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On rendezvous for visually-guided agents in a nonconvex polygon

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Abstract— **This paper presents coordination algorithms for mobile autonomous agents equipped with line-of-sight sensors in a nonconvex polygon. The objective of the proposed algorithms is to achieve rendezvous, that is, agreement over the location of the agents in the network, using only information from the line-of-sight sensors. Two key novel components of the algorithms are the notions of locally-cliqueless visibility graph and of convex continuous constraint set.**

I. Introduction

Consider a group of robotic agents moving in a nonconvex environment. For simplicity, we model the environment as a simple polygon and the agents as point masses. Assume that each member of the group is equipped with omnidirectional line-of-sight sensors. By a line-of-sight sensor, we mean any device or combination of devices that can be used to determine, in its line-of-sight, (i) the position or state of another agent, and (ii) the distance to the boundary of environment. By omnidirectional, we mean that the field-of-vision for the sensor is 2π radians. We assume that the algorithm regulating the agents' motion is memoryless, i.e., we consider static feedback laws. Given this model, the goal is to design a provably correct discrete-time algorithm which ensures that the agents converge to a common location within the environment. See Fig. 1 for a graphical description of our objective. Ideally, the algorithm would work asynchronously but here we confine ourselves to the synchronous case.

This work is motivated by the recent surge of interest in the study of groups of mobile autonomous robots. The "multi-agent rendezvous" problem and the first "circumcenter algorithm" have been introduced in [1]. The algorithm proposed in [1] has been extended to various asynchronous strategies in [2], [3]. A related algorithm, in which connectivity constraints are not imposed, is proposed in [4].

One important difference between these works and the present one is that we consider visually-guided robots. In fact, technical advancement in sensor technology and mobile robotics have facilitated the implementation of these algorithms on real systems. Examples of panoramic depth sensors relevant to our work are (1) omnidirectional cameras, e.g., [5], and (2) laser scanners with accurate distance measurements at high angular density. We conclude our literature review by mentioning that the problem of rendezvousing at a specified location for visually-guided agents was first introduced in [6]. However, the proposed solution was not distributed, in the sense that each agent required the knowledge of the locations of all other network agents.

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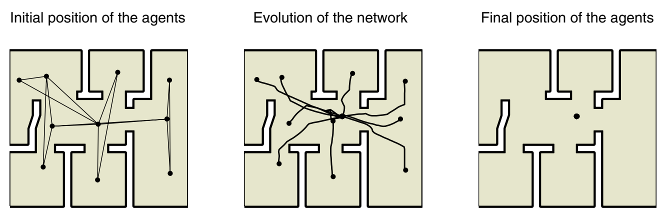

Fig. 1. Execution of the Circumcenter Algorithm described in Section IV-C on a network of agents distributed in a polygon shaped like a typical floor plan. The algorithm is run over the visibility graph $\mathcal{G}_{\text{vis},Q}$ (see Section III).

The contribution of this paper is threefold. First, we develop a geometric framework which makes it possible to apply recently developed results on convergence analysis of nonlinear systems, e.g., the LaSalle Invariance Principle for set-valued maps, on a network of visually-guided agents in a nonconvex environment. More explicitly, we constrain the motion of agents to sets that (i) ensure that the visibility between two agents is preserved, and (ii) changes continuously as a function of the position of the agents. We call such sets convex continuous constraint sets and characterize their properties. Second, based on a discussion on visibility graphs, we define a new proximity graph, called the locally-cliqueless visibility graph, which contains fewer edges than the visibility graph, and has the same connected components. This construction can be, in general, useful for any problem where the connectivity of the visibility graph is important and fewer constraints on the agents, in terms of number of neighbors, is beneficial. Examples of such problems might include line-of-sight wireless routing and consensus problems over line-of-sight wireless communication networks. Third, we propose a coordination algorithm to solve the rendezvous problem and provide a convergence proof.

II. Convex continuous constraint sets

Here we design motion constraint sets for pair of agents mutually visible to one another. By constraining the motion of agents, we aim to preserve the connectivity of the network. Additionally, we require that motion constraint sets change continuously as a function of the position of the agents. This turns out to be a critical property for the convergence analysis of algorithms based on these sets. We emphasize that the construction proposed here may be applied to *any distributed algorithm for a network of visually-guided agents.*

We begin by reviewing some notation for standard geometric objects. For $p \in \mathbb{R}^2$, let $\overline{B}(p,r)$ denote the *closed ball* centered at p of radius $r \in \mathbb{R}_+$. We let $\mathbb{R}_+$ and $\overline{\mathbb{R}}_+$ denote the

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positive and the nonnegative real numbers, respectively. For a bounded set $X \subset \mathbb{R}^2$, we let $\mathrm{co}(X)$ denote the convex hull of X. For $p, q \in \mathbb{R}^2$, we let $]p, q[= \{\lambda p + (1-\lambda)q \mid 0 < \lambda < 1\}$ and $[p, q] = \mathrm{co}(\{p, q\})$ denote the *open* and *closed segment* with extreme points p and q, respectively. For a closed convex set $X \subset \mathbb{R}^2$ and $q \in \mathbb{R}^2$, let $\mathrm{proj}_X(q)$ denote the orthogonal projection of q onto X. For a bounded set $X \subset \mathbb{R}^2$, we let $\mathrm{CC}(X)$ denote the *circumcenter* of X, i.e., the center of the smallest-radius circle enclosing X. The computation of the circumcenter is a strictly convex problem. Let $|X|$ denote the cardinality of a finite set X in $\mathbb{R}^2$. Next, we define continuous set-valued maps; see [7].

Definition II.1 *Let X and Y be topological vector spaces (real and Hausdorff). A set-valued map $f : X \to 2^Y$ with non-empty and compact values is continuous at a point $x_0 \in X$ if given any $\epsilon > 0$, there exists a $\delta > 0$ such that for all $x \in \overline{B}(x_0, \delta)$, we have*

$$f(x) \subset \bigcup_{y \in f(x_0)} \overline{B}(y, \epsilon) \quad \text{and} \quad f(x_0) \subset \bigcup_{y \in f(x)} \overline{B}(y, \epsilon)$$

Now let us turn our attention to the environment. A polygon is *simple* if its vertices are the only points in the plane common to two polygon edges and every vertex belongs to at most two polygon edges. Such a polygon has a well defined interior and exterior. *Note that a simple polygon can contain holes*. Let $\mathcal{Q}$ denote the set of all simple polygons. Let $Q \in \mathcal{Q}$ and let $\mathrm{Ve}(Q) = (v_1, \dots, v_n)$ be the list of vertices of Q ordered counterclockwise. The *interior angle of a vertex v* of Q is the angle formed inside Q by the two edges of the boundary of Q incident at v. The point $v \in \mathrm{Ve}(Q)$ is a *reflex vertex* if its interior angle is strictly greater than π radians. Let $\mathrm{Ve_r}(Q)$ denote the list of reflex vertices of Q ordered counterclockwise. Note that a reflex vertex may be defined even for polygons that are not simple but have a well-defined interior and exterior. If X is a finite set of points in Q^n, let $\mathrm{MPP}(X, Q)$ be the *minimal perimeter polygon* containing X which is a subset of Q (see Fig. 2 for an example). Note that Q does not necessarily have to be simple for the minimal perimeter polygon to be defined; it only needs to have a well defined interior and exterior.

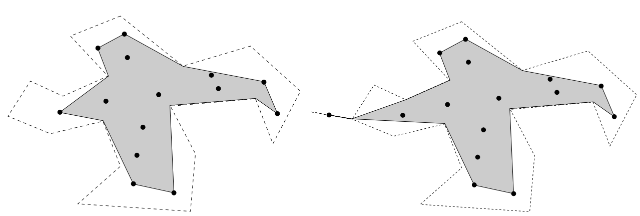

Fig. 2. Minimal perimeter polygon of a set of points inside nonconvex polygonal environments. The environments are represented by dashed lines, while the polygons represented by the solid lines are the minimal perimeter polygons of the points represented by the solid circles. On the left, the environment is a simple polygon whereas on the right the environment is polygonal, not simple, and still has a well-defined interior and exterior.

A point $q \in Q$ is *visible from* $p \in Q$ if $[p, q] \subset Q$. The *visibility polygon* $S(p) \subset Q$ from a point $p \in Q$ is the set of points in Q visible from p. We can also think of $p \mapsto S(p)$ as a map from Q to the set of polygons contained in Q.

Definition II.2 *Let v be a reflex vertex of Q, and let $w \in \mathrm{Ve}(Q)$ be visible from v. The (v, w)-*generalized inflection segment *$I(v, w)$ is the set*

$$I(v, w) = \{q \in S(v) \mid q = \lambda v + (1-\lambda)w, \lambda \geq 1\}.$$

If $w \in \mathrm{Ve_r}(Q)$, then we call $I(v, w)$ a bitangent *of Q. Let $\{I_\alpha\}_{\alpha \in \mathcal{A}}$ be the set of bitangents of Q. A reflex vertex v of Q is an* anchor *of $p \in Q$ if it is visible from p and $\{q \in S(v) \mid q = \lambda v + (1-\lambda)p, \lambda > 1\} \neq \emptyset$.*

In other words, a reflex vertex is an anchor of p if it occludes a portion of the environment from p. Next we define and characterize certain useful convex sets depicted in Fig. 3.

Definition II.3 *Given $Q \in \mathcal{Q}$, let $p, q \in Q$ such that $[p, q] \subset Q$. Let $v \in \mathrm{Ve_r}(Q)$. Let e'_v and e''_v be the edges of Q determining v. Then we define $H_v(p, q) \subset \mathbb{R}^2$ as follows:*

(i) *if $v \notin [p, q]$, then $H_v(p, q)$ is the half-plane with the following properties: (a) the boundary of $H_v(p, q)$ contains v and is perpendicular to the line passing through v and $\mathrm{proj}_{[p,q]} v$, and (b) p and q belong to the interior of $H_v(p, q)$;*

(ii) *if $v = p$ with $p \neq q$, then $H_v(p, q)$ is the half-plane with the following properties: (a) the boundary of $H_v(p, q)$ contains v and is perpendicular to the line passing through p and q, and (b) q belongs to the interior of $H_v(p, q)$ (Note: a similar definition holds when we interchange p and q);*

(iii) *if $v \in]p, q[$ with $p \neq q$, then $H_v(p, q)$ is the half-plane with the following properties: (a) the boundary of $H_v(p, q)$ contains the line passing through p and q, and (b) the interior of $H_v(p, q)$ intersected with e'_v or with e''_v is empty;*

(iv) *if $v = p = q$, then $H_v(p, q)$ is the set $H'_v \cap H''_v$. H'_v is a half-plane with the following properties: (a) the boundary of H'_v contains the edge e'_v, and (b) the interior of H'_v intersected with e''_v is empty. We define H''_v similarly with e''_v interchanged with e'_v.*

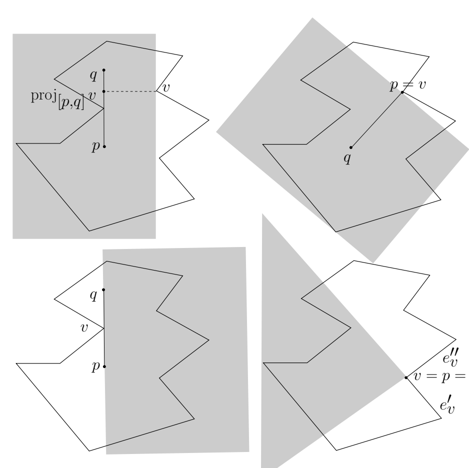

Fig. 3. Definition of the sets $H_v(p, q)$

Remark II.4 With the above definition, wherever defined, $H_v(p,q)$ is a closed and convex set containing p and q. Also, if $\mathcal{V} \subset Q$ is convex and compact, then $H_v(p,q)$ is well-defined everywhere in $(\mathcal{V})^2$ and $(p,q) \mapsto H_v(p,q)$ is a set-valued map over the domain $(\mathcal{V})^2$ with range $2^{(\mathbb{R}^2)}$.

Lemma II.5 *Given any $v \in \mathrm{Ve_r}(Q)$ and a convex and compact subset $\mathcal{V}$ of Q, the set-valued map $(p,q) \mapsto H_v(p,q) \cap Q$ restricted to $(\mathcal{V} \setminus \mathrm{Ve_r}(Q))^2$ is continuous.*

Lemma II.6 *Let $V \subset \mathrm{Ve_r}(Q)$ and $\mathcal{V}$ be a convex and compact subset of Q. The following statements are true:*

(i) *the set-valued map $(p,q) \mapsto \bigcap_{v \in V} S(p) \cap H_v(p,q)$ restricted to $(\mathcal{V} \setminus (\mathrm{Ve_r}(Q) \bigcup (\cup_{\alpha \in \mathcal{A}} I_\alpha)))^2$ is continuous;*

(ii) *the set-valued map $p \mapsto \bigcap_{v \in V} S(p) \cap H_v(p,p)$ restricted to $\mathcal{V} \setminus (\mathrm{Ve_r}(Q) \bigcup (\cup_{\alpha \in \mathcal{A}} I_\alpha))$ is continuous.*

Definition II.7 (Convex Continuous Constraint Sets) *Let $p, q \in Q$ have the property that $[p,q] \subset Q$ and let $I_Q(p,q) = \mathrm{Ve_r}(Q) \cap S(p) \cap S(q)$. The convex continuous constraint set between p and q is*

$$\mathcal{C}_Q(p,q) = \bigcap_{v \in I_Q(p,q)} S(p) \cap H_v(p,q).$$

Fig. 4 illustrates the constraint set.

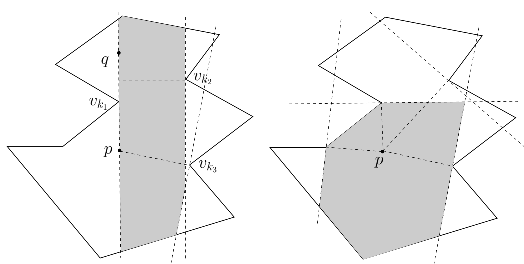

Fig. 4. The figure on the left is an example of the constraint set $\mathcal{C}_Q(p,q)$ where $I_Q(p,q) = \{v_{k_1}, v_{k_2}, v_{k_3}\}$. The figure on the right is an example of $\mathcal{C}_Q(p,p)$ where $I_Q(p,p) = \mathrm{Ve_r}(Q)$.

Theorem II.8 *Let $\mathcal{V} \subset Q$ be convex and compact. For any two points $p, q \in \mathcal{V}$, the following statements are true:*

(i) *$\mathcal{C}_Q(p,q)$ is convex, $\mathcal{C}_Q(p,q) = \mathcal{C}_Q(q,p)$, and*

(ii) *the set-valued map $(p,q) \mapsto \mathcal{C}_Q(p,p) \cap \mathcal{C}_Q(p,q)$ restricted to $(\mathcal{V} \setminus \mathrm{Ve_r}(Q))^2$ is continuous.*

III. The locally-cliqueless visibility graph

In Section II we proposed the construction of motion constraint sets to preserve the connectivity of the network. The number of such constraints for an agent is the number of the agents visible to it. It is intuitively clear that the lesser the number of such constraints, the faster will be the convergence of the algorithm. Here we introduce the notion of locally-cliqueless visibility graph, which is a subgraph of the visibility graph. In general, it contains fewer edges than the visibility graph but has the same connected components. In addition, we show that this graph can be computed based on the information obtained only from the visibility graph.

We begin by introducing some concepts regarding proximity graphs for point sets in $\mathbb{R}^2$. We assume the reader is familiar with the standard notions of graph theory. We recall that a *clique* of a graph is a complete subgraph of it. A *maximal clique of an edge* is a clique of the graph that (i) contains the edge and (ii) is not a strict subgraph of any other clique of the graph that also contains the edge.

Given a vector space $\mathbb{V}$, let $\mathbb{F}(\mathbb{V})$ be the collection of finite subsets of $\mathbb{V}$. Accordingly, $\mathbb{F}(\mathbb{R}^2)$ is the collection of finite point sets in $\mathbb{R}^2$; we shall denote an element of $\mathbb{F}(\mathbb{R}^2)$ by $\mathcal{P} = \{p_1, \dots, p_n\} \subset \mathbb{R}^2$, where $p_1, \dots, p_n$ are distinct points in $\mathbb{R}^2$. Let $\mathbb{G}(\mathbb{R}^2)$ be the set of undirected graphs whose vertex set is an element of $\mathbb{F}(\mathbb{R}^2)$. A *proximity graph function* $\mathcal{G} : \mathbb{F}(\mathbb{R}^2) \to \mathbb{G}(\mathbb{R}^2)$ associates to a point set $\mathcal{P}$ an undirected graph with vertex set $\mathcal{P}$ and edge set $\mathcal{E}_\mathcal{G}(\mathcal{P})$, with $\mathcal{E}_\mathcal{G} : \mathbb{F}(\mathbb{R}^2) \to \mathbb{F}(\mathbb{R}^2 \times \mathbb{R}^2)$ such that $\mathcal{E}_\mathcal{G}(\mathcal{P}) \subseteq \mathcal{P} \times \mathcal{P} \setminus \mathrm{diag}(\mathcal{P} \times \mathcal{P})$ for any $\mathcal{P}$. Here, $\mathrm{diag}(\mathcal{P} \times \mathcal{P}) = \{(p,p) \in \mathcal{P} \times \mathcal{P} \mid p \in \mathcal{P}\}$. In other words, the edge set of a proximity graph depends on the location of its vertices. General properties of proximity graphs are defined in [8], [9]. Here, we define:

(i) a *Euclidean Minimum Spanning Tree* of a proximity graph $\mathcal{G}$, denoted $\mathcal{G}_{\text{EMST},\mathcal{G}}$, assigns to each $\mathcal{P}$ a minimum-length spanning tree of $\mathcal{G}(\mathcal{P})$ whose edge (p_i, p_j) is assigned a length $\|p_i - p_j\|$. If $\mathcal{G}(\mathcal{P})$ is not connected, then $\mathcal{G}_{\text{EMST},\mathcal{G}}(P)$ is simply the union of Euclidean Minimum Spanning Trees of its connected components. For simplicity, when $\mathcal{G}$ is the complete graph $(\mathcal{P}, \mathcal{P} \times \mathcal{P} \setminus \mathrm{diag}(\mathcal{P} \times \mathcal{P}))$, we denote the Euclidean Minimum Spanning Tree by $\mathcal{G}_{\text{EMST}}$;

(ii) the *visibility graph* $\mathcal{G}_{\text{vis},Q}$, for $Q \in \mathcal{Q}$, with $(p_i, p_j) \in \mathcal{E}_{\mathcal{G}_{\text{vis},Q}}(\mathcal{P})$ if the line segment $[p_i, p_j] \in Q$;

(iii) the *locally-cliqueless visibility graph* $\mathcal{G}_{\text{lc-vis},Q}$, for $Q \in \mathcal{Q}$, with $(p_i, p_j) \in \mathcal{E}_{\mathcal{G}_{\text{lc-vis},Q}}(\mathcal{P})$ if $(p_i, p_j) \in \mathcal{E}_{\mathcal{G}_{\text{vis},Q}}(\mathcal{P})$ and (p_i, p_j) belongs to a set $\mathcal{E}_{\mathcal{G}_{\text{EMST}}}(\mathcal{P}')$ for any maximal clique $\mathcal{P}'$ of the edge (p_i, p_j) in $\mathcal{G}_{\text{vis},Q}$.

Fig. 5 contains some examples of proximity graphs in a nonconvex polygon Q shaped like a typical floor plan.

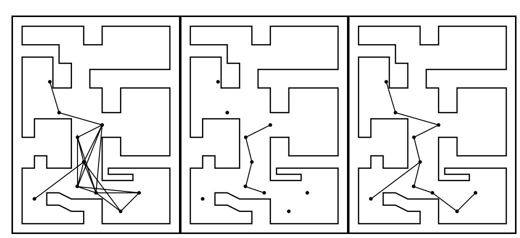

Fig. 5. From left to right, visibility graph, Euclidean Minimum Spanning Tree for the five agents in the center, and locally-cliqueless visibility graph.

To each proximity graph function $\mathcal{G}$, we associate the *set of neighbors map* $\mathcal{N}_\mathcal{G} : \mathbb{R}^2 \times \mathbb{F}(\mathbb{R}^2) \to \mathbb{F}(\mathbb{R}^2)$, defined by

$$\mathcal{N}_\mathcal{G}(p, \mathcal{P}) = \{q \in \mathcal{P} \mid (p,q) \in \mathcal{E}_\mathcal{G}(\mathcal{P} \cup \{p\})\}.$$

Also, for $p \in \mathbb{R}^2$, define $\mathcal{N}_{\mathcal{G},p} : \mathbb{F}(\mathbb{R}^2) \to \mathbb{F}(\mathbb{R}^2)$ by $\mathcal{N}_{\mathcal{G},p}(\mathcal{P}) = \mathcal{N}_\mathcal{G}(p, \mathcal{P})$. Let $\mathcal{G}_1$ and $\mathcal{G}_2$ be two proximity graph functions. $\mathcal{G}_1$ is *spatially distributed over* $\mathcal{G}_2$ if, for all $p \in \mathcal{P}$,

$$\mathcal{N}_{\mathcal{G}_1,p}(\mathcal{P}) = \mathcal{N}_{\mathcal{G}_1,p}\big(\mathcal{N}_{\mathcal{G}_2,p}(\mathcal{P})\big).$$

It is straightforward to deduce that if $\mathcal{G}_1$ is spatially distributed over $\mathcal{G}_2$, then $\mathcal{G}_1$ is a subgraph of $\mathcal{G}_2$, that is, $\mathcal{G}_1(\mathcal{P}) \subset \mathcal{G}_2(\mathcal{P})$ for all $\mathcal{P} \in \mathbb{F}(\mathbb{R}^2)$. Two proximity graph functions $\mathcal{G}_1$ and $\mathcal{G}_2$ *have the same connected components* if, for any $\mathcal{P} \in \mathbb{F}(\mathbb{R}^2)$, $\mathcal{G}_1(\mathcal{P})$ and $\mathcal{G}_2(\mathcal{P})$ have the same number of connected components consisting of the same vertices.

Theorem III.1 *For $Q \in \mathcal{Q}$, the following statements hold:*

(i) $\mathcal{G}_{\text{EMST},\mathcal{G}_{\text{vis},Q}} \subset \mathcal{G}_{\text{lc-vis},Q} \subset \mathcal{G}_{\text{vis},Q}$*;*
(ii) $\mathcal{G}_{\text{lc-vis},Q}$ *is spatially distributed over* $\mathcal{G}_{\text{vis},Q}$*, for the case when Q does not contain any hole;*
(iii) $\mathcal{G}_{\text{lc-vis},Q}$*,* $\mathcal{G}_{\text{vis},Q}$ *have the same connected components.*

In general, the inclusions in Theorem III.1(i) are strict. Fig. 6 shows an example where $\mathcal{G}_{\text{EMST},\mathcal{G}_{\text{vis},Q}} \subsetneq \mathcal{G}_{\text{lc-vis},Q} \subsetneq \mathcal{G}_{\text{vis},Q}$.

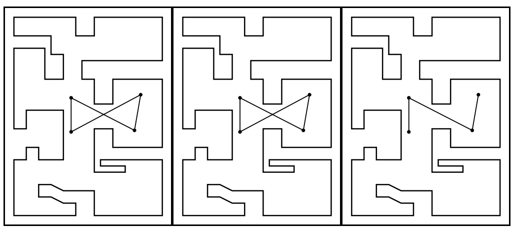

Fig. 6. From left to right, visibility graph, locally-cliqueless visibility graph and Euclidean Minimum Spanning Tree of the visibility graph.

IV. Rendezvous via proximity graphs

Here we state the model, the control objective, the coordination algorithm, and the closed-loop system properties.

A. A synchronous network of visually-guided agents

By a *visually-guided agent*, we refer to any agent, occupying a location in $Q \in \mathcal{Q}$, and capable of measuring the relative position of every other agent visible to it, i.e., within line-of-sight. In addition to this, it can also sense the boundary of Q. Each agent has a processor with the ability of allocating continuous and discrete states and performing operations on them. A collection of finite number, say n, of such agents form a network. Note that as a consequence of the above, whenever Q contains no hole, the processor on any agent has the capability to answer the query as to whether two agents visible to it are mutually visible to one another. The ith agent in such a network is capable of moving at any time $m \in \mathbb{N}$, for any unit period of time, according to the *synchronized* discrete-time control system

$$p_i(m+1) = p_i(m) + u_i. \tag{1}$$

We also assume that there is a maximum step size $s_{\text{max}} \in \mathbb{R}_+$ for all agents, that is, $\|u_i\| \leq s_{\text{max}}$, for $i \in \{1, \dots, n\}$.

B. The rendezvous motion coordination problem

We now state the control design problem for the network of visually-guided agents. The *rendezvous objective* is to steer each agent to a common location. This objective is to be achieved with the limited information flow described in the model above. Typically, it will be impossible to solve the rendezvous problem if the agents are placed in such a way that they do not form a connected graph. Arguably, a good property of any algorithm to rendezvous is that of maintaining some form of connectivity between agents.

C. The Circumcenter Algorithm

Here is an informal description of what we shall refer to as the *Circumcenter Algorithm over a proximity graph* $\mathcal{G}$:

> Each agent performs the following tasks: (i) it detects its neighbors according to $\mathcal{G}$; (ii) it computes the circumcenter of the point set comprised of its neighbors and of itself, and (iii) it moves toward this circumcenter while maintaining connectivity with its neighbors.

This algorithm is inspired by the one introduced in [1]. Let us clarify which proximity graphs are allowable and how connectivity is maintained. Firstly, we are allowed to design over any proximity graph $\mathcal{G}$ that is spatially distributed over $\mathcal{G}_{\text{vis},Q}$. This is a direct consequence of our modeling assumption that each agent can acquire the location of every other agent visible to it. Secondly, we maintain connectivity by restricting the allowable motion of each agent. In particular, if agents p_i and p_j are neighbors in the proximity graph $\mathcal{G}$, then their subsequent positions are required to belong to $\mathcal{C}_Q(p_i, p_j)$ as defined in Theorem II.8.

If an agent p_i has its neighbors at locations $\{q_1, \dots, q_l\}$, then define $\mathcal{M}_i = \{q_1, \dots, q_l\} \cup \{p_i\}$. We define the *constraint set* $C_{p_i,Q}(\mathcal{M}_i)$ by

$$C_{p_i,Q}(\mathcal{M}_i) = \bigcap_{q \in \mathcal{M}_i} \mathcal{C}_Q(p_i, q).$$

Remark IV.1
- $C_{p_i,Q}(\mathcal{M}_i)$ is a convex subset of Q containing p_i. This follows from the definition of $C_{p_i,Q}(\mathcal{M}_i)$ and Theorem II.8 (i).
- If $\mathcal{M}_i \cap \text{Ve}_\text{r}(Q)$ is empty and the set of neighbors of p_i is fixed, then $C_{p_i,Q}(\mathcal{M}_i)$ changes continuously as a function of p_i and of the positions of its neighbors. This follows from the fact that for each $p_j \in \mathcal{M}_i$, p_i is constrained to remain in $\mathcal{C}_Q(p_i, p_j)$ which is a convex and compact subset of Q. The statement is then a consequence of Theorem II.8 (iii) and the fact that $C_{p_i,Q}(\mathcal{M}_i)$ is an intersection of continuous maps.

With this, we are ready to formally describe the algorithm.

Name:	Circumcenter Algorithm over $\mathcal{G}$
Assumes:	(i) $s_{\text{max}} \in \mathbb{R}_+$ is maximum step size
	(ii) $Q \in \mathcal{Q}$
	(iii) $\mathcal{G}$ is a spatially distributed proximity graph over $\mathcal{G}_{\text{vis},Q}$

For $i \in \{1, \dots, n\}$, agent i executes the following at each time instant in $\mathbb{N}$:

1: acquire $\{q_1, \dots, q_k\} := \mathcal{N}_{\mathcal{G}_{\text{vis},Q}, p_i}(\mathcal{P})$
2: compute $\mathcal{M}_i := \mathcal{N}_{\mathcal{G},p_i}(\{q_1, \dots, q_k\}) \cup \{p_i\}$
3: compute $X_i := C_{p_i,Q}(\mathcal{M}_i) \cap \text{co}(\mathcal{M}_i)$
4: compute $q_i^* := \text{proj}_{X_i}(\text{CC}(\mathcal{M}_i))$
5: $u_i := \frac{\min(s_{\max}, \|q_i^* - p_i\|)}{\|q_i^* - p_i\|}(q_i^* - p_i)$

See Fig. 7 for examples of the constraint sets $C_{p_i,Q}(\mathcal{M}_i)$.

In what follows we shall refer to the Circumcenter Algorithm over the proximity graph $\mathcal{G}$ as the map $T_\mathcal{G} : Q^n \to Q^n$.

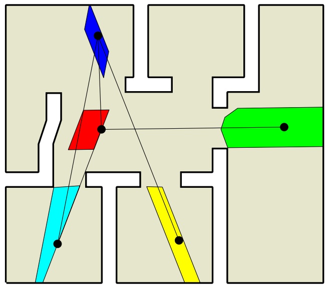

Fig. 7. Constraint sets $C_{p_i,Q}(\mathcal{M}_i)$ generated by the algorithm encoded as described in Section V

D. Asymptotic correctness of the Circumcenter Algorithm

Henceforth, P shall refer to tuples of elements in Q of the form $(p_1, \dots, p_n)$. With a slight abuse of notation, we shall use P interchangeably with a point set $\mathcal{P}$ of the form $\{p_1, \dots, p_n\}$. Before proceeding to analyze the convergence properties of the Circumcenter Algorithm, let us first define a candidate Lyapunov function $V_{\text{perim},Q} : Q^n \to \overline{\mathbb{R}}_+$, by

$$V_{\text{perim},Q}(P) = \text{perimeter}(\text{MPP}(P, Q)).$$

Lemma IV.2 *For any polygon Q with a well-defined interior and exterior, we have the following:*

(i) *for $P \in Q^n$, $\text{MPP}(P, Q)$ contains all the visibility edges of $\mathcal{G}_{\text{vis},Q}(P)$;*
(ii) *for $P_1 \in Q^n$ and $P_2 \in Q^m$, we have that $\text{MPP}(P_1, Q) \subset \text{MPP}(P_1 \cup P_2, Q)$;*
(iii) *for any polygon $X \subset Q$ with a well-defined interior and exterior and $P \in X^n$, we have that $\text{MPP}(P, X) \subset \text{MPP}(P, Q)$;*
(iv) $C_{p_i,Q}(\mathcal{M}_i) \cap \text{MPP}(\mathcal{M}_i, S(p_i)) = C_{p_i,Q}(\mathcal{M}_i) \cap \text{co}(\mathcal{M}_i)$*, where p_i and $\mathcal{M}_i$ are as in the description of the Circumcenter Algorithm in Section IV-C;*
(v) $C_{p_i,Q}(\mathcal{M}_i) \cap \text{MPP}(\mathcal{M}_i, S(p_i))$ *is convex;*
(vi) *if $\text{MPP}(P', Q)$ is a strict subset of $\text{MPP}(P'', Q)$, then $V_{\text{perim},Q}(P') < V_{\text{perim},Q}(P'')$.*

Finally, we state an important lemma that is crucial in characterizing the set to which the sequence of the positions of the agents converges.

Lemma IV.3 *Let $P \in (Q \setminus \text{Ve}_\text{r}(Q))^n$. Let $\mathcal{G}(P)$ be any graph spatially distributed over $\mathcal{G}_{\text{vis},Q}(P)$. There exists at least one agent i with $p_i \in \text{Ve}(\text{MPP}(P, Q)) \setminus \text{Ve}_\text{r}(\text{MPP}(P, Q))$ such that the following are true:*

(i) *there exists $p \in X_i$ such that $p \neq p_i$ and $[p_i, p] \subset X_i$;*
(ii) $\|p_i - \text{proj}_{X_i} \text{CC}(\mathcal{M}_i)\| > 0$.

We shall also require, at some times, to make the following assumption on a sequence $\{P_m\}_{m \in \mathbb{N} \cup \{0\}} \subset Q^n$:

(A) There exists a compact set $\mathcal{X} \subset (Q \setminus \text{Ve}_\text{r}(Q))$ such that $\{P_m\}_{m \in \mathbb{N} \cup \{0\}} \subset \mathcal{X}^n$.

We are now ready to state the following convergence result.

Theorem IV.4 *Let $p_1, \dots, p_n$ be a network of visually-guided agents in $Q \in \mathcal{Q}$, with maximum step size $s_{\max} \in \mathbb{R}_+$. Assume that Q does not contain any holes, and that the proximity graph $\mathcal{G}$ is spatially distributed over $\mathcal{G}_{\text{vis},Q}$ and has the same connected components as $\mathcal{G}_{\text{vis},Q}$. Then, any trajectory $\{P_m\}_{m \in \mathbb{N} \cup \{0\}}$ of $T_\mathcal{G}$ has the following properties:*

(i) *if the locations of two agents belong to the same connected component of $\mathcal{G}_{\text{vis},Q}(P_k)$ for some $k \in \mathbb{N} \cup \{0\}$, then they remain in the same connected component of $\mathcal{G}_{\text{vis},Q}(P_m)$ for all $m \geq k$,*
(ii) $V_{\text{perim},Q}(P_{m+1}) \leq V_{\text{perim},Q}(P_m)$*, for all $m \in \mathbb{N} \cup \{0\}$,*
(iii) *if $\{P_m\}_{m \in \mathbb{N} \cup \{0\}}$ satisfies (A), then $\{P_m\}_{m \in \mathbb{N} \cup \{0\}}$ converges to a point $P^* \in \mathcal{X}^n$ such that either $p_i^* = p_j^*$ or $[p_i^*, p_j^*] \not\subset Q$ for all $i, j \in \{1, \dots, n\}$.*

The proof for Theorem IV.4 is based on the following useful results. The technical approach in what follows is similar to the one in [9].

To a proximity graph function $\mathcal{G}$ that is spatially distributed over $\mathcal{G}_{\text{vis},Q}$, and a configuration $P \in Q^n$, one may associate a graph $G_{\mathcal{G}(P)} = (\{1, \dots, n\}, E)$ by defining $(i, j) \in E$ if (p_i, p_j) is an edge of $\mathcal{G}(P)$. Clearly, for each $P \in Q^n$, $\mathcal{N}_{G_{\mathcal{G}(P)}}(i)$ is equal to the set of neighbors of p_i with respect to the graph $\mathcal{G}(P)$. Given an undirected graph $G = (\{1, \dots, n\}, E)$, define the *Circumcenter Algorithm at Fixed Topology* $T_G : Q^n \to Q^n$ whose ith component is

$$(T_G)_i(p_1, \dots, p_n) = (T_\mathcal{G})_i(p_1, \dots, p_n).$$

Lemma IV.5 *For $G = (\{1, \dots, n\}, E)$, the map $T_G : Q^n \to Q^n$ has the following properties:*

(i) *The map $P \mapsto T_G(P)$ restricted to $(Q \setminus \text{Ve}_\text{r}(Q))^n$ is continuous, and*
(ii) $\text{MPP}(T_G(P), Q) \subseteq \text{MPP}(P, Q)$*, for $P \in Q^n$.*

Given $Q \in \mathcal{Q}$, define the *Circumcenter Algorithm at All Connected Topologies* $T : Q^n \to 2^{(Q^n)}$ by

$$T(P) = \{T_G(P) \in Q^n \mid G = (\{1, \dots, n\}, E) \text{ is connected}\}.$$

Proposition IV.6 *For $Q \in \mathcal{Q}$, the map $T : Q^n \to 2^{(Q^n)}$ has the following properties:*

(i) *the map $P \mapsto T(P)$ restricted to $\mathcal{X}$, a compact subset of $(Q \setminus \text{Ve}_\text{r}(Q))^n$, is upper semicontinuous, and*
(ii) $\text{MPP}(T(P), Q) \subset \text{MPP}(P, Q)$*, for $P \in Q^n$ if there exists $p_i, p_j \in P$ such that $.p_i \neq p_j$.*

Now that we have analyzed the smoothness of T, let us study the properties of the function $V_{\text{perim},:} Q^n \to \overline{\mathbb{R}}_+$.

Lemma IV.7 *The function $V_{\text{perim},:} Q^n \to \overline{\mathbb{R}}_+$ has the following properties:*

(i) *$V_{\text{perim},Q}$ is continuous, and is invariant under permutations of its arguments;*
(ii) *$V_{\text{perim},Q}(P) = 0$ if and only if $p_i = p_j$ for all $p_i \in P, i \in \{1, \dots, n\}$;*
(iii) *$V_{\text{perim},Q}$ is strictly decreasing along T as long as $V_{\text{perim},Q}(P) > 0$.*

We now present the asymptotic convergence properties of the algorithm T. The proof of this relies on a discrete-time LaSalle Invariance Principle for set-valued maps; see [9].

Lemma IV.8 *Let $Q \in \mathcal{Q}$. Assume that Q does not contain any holes, and that the proximity graph $\mathcal{G}$ is spatially distributed over $\mathcal{G}_{\text{vis},Q}$ and has the same connected components as $\mathcal{G}_{\text{vis},Q}$. Then, any sequence $\{P_m\}_{m \in \mathbb{N}\cup\{0\}}$, defined by $P_{m+1} \in T(P_m)$ and satisfying Assumption (A), converges to a point $P^* \in \mathcal{X}^n$ such that $p_i^* = p_j^*$ for all $i, j \in \{1, \dots, n\}$.*

E. A variant of the Circumcenter Algorithm

In Section IV-D, we conjecture that the Circumcenter Algorithm solves the rendezvous problem for visually-guided agents if the network evolves in a compact subset of $Q \setminus \text{Ve}_\text{r}(Q)$. In what follows we describe an algorithm that we conjecture guarantees convergence without this assumption.

Name: Modified Circumcenter Algorithm over $\mathcal{G}$
Assumes:
(i) $s_{\max} \in \mathbb{R}_+$ is maximum step size
(ii) $Q \in \mathcal{Q}$
(iii) $\mathcal{G}$ is a spatially distributed proximity graph over $\mathcal{G}_{\text{vis},Q}$ with the property that two agents at the same location have identical sets of neighbors.

For $i \in \{1, \dots, n\}$, agent i executes the following at each time instant in $\mathbb{N}$:

acquire $\{q_1, \dots, q_k\} := \mathcal{N}_{\mathcal{G}_{\text{vis},Q}, p_i}(\mathcal{P})$
compute $\mathcal{W}_i := \{q_j \mid q_j = p_i,\ j \in \{1, \dots, n\}\}$
compute $\mathcal{B}_i := (\mathcal{N}_{\mathcal{G}, p_i}(\{q_1, \dots, q_k\}) \setminus \mathcal{W}_i)$
compute $\mathcal{M}_i := \mathcal{B}_i \cup \{p_i\}$
if $\mathcal{B}_i = \{v\}$, for $v \in \text{Ve}_\text{r}(Q)$, and $p_i \notin \text{Ve}_\text{r}(Q)$ **then**
  compute $q_i^* := v$
else
  compute $X_i := C_{p_i,Q}(\mathcal{M}_i) \cap \text{MPP}(\mathcal{M}_i)$
  compute $q_i^* := \text{proj}_{X_i}(\text{CC}(\mathcal{M}_i))$
end if
$u_i := \frac{\min(s_{\max}, \|q_i^* - p_i\|)}{\|q_i^* - p_i\|}(q_i^* - p_i)$

Remark IV.9 The graph $\mathcal{G}_{\text{lc-vis},Q}$ fulfills assumption (iii) in the statement of the Modified Circumcenter Algorithm.

V. Simulation results

To conduct experiments, a two-layer simulation environment has been developed in `Matlab`®. Figs. 1, 8 and 9 illustrate the performance of the Circumcenter Algorithm in Section IV-C.

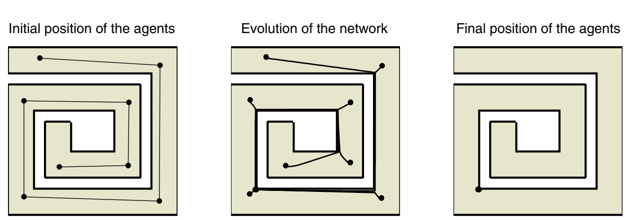

Fig. 8. Simulation results of the Circumcenter Algorithm on a network of agents distributed in a spiral polygon. The locations of the agents, at all times, do not belong to reflex vertices. However, at some instants, reflex vertices are approached very closely. The algorithm is run over $\mathcal{G}_{\text{vis},Q}$.

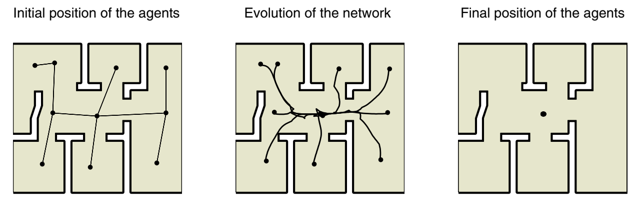

Fig. 9. Simulation results of the Circumcenter Algorithm on a network of agents distributed in a polygon shaped like a typical floor plan. The algorithm is run over $\mathcal{G}_{\text{lc-vis},Q}$.

VI. Conclusions

This paper focuses on the distributed control of synchronous networks of visually-guided robotic agents. We have defined some useful geometric quantities, such as continuous constraint sets and generalized visibility graphs, and studied circumcenter algorithms for rendezvous. We have provided a convergence proof as well as successful numerical simulations. Possible future work involves coordination algorithms for deployment and search for visuall-guided agents.

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