Proceedings of the
44th IEEE Conference on Decision and Control, and
the European Control Conference 2005
Seville, Spain, December 12-15, 2005

Optimization of fresh–food supply chains in uncertain environments: an application to the meat-refrigeration process

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Abstract— **In this paper we present a novel approach for the optimization of fresh-food supply chain by managing a trade-off between logistics and some index measuring the quality of the food itself. We propose an hybrid model of the chain, with a part constituted by a event–driven dynamics (the supply chain itself) and a part with time–driven dynamics (the dynamics of some parameters characterizing the jobs in the supply chain). The proposed framework is applied to a real–world example relative to the process of beef–carcasses refrigeration and distribution.**

I. INTRODUCTION

The distribution of fresh food products such as meat, vegetables, fruits and dairy from producer to vendor is in general a complex process, owing to the perishable nature of these agricultural products.

A distribution (or supply) chain is a sequence of activities performed in order to deliver a product to a destination with the highest possible quality. Any activity has a potential impact on the product due to the interaction between the action environment and the product.

In some cases, i.e. when a product is stored into a cell together with other products, not only the interaction between the product and the surrounding environment is important, but also the interactions between the various products. Each product can be seen as an "object" described by a dynamical model which takes into account the physiological processes occurring in the product. These processes are generally affected by the conditions in the immediate environment of the product. On the other hand, the processes may themselves affect the immediate environment.

The objective is to describe the product behavior as a collection of interacting processes, such that their combined action describes the observed phenomenon and such that each subprocess can be fully understood in its description. The nature of the subprocess is largely defined by means of fundamental physical laws and the generally accepted rules in a particular discipline.

A typical approach, see e.g. [16], consists in separating high–level processes analyzing intrinsic product properties that, in general, correspond to quality attributes in the phenomenon under study.

As well explained in [8], the first step in the design of a node of a supply chain (e.g. a chiller or a storage cell) is for the user to draw up specifications. In fact, in many cases, poor design in existing networks is mainly due to a mismatch between the purpose the nodes were originally designed for and how they are actually used. A proper sizing of network throughput and node loads could be in fact a difficult task without the aide of ad-hoc simulation tools.

In this paper we present a novel approach for the optimization of fresh-food supply chain by managing a trade-off between the logistics of the food treatment, for instance the food delivery date against the quality of the food itself. A quite similar approach can be found for instance in [4], where the authors optimize a manufacturing process that performs a sequence of operations on a set of jobs. Such operations modify some physical characteristic of the jobs. However, the focus of this paper is somewhat different, and there are some distinctive features that distinguish the two works. In fact, in our approach, the single nodes can process more than one job at a time and are supposed to have infinite capacity, and therefore we do not consider queues before the single nodes. More importantly, we consider the situation in which not only the operations performed in the nodes modify some physical characteristic of the single job, but the jobs may influence each other and also the way the nodes process the jobs themselves.

II. PROBLEM DESCRIPTION AND NOTATION

In this section, we present the formulation of the general problem of optimizing a food supply chain. The first step consists in constructing a mathematical model of the chain. The model we propose is an hybrid model, with a part constituted by a event–driven dynamics (i.e., the supply chain itself) and a part with time–driven dynamics (i.e., the dynamics of some parameters characterizing the jobs in the supply chain).

We consider a supply-chain configuration depicted in Fig. 1, consisting in a network formed by n successive nodes in which the different jobs are processed in a sequential way. To keep the formalism at a general level, we will use the term

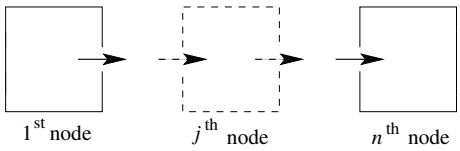

Fig. 1. General structure of the problem: n sequential nodes.

"job" to refer to the generic portion of food to be treated (which can be for example a pallet of fresh-fruits, a meat

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carcass, etc.), and the term "nodes" to refer to the different servers in which different activities are performed on the product (e.g. refrigeration or storage cells, transportation, etc.).

The total number of jobs that enter the chain is m. The jobs enter in the first cell at different time instants (and with possibly different initial conditions) and are processed sequentially from node 1 to node n. Every node has the possibility to process more than one job at a time and, for the sake of simplicity, it is supposed to possess infinite capacity. Hence, when a job has been processed by one node, it immediately leaves it and enters the next one.

To the i^{th} job we associate a measure of goodness y_i, that we call *attribute*, and which represents some characteristics (e.g. temperature, firmness, ripening, microbial charge, etc.) that we choose as representative of the goodness of the product. The attributes $y_i(t)$ of the different jobs vary in time, according to a differential equation which depends on the operating conditions of the network and the different cell j where the job is currently processed.

On the other hand, the supply-chain is characterized by a vector of parameters $\theta \in \mathbb{R}^q$ that represent the operating conditions under which the network is running. Some of these parameters may be directly imposed by the network manager, and can therefore be considered as *control variables*. Other parameters are instead not directly accessible. In a general setting, these uncontrollable parameters may be either fixed (deterministic) and possibly not perfectly known, or stochastic parameters subject to random variations. Examples of such parameters may be transportation times, power of already existing plants, etc.. Hence, we partition the parameter vector θ into two vectors, i.e.

$$\theta = \begin{bmatrix} \theta_C \\ \theta_{NC} \end{bmatrix}$$

where $\theta_C \in \mathbb{R}^{q_C}$ contains all the controllable parameters, while $\theta_{NC} \in \mathbb{R}^{q_{NC}}$ contains the remaining (uncontrollable) ones.

Summarizing, the system under consideration results is an intrinsically hybrid system, which sees the co–existence of the time-driven dynamics of the parameters $y_i(t)$ and the event–driven dynamics of jobs which move in the supply chain.

Discrete–events dynamics.

If we denote by τ_i^j the time instant in which the i^{th} job departs from the j^{th} node, then we can formalize the discrete–event dynamics by means of the following recursive equation, for $i = 1, \ldots, m$ and $j = 1, \ldots, n$

$$\begin{aligned} \tau_i^j &= \tau_i^{j-1} + s^j(\theta), \\ \tau_i^0 &= a_i \end{aligned} \tag{1}$$

where a_i denotes the arrival time of the i^{th} job in the first node. The function $s^j(\theta)$ gives the time the job i stand in the node j; this time is a function of the parameters θ.

Hence, the vectors $\tau_i \in \mathbb{R}^n$, $i = 1, \ldots, m$,

$$\tau_i \doteq [\tau_i^1 \quad \tau_i^2 \quad \cdots \quad \tau_i^n]^T$$

contain the times when the i^{th} job switches between nodes. For notation ease, we also define the composite vector $\tau \doteq [\tau_1^T \quad \cdots \quad \tau_m^T]^T$ that we call the *vector of events*.

The position (node) $p_i(t)$ in which the i^{th} job is at time t can be therefore obtained as a function of τ_i. In particular, we may represent the evolution of the variables $p_i(t)$, $i = 1, \ldots, m$ using a simple differential equation

$$\begin{aligned} \dot{p}_i(t) &= \sum_{j=0}^{n-1} \delta(t - \tau_i^j) \\ p_i(0) &= 0 \end{aligned} \tag{2}$$

where $\delta(\cdot)$ is the Dirac delta function.

Clearly, the variable p_i can assume only the values $0, 1, 2, \ldots, n$ and is a monotonically nondecreasing function of time. An example of this situation is depicted in Fig. 2.

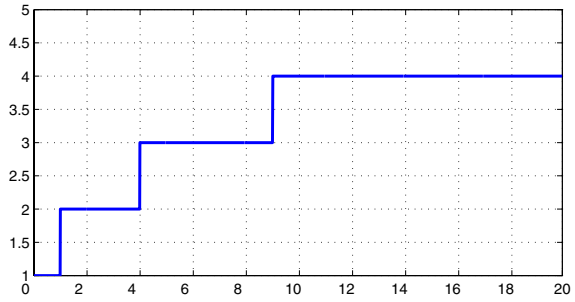

Fig. 2. A generic trajectory of $p_i(t)$.

Then, at each time instant t, the number of jobs in the j^{th} node can be computed as

$$m^j(t) = \sum_{i=1}^{m} \delta_K(j, p_i(t)) \tag{3}$$

where $\delta_K(i, j)$ is the Kronecker delta function[1]. Equation (3) indicates that the number of jobs in the j^{th} node increases when a generic job i leaves the node $j-1$ at τ_i^{j-1} and enters the node j and decreases when a jobs leaves the j^{th}cell at τ_i^j.

Time–Driven Dynamics

We denote with $y = [y_1 \quad \cdots y_m]^T$ the vector of attributes. As already mentioned, the attribute $y_i(t)$ relative to the i^{th} job evolves in time according to a differential equation. More precisely, $y(t)$ can be seen as the output of a system of differential equations

$$\begin{aligned} \dot{x}(t) &= f(x(t), \tau, \theta) \\ y(t) &= g(x(t), \tau, \theta) \end{aligned} \tag{4}$$

with initial conditions $x(0) = \zeta_0$, being x a vector gathering the state variables of the different products and the states variables that describe the interaction with the surrounding

[1]The Kronecker delta function is defined as

$$\delta_K(i, j) = \begin{cases} 1 \text{ if } i = j \\ 0 \text{ ow.} \end{cases}$$

environment. Notice that equations (1) and (4) define in all aspects an hybrid system, where the time–driven dynamics of (4) depend on the vector of events τ, whose dynamics are expressed by the recursion (1). We remark also that, looking at the problem from a different viewpoint, (4) is in effect a switching system, in which the switching times are regulated by the recursion (1). This formulation will be made more clear in the example of Section IV.

III. OPTIMIZATION

The goal of the optimization algorithm is to choose the controllable parameters θ_C in order to minimize a performance function that takes into account a desired behavior of the network. In particular, we consider a performance function J that is a sum of three terms

$$J(\theta) = J(\theta_C, \theta_{NC}) = \mathcal{C}(\theta) + \mathcal{P}(\theta) + \mathcal{D}(\theta).$$

The first term takes into account the cost related to the particular operating condition θ_C (e.g. power consumption, transport costs, etc.). The second term contains the performance requirement relative to the product attributes, that could be expressed in different ways depending on the specific product. For instance, we may consider

1. trajectory tracking: $\|y_i(t) - \tilde{y}(t)\|$,
2. final value objective: $\|y_i(\tau_i^n) - \bar{y}_i\|$,

where $\tilde{y}(t)$ and $\bar{y}_i$ are respectively the desired trajectory and the final value references. Since the behavior of $y(t)$ depends on the operating conditions θ, requirements 1. e 2. are traduced in a cost term $\mathcal{P}(\theta)$. The third term $\mathcal{D}(\theta)$ is related to the logistic aspects of the chain, such as due date (which measures the difference between the actual final time τ_i^n and the desired one $\bar{\tau}_i$) or deadlines.

From a practical point of view, the operating conditions in general are constrained and bounded in specific intervals. This corresponds to assume the variable θ_C to be belong to a compact set Θ_C, i.e. $\theta_C \in \Theta_C$.

In the case when the noncontrollable parameters θ_{NC} assume fixed values, the optimization problem simply writes

$$\min_{\theta_C \in \Theta_C} J(\theta_C, \theta_{NC}). \tag{5}$$

However, in a more general setting, θ_{NC} may be affected by random uncertainty, that is

$$\theta_{NC} = \bar{\theta}_{NC} + \Delta_\theta$$

where $\bar{\theta}_{NC}$ represents a "nominal" value and Δ_θ is a random variable with zero mean and associated probability measure $\mathbb{P}$. Hence, the problem needs to be formulated in a stochastic framework. A frequently used requirement in this context is to optimize an "average" instance of the problem: in other words, one asks to minimize the expected value of the objective function taken with respect to the random uncertain parameters θ_{NC} (see for instance [13]), that is

$$\min_{\theta_C \in \Theta} E(\theta_C), \quad E(\theta_C) \doteq \mathbb{E}_{\theta_{NC}}[J(\theta_C, \theta_{NC})] \tag{6}$$

where $E(\theta_C)$ is the expectation of $J(\theta_C, \theta_{NC})$ taken with respect to the probability measure $\mathbb{P}$.

The above stochastic optimization problem is in general very hard to solve, since the mere evaluation of the expected value $E(\theta_C)$ (even if an analytical expression of the cost function was available, which is not our case), would require the solution of a multiple integral. Recently however, approaches based on uncertainty randomization have proven their efficacy, see e.g. [19], [20]. Here, following a similar philosophy, we consider a "empirical" version of the expectation

$$\hat{E}(\theta_C) \doteq \frac{1}{N} \sum_{i=1}^{N} J(\theta_C, \bar{\theta}_{NC} + \Delta_\theta^{(i)})$$

where $\Delta_\theta^{(i)}$ are uncertainty samples drawn according to the probability measure $\mathbb{P}$, and make use of this approximation for the solution of the optimization problem. In particular, we follow the solution approach proposed by Spall (see e.g. [17], [18]) and tackle the problem via a simultaneous–perturbations stochastic approximation (SPSA) approach. In details, the SPSA algorithm is based on a recursive algorithm in which successive approximations of the optimal value

$$\theta_C^* \doteq \arg\min_{\theta_C \in \Theta} E(\theta_C)$$

are recursively constructed based on noisy observations of the cost function.[2] The recursion mimics a classical gradient descent method, in which the gradient w.r.t θ_C of the functional $E(\theta_C)$ is approximated at each step using only two (possibly noisy) evaluations of the cost function. Formally, let $\theta_C^{(k)}$ denote the k^{th} estimate of the minimum, and let $\{\eta^{(k)}\}$ be a random sequence of column random vectors where $\eta^{(k)} = [\eta_1^{(k)} \quad \eta_2^{(k)} \quad \cdots \eta_{n_C}^{(k)}]^T$ are not necessary identically distributed. The two-sided SPSA algorithm to update $\theta_C^{(k)}$ is constructed as follows

$$\theta_C^{(k+1)} = \Pi_\Theta \left[\theta_C^{(k)} - \alpha^{(k)} \left[{\eta^{(k)}}^{-1} \right] \frac{\hat{E}_+^{(k)} - \hat{E}_-^{(k)}}{c^{(k)}} \right]$$

where $\Pi_\Theta[\cdot]$ is the projection operator on the set Θ_C, $c^{(k)}$, is a positive sequence converging to zero, $\alpha^{(k)}$ is the step–size multiplier, and $\left[{\eta^{(k)}}^{-1} \right]$ is defined as the vector containing the inverses of the elements of $\eta^{(k)}$. The values $\hat{E}_\pm^{(k)}$ represent the empirical cost function taken at parameter values $\theta_C^k \pm c^{(k)}\eta^{(k)}$, i.e.

$$\hat{E}_\pm^{(k)} \doteq \hat{E}(\theta_C^k \pm c^{(k)}\eta^{(k)}).$$

Various convergence results of this algorithm have been proven under different hypothesis, see for instance [18], [7], [9] and references therein. In particular, it can be shown that the algorithm still converges when the empirical mean is constructed with a very small number of samples. Indeed, even a single sample is sufficient, thus allowing to

[2] In other words, we assume to have "noisy measurements" of our cost function $E(\theta_C)$, i.e. $E(\theta_C) = \hat{E}(\theta_C) + \nu$, where ν is a random variable with zero mean.

significantly simplify the algorithm, generating at each step a single instance $\Delta_\theta^{(k)}$ of the uncertainty and letting

$$\hat{E}_\pm^{(k)} = J(\theta_C^k \pm c^{(k)}\eta^{(k)}, \bar{\theta}_{NC} + \Delta_\theta^{(k)}).$$

In order to illustrate the advantages of the proposed framework, in the next section we propose a case study based on a real-world example.

IV. A CASE STUDY: THE MEAT SUPPLY CHAIN

The example we consider comes from a real case study, related to the process of the beef–carcasses refrigeration and distribution in the post-slaughter supply chain. The example is based on real data collected in the year 2001 in the North of Italy.

The beef carcasses, after weighting and labelling, stay in a pre–chilling tunnel, followed by storage cells where they complete refrigeration. After storage, carcasses are transferred in the loading area and transported in a refrigerated vehicle. After transport, the meat reaches the retailer, see [10].

Careful control is required to achieve conditions that will reduce the carcass temperature in the designed time cycle. This has to be carried out in the most economic manner taking into account energy consumption, being aware that the temperatures assumed by the carcass during the refrigeration and distribution phases highly influences the final quality of the meat, as shown for instance in [3], [12] and [14].

To this extent, EU regulations require that all meat temperatures within the carcass must be reduced below 7°C before the carcass is further processed or moved from the chiller. However, according a national dispensation [21], the product can be transported before it reaches the internal temperature of 7°C for short and fast displacements. As a consequence of this regulation, chilling can take place also on trucks and in the final destination cell.

To summarize, the supply–chain considered in this example is depicted in Fig. 3 and consists in a sequence of four nodes:

- the pre–chilling tunnel,
- the refrigeration cell,
- the transportation,
- the destination (or retailer) cell.

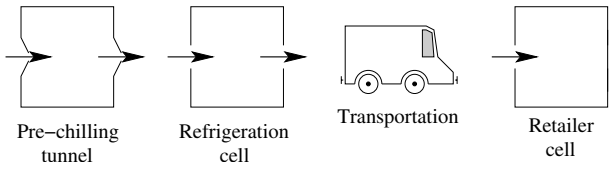

Fig. 3. Case study: the meat refrigeration supply chain.

The pre–chilling tunnel is used to promote a rapid drop in surface temperature, while the other subsequent stages are necessary to carry-on refrigeration avoiding surface freezing, eventually allowing for product displacement. When the product reaches the destination cell (the retailer), it stays there until it is sold. In our case, the slaughter, i.e. the pre–chilling tunnel and the first refrigeration cell, was situated in the Cuneo province, Piemonte, while the destination cell was situated in the Liguria region, Italy.

To follow the formalism introduced in Section II, we modelled the single beef carcass as a distinct job, and assumed as attribute parameter $y_i(t)$ the internal temperature of the i^{th} job. In general, it is very difficult to relate temperature measurements obtained in a non–destructive way with the internal meat temperature, especially when considering real carcasses and not meat-samples with a simplified geometry, see for instance [11], Chapter 14. To overcome this difficulty, we adopted a single temperature measurement for each beef–carcasse, obtained inserting a 10 cm long probe orthogonally into the muscle[3].

The data were collected using Hobo data-loggers (Onset Computer Corporation, CA) equipped by two external probes. The first probe was used to measure the room temperature $T_C^j(t)$, while the second one was used to determine the mean meat temperature in a muscle from the surface to a 10 cm depth. Being the Hobo data-logger small, equipped with autonomous battery, it was easy to hook it on a single meat mass. This instrument set-up did not yield any inconvenience for the operators during transport and storage.

The j^{th} refrigeration cell is powered by a chiller unit which ensures an energy flux Φ^j [W]. We assume that the air flow conditions inside the refrigeration cell ensure an even air distribution and an homogeneous inside air temperature. This should be the case of any well designed cell. The consequence of uneven air distribution has been outlined in [5] and [15]. In Fig. 4 we depict the typical situation of the

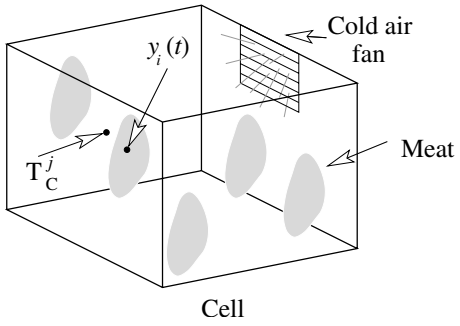

Fig. 4. Schematic representation of a refrigeration cell.

j^{th} cell. We denote by $T_C^j(t)$ the (homogeneous) temperature inside the cell.

To model the interaction between the different carcasses and the surrounding environment, we introduce the thermal flux from the carcasses to the air as

$$\Phi_T^j = \sum_{i=1}^{m} hS(y_i - T_C{}^j)\delta_K(j, p_i(t))$$

where h is the surface heat transfer coefficient [W/m^2°C] and S [m^2] is the averaged surface of a carcass. The effects of air

[3]This corresponds in practice to consider an average temperature of the carcass.

speed and turbolence are taken into account via the parameter h. We also remark that, to be rigorous, in this formula the temperature on the surface of the carcass should be used. Being this information unavailable, we use instead the average temperature y_i. Being these two temperatures very close, we assume the error introduced by this approximation to be negligible. Assuming the volume V [m^3] of the air in the cell to be greater than the volume occupied by the carcasses[4], we have

$$\rho V C_p \frac{\partial T_C^j}{\partial t} = \Phi_T^j - r(T_C^j)\Phi^j$$

where ρ and C_p are, respectively, the density [kg/m^3] and the specific heat capacity [J/kg°C] of the air inside the cell and Φ^j is the power of the j^{th} refrigerator, and the binary function

$$r(T_C^j) = \begin{cases} 1 & \text{if } T_C^j \geq T_{min} \\ 0 & \text{if } T_C^j < T_{min} \end{cases}$$

switches-off the power whenever the cell temperature drops beneath the value T_{min}. The relationship between the air temperature $T_C^j(t)$ and the (average) internal temperature $y_i(t)$ of the i^{th} carcasse was directly estimated from the available measurements. This allowed to reduce the number of physical parameters to be estimated. In the work [6], this relation has been estimated in terms of a discrete–time ARX model. Here, we used the same data to identify a continuous–time second order state–space model describing the transfer function between $T_C^j(t)$ and $y_i(t)$

$$\begin{aligned} \dot{z}_i(t) &= Az_i(t) + b_i T_C^j(t) \quad i = 1, \ldots, m \\ y_i(t) &= Cz_i(t) \quad i = 1, \ldots, m \end{aligned}$$

where

$$A = \begin{bmatrix} -1.897e-5 & 8.056e-5 \\ -1.45e-5 & -2.348e-3 \end{bmatrix},$$

$$b = \begin{bmatrix} -3.160e-8 \\ 2.504e-6 \end{bmatrix}, \quad C = \begin{bmatrix} 355.4916 & -1.6368 \end{bmatrix}.$$

The above equations lead to the following system of differential equations, for $i = 1, \ldots, m$ and $j = 1, \ldots, n$

$$\begin{cases} \dot{p}_i(t) = \sum_{j=0}^{n-1} \delta(t - \tau_i^j) \\ \dot{T}_C^j(t) = \frac{1}{\alpha}\left[hS\sum_{i=1}^{m}(y_i(t) - T_C^j(t))\delta_K(j, p_i(t)) - r(T_C^j)\Phi^j\right] \\ \dot{z}_i(t) = Az_i(t) + B_i T_C(t) \\ y_i(t) = Cz_i(t) \end{cases} \tag{7}$$

where $\alpha = \rho V C_p$, and

$$\begin{aligned} T_C &= [T_C^1 \quad T_C^2 \quad \cdots T_C^n]^T \\ B_i &= b[\delta_K(1, p_i(t)) \quad \delta_K(2, p_i(t)) \quad \cdots \quad \delta_K(n, p_i(t))]. \end{aligned}$$

The (switching) times τ_i^j evolve according to the recursion (1), that is

$$\begin{aligned} \tau_i^j &= \tau_i^{j-1} + t_i^j \\ \tau_i^0 &= a_i \end{aligned} \tag{8}$$

[4]In this way, the total volume of air can be assumed constant even when the number of carcasses in the different cells varies.

where t_i^j is the time the job i stays in the cell j.

The hybrid structure of the model results clearly from the above equations: the time–driven equation (7) is a time–varying linear model whose dynamics change in correspondence of the events governed by the recursion (8). Notice also that both equations (7) and (8) depend on the operating conditions, namely the power of the cells Φ^j and the time t_i^j spent in the different cells by the single jobs.

Figure 5 exemplifies the features of the system, depicting the behavior of a single cell and two carcasses. Initially, a single carcass is present in the cell. After 12 hours a second carcass enters the cell and after 24 hours the first one leaves the cell. It can be seen that the temperature behavior of a carcass and of the cell itself is influenced[5] by the entrance/exit of another element in the cell. Notice also that in the real simulation, in order to reduce chattering effect the function $r(T_C^j)$ was substituted by an hysteresis function.

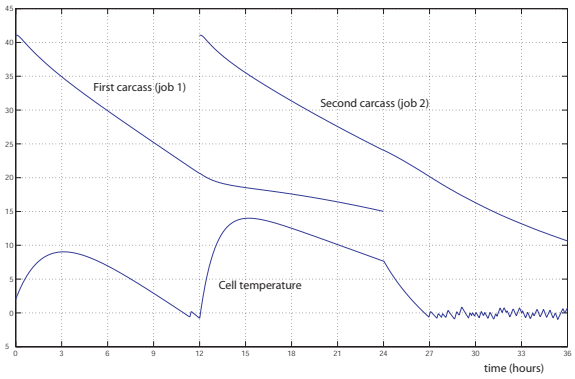

Fig. 5. Simulation showing the multibody nature of the system.

Following the framework introduced in Section II, we collect the operating conditions of the entire chain in a single vector θ. Some elements of the vector θ can be manipulated whereas the others represents intrinsic characteristics of the system. In the case under study, the network manager has the possibility to control the power in the refrigeration cell (node 2), and the time the single products spend in the pre–chiller and in the refrigeration cell (nodes 1, 2). Therefore, the vector of controlled variables becomes

$$\theta_C = \begin{bmatrix} \Phi^2 & t_1^1 \cdots t_m^1 & t_1^2 \cdots t_m^2 & t_1^4 \cdots t_m^4 \end{bmatrix}^T.$$

The other components of θ are not controllable by the manager of the chain. In particular, in our case, the power of the different refrigeration was fixed due to technological constraints. Transportation times $t_1^3 \cdots t_m^3$ are instead modeled as random variables whose statistics has been derived from measurements collected during experimentation

$$\theta_{NC} = \begin{bmatrix} \Phi^1 & \Phi^3 & \Phi^4 & t_1^3 \cdots t_m^3 \end{bmatrix}^T.$$

The cost function of this example was chosen to take into account processing costs, product attributes and due-

[5]To enhance this fact in this simulation a low power for the cell was chosen.

date specifications:

$$\mathcal{C}(\theta) = \beta^{\mathrm{T}}\theta_C + \gamma_1 f(y_i(\tau_i^4)) + \gamma_2 \sum_{i=1}^{m} (\tau_i^n - d_i)^2$$

where $\beta \in \mathbb{R}^{qC}$, γ_1 and $\gamma_2 \in \mathbb{R}$ are appropriately chosen weights and the function

$$f(x) = \begin{cases} -x - 2 & \text{if } x \leq -2 \\ 0 & \text{if } -2 < x < 7 \\ (x-7)^2 & \text{if } x \geq 7, \end{cases}$$

reflect the requirement that final temperature of each carcass should be between 2°C and −7°C. In this study we neglect costs related to the weight loss due to water evaporation at the meat surface.

Optimization results

To solve the optimization problem we applied the algorithm described in Section III starting from a random initial point $\theta^{(0)}$. The parameters of the algorithms required a tuning process that lead to the values

$$\alpha^{(k)} = \left(\frac{1}{k+10}\right)^{0.45}, \quad c^{(k)} = \left(\frac{1}{k}\right)^{1.1}.$$

The vectors $\eta^{(k)}$ where chosen as Bernoulli processes.

For visualization purposes, we present here the results of an examples consisting of 5 different carcasses entering the first cell at different randomly generated times. The SPSA algorithm converged after $K = 1000$ iterations to an optimal configuration $\theta^{(K)}$. In Figure 6 we show the simulation results relative to this configuration.

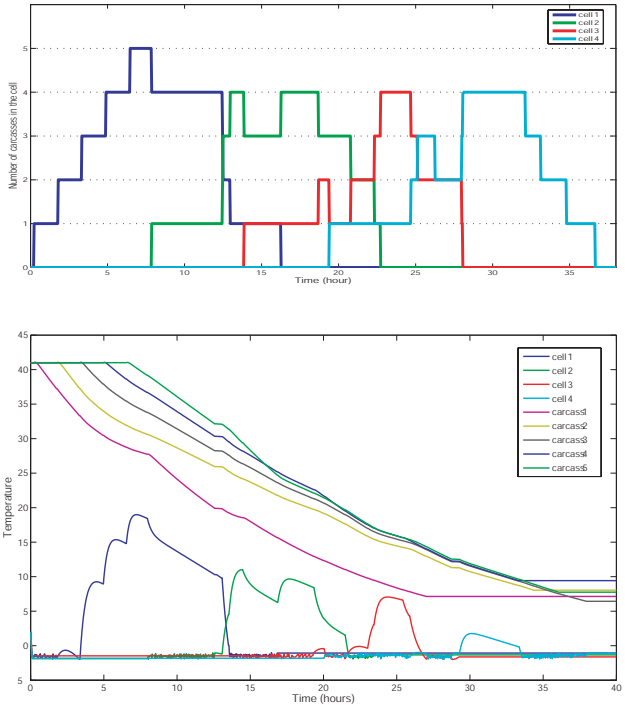

Fig. 6. Simulation of the optimized chain. (a) Number of carcasses in the different cells. (b) Temperature behavior of the different jobs and cells. The straight lines indicate that the job has leaved the last cell.

V. CONCLUSIONS

We introduced a general framework for describing a fresh–food supply chain, and proposed an optimization methodology to improve the performances of the network preserving the quality of the product. This approach has been illustrated on a case–study relevant to a beef–meat refrigeration and distribution chain.

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