Proceedings of the
44th IEEE Conference on Decision and Control, and
the European Control Conference 2005
Seville, Spain, December 12-15, 2005

Planning Remanufacturing Systems by Constrained Ordinal Optimization Method with Feasibility Model

Chen Song, *Student member, IEEE*, Xiaohong Guan, *Senior Member, IEEE*,
Qianchuan Zhao, Qingshan Jia, *Student member, IEEE*

***Abstract*—Planning for a complex remanufacturing systems is often an NP-hard problem in terms of computational complexity and simulation is usually the only available but very time-consuming approach in many cases. Ordinal optimization offers an efficient framework for simulation based optimization approaches. In this paper, a new constrained ordinal optimization method is presented for solving remanufacturing planning problems. The scheme of *"Horse Race" with Feasibility Model (HRFM)* is developed to select the set of good enough plans. The rough set method in machine learning and knowledge discovery is applied to generate rules for feasibility determination. This method is compared with the *Blind Picking with Feasibility Model (BPFM)* method. Numerical testing of a practical remanufacturing system shows that the HRFM method presented in this paper is more efficient to meet the same required alignment probability.**

I. INTRODUCTION

Planning for a complex remanufacturing systems is often an NP-hard problem in terms of computational complexity, including combinatorial sizes of huge discrete state space, lack of analytical model, uncertainties, etc. The basic activities in a practical remanufacturing system for a repairing factory considered in this paper are demonstrated in Figure 1 and described in detail in [1]. In this system, ensembles or assets to be repaired from various customers arrive at an overhaul center and queued for maintenance. Assets are accepted by the overhaul center if the capacity is available and disassembled into modules with variable rates on different types and conditions, and the modules are further disassembled into parts. The condition of the parts is inspected to determine if overhauling is needed. The modules and parts without the need of service are organized and put aside for reassembling. Repairable parts are transported to the repair shops according to their types. New parts are replenished by placing an order to meet the demand of assembly. At the end of the repair windows, the parts are assembled into modules and the finished modules are stored and used for asset assembly.

The planning problem of the remanufacturing system is to allocate appropriate capacity, inventory, etc to minimize the cost while satisfying constraints such that the repair cycling

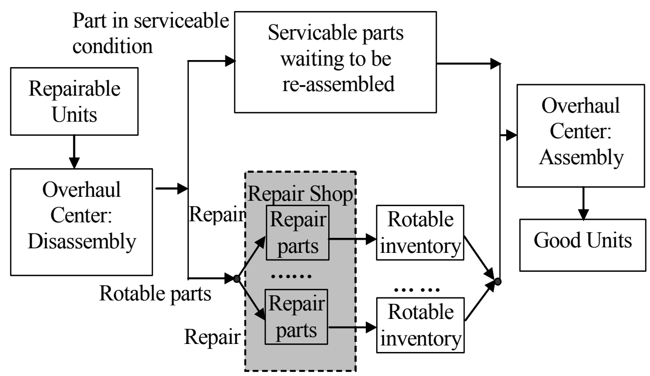


Figure 1 Structure of the remanufacturing system

time with certain probability must be within the required due date. The research includes capacity planning ([2], [3]), inventory planning ([1]), etc. One or more segments of a remanufacturing system such as disassembly were analyzed and optimized in [5] and [6], or some important factors of a disassembly line were discussed in [7]. However few existing methods addressed the planning and operation control of a remanufacturing system as a whole and the interactions among the activities of the segments and dealt with the systems with complicated dynamics and uncertainties. Because of discrete states, uncertainties, complicated stochastic constraints ([8]), analytical methods are difficult to apply and simulation is usually the only available but very time-consuming approach in many cases.

Ordinal optimization (OO) method offers an efficient way to simulation based optimization approach ([9]). It intends to find a good or satisfying solution among a large number of candidates rather than the true optimum by accurately estimating the performance value. OO is particularly attractive for stochastic discrete optimization since it is immune to large noise with affordable computational complexity.

However, the regular OO method would encounter serious challenges when there exist complicated stochastic constraints since many infeasible decisions cannot be excluded from ordinal comparison without extensive computation involving the expectation operation. Determining feasibility beforehand is extremely difficult by analytical or numerical methods for the same reason that evaluating stochastic constraints may be just as complicated as evaluating the cost function. In fact incorporating

The research presented in this paper is supported in part by the National Outstanding Young Investigator Grant (6970025), National Science Foundation (60243001, 60274011) and 863 High Tech Development Plan (2001AA413910) of China.

Chen Song (csong@sei.xjtu.edu.cn) and Xiaohong Guan (xhguan@sei.xjtu.edu.cn) are with SKLMS Lab and System Engineering Institute, Xian Jiaotong University, China 710049. Chen Song is also with NEC Labs, China 100084. Qianchuan Zhao (zhaoqc@tsinghua.edu.cn), Qingshan Jia (jiaqingshan98@mails.tsinghua.edu.cn), and Xiaohong Guan also, are with Center for Intelligent and Networked Systems, Tsinghua University, China 100084.

constraints efficiently is one of the major challenges in developing any simulation-based optimization method.

The OO for constrained problem is first studied in [10] where the constraints are converted into additional objectives to form a multi-objective optimization problem. Another is followed by [11] as the *vector ordinal optimization* method that uses a regression function to estimate how many number of observed "Pareto Layers" to be selected in order to contain at least k decisions in true Pareto Frontier with high probability. The concept of "order-constraint" is developed in [8] to estimate the lower bound of *Alignment Probability* in *constraint ordinal optimization*. However, neither approach offers a complete solution to the core difficulty of the constrained problems since constraint satisfaction is basically a cardinal notion contrary to ordinal comparison that OO tries to exploit.

The feasibility model developed for remanufacturing planning in our previous work provides a new framework for developing constrained ordinal optimization methods ([12]). The basic idea is to combine the feasibility estimation with ordinal selection of good enough plans. The feasibility of a plan is estimated by the feasibility model without simulation and infeasible plans are excluded with certain confidence. The feasibility model may not perfect. However, as long as the estimation is more correct than wrong, no costly simulation is needed to exclude infeasible plans.

In this paper, we present a new constrained ordinal optimization method for remanufacturing planning. The scheme of "Horse Race" with Feasibility Model (HRFM) is developed to select the set of good enough plans. The rough set method in machine learning and knowledge discovery is applied to generate rules for feasibility determination ([1]). In fact any crude method such as the one based on rough set theory developed in our previous work can be applied to determine the decision feasibility efficiently. This method is compared with the *Blind Picking with Feasibility Model* (BPFM) method developed in our previous work ([12]). Numerical testing of a practical remanufacturing system shows that the HRFM method presented in this paper is more efficient to meet the same required alignment probability.

The rest of the paper is organized as follows. The problem formulation of the remanufacturing system is presented in Section II. In Section III, the basic principle and methodology of COO are presented firstly, then the subset selection rule: *Horse Racing with feasibility model* (HRFM) is presented. Numerical testing results and analysis on a remanufacturing planning are shown in Section IV and concluding remarks are given in section V.

II. Problem formulation of the remanufacturing problem

Suppose in the remanufacturing system, the cycle time T_c of repairing asset i is considered as the time completing assembly η_k (initial time assumed as 0). The required cycle time is d days and planning horizon, i.e., the time span along which the planning problem is defined (can be considered as the longest possible cycle time), is T_t days. The probability $P_b(\cdot)$ of cycle time for a feasible plan with suitable capacity and inventory configuration should be no less than the required probability P_{rc}, i.e.

$$P_b(T_c(i) \le d \mid \eta_k < T_t) \ge P_{rc} \quad (1)$$

The cost of a remanufacturing process is related to the allocated processing capacity and the inventory level, orders for new parts, etc. Suppose there are n part types and planning horizon is m quarters. The goal of the planning problem is to allocate resources among repair shops to minimize the remanufacturing cost subject to the cycle time less than certain days satisfied with certain probability. It is formulated as a stochastic optimization problem in (2) subject to (1):

$$\min_{C_c, I} \quad Cost = \tau C_c + V(I, \varpi), \quad (2)$$

where

Cost: total cost consisting of capacity cost τ C_c and inventory cost $V(I, \varpi)$;

C_c: total allocated capacity;

$V(I, \varpi)$: inventory cost related to the inventory I and uncertain difference ϖ;

τ: non-negative cost coefficients;

ϖ: total part surplus;

The decision variables in this problem are reserved capacity C_c and inventory level I, which are referred for convenience as decision θ in the search space Θ. The above problem formulation is a much simplified. The detailed descriptions of the problem are presented in [1].

III. Constrained Ordinal Optimization Approach

III.1. Basic Principle of COO

OO is based on two tenets: (a) it is much easier to determine "order" than "value." To determine whether A is better or worse than B is a simpler task than to determine how much A is better than B (i.e., the *value* of A-B). This is true especially when uncertainties exist. (b) Instead of asking the "best for sure," we seek the "good enough with high probability."

In OO, the good enough decisions with high probability is quantified by the probability of "matching" between the observed "good enough" set called *selected subset S*, and the true *good enough subset G*. The set G is a subset of the search space Θ satisfying some desirable criteria, say the top-r% best decisions in Θ. The true good enough subset G is generally not obtainable. A selection rule or method is needed to select S with certain evaluation scheme (crude algorithms, heuristics and even blind pick) to guarantee the desired degree of "matching" called *alignment level*, and the confidence of achieving a certain alignment level is referred to as the *alignment probability*. It is generally defined as the probability of the observed top-r decisions actually contained at least k of actual top-r decisions (described in detail in Section III). After S is obtained, detailed simulations may be applied to its members to select the best solution within S as required in many practical problems.

The relationship among decision space Θ, good enough subset G and selected subset S is shown in Figure 2. The alignment between G and S is Region 1. The alignment level depends on how many good enough decisions are selected in S.

Direct selection of S in Θ for constrained problems is inappropriate or at least inefficient because there are many infeasible decisions (black spots) that cannot be excluded beforehand. Consequently, the alignment level (region 1 in Figure 2) cannot be estimated by the regular OO method since the quantity of infeasible decisions in S is unknown. In other words, the uncertain number of infeasible decisions in Θ and S is the main difficulty of COO.

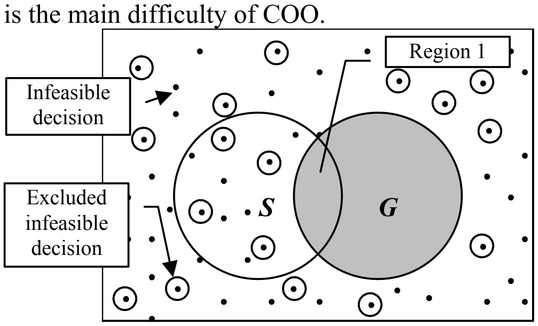


Figure 2 Concept of COO

In order to overcome the above difficulty, we apply the feasibility model in [1] to exclude infeasible decisions (black spots with circle) and obtain an approximate feasible region Θ_f in the search space Θ. The probability that a decision is correctly determined by the feasible model as feasible is P_f. Although some infeasible decisions in Θ_f and S_f may be incorrectly determined as feasible, the densities of feasible decisions in S_f (or Θ_f) are more than S (or Θ) if the feasibility probability P_f is more than 50%. This means that there are more good plans in S_f than in S if their sizes are the same since there are less infeasible plans in S_f.

It is noted that that determination of "*Feasible* vs *infeasible*" is ***ordinal***. All the advantages of OO apply here, i.e., determination of Θ_f from Θ can be very effective since only approximate feasibilities are required. In addition, "*imperfectness*" of the feasibility model is also in tune with the "goal softening" concept in OO. Although individual determination of feasibility using a crude model may give erroneous results, the model could be very robust with respect to a group of candidates overall. The above approach is an evolution of the OO methodology amenable to constrained optimization problems with a complete ordinal concept. The tenets of "*ordinal comparison*" and "*goal softening*" are reflected by the integration of "imperfect feasibility model" and "feasibility determination."

III.2. Horse Racing in COO

In OO theory, the effectiveness of the approach depends on how to select subset S. The simplest selection method that requires no performance sampling or estimate is to blindly or randomly select S. The BPFM method that combine Blind Picking (BP) with the Feasibility Model for this case has been presented in our previous work ([12]). In this paper, we will concentrate on developing the HRFM method that integrate Horse Racing (HR) and the Feasibility Model, applying it to the practical remanufacturing system and compare the results with those obtained by BPFM.

Different from BP, HR method is based on the performance estimates in decision space, possibly inaccurate and crude. The decisions or plans selected by the HR rule can be considered as having all decisions with noisy performance competing to be selected, very much similar to N horses running in a race with the current leading being selected but possibly falling behind later. The selection of good decisions or plans in S is determined by their estimated performance values ([13]).

III.2.1 Horse Racing for COO with Feasibility Model

With feasibility model, the infeasible decisions are excluded and the feasible decisions are selected at the beginning of the HR process. The process is described in detail in Table 1. It should be noted that the approximately feasible decisions are possibly not consistent to the actually feasible decisions because of the imperfectness of feasibility model.

TABLE 1 HORSE RACING FOR THE COO PROBLEM

Step	*G* selection	*S* selection (HRFM)
1	Decision samples in Θ, $\{\theta_1,\ldots,\theta_N\}$	Approximately feasible decision samples, $\{\theta_1,\ldots,\theta_{N_2}\}$
2	Feasible decision samples, $\{\theta_1,\ldots,\theta_{N_1}\}$	Noise samples $\sim\xi(\cdot)$, $\{\omega_1,\ldots,\omega_{N_2}\}$
3	Performances $J(\theta_i)$, $\{J_1,\cdots,J_{N_1}\}$	Noisy performances $\bar{J}_i = J_i + \omega_i$, $\{\bar{J}_1,\cdots,\bar{J}_{N_2}\}$
4	Reordering	Reordering
5	Ordered Performances $\{J_{[1]},\cdots,J_{[N_1]}\}$	Ordered Noisy Performances, $\{\bar{J}_{[1]},\cdots,\bar{J}_{[N_2]}\}$
6	Ordered decision samples, $\{\theta_{[1]},\cdots,\theta_{[N_1]}\}$	Observed decision samples, $\{\bar{\theta}_{[1]},\cdots,\bar{\theta}_{[N_2]}\}$
7	Good enough decisions *G*, $\{\theta_{[1]},\cdots,\theta_{[g]}\}$	Selected decisions *S*, $\{\bar{\theta}_{[1]},\cdots,\bar{\theta}_{[s]}\}$

The alignment probability (AP) for HR rules is defined as:

$$P_A = \Pr ob\{|G \cap S| \geq k\} = P_A(k, s, g|HR) \qquad (5)$$

where k is alignment level.

For unconstrained problems, the size s of HR method depend on ([13]):

1) alignment level, k;
2) size of the good enough subset, $|G|=g$;
3) size of decision space, N;
4) value of the required AP, P_A.
5) noise level, $\xi(\cdot)$;

6) type of Ordered Performance Curve (OPC) reflecting the ordered performance values from the smallest to largest versus corresponding decisions.

For COO problems, the size s_f of HRFM method also depends on additional factors:

7) feasibility probability reflecting the accuracy of the feasibility model, P_f;
8) density of feasible decisions in decision space, $P=F/N$. where F is the number of feasible decisions discerned by the feasibility model. We define the ***Feasibility Curve*** (FC) to reflect the feasibilities density with 1 representing a feasible decision and 0 an infeasible decision in Figure 4. The size of subset selection depends on the density of feasible decisions. The required size of selected subset is smaller when there exist a larger number of feasible decisions.
9) distribution of feasible decisions in decision space. We define ***Correlation Coefficient*** (ρ_{FO}) between the FC and the OPC to reflect the distribution of feasibility for ordered decisions.

$$\rho_{FO} = \left\{ \sum_{i=1}^{N} [F_i \cdot O_i]/N - [\sum_{i=1}^{N} F_i / N] \cdot [\sum_{i=1}^{N} O_i / N] \right\} \Bigg/ \sqrt{(\sum_{i=1}^{N} F_i^2)/N - (\sum_{i=1}^{N} F_i / N)^2} \sqrt{(\sum_{i=1}^{N} O_i^2)/N - (\sum_{i=1}^{N} O_i / N)^2} \quad (6)$$

where F_i and O_i represent the value of decision i for FC and OPC respectively.

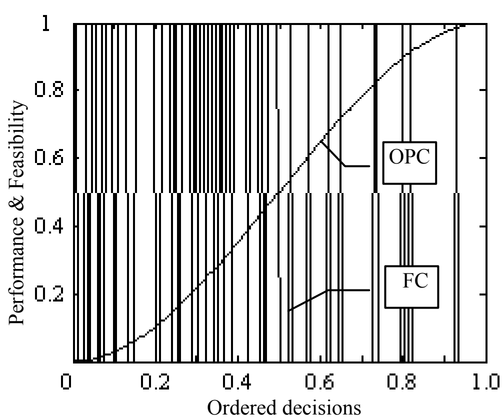


Figure 4 Relationship between FC and OPC

An example of the relationship between the FC and the OPC is shown in Figure 4. The size of the selection subset can be considered as a function of the above factors:

$$s_f(g,k; N, C, \xi(\cdot), P_A, P, P_f, \rho_{FO}) \quad (7)$$

III.2.2. Obtaining the size of the selected subset by regression

The regression process for determining the size of the selected subset in the HR method for unconstrained problems is listed in Table 2 ([13]).

TABLE 2 DETERMINING *S* BY REGRESSION IN HR

Step	Process
1	Generate standard OPC's;
2	Add noise on standard OPC's;
3	Record selected subset sizes that satisfy required P_A for different g and k through horse racing;
4	Determine Z_0, ρ, γ and η based in (8) by regression;
5	Calculate *s* based on (8) for required P_A, g and k;

In step 1 of the above process, the function $\Lambda(\cdot)$ defined in [13]* is applied to generate standard OPC's. Uniform noise density $\xi(\cdot) \sim [-W, W]$ is assumed in step 2. In step 4 and 5, the form of function is used as (8) in [13]:

$$s(g,k) = e^{Z_0} k^{\rho} g^{r} + \eta \quad (8)$$

where Z_0, ρ, γ and η are coefficients depending on N, C, P_A and $\xi(\cdot)$.

For constrained problems, the size of the selected subset is related to more factors such as P, P_f and ρ_{FO}. These factors must be embedded in the procedure to obtain (8) by regression. This procedure is listed in Table 3:

TABLE 3 DETERMINING S_f BY REGRESSION IN HRFM

Step	Process
1	Generate a corresponding standard OPC for the given model;
2	Add noise on the OPC;
3	Generate $N \times P$ feasible decisions that comply with the value of ρ_{FO} between the FC and the OPC;
4	Pick feasible decisions from the result of step 3 based on probability P_f;
5	Observe the size of subset selection that meet the required P_A for *g* and *k* through horse racing;
6	Replicate above steps 3-6 for different P_A, *g* and *k*;
7	Record all result of subset selection and determine Z_0, ρ, γ and η in (8) by regression.
8	Calculate s_f based on (8) for the required P_A, *g* and *k*

After the values of Z_0, ρ, γ and η are determined in Step 7, the subset selection of HRFM for the constrained problem then can be calculated by (8) for any required P_A, *g* and *k*.

* The standardized OPC is approximated by ([13]):

$$\Lambda(x|\alpha,\beta) = F^{-1}(x|\alpha,\beta) = F(x|\frac{1}{\alpha},\frac{1}{\beta})$$

where

$$F(x|\alpha,\beta) = \int_0^y f(z|\alpha,\beta)dz \,,\; f(y|\alpha,\beta) = Cy^{\alpha-1}(1-y)^{\beta-1},$$

$$C = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \quad \Gamma(\cdot) \text{ is gamma function.}$$

where C is one of the five OPC classes. The two-parameter model provide us the flexibility in describing the five OPC categories by varying α and β.

IV. Numerical Testing Results and Analysis

Numerical testing is performed for the remanufacturing system described in Section I. The parameters of the system are listed in Table 4. Allocated processing capacity $\vec{C}_i$ and inventory level $\vec{I}_i$ range between [10, 25] and [0, 10] respective in every quarter, and initialized to their minimum values, i.e., A = {(10, 10, …, 10), (0, 0, …, 0)} at start.

TABLE 4 THE PARAMETERS OF THE SYSTEM

Item	Value
Planning horizon	8 (quarters), i.e., 720 (days)
Arrival rate of assets	(2.5, 3, 2.5, 3, 3.5, 2.5, 2.5, 2.5, 3)
Disassembling time	5 (days)
Assembling time	7 (days)
Service time	Triangular Distribution with Parameters {30,60,90} (days)
Required cycle time	100 (days)
Required probability of cycle time	95%

Suppose the accuracy of the feasibility model is P_f = 80%, which is lower than the actual accuracy obtained in [1] for testing purpose. The FC curve is shown as the "0-1" curve in Figure 5. There are 654 feasible decisions in 1000 decisions with P = 0.654.

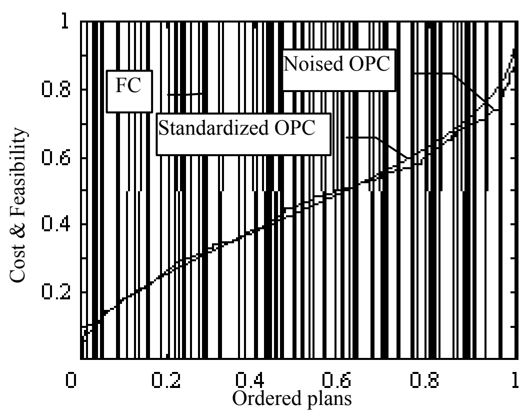


Figure 5 FC and OPC of the remanufacturing plans

The noisy OPC of the remanufacturing system causes the unsmooth non-decreasing curve in Figure 5. To obtain the value of coefficients in (8), the OPC is smoothed by standardized OPC $\Lambda(\cdot)$ with parameters $\alpha = 2$ and $\beta = 2.5$ as the smooth non-decreasing curve. Then the correlation coefficient between FC and OPC is calculated as ρ_{FO} = 0.23. When we run short simulation for HRFM, and the number of replications of the Monte Carlo simulation is 100, the noise of the cost evaluation $\xi(\cdot) \in [-0.01, 0.01]$ is low ([13]). Suppose the required AP = 95%. The size of selected subset is observed when g changed from 20 to 200 at intervals of 10 and k changed from 1 to 10. The given parameters and requirements, and the coefficients obtained by regression are listed in Table 5.

TABLE 5 COEFFICIENTS Z_0, ρ, γ, η BY REGRESSION IN HRFM

Parameters	Coefficients	Value by regression
N=1000, P_A=95%, $\alpha = 2$, $\beta = 2.5$, $\xi(\cdot) \in [-0.01, 0.01]$, P=0.654, P_f=80%, ρ_{FO} =0.23	Z_0	0.2172
	ρ	1.1347
	γ	0.5027
	η	5.6115

Once the value of Z_0, ρ, γ and η are obtained, the size of subset selection is calculated by (8). For example, when g = 50, k = 1, the calculated value of S_f should be at least 15.

The size of the selected subset by HRFM method is smaller than that by BPFM since the information of the current "leading horses" is incorporated in the model. Comparison of the two methods is performed. For BPFM method, the size of selected subset is given by (9) ([12]):

$$AP(|G \cap S_f| \geq k) = \sum_{j=k}^{\min(g,s_f)} \sum_{i=0}^{s_f-j} \frac{\binom{g}{j}\binom{F-g}{s_f-i-j}}{\binom{F}{s_f-i}} \binom{s_f}{i} (P_f)^{s_f-i} (1-P_f)^{i} \qquad (9)$$

where F is the size of feasible decision space Θ_f. The sizes of the selected subset for different alignment level k obtained by HRFM with N=1000, $\alpha = 2$, $\beta = 2.5$, P_A=95%, P=0.654, P_f=80%, $\xi(\cdot) \in [-0.01, 0.01]$, ρ_{FO} =0.23, g=50 are listed in Table 6, together with those obtained by BPFM. It is seen that for same alignment level, the required size of the selected subset by HRFM is smaller than that by BPFM.

TABLE 6 SIZES OF THE SELECTED SUBSETS

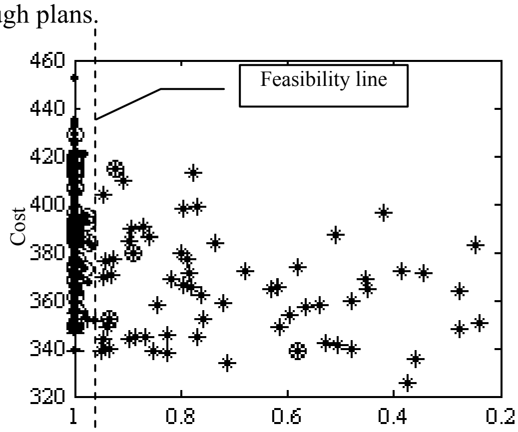
k	s_f	
	BPFM	**HRFM**
1	46	15
2	73	26
3	97	37
4	119	49
5	140	61

The alignment between S_f and G by HRFM with k=1 is shown in Table 7. It is shown that **Plan 90, 270, 450** in the selected subset belong to the good enough set G.

TABLE 7 GOOD ENOUGH AND SELECTED PLANS BY HRFM

Set	Plans	Alignment
S_f	{18, 198, 378, 558, 738, 918, 66, 246, 426, 606, 786, 966, ***90***, ***270***, ***450***}	{***90***, ***270***, ***450***}
G	{***90***, ***270***, ***450***, 630, 810, 990, 157, 337, 517, 697, 877, 1, 194, 374, 554, 734, 914, 136, 316, 496, 676, 856, 29, 209, 389, 569, 749, 929, 184, 364, 544, 724, 904, 43, 223, 403, 583, 763, 943, 146, 326, 506, 686, 866, 137, 317, 497, 677, 857, 143}	

The distributions of the selected plans of two methods are shown in Figure 6 and 7 respectively. It is seen that the selected plans by HRFM are better (lower costs) on average than those by BPFM. Therefore, the HRFM method is more effective to select the subset with smaller size in finding good enough plans.



Legends:

1. Asterisk ------ infeasible plans
2. Solid dots ------- feasible plans
3. Plus ------- good enough plans
4. Circle------------ selected plans

Figure 6 Distribution of the plans of BPFM

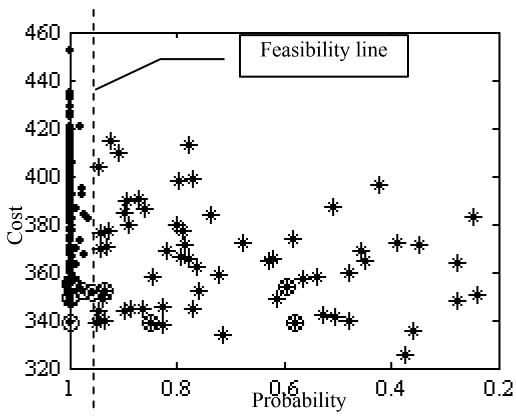


Figure 7 Distribution of plans in HRFM

V. CONCLUSION

Remanufacturing planning with complicated stochastic constraints is generally very difficult and simulation is usually the only way available. Combining the feasibility estimation with ordinal selection of good enough plans is an efficient and effective approach for solving this problem and the new HRFM method of COO approach is developed to solve this problem. Numerical testing for the planning problem of a practical remanufacturing system shows that to meet the same required alignment probability, HRFM requires a smaller size of the selected subset in comparison with BPFM and is more efficient. Furthermore, the COO method presented in the paper is a general approach can be applied in many discrete and hybrid optimization problems with complicated stochastic constraints.

REFERENCES

[1] C. Song, X. H. Guan, Q. C. Zhao and Y. C. Ho, "Machine Learning Approach for Determining Feasible Plans of a Remanufacturing System," to appear in *IEEE Transaction on Automation Science and Engineering*, 2004.

[2] V. D. R. Guide, "Technical note: Production planning and control for remanufacturing: industry practice and research needs," *Journal of Operations Management*, vol. 18, pp. 467–483, 2000.

[3] V.D.R. Guide, R. Srivastava, and M.S. Spencer, "An evaluation of capacity planning techniques in a remanufacturing environment," *Int. J. Prod. Res*, vol. 35, no. 1, pp. 67-82, 1997.

[4] B. Okubo, "Reserve production capacity in a production inventory system," *Int. J. of Production Economics*, vol. 44, pp. 159-166, 1996.

[5] A. J. D. Lambert, "Determining optimum disassembly sequences in electronic equipment," *Computers & Industrial Engineering*, vol. 43, pp. 553–575, 2002.

[6] G. P. Kiesmuller, "Optimal control of a one product recovery system with lead times," *Int. J. Production Economics*, vol. 81-81, pp. 333–340, 2003

[7] A. Gungor and S. M. Gupta, "Disassembly Line in Product Recovery," *International Journal of Production Research*, vol. 40, no. 11, pp. 2569-2589, 2002.

[8] D. Li, L. H. Lee and Y. C. Ho, "Constraint Ordinal Optimization," *Information Sciences*, vol. 148, pp. 201-220, 2002.

[9] Y. C. Ho and R. S. Sreenivas, "Ordinal Optimization of Discrete Event Dynamic Systems," *Journal of DEDS,* vol. 2, no. 2, pp. 61-88, 1991.

[10] L. H. Lee, W. G. Li , and Y. C. Ho, "Vector Ordinal Optimization - a New Heuristic Approach and Its Application to Computer Network Routing Design Problems,", *International Journal of Operations and Quantitative Management* , vol. 5, no. 3, pp 211-230, 1999.

[11] Q. C. Zhao, Y. C. Ho and Q. S. Jia, "Vector Ordinal Optimization," to appear in *Journal of Optimization Theory and Applications*.

[12] X. H. Guan, C. Song, Y. C. Ho and Q. C. Zhao, "Constrained Ordinal Optimization – A Feasibility Model Based Approach", submitted.

[13] T. W. E. Lau and Y. C. Ho, "Universal Alignment Probabilities and Subset Selection for Ordinal Optimization," *Journal of Optimization and Theory*, vol. 39, no. 3, 1997.

[14] Y. C. Ho, "Overview of Ordinal Optimization," in *Proceedings of 33rd conference on decision and control,, Lake Buena Vista*, FL, December 1994, pp. 1975-1977.

Nomenclature

Notation	Definition	Notation	Definition
$P_b(\cdot)$	probability of satisfying required cycle time	s_f	size of S_f
$T_c(i)$	cycle time of repairing asset *i*	$J(\theta_i)$	performance of decision *i*
d	required cycle time	J_i	performance of decision *i*
η_k	time completing assembly	$J_{[i]}$	*i*th smallest performance
T_t	time span	ω_i	noise of decision *i*
P_{rc}	required probability of cycle time less than requirement	$\bar{J}_i$	noisy performance of decision *i*
$Cost$	total cost	$\bar{J}_{[i]}$	*i*th smallest noisy performance
C_c	total allocated capacity	θ_i	decision *i*
$V(I,\varpi)$	inventory cost related to the inventory *I* and uncertain difference ϖ	$\theta_{[i]}$	Ordered decision *i*
τ	non-negative cost coefficients	$\bar{\theta}_i$	noisy decision *i*
ϖ	total part surplus	$\bar{\theta}_{[i]}$	ordered noisy decision *i*
θ	decision	P_A	alignment probability
Θ	decision space	k	alignment level
N	size of decision space	$\xi(\cdot)$	noise
G	good enough subset	P_f	accuracy probability of the feasibility model
g	size of *G*	P	density of feasible decisions
S	selected subset	ρ_{FO}	correlation coefficient between the FC and the OPC
s	size of *S*	Z_0, ρ, γ, η	coefficients of function *s*(*k, g*)
Θ_f	approximately feasible decision space	α, β	coefficients of function $\Lambda(\cdot)$
S_f	selected subset with feasibility model	F	size of Θ_f in BPFM method