Proceedings of the
44th IEEE Conference on Decision and Control, and
the European Control Conference 2005
Seville, Spain, December 12-15, 2005

Robotic Interaction Through Compliant Interfaces: Modelling and Identification

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***Abstract*— In this work, a new model, able to emulate the interaction of a robot with the environment when viscoelastic phenomena occur, is presented. The starting point of this research activity has been the study of soft pads for dexterous manipulation but the results obtained are applicable to all situations which foresee the interaction of a robotic manipulator through/with a viscoelastic medium (e.g. robotic surgery). Based on the so called quasi-linear model, adopted to describe the behavior of human hand pads and, more generally, of biological tissues, this model allows to overcome the problems tied to classical (linear) models, often used in the robotic field. A procedure aiming to obtain the parameters of this nonlinear model is developed on the basis of classical identification methods and of physical insights. Finally, through experimental tests on some viscoelastic materials (a polyurethane gel and a silicon rubber) the model is validated.**

I. INTRODUCTION

The problems of viscoelasticity and of nonlinear elasticity have been neglected in the robotic field for many years. The reason is straightforward; in the recent past, robotic manipulators were very rigid structures interacting with a world rigid as well.

Nowadays, the design of robotic systems (and in particular of robot hands) inspired to human beings or to other biological examples, leads to the adoption of soft covers over endoskeletal rigid structures [1]. This makes the modelling of the robotic hand interacting with the environment (through the compliant layer) quite complex. A number of analytical and computational models have been proposed in literature in order to describe the behavior of soft materials, but design and simulation of controllers for robotic manipulation are still tied to classical linear spring-damper models, e.g. [2], [3] among many others.

Previous experimental activities on materials for robotic finger-pads [4] and human tissues as well [5] have demonstrated that they are strongly nonlinear and characterized by dynamical effects (relaxation and creep phenomena), which can not be taken into account by means of linear tools. Aim of this paper is to develop a viscoelastic model of such pads, which exhibits a behavior quite close to the real one but, at the same time, is enough simple for real-time simulation and theoretical analyses.

Another promising area of application of the proposed model is that of the robotic medicine (and in particular surgery), where a standard rigid robot interacts with compliant viscoelastic tissues. The use of such a model could be used to improve the rendering of biological tissues by means of haptic interfaces.

In this paper, after an initial introduction about viscoelasticity and characterization of nonlinear compliance, a model (belonging to the class of Hammerstein models) is developed and used to characterize some viscoelastic pads. From experimental tests, a procedure aiming to estimate the parameters model is devised and validated.

II. THE NONLINEAR VISCOELASTICITY

Many materials (e.g. polymers) exhibit hysteresis, relaxation and creep phenomena when they are deformed and loaded by external forces [6]. Such a behavior is defined viscoelastic since the elastic response is affected by time-dependent phenomena that are characteristic of viscous materials. The constitutive equations, suitable to describe this behavior, are usually used to relate strain, stress and time in uniaxial state of stress; nevertheless, for the purposes of this work, it is more convenient to refer to force and displacement, instead of stress (force per unit area) and strain (deformation per unit length), making still reference to uniaxial state. In the general case the material behaves nonlinearly and force (F), displacement (δ) and time (t) are involved in extremely complex mathematical representations.

A general relation is the integral representation, which, in the linear case, relates the input displacement function $\delta(t)$ and the output force $F(t)$, according to

$$F(t) = \int_0^t \Psi(t-\tau) \cdot d[\delta(\tau)] \qquad (1)$$

where Ψ is called relaxation function. Note that, in general, Ψ is a function of both displacement and time $\Psi(\delta, t)$. In order to extend the use of (1) to nonlinear systems, a significant hypothesis has been formulated by Fung [7]. He assumes that the relaxation function has the form

$$\Psi(\delta, t) = F^{(e)}(\delta) \cdot g(t) \quad \text{with} \quad g(0) = 1 \qquad (2)$$

where $F^{(e)}(\delta)$ is the *elastic response*, which is the amplitude of the force instantaneously generated by a displacement δ, while $g(t)$, called *reduced relaxation function*, describes the time-dependant behavior of the material.

The force produced by an infinitesimal displacement $d\delta(\tau)$, superposed in a state of displacement δ at an instant of time τ is, for $t > \tau$,

$$dF(t) = g(t-\tau)\frac{\partial F^{(e)}[\delta(\tau)]}{\partial \delta}d\delta(\tau) \tag{3}$$

By applying a modified superposition principle, discussed in [6], the total force at the instant t is the sum of contribution of all the past changes, i.e.

$$F(t) = \int_{-\infty}^{t} g(t-\tau)\frac{\partial F^{(e)}[\delta(\tau)]}{\partial \delta}\frac{\partial \delta(\tau)}{\partial \tau}d\tau \tag{4}$$

Equation (4) can be rewritten in the form

$$F(t) = \int_{0}^{t} g(t-\tau)\ K^{(e)}[\delta(\tau)]\ \dot{\delta}(\tau)d\tau \tag{5}$$

where the lower limit of integral has been changed assuming that the motion starts at time t=0 and $F^{(e)} = 0$, $\delta = 0$ for $t < 0$; the term $K^{(e)}(\delta) = \frac{\partial F^{(e)}[\delta]}{\partial \delta}$ is the elastic stiffness and $\dot{\delta}(\tau)$ is the rate of displacement.

A. The reduced relaxation function $g(t)$

Assuming the Fung's hypotheses, expressed by (2), the relaxation function $g(t)$ is a decreasing continuous function of the time, normalized to 1 at $t = 0$. It is composed by a linear combination (with the coefficient c_i depending on the material) of exponential functions, whose exponents ν_i identify the rate of the relaxation phenomena, i.e.

$$g(t) = \sum_{i=0}^{r} c_i e^{-\nu_i \cdot t} \quad \text{with} \quad \sum_{i=0}^{r} c_i = 1 \tag{6}$$

The parameters ν_i ($i = 1 \ldots r$) depend on the behavior of the system under analysis, while $\nu_0 = 0$.

B. The elastic response $F^{(e)}(\delta)$

From the definition of the reduced relaxation function $g(t)$, $F^{(e)}$ can be approximated by the force response in a loading experiment with a sufficiently high rate of displacement, without inducing shock waves.

In Fig. 1 the force responses of two pads of different materials to constant speed deformations $\delta(t)$ are reported. When the speed grows such responses tend to converge to a unique characteristic: the elastic response.

The nonlinear elastic response can be modelled through different analytical expressions. Two are the most significant models of the elastic stiffness adopted in literature:

$$K^{(e)}(\delta) = m \cdot e^{b\delta} \tag{7}$$

$$K^{(e)}(\delta) = p \cdot \delta^{q} \tag{8}$$

where (m, b) and (p, q) are parameters which depend on the material and the geometry taken into account. The expression of $F^{(e)}(\delta)$ directly descends from the integration of (7) or (8), with respect to δ:

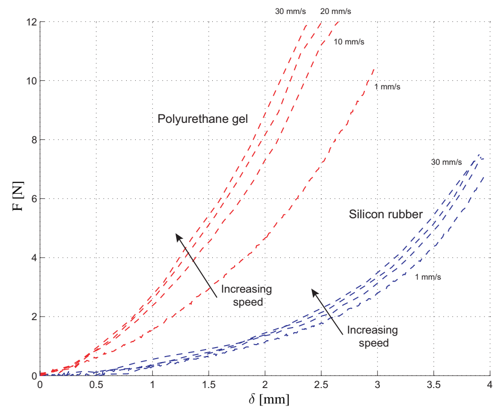

Fig. 1. Force response F vs. indentation depth δ for silicon rubber and polyurethane gel, at different velocities.

$$F^{(e)}(\delta) = \frac{m}{b}(e^{b\delta} - 1) \tag{9}$$

$$F^{(e)}(\delta) = \frac{p}{q+1}\delta^{q+1} \tag{10}$$

Pawluk and Howe [5] exploit (7) to model the relationship between normal force and normal displacement for human finger indentation. Han and Kawamura [8] compared the human finger stiffness with that of artificial fingers using both (7) and (8). Kao and Yang [4], starting from previous research results, derived an expression for nonlinear stiffness of soft contact that can be associated with (8) [9].

III. The quasi-linear model: a block-scheme representation

The quasi-linear model, introduced in the previous section, has a simple and meaningful interpretation. Equation (4) describes the signal $K^{(e)}(\delta)\ \dot{\delta}$ filtered by a linear system represented by $G(s) = \mathcal{L}\{g(t)\}$, where $\mathcal{L}$ denotes the Laplace transform.

By noticing that the input signal of the linear filter is rewritable in the form

$$K^{(e)}(\delta)\ \dot{\delta} = \frac{\partial F^{(e)}(\delta)}{\partial \delta}\dot{\delta} = \frac{dF^{(e)}(\delta(t))}{dt} \tag{11}$$

being the derivative a linear operator, from (5) and (11) it comes out that it exists a linear relation between the elastic response $F^{(e)}(\delta)$ and the force produced by the system. Such a relationship is given by the transfer function

$$\begin{aligned} G_L(s) &= s\left(\frac{c_0}{s} + \frac{c_1}{s+\nu_1} + \frac{c_2}{s+\nu_2} + \ldots + \frac{c_r}{s+\nu_r}\right) \\ &= \frac{\left(\sum_{i=0}^{r} c_i\right)\ s^r + \ldots + c_0\left(\prod_{i=0}^{r}\nu_i\right)}{s^r + \ldots + \left(\prod_{i=0}^{r}\nu_i\right)} \end{aligned} \tag{12}$$

where, for both the numerator and the denominator, only the coefficients of terms of maximum and minimum degree are explicitly reported. It is worth to notice that the relative

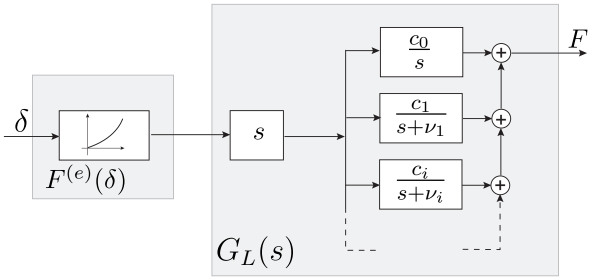

Fig. 2. Block diagram of the quasi-linear model: static nonlinear part and dynamic linear function.

degree of the function $G_L(s)$ is zero, therefore, when a step (of amplitude $\widehat{F^e}$) is applied to the system, its response will be discontinuous and will start from $\widehat{F^e}$ (in (12) note that $\sum_{i=0}^{r} c_i = 1$).

In Fig. 2 the complete model of a viscoelastic material is reported; it is composed by two elements, connected in cascade, which are representative of the two main phenomena characterizing a viscoelastic pad:

- a nonlinear static block which has the displacement δ as input and provides the instantaneous elastic response $F^{(e)}$;
- a linear dynamic block $G_L(s)$, which takes into account the (typically slow) dynamic behavior of the material.

IV. From the quasi-linear model to the Hammerstein model

The response of quasi-linear model and its capabilities of reproducing the actual behavior of viscoelastic pads, depends on the order r of the filter $G_L(s)$. A large value for r produces an estimation of the force $F(t)$ due to the displacement $\delta(t)$ very close to the real one, but, on the other hand, requires the identification of a large number of parameters (c_i, ν_i, $i = 1 \ldots r$). In practice, when r is greater than 2, a direct estimation of such parameters results prohibitive, and the use of a digital filter, whose general form is

$$G_L(z^{-1}) = z^{-k} \frac{b_0 + b_1 z^{-1} + \ldots + b_l z^{-l}}{1 + a_1 z^{-1} + \ldots + a_r z^{-r}} \tag{13}$$

seems preferable.

Therefore, from the scheme of Fig. 2 descends the model reported in Fig. 3, where the continuous transfer function $G_L(s)$ has been replaced by the digital filter $G_L(z^{-1})$. Note that, from the consideration reported in Sec. III, it follows that:

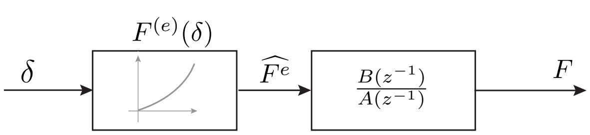

Fig. 3. Hammerstein model of the pads.

- no pure delay exists in the systems between the input (of the linear systems) $\widehat{F^e}$ and the output F ($k = 0$);
- the response is depending on the input at the same instant; therefore the orders of the numerator of $G_L(z)$ and that of the denominator must be assumed equal ($l = r$).

Finally, it is worth to notice that in the general case the model, composed by a cascade of a nonlinear block and of a linear one, includes a redundancy in the parameters definition. As a matter of fact, the gain of overall system results from the product of the gain of nonlinear characteristic and of that of the linear part. In our case, this ambiguity has been solved by assuming that in (13) the coefficient b_0 is equal to 1. In this way, the initial value of F to a unit step input

$$F(0^+) = \lim_{z \to +\infty} G_L(z) \frac{1}{1 - z^{-1}} = b_0$$

is the same of that produced by $G_L(s)$

$$\lim_{s \to +\infty} s(G_L(s)\frac{1}{s}) = \lim_{s \to +\infty} sG(s) = g(0^+) = 1$$

and, accordingly, the (physical) meaning of the elastic response $F^{(e)}$ remain unchanged.

The scheme of Fig. 3 reproduces the well-known Hammerstein model, frequently adopted to model and make the identification of nonlinear systems [10]. This structure is quite interesting, since it is composed by a no-memory nonlinear gain and a dynamic linear block, resulting particularly suitable for identification. To this purpose a number of techniques have been proposed in the literature [11], [12], even if they mostly consider the nonlinear characteristic expressed in a polynomial form. Therefore, besides the expressions of $F^{(e)}$ given by (9) and (10), another relation, which is worth to take into account, is:

$$F^{(e)} = \sum_{i=0}^{m} \alpha_i \delta^i \tag{14}$$

The techniques used to determine the parameters of an Hammerstein model can be classified in two main categories:

- noniterative methods (including overparameterization and stochastic methods) [13], [14];
- iterative methods [12].

A. Noniterative methods

These identification techniques are based on a minimization of the prediction error. By assuming that the output of the Hammerstein model is given by:

$$F(t) = \frac{B(z^{-1})}{A(z^{-1})}\ F^{(e)}(\delta) + e(t) \tag{15}$$

where, with a little abuse of notation, $B(z^{-1})$ and $A(z^{-1})$ denotes a linear combination of backward shift operators (e.g. $z^{-1}u(t) = u(t-1)$), and the error $e(t)$, which model the disturbances, is assumed to be independent (white) noise. The prediction of the output is given by:

$$\overline{F(t,\theta)} = \frac{B(z^{-1})}{A(z^{-1})}\ F^{(e)}(\delta)$$

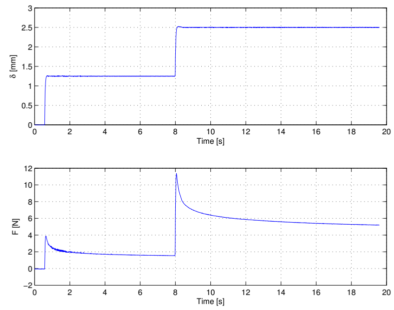

Fig. 4. Data samples used for the identification process of polyurethane model.

with the unknown parameters

$$\theta = [a_1, \ldots, a_r, b_1, \ldots, b_r, \gamma_1, \gamma_2]^T$$

where a_i and b_i are shown in (13), while γ_i characterize the nonlinear elastic response $F^{(e)}$, which can be represented by (10) (in this case, $\gamma_1 = \frac{p}{q+1}$, $\gamma_2 = q + 1$), or (9) ($\gamma_1 = m$, $\gamma_2 = b$), or (14) ($\gamma_1 = \alpha_0$, $\gamma_2 = \alpha_1$).
Given a data set Z^N properly chosen (in our case, two vectors containing N samples of δ and F) the goal is to select $\theta^\star$ so that the prediction error

$$\varepsilon(t, \theta) = F(t) - \overline{F(t, \theta)}$$

for $t = 1 \ldots N$, become as small as possible.
To this purpose we define the (quite standard) criterion function

$$V(\theta, Z^N) = \frac{1}{N} \sum_{t=1}^{N} \varepsilon^2(t, \theta) \tag{16}$$

and we search for the value of $\theta^\star$ by minimizing (16), with respect to θ:

$$\theta^\star = \arg\min_\theta V(\theta, Z^N) \tag{17}$$

The nonlinear least-square problem above explained cannot be solved by analytical methods, but requires the use of iterative numerical techniques, which starting from an initial guess of θ try to find the value $\theta^\star$ that provides the minimum $V(\theta, Z^N)$ (for a detailed overview of the techniques available in the literature refer to [15]).
In Fig. 4 the data samples exploited for the identification process are shown. In particular, in this and in the following experiments two finger pads of different materials (a silicon rubber and a polyurethane gel) are considered. They are excited by means of a sequence of steps, or better, by the approximation which a linear motor makes possible, i.e. a sequence of very fast ramps followed by constant position phases (see Fig. 5).

The identification is performed by considering the expression of the elastic response given by (10) and with linear filters $G_L(z)$ of different orders r. The results are shown in Fig. 6, while the parameters achieved are summarized in Tab.I. It is worth to notice that in the case $r = 3$, the identification process converges to a local minimum and

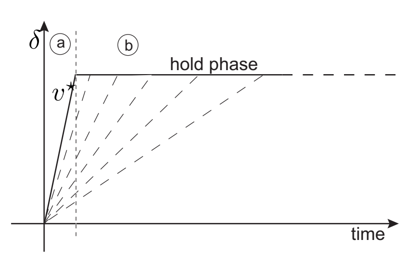

Fig. 5. Deformation trajectory used in the experiments.

the response of the model is quite different from the real one. Moreover, it comes out that a model with a linear filter $G_L(z)$ of the first order is not adequate to describe the behavior of this strongly viscoelastic material.
This iterative method, which considers at the same time both the nonlinear static part and the linear dynamic one, leads to a loss of the physical meaning of the model. As a matter of fact the elastic response $F^{(e)}$, which represents the instantaneous response of the pad to a some displacement, should be the same in any case ($r = 1, \ldots, 4$). But, if we consider the values of $F^{(e)}$ achievable with the parameters γ_i reported in Tab. I, the result is quite different. The identification process leads to considerably different estimations of the elastic response even if the global estimation of the overall response is very satisfactory.

B. Iterative methods

In order to overcome the problems tied to a noniterative estimation methods (i.e. convergency to local minima, loss of physical meaning) iterative procedures seem more adequate. These methods search for the parameters of the model, by separating the estimation problem of the linear and of the nonlinear static parts. In this way, the nonlinearity are lumped in reduced dimension problem (easily solvable),

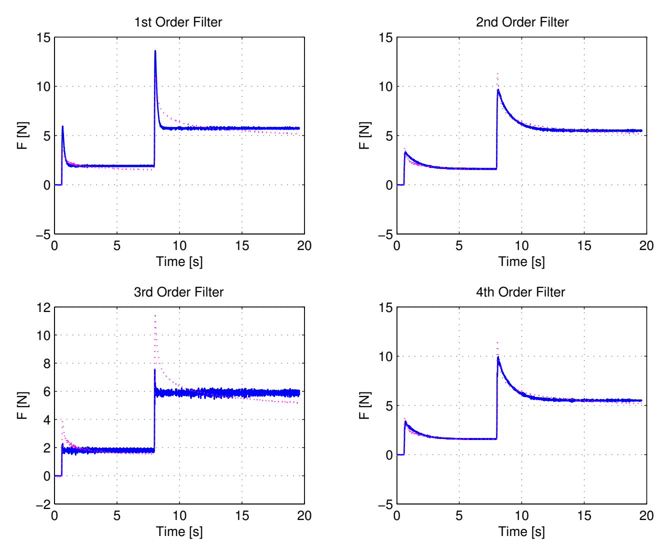

Fig. 6. Responses of estimated polyurethane models (solid) of order r ($r = 1, \ldots, 4$) compared to the real one (dashed). The nonlinear function (10) is adopted.

r	a_1	a_2	a_3	a_4	b_1	b_2	b_3	b_4	γ_1	γ_2	$V(\theta^\star, Z^N)$
1	-0.9261	-	-	-	-0.9823	-	-	-	5.6118	1.5845	0.2703
2	-0.0874	-0.8945	-	-	1.0084	-1.9949	-	-	1.4559	1.7708	0.0759
3	1.7249	0 .6706	-0.0526	-	2.3938	-0.2924	-1.7025	-	2.9531	1.7036	0.5074
4	-0.8787	0.4685	-0.1295	-0.4400	-0.3142	3.1638	2.7338	-6.5291	0.3939	1.8003	0.1079

TABLE I

COEFFICIENT VALUES OF THE HAMMERSTEIN MODEL FOR POLYURETHANE GEL, DEDUCED BY MEANS OF A NONITERATIVE METHOD. THE NONLINEAR FUNCTION (10) IS ADOPTED.

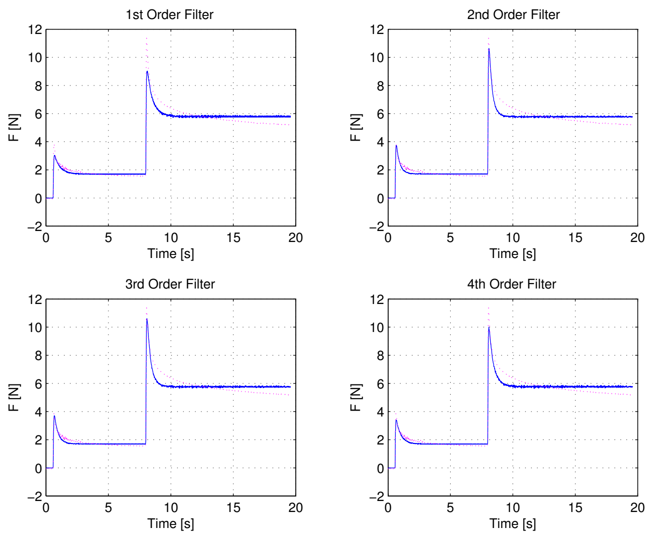

Fig. 7. Responses of of polyurethane models estimated by iteration (solid) compared to the real one (dashed). The nonlinear function (9) is adopted.

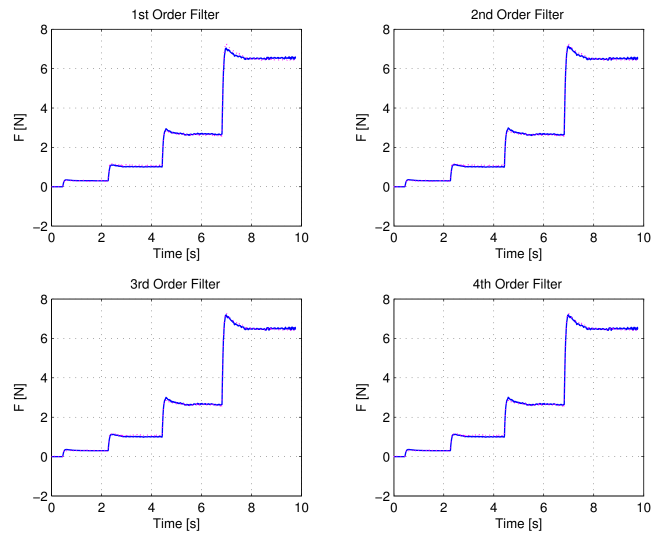

Fig. 8. Responses of silicon rubber models estimated by iteration (solid) compared to the real one (dashed). The nonlinear function (9) is adopted.

while the identification of the linear subsystem can be treated with standard methods, i.e. by modelling this dynamic parts as an ARX system described by:

$$A(z^{-1})F(t) = B(z^{-1})\ \widehat{F^e}(t) + e(t)$$

where $e(t)$ is a disturbance, represented by a zero-mean white noise signal.

The methods is composed by two main steps which have to be iterated:

- By supposing that the coefficients of the two polynomials $A(z^{-1})$ and $B(z^{-1})$ are known, the criterion function (16) is minimized with respect to the parameters γ_i which characterize the nonlinear elastic response $F^{(e)}$ and the optimal values $\gamma_i^\star$ are found.
- On the basis of $\gamma_i^\star$ and of the adopted expression of the elastic response, the input $\widehat{F^e}$ (corresponding to the data samples $\delta(t)$) for the linear system is computed. From $\widehat{F^e}$ and the actual output of the system $F(t)$ the coefficient a_i, b_i of the ARX system are estimated (by standard linear least-square methods).

This procedure is iterated until the values of the parameters converge.

The results (corresponding to the coefficient values reported in Tab. II) obtained for the polyurethane gel are reported in Fig. 7. Note that, in this case, the identification procedure converges to satisfactory values of the coefficient for every r. Moreover, it is worth to underline that the parameters γ_i are very similar and, accordingly, the elastic response $F^{(e)}$ is correctly identified.

The same procedure has been applied to the identification of the model parameters for silicon rubber. The results, shown in Fig.8 are appreciable also in this case. In particular, it is interesting to observe that a first order linear system provides a very good estimation of the pad response. This was predictable, since the behavior of such material is only lightly viscoelastic.

The iterative identification procedure, can be optimized from a computational point of view, by noticing that, with reference to the trajectory reported in Fig. 5 (and supposed enough fast), the response of the system coincides with the elastic response $F^{(e)}$ during the rising ramp (phase (a)) while depends only on the linear filter (the output of the nonlinear static part remains unchanged) during the hold phase (b). It is therefore possible to use the iterative methods above described, limiting the used data set to the (very short) phase (a) for the non linear part identification (and considering $b_i = 0$ and $a_i = 0$ for $i = 1, \ldots, r$) and using the overall period (a)+(b) only for the estimation of the linear subsystem parameters (in this case the identification procedure is available in a closed form with a limited

r	a_1	a_2	a_3	a_4	b_1	b_2	b_3	b_4	γ_1	γ_2
1	-0.9786	-	-	-	-0.9889	-	-	-	2.0766	0.7029
2	-1.0662	0.0984	-	-	-0.5556	-0.4243	-	-	2.2445	0.6911
3	-1.0907	0.2616	-0.1385	-	-0.4918	-0.4854	-0.0037	-	2.1199	0.6948
4	-1.0061	0.1043	-0.1735	0.0992	-0.4546	-0.6155	-0.2687	0.3513	1.9471	0.7038

TABLE II

COEFFICIENT VALUES OF THE HAMMERSTEIN MODEL FOR POLYURETHANE GEL, DEDUCED BY MEANS OF AN ITERATIVE METHOD.THE NONLINEAR FUNCTION (9) IS ADOPTED.

r	a_1	a_2	a_3	a_4	b_1	b_2	b_3	b_4	γ_1	γ_2
4	-1.1942	0.2149	0.0257	-0.0261	-0.8502	-0.2029	-0.0844	0.1464	2.4507	0.5439

TABLE III

COEFFICIENT VALUES OF THE HAMMERSTEIN MODEL FOR POLYURETHANE GEL, DEDUCED BY MEANS OF A FAST ITERATIVE METHOD. THE NONLINEAR FUNCTION (9) IS ADOPTED.

computational burden). This procedure has been applied to the polyurethane gel pad, and the results are reported in Fig. 9 (while the values of the coefficients are shown in Tab. III). In this case, only one iteration has been performed; as a matter of fact, the hypothesis on the initial value of the coefficients of the linear filter ($b_i = 0$ and $a_i = 0$ for $i = 1, \ldots, r$) does not affect the estimation of the elastic response parameters since, as highlight in Sec. II, for enough high velocity, $F^{(e)}$ is not influenced by the dynamic behavior of the system.

V. CONCLUSIONS

In this work, a dynamic model for the interaction of robots and the environment through a viscoelastic interfaces is developed. The model can be used to characterize, in a precise and quantitative way, the behavior of compliant pads, which can be adopted for robotic devices and, in particular, for bio-inspired robotic hands. Moreover, it provides a valid (simple, but, at the same time precise) simulation tool.

Some procedures for the estimation of model parameters are proposed and tested. They are suitable for an online implementation of the estimator, and can be profitably used in telemanipulation and haptic rendering.

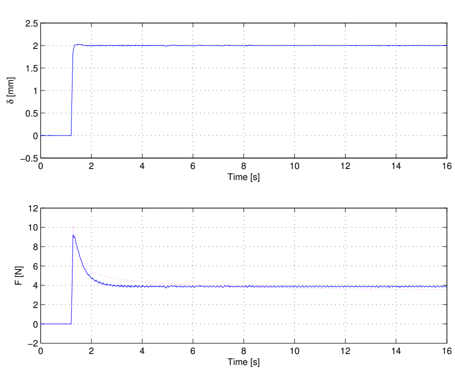

Fig. 9. Responses of polyurethane models estimated by a fast iterative method (solid) compared to the real one (dashed). The nonlinear function (9) is adopted.

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