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Sensor fusion by using a sliding observer for an underwater breathing system

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Abstract— **This paper addresses the sensor fusion problem applied to an underwater breathing apparatus. In such systems hardware redundancy is used for safety reasons: in order to overcome any dangerous situations eventually coming from sensor faults, the measurement is taken by using three sensors that measure the same variable (oxygen partial pressure). Thus a sensor fusion scheme is needed in order to feed-back the measurement for controlling purposes. The work presents a fusion algorithm that uses a sliding mode observer for taking into account model information, and combine the different measurements by looking at how they behave with respect to the observer prediction. A forgetting factor approach is used for making more reactive the algorithm. Extensive simulations show the effectiveness of the proposed solution comparing results with a more traditional approach like the voting logic.**

I. Introduction and Problem Motivation

Closed-circuit underwater breathing apparatus, commonly known as ‘rebreather’, is a device that permits completely autonomous diver operations at a very deep depth without an umbilical.

The term closed-circuit rebreather (CCR) refers to the recirculation of the breathable mixture. In order to recirculate gasses, all rebreather concepts include a mouthpiece, through which the diver breathes, connected with a collapsible bag that inflates when he exhales, and deflates when he inhales. This bag is usually called counterlung.

The macroscopic chemical effects on breathed gasses are (partial) oxygen subtraction for metabolic use with a consequent carbon dioxide increase. All other gasses different from oxygen are inert with respect to the respiration process and flow through the lungs without being chemically transformed. This means that exhaled gasses can be recycled (or better re-breathed), provided that oxygen content is restored and carbon dioxide is removed.

To reestablish a breathable mixture, rebreathers must be equipped with a device (usually a chemical scrubber) for CO_2 removal and a supply valve for O_2 injection into the breathing loop. CCR has a feed-back electronic controller that, based on the measure of the oxygen level in the counterlung, injects pure oxygen by operating a solenoid supply valve, so as to regulate oxygen partial pressure to a given set-point value during the entire dive.

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Controlling the oxygen partial pressure during all phases of a dive is a crucial task for the correct and safe use of a CCR. The actual oxygen content in the breathed mixture must be within specific limits by considering that *(i)* the oxygen partial pressure inside the breathing loop should never fall below 0.16 [atm] to avoid hypoxia (it can rapidly bring the diver to unconsciousness [1]); *(ii)* breathing oxygen at high partial pressure (greater than 0.5 [atm]) can be toxic [2] (Central Nervous System (CNS) oxygen toxicity is related to both oxygen pressure level and duration of the exposure [3]).

However O_2 partial pressure in the breathing loop is subject to variations due to disturbances such as the diver individual metabolism, which depends on the workload and the internal pressure of the counterlung which changes with the dive profile.

While variations of O_2 partial pressure due to the depth can be easily predicted, metabolic oxygen consumption rate can vary from person to person by a factor of 6 (or more) in normal conditions, and as much as 10-fold in extreme conditions, depending on the activity level.

In order to keep the oxygen partial pressure at a desired value, it is important to design a controller that is robust to such disturbances. This is crucial since the oxygen set point is chosen as the maximum safe value throughout the dive, thus the non-oxygen portion of the breathing gas (the part that determines decompression obligations) is kept at a minimum. This allows the diver to stay longer at depth without incurring a decompression obligation [4], and also to speed up the decompression process whenever an obligation is incurred.

When working in critical conditions or environments, such as under the sea, the control-loop has to be fault-tolerant. In other words the control strategy of the rebreather system has to be ‘robust’ against sensor failures. An error on the actual oxygen level measurement, especially in the set point proximity, can cause a disease to the diver and may have fatal consequences. Thus, in the counterlung control-loop some hardware redundancy is desired: usually the system output (oxygen partial pressure) is measured through three sensors so that eventual sensor faults can be detected and isolated for the diver safety. Of course, by introducing this redundancy, an algorithm that decides which measurement value to give to the controller is needed. The problem of designing this algorithm is called ‘sensor fusion’ problem.

In this paper a sensor fusion algorithm that uses sensors information as well as model information, is proposed. The scheme is based on a sliding mode observer [5]. The highly nonlinear nature of the plant and the necessity to further implement the sensor fusion scheme in a low price microcontroller suggested to investigate about the use of a deterministic sliding observer instead of a classical Kalman filter (see for example [6] and reference therein).

The performance of the proposed strategy are discussed comparing it with a more traditional approach based on a *decision by majority* criteria [7], [8], usually named as *voting logic*, which is currently implemented in some commercial rebreathers and is a proven technology in some applicative areas as fly-by-wire control [9].

The paper is structured as follows: in Section II the rebreather system model is presented; Section III discusses the fusion problem; in Section IV a sliding mode observer is proposed; in Section V simulations for different and typical fault scenarios show the effectiveness of the approach, and finally Section VI gives conclusions.

II. THE REBREATHER SYSTEM

In this section a physics based analytical model is presented for predicting the oxygen level in rebreather counterlung for various dive profiles and diver activity levels. This open-loop model will be useful to design the state observer that will compute the estimated output and the corresponding measurement residuals.

The model considers the counterlung as an adiabatic collapsible recipient which contains the breathable composed by inert gas (usually nitrogen and/or helium) and oxygen. Notice that, in the rebreather system, complete collapse of the counterlung when external pressure increases is avoided by the action of a demand valve which is triggered when the counterlung is almost completely collapsed. This always guarantees the availability of a minimum breathable gas volume at any depth by the injection of a fresh gas mixture usually called 'diluent gas'. Similarly over-expansion of the counterlung when external pressure decreases is avoided by an overpressure relief valve.

The respiration process is simply modelled as a (partial) oxygen subtraction from the collapsible bag at a time varying rate. No carbon dioxide presence is assumed in the breathable gas, *i.e.* perfect $\mathrm{CO_2}$ adsorption is considered.

According to these assumptions, it is possible to derive the counterlung dynamic model as a balance of volume flow rates [liter/min] (see [10] and [11] for details):

$$\dot{p}_{\mathrm{pO_2}} = \left(\frac{p_{\mathrm{a}}}{V} - \frac{p_{\mathrm{pO_2}} p_{\mathrm{a}}}{p_{\mathrm{e}} V}\right)(s-m) + \frac{p_{\mathrm{pO_2}}}{p_{\mathrm{e}}}\dot{p}_{\mathrm{e}} + + \left(\frac{p_{\mathrm{e}}\beta}{V} - \frac{p_{\mathrm{pO_2}}}{V}\right)s_{\mathrm{dv}}, \tag{1a}$$

$$\dot{V} = \frac{p_{\mathrm{a}}}{p_{\mathrm{e}}}(s-m) - \frac{V}{p_{\mathrm{e}}}\dot{p}_{\mathrm{e}} + s_{\mathrm{dv}} - q, \tag{1b}$$

where $p_{\mathrm{pO_2}}$ is the oxygen partial pressure level [atm], V is the counterlung total volume defined as the sum of the oxygen and inert partial volumes, p_{e} is the hydrostatic pressure [atm], s is the flow of the supply valve [liter/min], m is the oxygen metabolic volume rate consumption [liter/min], p_{a} is the pressure at the sea level [atm], s_{dv} is the demand valve flow [liter/min], q is the exhaust flow through the relief valve [liter/min] and β is the oxygen fraction of the gas mixture supplied by the demand valve. See also Figure 1 for the schematics of a rebreather.

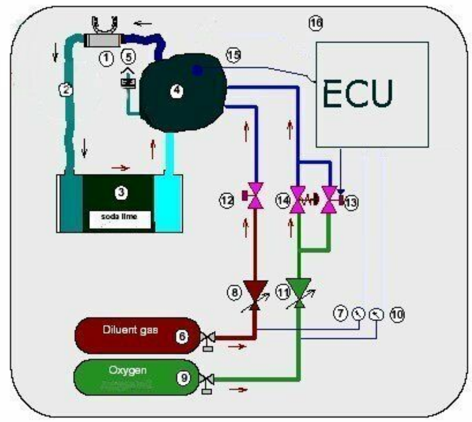

Fig. 1. Schematics of a rebreather. 1) mouthpiece; 2) connecting hose; 3) $\mathrm{CO_2}$ scrubber; 4) inhalation breathing bag; 5) relief valve; 6) gas cylinder diluent; 7) pressure gauge diluent; 8) pressure regulator diluent; 9) gas cylinder $\mathrm{O_2}$; 10) pressure gauge $\mathrm{O_2}$; 11) pressure regulator $\mathrm{O_2}$ 12) demand valve; 13) electrically actuated solenoid valve; 14) bypass valve $\mathrm{O_2}$; 15) $\mathrm{O_2}$ sensors; 16) electronic control unit.

Equations (1) show that the simplified model of the rebreather is nonlinear with respect to state variables and control input. They also show, as expected, that counterlung volume dynamics do affect oxygen partial pressure dynamics while the converse is not true.

Model (1) has been validated through extensive simulations on experimental data collected during different dives. It shows a good agreement with the real scenario and some validation results can be found in [10] and [11].

A. The Closed Loop-Plant

The CCR has to be controlled so that the counterlung total volume excursion is limited in an interval $[\mathrm{V_m}, \mathrm{V_M}]$ and the oxygen partial pressure is around a desired set value.

The first control requirement is achieved by the 'demand valve-relief valve', a pneumatic system which behaves as static nonlinear feedback of the actual volume that is capable of bounding the counterlung volume in spite of external pressure variation and oxygen subtraction/injection.

The second requirement, *i.e.* the regulation of the $p_{\mathrm{pO_2}}$, is achieved by controlling the oxygen flow by the solenoidal valve, thus compensating for metabolic oxygen consumption and variations of external pressure (see equation (1a)). The solenoid valve is an on-off actuator and the maximum value of the supplied flow is a nonlinear function of the external pressure p_{e} (see [11] for details).

The closed-loop plant consists of a PI control law on the $p_{\mathrm{pO_2}}$ measurement, implemented by a pulse width modulation (PWM) technique that takes into account the on-off behaviour of the actuator. It is well known that the integral action of the controller guarantees that, at the steady state behaviour, constant disturbances are fully rejected so when the external pressure and the metabolic consumption rate are constant, the desired set-point is tracked. For the considered plant (see equations (1)) this implies that the control signal balances the metabolic consumption rate in an average sense (considering the PWM implementation).

III. SENSOR FUSION

An accurate determination of oxygen partial pressure is fundamental in underwater breathing apparatus with characteristics of partially or totally recycling the breathed gasses.

Due to the criticity of the application and to the possible high uncertainty on sensor signal and/or sensor failures, rebreather apparatus present usually multiple oxygen sensors. Different measurements are then fused together to realize a more accurate estimation of oxygen partial pressure. In particular the considered rebreather uses three oxygen partial pressure sensors.

One of the simplest and most intuitive general methods of fusion is to take the measurement that is in the middle, *i.e.* $y_1 \leq y = y_2 \leq y_3$ where y is the fused value and y_i are the measured values coming from the sensors and ordered from the minimum to the maximum value. This approach is usually called *voting logic* [7].

In literature other fusion algorithms have been proposed in order to cope with problems that voting logic cannot overcome. Many of these algorithms use model information in order to improve the fusion algorithm and for excluding faulted sensors and avoiding false fault detections.

The model information is usually embedded in a state observer. So the approach is the following: a state observer uses the input signal to the plant and the fused measurement in order to estimate the plant state. Then the observer estimates the output. This estimation is used in order to compute the residuals (difference between the estimated output and the measured value) for each measurement. From residuals the fusion algorithm computes the fused measurement that will be fed-back to the controller and to the observer. Figure 2 explains the proposed approach.

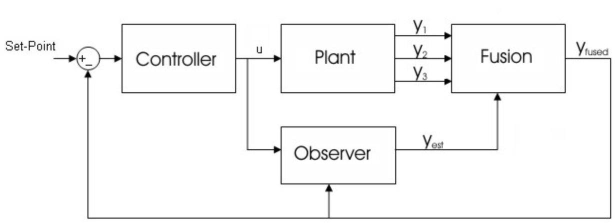

Fig. 2. Sensor fusion scheme.

Usually a Kalman Filter (or Extended Kalman Filter in the case of nonlinear plants) is used as state observer. In this case fusion algorithms use error covariances estimated by the KF in order to compute the fused measurement. In this paper, instead, a deterministic observer is proposed. In this way all the assumptions on the stochastic nature of the noise and the estimation errors are not needed and, moreover, heavy computation requirements coming from the Kalman Filter algorithm (in particular linearization and matrix inversions) are avoided. Due to the nonlinearity of the system the fusion scheme will be implemented by using a deterministic observer that, in particular, is a sliding mode observer as discussed in Section IV.

The fusion algorithm (discrete time) is described in the following. Let $y = \begin{bmatrix} y_1 & y_2 & y_3 \end{bmatrix}^T$ be the measurements vector. The fused value corresponding to such measurements will be a linear combination of them:

$$\tilde{y} = \alpha_1 y_1 + \alpha_2 y_2 + \alpha_3 y_3 = \alpha^T y. \tag{2}$$

Usually the aimed task is to derive a fusion algorithm that optimizes some quality index. For example, in a stochastic framework the objective might be to minimize the variance of the fused measurement and to have an unbiased estimator, as well. In that case it is not difficult to show that the optimal linear estimator correspond to the case in which the weights are chosen as

$$\alpha_j = \frac{\frac{1}{\sigma_j^2}}{\sum_{i=1}^{3} \frac{1}{\sigma_i^2}}, \tag{3}$$

where σ_i^2 is the variance corresponding to the i-th measurement [8], [12].

This paper elaborates such approach and, moving within a deterministic framework rather than a stochastic one, uses the same fusion rule (3) by replacing the uncertainty σ_i with the time average of the so called residual $r_i(k) \triangleq \hat{y}(k) - y_i(k)$ where $\hat{y}(k)$ is the system output estimated by the state observer that uses the system input u and the fused measurement $\tilde{y}$ for solving the estimation problem (see Figure 2). The state observer is interlaced with the fusion algorithm and the estimated output is used as 'true' measurement for computing residuals: the weights are computed as

$$\alpha_j(k) = \frac{\frac{1}{\frac{1}{k}\sum_i^k r_j^2(i)}}{\frac{1}{\frac{1}{k}\sum_i^k r_1^2(i)} + \frac{1}{\frac{1}{k}\sum_i^k r_2^2(i)} + \frac{1}{\frac{1}{k}\sum_i^k r_3^2(i)}}. \tag{4}$$

As a further step, a forgetting factor approach [13] is introduced in order to weight in different way past samples and recent samples. The idea is to derive an algorithm more reactive to sensor changes thus allowing fault detection. In conclusion the weights are computed as

$$\alpha_j(k) = \frac{\frac{k}{\sum_i^k r_j^2(i)\mu^{k-i}}}{\frac{k}{\sum_i^k r_1^2(i)\mu^{k-i}} + \frac{k}{\sum_i^k r_2^2(i)\mu^{k-i}} + \frac{k}{\sum_i^k r_3^2(i)\mu^{k-i}}}, \tag{5}$$

where $0 < \mu \leq 1$ is the forgetting factor.

IV. Sliding Observer

The observer is designed by neglecting the passive non-linear feed-back of the actual volume realized through the 'demand valve- relief valve' actuation (s_{dv} and q terms in (1), see [14] for further details).

Letting $x = [p_{\mathrm{pO_2}}, \mathrm{V}]$ be the state vector and $u = s$ be the control input: the open-loop model equations (1) can be rewritten as

$$\dot{x}_1 = \left(\frac{p_{\mathrm{a}}}{x_2} - \frac{x_1 p_{\mathrm{a}}}{x_2 p_{\mathrm{e}}}\right)(u-m) + \frac{x_1}{p_{\mathrm{e}}}\dot{p}_{\mathrm{e}} \tag{6a}$$

$$\dot{x}_2 = \frac{p_{\mathrm{a}}}{p_{\mathrm{e}}}(u-m) - \frac{x_2}{p_{\mathrm{e}}}\dot{p}_{\mathrm{e}}. \tag{6b}$$

Note that p_{e}, $\dot{p}_{\mathrm{e}}$ and m act as disturbances for the plant. The variables $p_{\mathrm{pO_2}}$ and p_{e} are measured and $\dot{p}_{\mathrm{e}}$ can be computed on-line while m is unknown.

In order to estimate the disturbance m, it is convenient to enlarge the state vector and rewrite the equations (6) as

$$\dot{x}_1 = \frac{p_{\mathrm{a}}}{p_{\mathrm{e}}}\frac{(p_{\mathrm{e}} - x_1)}{x_2}(u - x_3) + \frac{x_1}{p_{\mathrm{e}}}\dot{p}_{\mathrm{e}}, \tag{7a}$$

$$\dot{x}_2 = \frac{p_{\mathrm{a}}}{p_{\mathrm{e}}}(u - x_3) - \frac{x_2}{p_{\mathrm{e}}}\dot{p}_{\mathrm{e}}, \tag{7b}$$

$$\dot{x}_3 = 0, \tag{7c}$$

by assuming a slowly time-varying metabolism.

Starting from equations (7) the sliding mode asymptotic observer uses $p_{\mathrm{pO_2}}$ and p_{e} measurements for estimating the state and can be written as [5]

$$\dot{\hat{x}}_1 = \frac{p_{\mathrm{a}}}{p_{\mathrm{e}}}\frac{(p_{\mathrm{e}} - \hat{x}_1)}{\hat{x}_2}(u - \hat{x}_3) + \frac{\hat{x}_1}{p_{\mathrm{e}}}\dot{p}_{\mathrm{e}} + \\ + k_1 \operatorname{sgn}(x_1 - \hat{x}_1), \tag{8a}$$

$$\dot{\hat{x}}_2 = \frac{p_{\mathrm{a}}}{p_{\mathrm{e}}}(u - \hat{x}_3) - \frac{\hat{x}_2}{p_{\mathrm{e}}}\dot{p}_{\mathrm{e}} + \\ + k_2 \operatorname{sgn}(x_1 - \hat{x}_1), \tag{8b}$$

$$\dot{\hat{x}}_3 = k_3 \operatorname{sgn}(x_1 - \hat{x}_1). \tag{8c}$$

From equations (7) and (8) the error dynamics are

$$\dot{e}_1 = (\dot{x}_1 - \dot{\hat{x}}_1) = \Delta(x, \hat{x}, u) - k_1 \operatorname{sgn}(e_1), \tag{9a}$$

$$\dot{e}_2 = (\dot{x}_2 - \dot{\hat{x}}_2) = -\frac{p_{\mathrm{a}}}{p_{\mathrm{e}}}e_3 - \frac{\dot{p}_{\mathrm{e}}}{p_{\mathrm{e}}}e_2 + \\ - k_2 \operatorname{sgn}(e_1), \tag{9b}$$

$$\dot{e}_3 = (\dot{x}_3 - \dot{\hat{x}}_3) = -k_3 \operatorname{sgn}(e_1), \tag{9c}$$

with

$$\Delta(x, \hat{x}, u) = -\tfrac{p_{\mathrm{a}}}{p_{\mathrm{e}}}\left[\tfrac{e_3}{\hat{x}_2}(p_{\mathrm{e}} - \hat{x}_1)\right] + \tfrac{\dot{p}_{\mathrm{e}}}{p_{\mathrm{e}}}e_1 + \\ - \tfrac{p_{\mathrm{a}}}{p_{\mathrm{e}}}\left[\tfrac{e_1}{\hat{x}_2}(u - x_3) + \tfrac{e_2}{x_2\hat{x}_2}(p_{\mathrm{e}} - x_1)(u - x_3)\right]. \tag{10}$$

The sliding manifold is $e_1 = 0$, while the reaching condition can be written as

$$\eta + |\Delta(x, \hat{x}, u)| \leq k_1 \tag{11}$$

with an arbitrary constant $\eta > 0$. Along the sliding manifold it holds $\dot{e}_1 = 0$. In this situation, according to the Filippov conditions [15], from equation (9a) $\Delta(x, \hat{x}, u)$ can be written as a convex combination of the values $+k_1$ and $-k_1$ and then, on the sliding manifold, it follows

$$\operatorname{sgn}(e_1) = \frac{\Delta(x, \hat{x}, u)}{k_1}. \tag{12}$$

Taking into account equation (12), during the sliding motion the following error dynamics occur

$$\dot{e}_2 = -\frac{p_{\mathrm{a}}}{p_{\mathrm{e}}}e_3 - \frac{\dot{p}_{\mathrm{e}}}{p_{\mathrm{e}}}e_2 - \frac{k_2}{k_1}\Delta(x, \hat{x}, u)|_{e_1=0}, \tag{13a}$$

$$\dot{e}_3 = -\frac{k_3}{k_1}\Delta(x, \hat{x}, u)|_{e_1=0}. \tag{13b}$$

In order to give a constant convergence rate to the error dynamics, the observer gains k_2 and k_3 can be chosen in the following suitable way

$$k_2 = \frac{k_1}{p_{\mathrm{a}}}\frac{\hat{x}_2}{p_{\mathrm{e}} - \hat{x}_1}\gamma_2(\hat{x}, u), \tag{14a}$$

$$k_3 = -\frac{k_1}{p_{\mathrm{a}}}\frac{\hat{x}_2}{p_{\mathrm{e}} - \hat{x}_1}\gamma_3(\hat{x}, u). \tag{14b}$$

These gains are functions of the estimated state and the known and measurable signals like the control input and the external pressure. Furthermore the γ functions will be designed as in Appendix A for coping with the dependence of the error dynamics from the input signal u.

Inserting equations (14) in (13), finally it comes out

$$\dot{e}_2 = -\frac{p_{\mathrm{a}}}{p_{\mathrm{e}}}e_3 - \frac{\dot{p}_{\mathrm{e}}}{p_{\mathrm{e}}}e_2 + \frac{\gamma_2(\hat{x}, u)}{p_{\mathrm{e}}} \times \\ \times \left[\frac{e_2}{x_2}(u - x_3) + e_3\right], \tag{15a}$$

$$\dot{e}_3 = -\frac{\gamma_3(\hat{x}, u)}{p_{\mathrm{e}}}\left[\frac{e_2}{x_2}(u - x_3) + e_3\right]. \tag{15b}$$

The convergence of the observation error has been proven in [14] and is briefly reported in the Appendix for sake of completeness.

V. Simulations

In order to validate the proposed fusion scheme, three scenarios are considered:

- **Saturation**: one or two sensors have a saturation behaviour at a some given value (critical values close to the set-point have been chosen);
- **Gain**: one of the sensors has a gain error;
- **Peak**: one sensor gives a peak value as measurement and then it always measures zero value. Immediately after that one more sensor presents the same fault. In practice only one sensor works within this scenario fault.

Simulations are carried out after discretizing the observer described in Section IV and simulating it with the fusion algorithm on the closed loop rebreather system. In the case of one sensor fault (gain scenario and saturation scenario) both the proposed fusion algorithm and a more traditional voting logic work quite well. In the case of twofold fault, when only one sensor works (peak scenario and two sensors saturation scenario) the voting logic for its own nature cannot but fail. In fact, in the case of two sensors that saturate the 'good' measurement is always the maximum value that is discarded by the logic and the control does not achieve its task (see Figure 3). On the other hand the fusion algorithm works well detecting and rejecting the faulted measurements, so that the control objective is fulfilled (see Figure 4).

This behaviour repeats in the peak scenario when only one sensor works well and the voting logic fails as reported in Figure 5. Moreover the proposed fusion algorithm continues to work well (see Figure 6). In Figure 7 the weights α_i corresponding to the different sensors are reported and it appears clear how the fusion algorithm detects which sensor is faulted and removes it from the fusion scheme by putting to zero the corresponding weight.

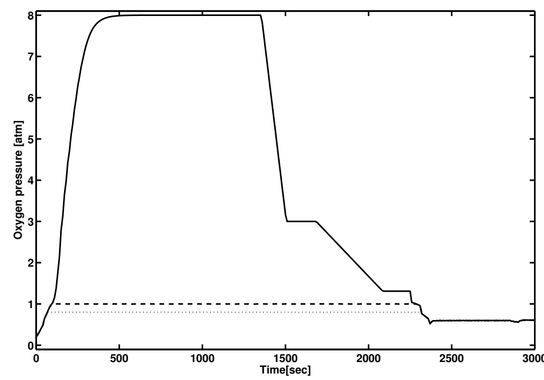

Fig. 3. Saturation scenario (two faulted sensors) with voting logic: output of faulted sensors (dotted and dashed line); true value (solid) and fused value that follows one faulted sensor (dotted).

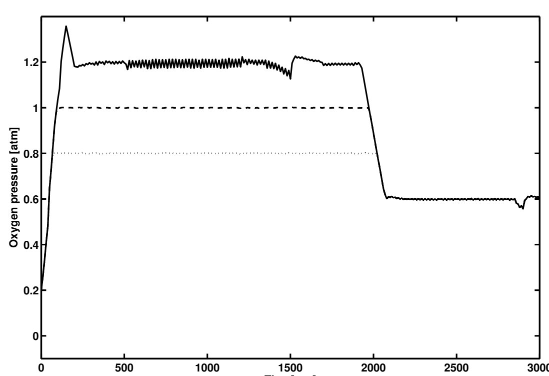

Fig. 4. Saturation scenario (two faulted sensors) with fusion algorithm: output of faulted sensors (dotted and dashed line); true value (solid) and fused value that follows the true value.

VI. Conclusions

For safety reasons, in underwater breathing system it is mandatory that the control strategy, which regulates the oxygen content into the breathable mixture, is 'robust' against sensor failures, or in other words, fault-tolerant. In fact an error on the actual oxygen level measurement not simply deteriorates the control performances, but can cause a disease to the diver and may have fatal consequences.

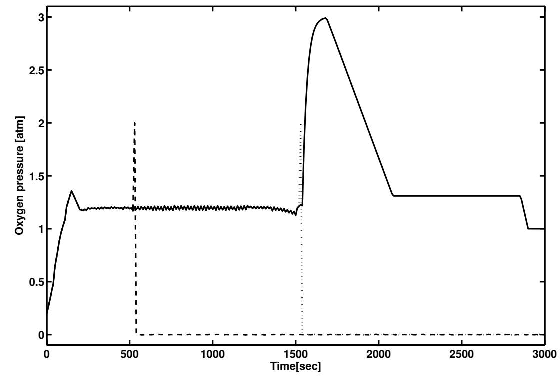

Fig. 5. Peaks and death scenario with voting logic: output of faulted sensors (dotted and dashed line); true value (solid) and fused value that follows one faulted sensor (dotted).

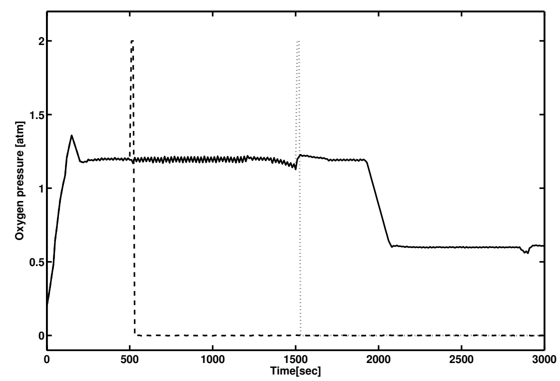

Fig. 6. Peaks and death scenario with fusion algorithm: output of faulted sensors (dotted and dashed line); true value (solid) and fused value that follows the true value.

To this aim in this paper it has been investigated the use of a sliding observer in a sensor fusion scheme that merges redundant sensors measurements as well as model information.

The effectiveness of the approach has been shown through simulations. The proposed strategy has been also compared with a more traditional approach based on a *decision by majority* criterion, showing a significant improvement. On such premises hardware in the loop simulations are currently under investigation aimed to the final implementation of the algorithm on a prototipal rebreather system.

Acknowledgments

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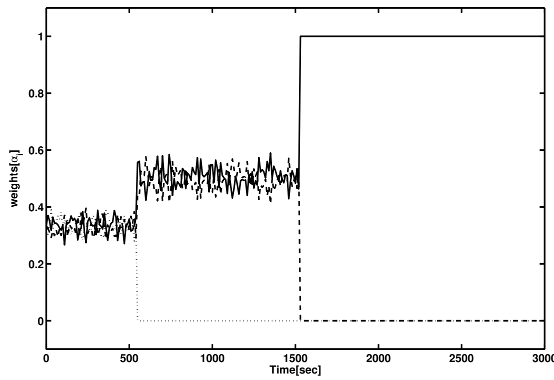

Fig. 7. Peaks and death scenario with fusion algorithm: weights corresponding to the faulted sensors (dotted and dashed line); weight corresponding to the working sensor (solid).

APPENDIX

A. Convergence of the observation error

Once the sliding manifold $e_1 = 0$ is reached, the reduced dynamics are given by equations (15). A stability analysis aimed to guaranteeing the asymptotic stability of the reduced error system will be carried out by neglecting environmental disturbance. In fact, letting $\dot{p}_{\mathrm{e}} = 0$ in (15), the reduced error system is

$$\begin{aligned} \dot{e}_2 &= -\frac{p_{\mathrm{a}}}{p_{\mathrm{e}}} e_3 + \frac{\gamma_2(\hat{x}, u)}{p_{\mathrm{e}}} \left[\frac{e_2}{x_2}(u - x_3) + e_3 \right] && \text{(16a)} \\ \dot{e}_3 &= -\frac{\gamma_3(\hat{x}, u)}{p_{\mathrm{e}}} \left[\frac{e_2}{x_2}(u - x_3) + e_3 \right]. && \text{(16b)} \end{aligned}$$

For this addressed problem the following Lyapunov function has been chosen

$$V(e_2, e_3) = \frac{1}{2} c_2 e_2^2 + \frac{1}{2} c_3 e_3^2, \tag{17}$$

giving a time derivative

$$\begin{aligned} &\dot{V}(e_2, e_3) = -e_2^2 \left[-c_2 \gamma_2(\hat{x}, u) \frac{(u - x_3)}{p_{\mathrm{e}} x_2} \right] + \\ &-e_3^2 \left[c_3 \frac{\gamma_3(\hat{x}, u)}{p_{\mathrm{e}}} \right] + \\ &\underbrace{-e_2 e_3 \left[c_2 \frac{p_{\mathrm{a}}}{p_{\mathrm{e}}} - c_2 \frac{\gamma_2(\hat{x}, u)}{p_{\mathrm{e}}} + c_3 \gamma_3(\hat{x}, u) \frac{(u - x_3)}{p_{\mathrm{e}} x_2} \right]}_{\Phi}. \end{aligned} \tag{18}$$

In order to show that $\dot{V}(e_2, e_3)$ is negative definite, firstly it will be neglected the term Φ in (18). It has to be remarked that in closed-loop plant, for the actuator characteristics, the input signal $u(t)$ can assume only two values $u = 0$ or $u = u_{\max} > x_3(t) > 0$. In this way, by choosing a proper switching law for γ_2 as

$$\gamma_2(\hat{x}, u) = \begin{cases} \bar{\gamma}_2 > 0 & u = 0, \\ -\bar{\gamma}_2 < 0 & u = u_{\max}, \end{cases} \tag{19}$$

the term multiplying e_2^2 in equation (18) is always positive. Now, setting $\gamma_3(\hat{x}, u) = \bar{\gamma}_3 > 0$, the second term in the right hand side of equation (18) is always negative. By these choices of the γ functions, the observer stability is showed when the hypothesis of the negligibility of the Φ term holds.

Some engineering considerations can be made in order to validate such assumption. The PI controller output described in Section II-A is modulated by using a PWM technique. As previously highlighted, in steady state behaviour (at a fixed depth) the effectiveness of the control action guarantees $u = x_3$ in an average sense. Taking into account this feature, equation (16b) shows that $e_3(t)$ converges to zero independently on the volume estimation error $e_2(t)$. Consequently the term Φ in equation (18) is negligible after a transient depending on the observer gain γ_3 (see equation (16b)).

Note that the stability analysis has been investigated in the absence of environmental disturbances. The robustness of the observer with respect to depth variations has been checked through extensive simulations and some examples are showed in [14].

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