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Predictive Control of a Complex District Heating Network

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Abstract—**The short term optimization and control of district heating networks is of great interest for Energy Industries because of the technical, economical and environmental benefits which could be earned from an appropriate management. However, models of such complicated systems are strongly non linear and suffer from important uncertainties. In this article, models well suited to industrial issues are first designed. The whole technological string "production – distribution – consumption" is taken into account. The aim of this study is then to compute an optimal and robust control law for the network. Because of the errors in consumers' demand prediction and modelling uncertainties, a closed loop strategy has to be used to compute a robust control law for the district heating network. In this paper, a robust predictive control strategy of the network is thus developed. The method has been successfully tested on a benchmark network created by EDF ('Electricité de France') and some results are presented here.**

I. INTRODUCTION

The short term optimal scheduling and control of power systems has become a crucial point. Indeed, energy markets have become more and more competitive. Producers and network managers have to drive their power systems, which are more and more complicated, to fulfill consumers' power demands with the lowest global costs. Producers are also made to be aware of environmental issues by environmental laws. They are compelled to reduce their rate of polluting emissions. Thus, technical, economical and environmental constraints have to be simultaneously dealt with.

The optimization problem stated from this multi field area can hardly be solved as it is a non linear programming problem, made of numerous variables. The optimal control of district heating networks, for which propagation delays can not be neglected and mechanical and thermal losses have non linear expressions, picks up all these harsh difficulties.

'Electricité de France' is used to developing power plants and network control methods. In cooperation with Supelec, new research areas are investigated, for which this experience is an advantage: modelling, design, optimization and control of multi energy, multi domain networks. Among them are district heating networks. This study aims to take into account the whole technological string "Production – distribution – consumption" to design a control law which has to be robust against load prediction errors.

In most studies, consumers' demands are considered as given and perfectly known data; see [1]-[3]. In this case the minimization of operational costs is an ideal reference trajectory for the district heating system, but is not robust against load prediction errors. In [4] and [5], a closed loop control strategy is depicted for district heating networks with one thermal power production point and leads to very coherent behaviours: as heat losses in the distribution network increase with the supply temperature, the optimal strategy is to keep this temperature as low as possible while satisfying consumers' demands. However, in the general case, this control approach may fail as supply temperature is difficult to compute. This difficulty occurs for multi supply point networks, time varying operational costs or networks with heat storage tanks. For these cases, it may be economically interesting to produce more power than requested to reduce global costs. The supply temperature may be temporary higher than supposed to achieve better global efficiencies. In this paper a new approach is presented, based on predictive control principle depicted in [6] and [7]. This approach aims to be quite versatile and could be apply to various kinds of district networks. It aims also to be robust against load prediction errors and model uncertainties.

The model, designed for simulation purposes, is depicted in section II. It appears to be well representative of district heating networks and remains tractable for simulation. However, because of computation times, it can hardly be used for optimization purposes. Thus, a model for optimization has been developed and is presented in section III. Based on load predictions, the scheduling planning can be computed with this optimization model, leading to an ideal open loop control. Because of load prediction errors and modelling uncertainties of the optimization model, this open loop control can not be applied to the network. A closed loop control is necessary, and its design is the subject of section IV. Numerical results are given in section V for a benchmark district network designed by 'Electricité de France', showing that the control method leads to a robust behaviour of the network. A discussion about the predictive control strategy and forthcoming works is given in section VI. Finally, conclusions are drawn in section VII.

II. MODEL FOR SIMULATION

The model for simulation has been fully defined in previous work [8]. Some results are here called up. The time

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range has been discretized with a sampling period of one hour for the modification of network inputs (produced power decisions and opening degrees of valves). This value allows to make the hypothesis that there is no need to consider dynamic on pressures and mass flows.

A. Production model

Production sites are made of several production units. Production models are aggregated ones: production site k can be globally modelled by a non dynamic characteristic, identified from technical data with a least square method. For hour n, production costs can be derived from produced thermal power Q_n^k [W]:

$$c_{prod}^k\left(Q_n^k\right)=a_2\left(Q_n^k\right)^2+a_1Q_n^k+a_0 \tag{1}$$

Note that the coefficients of this characteristic can be time varying: this is typically true for a cogeneration site. To take into account dynamics of production units in site k, a penalization on power increments is added:

$$c_{pen}^k\left(Q_{n-1}^k,Q_n^k\right)=\lambda\left(Q_n^k-Q_{n-1}^k\right)^2 \tag{2}$$

The thermal power given to primary network is related to network temperatures by:

$$Q_n^k=c_p m_s\left(T_s^k-T_r^k\right) \tag{3}$$

where m_s [kg.s^{-1}] is the mass flow, T_s [K] the supply temperature and T_r [K] the return temperature in primary network; c_p [J.kg^{-1}.K^{-1}] is the specific heat of water.

B. Energy supply network model

1) Mass flows and pressures: mass flows and pressures are related to the modelling of four kinds of components: pipes, valves, nodes and pumps.

Mechanical losses in pipes can be expressed by:

$$H_{out}=H_{in}-Z_p{m_p}^2 \tag{4}$$

with m_p [kg.s^{-1}] is the mass flow in pipe, H_{in} (resp. H_{out}) [m] the pressure at the beginning (resp. the end) of the pipe, and Z_p [m.kg^{-2}.s^2] the friction coefficient. For a valve, this coefficient becomes Z_p/d, where d is the opening degree of the valve (from 0 for a closed valve to 1 for an open one).

To counterbalance those mechanical losses, pumps are installed in the network leading to an increase of pressure:

$$\Delta H=a_2\left(m\frac{\omega_0}{\omega}\right)^2+a_1\left(m\frac{\omega_0}{\omega}\right)+a_0 \tag{5}$$

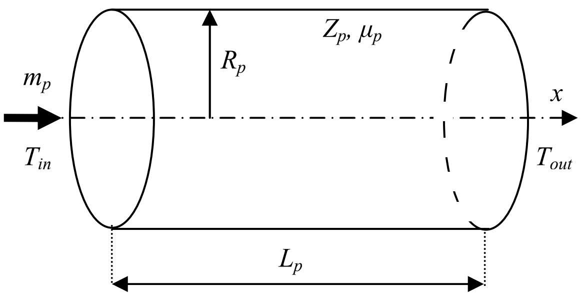

Fig. 1. Notations for energy propagation

m [kg.s^{-1}] is the mass flow through the pump, ω [rad.s^{-1}] its rotation speed and ω_0 its nominal rotation speed.

For the mass flows computation, nodes are modeled by mass flows balance equations.

Finally, mass flows and pressures have to be computed from an important non linear system of algebraic equations obtained from all these static equations modelling pipes, valves, pumps and nodes. An efficient dedicated Newton-Raphson method has been developed for this purpose.

2) Thermal energy propagation: Notations are shown in fig. 1; μ_p [J.m^{-2}.s^{-1}.K^{-1}] is the thermal loss coefficient, ρ [kg.m^{-3}] the relative density of water, T_0 [K] the external temperature, and $T(x,t)$ the temperature in the pipe. The thermal energy propagation in pipes can then be modeled by a partial differential equation ([9]):

$$\frac{\partial T}{\partial t}(x,t)+\frac{m_p(t)}{\pi\rho R_p^2}\frac{\partial T}{\partial x}(x,t)+\frac{2\mu_p}{c_p\rho R_p}\left(T(x,t)-T_0\right)=0 \tag{6}$$

This equation leads to the following solution ([8]):

$$T_{out}(t)=T_0+\left(T_{in}\left(t-t_0(t)\right)-T_0\right)e^{\left(-\frac{2\mu_p}{c_pR_p\rho}\left(t-t_0(t)\right)\right)} \tag{7}$$

where the varying time delay $t-t_0(t)$ is defined by:

$$\int_{t_0(t)}^{t}\frac{m_p(\tau)}{\pi R_p^2\rho}d\tau=L_p \tag{8}$$

For the energy propagation point of view, nodes are modelled by the help of an energy balance equation.

C. Consumer model

Secondary networks of consumers are connected to the primary network by way of a heat exchanger. Notations are those of fig. 2. The following equation is the classical equation for a counter flow heat exchanger with S [m^2] the surface of the heat exchanger, and e [W.K^{-1}.m^{-2}] its efficiency:

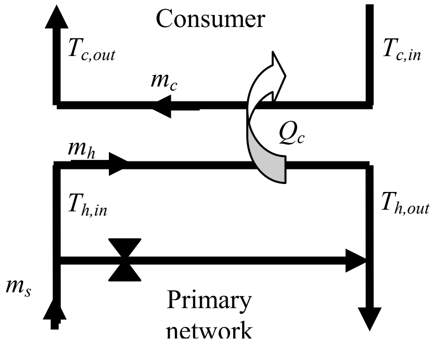

Fig 2. Notations for consumer modelling

$$Q_c = eS\frac{\left(T_{h,in}-T_{c,out}\right)-\left(T_{h,out}-T_{c,in}\right)}{\ln\left(T_{h,in}-T_{c,out}\right)-\ln\left(T_{h,out}-T_{c,in}\right)} \tag{9}$$

Assuming no thermal energy loss between primary and secondary networks, the thermal power given by the primary network can be also expressed by:

$$Q_c = c_p m_h\left(T_{h,in}-T_{h,out}\right) \tag{10}$$

Finally, the power received by the secondary network is :

$$Q_c = c_p m_c\left(T_{c,out}-T_{c,in}\right) \tag{11}$$

Assuming that m_c and $T_{c,out}$ are given, and that mass flow m_h is determined by the opening degree of the valve, then $T_{c,in}$, Q_c and $T_{h,out}$ can be computed from $T_{h,in}$. Q_c is an increasing function of m_h: the maximal thermal power which can be given to a consumer is obtained for $m_h = m_s$. There is a local regulation, which is not of interest in this study, so that the consumer can choose the value of m_h in the possible range, by controlling the opening degree of the valve. Consequently, the given power is finally expressed by:

$$Q_c = \min\left(Q_{dem}, Q_{\max}\right) \tag{12}$$

where Q_{dem} is the heat demand of the consumer, and Q_{max} is the maximum power that can be given by the primary network. Q_{max} is computed by solving the system made of (9), (10) and (11), in the particular case $m_h = m_s$.

III. MODEL FOR OPTIMIZATION

A. Necessity of a tractable optimization model

Assuming constant rotation speed of pumps, an ideal open loop control can be computed from the solution of the following optimization problem, where Q_n^k is the thermal power produced by the k^{th} site during the n^{th} hour, and d_n^v the opening degree of valve v:

$$\min_{\left\{\substack{Q_n^k, d_n^v \\ n\in[1:N], k\in[1:K], v\in[1:V]}\right\}} \sum_{k=1}^{K}\sum_{n=1}^{N}\left(\begin{array}{l} c_{prod}^k\left(Q_n^k\right) \\ + c_{pen}^k\left(Q_{n-1}^k, Q_n^k\right)\end{array}\right) \tag{13}$$

The constraints are those of the district heating network. In particular, consumers' demands, which are predicted variables, have to be satisfied.

The developed simulation model is fully representative of a district network and the objective function and constraints could be computed using this model. However, as the simulation of the district network is about several minutes, depending on the size of the network, it is not tractable for optimization and control purposes. So, a simplified model, well suited to optimization issues, has been developed.

B. Production and consumers model

Production and consumers models which have been developed for simulation purposes are highly tractable: they can also be used for the optimization and control procedure.

C. Energy supply network model

1) Mass flows and pressures: there is no use considering dynamic on mass flows and pressures. Thus, their values at hour *n* depend only on opening degrees of valves at hour *n*. The solution of the non linear system of algebraic equations is time consuming, and should be avoided during the optimization stage.

It is possible to compute off-line mass flows and pressures for few values of opening degrees. Then, during optimization, mass flows are computed on-line using a linear interpolation.

2) Thermal energy propagation: thermal losses in pipes are very low. Thus, (7) can be approximated by:

$$\begin{aligned} T_{out}(t) &\approx T_0 + \left(T_{in}\left(t-t_0(t)\right)-T_0\right)\left(1-\frac{2\mu_p}{c_p R_p \rho}\left(t-t_0(t)\right)\right) \\ &\approx T_{in}\left(t-t_0(t)\right)\left(1-\frac{2\mu_p}{c_p R_p \rho}\left(t-t_0(t)\right)\right)\end{aligned} \tag{14}$$

The computation of varying time delays is time consuming. That is why constant (and for instance nominal) time delays have been considered in (14) for the optimization model. This hypothesis allows to model thermal propagation as a simple non linear dynamic system, which can be quickly solved. A sampling period of 1/6 hour has been chosen for the optimization model. This value is compatible with a representative and tractable model. The globally simplified model becomes highly tractable for optimization purposes, and finally optimization problem (13) can be quickly solved.

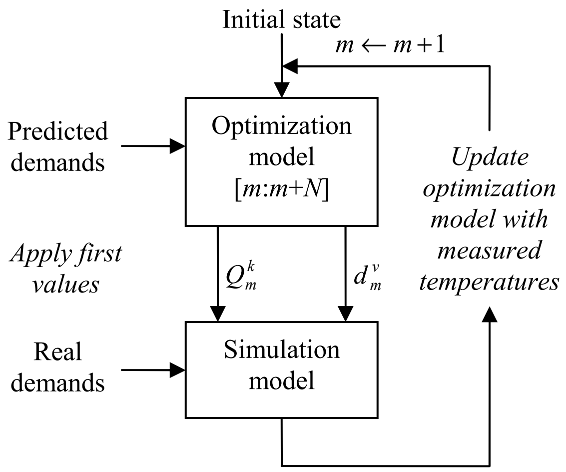

Fig. 3. Predictive control strategy

IV. PREDICTIVE CONTROL OF THE NETWORK

A. Network predictive control

The ideal open loop control computed with the help of the optimization model can not be directly applied on the simulation model because of model uncertainties and load prediction errors. To circumvent this problem, a closed loop control is defined, based on predictive control principle. The approach is depicted on fig. 3.

The idea is to compute the optimal scheduling on time interval [*m*:*m*+*N*], considering predicted demands and the simplified model. The first values (control values at time *m*) are then applied to the "real system" (in this case the simulation model), with the real consumers' demands.

Network temperatures (model states) are then "measured", and the optimization model can be updated for time interval [*m*+1:*m*+*N*+1] and the next optimization procedure.

B. Robustification of the control strategy

As a consequence of optimality, the short term scheduling of the optimization model is a limit scheduling: the network manager tries to produce as less thermal power as possible, while satisfying consumers' demands. This may lead to unfeasibility of next optimization problems.

Indeed, some consumers are installed in a cascading way in the district heating network. If the first consumers take more power than they were expected to, the demands of the following consumers may not be fulfilled, as there is not enough energy in the network anymore. Because of time delays, it is of course too late to react and then to produce the missing thermal power. In this approach, the classical notion of robustness is strongly related to the notion of unfeasibility.

Thus, it clearly appears that control procedures of district heating networks are very sensible to load prediction errors and model uncertainties. The robustness will be obtained by using the thermal storage property of the distribution network: it is possible to produce a little bit more than requested and to use the network as a storage tank. Indeed, if production controls Q_m^k are increased, thermal losses will also increase, but most of the extra power supply will remain in the network and will be usable later. Thus, the increase in production costs over the whole time interval is kept slight, and a robust behaviour will be obtained. Two facts have to be considered to choose the value of the extra amount of energy. On one hand, the higher the extra amount value is, the more robust against error predictions the control strategy will be. But, on the other hand, as heat losses increase with temperature, a high value will increase operation costs. Thus, this value should be carefully chosen, considering for example the variance of load predictions, see [10].

V. NUMERICAL RESULTS

The control strategy has been implemented with Matlab$^{\text{TM}}$ 6.1 and its optimization toolbox 2.1.1. The optimization algorithm is Sequential Quadratic Programming. The predictive horizon *N* is 12 hours. This value has to be greater than the maximum loop time delay in the network so as to get a satisfying behaviour of the control procedure. A two days length simulation will be presented.

The developed control method has been applied on the district heating network depicted on fig. 4. This network is a sub-problem of the network benchmark, designed by EDF and Supelec, which has been completely defined, modelled and simulated in [8]. This network is made of two interconnected small networks. Each of them is supplied by a production site. It is assumed that production cost characteristics of these production sites are not time varying. 2 valves allow the interconnection between both networks. 6 consumers' demands have to be satisfied.

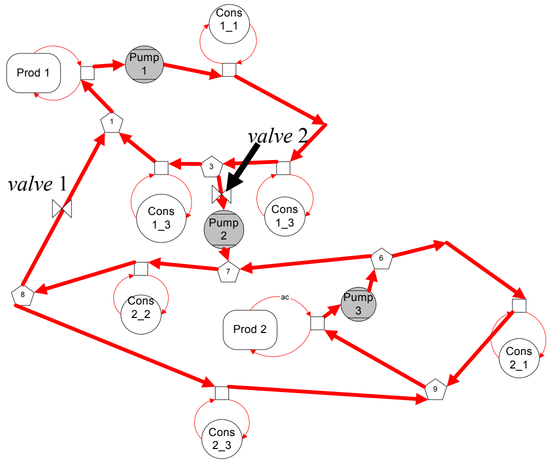

Fig. 4. District heating network benchmark

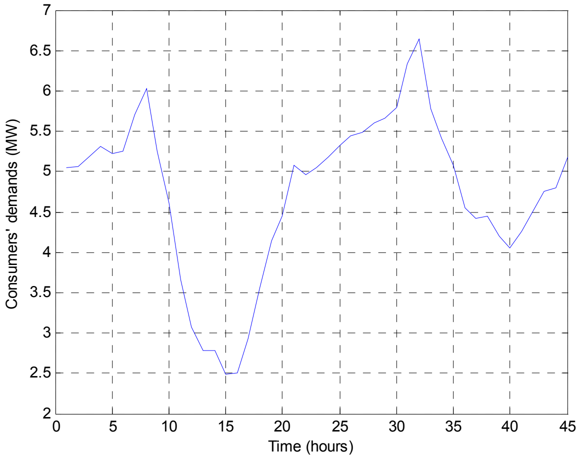

Fig. 5. Consumers' demands

These consumers are assumed to be blocks of flats, and demands are assumed to have the shape of fig. 5. A daily oscillation, due to the alternation between night and day, can be observed. To illustrate the robustness of the control procedure, it was also assumed that all consumers' demands have been underestimated. This is the worst case situation, which could quickly lead to unfeasibility. In the general case predictions errors can partially compensate each other.

An iteration of the scheduling algorithm is made in about 5 minutes on a Pentium IV, 2.5 GHz. These computation times allow the use of the control law in a real time context.

The control method leads to a very satisfying behaviour as no unfeasibility has occurred during the simulation: all consumers' demands have been perfectly fulfilled. Produced powers are depicted on fig. 6a, and opening degrees of valves on fig. 6b. It can be observed that the shape of produced powers is globally quite similar to consumers' demands shape: as production cost characteristics are constant, it is not economically interesting to use the network as a storage tank. Thus, the optimal solution is to produce as less thermal power as possible, while satisfying consumers' demands and respecting robustness margins. Time delays can also be observed: produced power peaks occur before demand peaks, according to propagation time delays.

The scheduling of valves 1 and 2 are quite similar. This can be explained by the fact that the mass flow going from network 1 to network 2 (valve 2), and the mass flow going from network 2 to network 1 (valve 1) have to be equal to satisfy mass flow balance equations.

Results show that opening degrees of valves are sometimes close to 1, so the interconnection of networks is of great interest to decrease global costs of district heating networks. A posteriori analysis shows that producer 1 is a little bit more profitable than producer 2. Thus, it is economically interesting to produce more power than requested with producer 1 and to use this extra power supply for network 2.

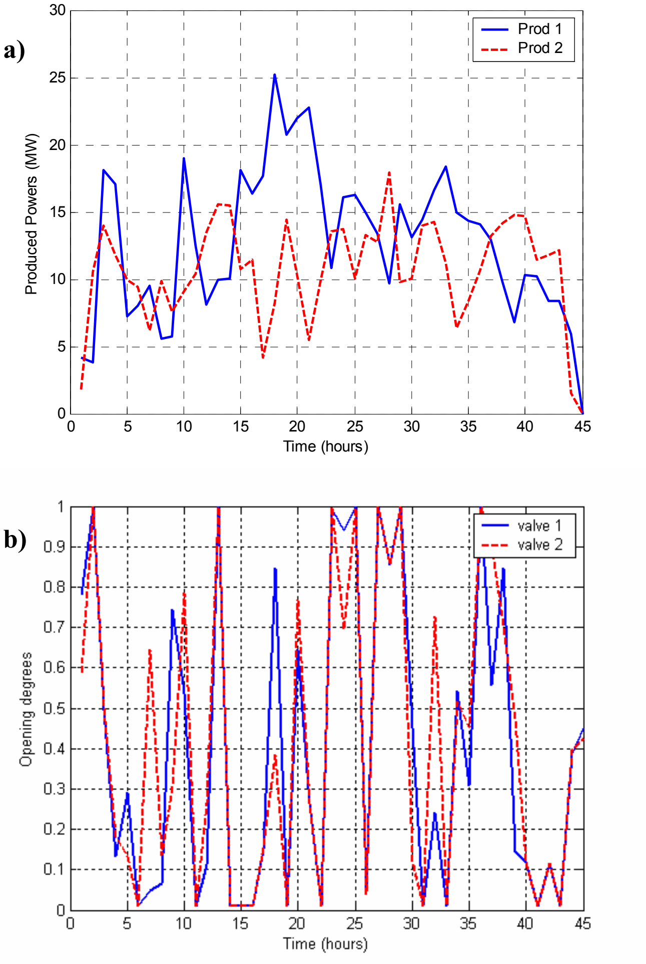

Fig. 6. a) Produced thermal powers b) Opening degrees of valves

VI. DISCUSSION AND FORTHCOMING WORKS

A. *Polluting emissions*

Polluting emissions have not explicitly been taken into account in the example. However, they could easily be, either by additional constraints on Q_n^k variables or by a penalization term in the objective function.

B. *Non linear estimator*

The control procedure assumes that all temperatures in the district heating network can be measured, so as to update the optimization model (see fig. 3). Of course, this may not be true for a real network, as there are often just a few temperature sensors in the district heating network. Thus it will be necessary to develop local non linear estimators so as to estimate useful temperatures and to update the optimization model.

C. Local optimization of producers

Aggregated models of consumers have been considered in this study. In fact, production sites are made of several production units which have been brought together. The local management of such production sites is in fact computed via the minimization of local production costs. This is a classical problem referred to "thermal Unit Commitment". Aggregated models represent an estimation of the generic solution of this "Unit Commitment" problem. Although the method is quite versatile and could be used for many kinds of district heating network, the determination of those aggregated models is an important but non systematic step, which may depend on the type of considered production site.

When Q_n^k variables have been computed, the "Unit Commitment" problem has to be solved for each production site. This can be done with the help of one of the classical optimization methods listed in [11]. This solution can be an opportunity to define a two level control approach. If the solution of production site optimization leads to production costs which are quite different from the estimated costs, the optimization problem (13) may be solved again with an updated estimated objective function to refine the global result.

D. Large scale cases

The feasibility of the methodology has been presented in this paper. The application is a medium scale case made of the district heating network of fig. 4. For large scale cases, the control procedure may require more computer time.

The solution of the non linear algebraic systems is time consuming, particularly for multi loop networks. This is the main point to reduce computation times and numeric methods may have to be developed for large scale cases.

VII. CONCLUSION

Energy systems, and particularly district heating networks, become an important stake for Energy Industries. In a competitive context, a suitable management of such systems could be an opportunity to fulfill consumers' demands with the lowest costs and the lowest rate of polluting emissions.

In this paper, a control procedure is presented which aims to define a robust control law which takes into account the whole district heating network, from producers to consumers. This control law is based on predictive control principles. Two models have been used: one for optimization and one for simulation. The optimization model allows the quick computation of an ideal open loop control. The simulation model is then used to update the optimization model leading to a robust closed loop structure of the control law.

The developed control law has been robustified against model uncertainties and demands prediction errors by using the storage tank effect of the distribution network.

Several research axes will shortly be investigated. Among them is the creation of non linear estimators. They will be used to update optimization model in the case of a real network. Another axe will consider local optimization: the control method has to be coupled with a "unit commitment" solution algorithm, to compute the scheduling of production sites. This may be an opportunity of updating the estimated cost function.

Finally results, obtained a district heating network designed by 'Electricité de France' and Supélec, can be obtained even in presence of underestimated consumers' demands. The control strategy which has been proposed here appears to be a robust and computationally efficient approach for large networks.

ACKNOWLEDGMENT

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