located at the mid-axis of the rear wheels of the robot, θ is the heading angle of the robot with respect to the positive x-axis and v and ω are the linear and angular velocity of the robot, respectively. The dynamic equations of the mobile robot can be described as

$$\begin{aligned}M\dot{v} &= F \\ J\dot{\omega} &= \tau\end{aligned} \tag{19}$$

where F and τ represent the force and torque exerted on the robot (control inputs), respectively, and M and J are the mass and moment of inertia of the robot, respectively. A schematic diagram of a mobile robot is shown in Figure 1. Equations (24) and (25), which can also serve as a description for a rotorcraft-like UAV flying at constant altitude, can be transformed into a two-dimensional double integrator using feedback linearization [13].

We consider the coordinates of a point off the centre of the wheel axis, (x_1,x_3), as the output (*e.g.* centre of mass of the robot, see Figure 1). Following a series of manipulations, we end up with the following system

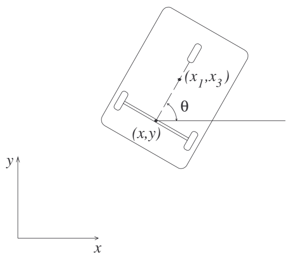

Figure 1. A schematic diagram of a mobile robot of unicycle type

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= u_1 \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= u_2 \end{aligned} \tag{20}$$

where the relationship between the new inputs, i.e. u_1 and u_2 and the actual inputs to the system, i.e. F and τ is given by

$$\begin{bmatrix} \frac{1}{m}F \\ \frac{L}{J}\tau \end{bmatrix} = \begin{bmatrix} L_a\omega^2 \\ -v\omega \end{bmatrix} + \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}^{-1} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{21}$$

In (27), L_a is the distance from the middle of the wheel axis of the robot to the chosen point (x_1,x_3). We take the actual uncertain system under control as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= u_1 + 0.5\sin(tx_1) \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= u_2 + 0.5\sin(tx_3) \end{aligned} \tag{22}$$

with initial conditions chosen to be $\boldsymbol{x_0}$=[6 2 5 -4]^T. We use the system described by (20) as the nominal system for the RHC synthesis and select the execution and prediction horizons to be 0.5 and 5 seconds, respectively. Weighting matrices **Q** and **R** are taken as identity matrices of appropriate dimension. Matrix **P** in (4) is found by solving the Lyapunov equation, (9) in [10].

As can be seen in Figures 2 to 5, the proposed modification in the generated control signal has improved the performance of the system considerably compared to the method pointed out in [5] (referred in the figures as *unmodified*). The introduced disturbances have resulted in some oscillations in the states in the unmodified case using the algorithm described in [5], whereas no oscillation is present when the proposed method of Algorithm 1 was used. In Figure 5 the magnitude of oscillation is growing, which shows that the existing method presented in [5] is not successful in stabilizing the states of the system. The value of $(bh)^2$=0.125 was determined to be negligible compared to the values selected above as required by Proposition 1. As the nominal system (20) is a linear system, Assumption 1 holds. Since the problem is linear with no constraints on the input, the RHC scheme described in [10] can be used, consequently, Assumptions 2 and 3 are satisfied.

The finite horizon open loop optimal control problem was solved numerically using MATLAB, where collocation method (see reference [16]) was utilized. The TPBVP was solved using backward sweep method [9].

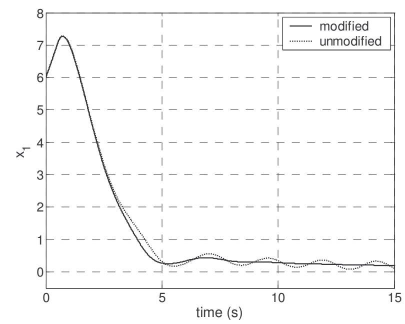

Figure 2. Time history of x_1

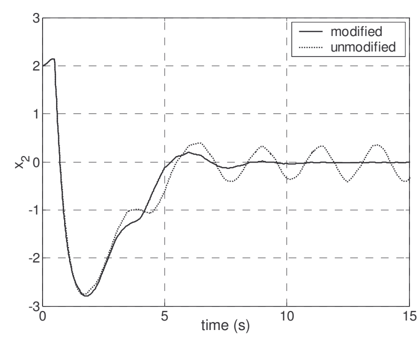

Figure 3. Time history of x_2

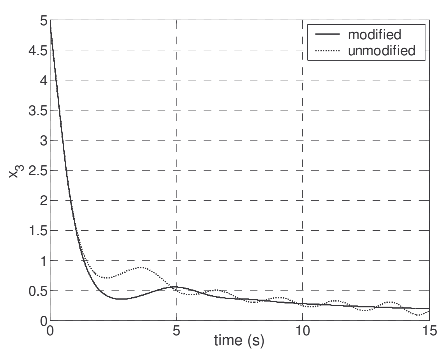

Figure 4. Time history of x_3

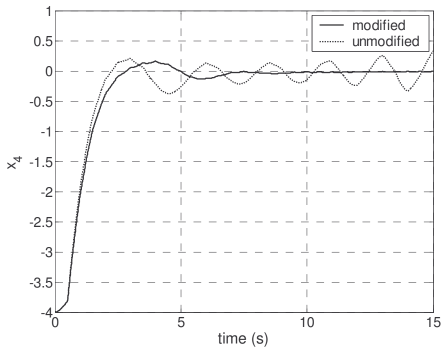

Figure 5. Time history of x_4

VII. Conclusion

A novel Receding Horizon Control strategy for uncertain nonlinear systems was proposed considering the effect of computational delay allowing theories developed with a zero computation time assumption to be used in the practical situations involving computational delays. The approach is composed of state prediction, trajectory generation, and trajectory correction. To allow for the computation of the optimal trajectories, states are predicted at the next sampling time. The predicted values of the states are used as initial conditions for the finite horizon open loop optimal control problem, allowing the optimal input profiles to be computed one sampling time in advance. At the time of implementation, when the new data from the states becomes available, the pre-computed input is modified using the perturbation analysis done off-line. The on-line modification analysis is composed of the solution to two initial value problems, assumed to be computed in negligible time. The proposed method is valid as long as the perturbation in the states and the sampling time are sufficiently small. The method was applied to simulations of a mobile robot of unicycle type, where the proposed method shows significant improvement in performance compared to existing methods.

It is anticipated that these new results will find significant utility for control design of mobile robot and unmanned aerial vehicle (UAV) systems. Future directions for the approach include experimental applications to mobile robot and UAV systems, and the addition of sufficient conditions for robust stability. The generalization of the method for RHC schemes requiring terminal inequality constraints and input constraints will also be addressed.

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