

Trajectory Manifold Exploration for the PVTOL aircraft

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Abstract—In this paper, we study the trajectory space of the PVTOL aircraft. We show that, due to the non-minimum phase nature of the system, more aggressive trajectories may be tracked with respect to the simplified differentially flat model. Given bounded C^2 trajectories of the center of gravity, we show that there exists a bounded roll trajectory which implements them. We compute an approximation of such roll trajectory using a Newton method for nonlinear optimization based on a trajectory tracking projection operator.

I. INTRODUCTION

Nonlinear systems characterized by the presence of unstable internal dynamics are called non-minimum phase [9]. These represent the extension to the nonlinear case of linear systems with positive transmission zeros. Trajectory tracking keeping bounded internal states is, for such systems, a very complex problem and so it is of interest to many researchers.

A large class of non-minimum phase mechanical systems is characterized by the presence of a pendulum like unstable dynamics. In this paper, we analyze the trajectory manifold of this class of non-minimum phase systems which we call “Driven Pushed Pendulums” (DPPs). It can be easily shown that the PVTOL aircraft is an important example of a DPP in its most general definition and that many other mechanical systems, e.g. the cart-pole system [1], the pendubot [14], the bicycle model [2], [3], can be seen as particular cases. PVTOL aircraft is a simplified model introduced by Hauser et al. in [6] in order to capture the non-minimum phase behavior of a real aircraft. Thenceforth many researchers have studied this system providing different solutions for trajectory tracking [10], [11], [13], [12].

In [6] and in the following papers, the non-minimum phase characteristic of the system was considered as an undesired effect which was preferred to be small. Here, we reverse that point of view and, conversely, we consider instability of the internal dynamics as a benefic effect which augments the maneuverability of the system.

We show that for any bounded C^2 assigned trajectory of the center of gravity we can find a bounded roll trajectory that (approximately) implements it. While the existence of the solution is proved by using results of Hauser et al. in [3] and [5] based on dichotomy, we use a different approach for computing it, based on an optimal control problem

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which guaranties a (fast) quadratic rate of convergence.

II. PVTOL AIRCRAFT MODEL

In [6] the model of the PVTOL aircraft was introduced. Using standard aeronautic conventions the equations of motion are given by

$$\begin{aligned}\ddot{y} &= u_1 \sin \varphi - \epsilon u_2 \cos \varphi, \\ \ddot{z} &= -u_1 \cos \varphi - \epsilon u_2 \sin \varphi + g, \\ \ddot{\varphi} &= u_2.\end{aligned}\quad (1)$$

The aircraft state is given by the position (y, z) of the center of gravity, the roll angle φ and the respective velocities \dot{y} , \dot{z} and $\dot{\varphi}$. The control inputs u_1 and u_2 are respectively the vertical thrust force and the rolling moment. The rolling moment u_2 generates also a lateral force because the lift forces are not perpendicular to the wings, ϵ is the coupling coefficient. Finally g is the acceleration of gravity. In fig. 1 the PVTOL aircraft with the reference system and the inputs is shown.

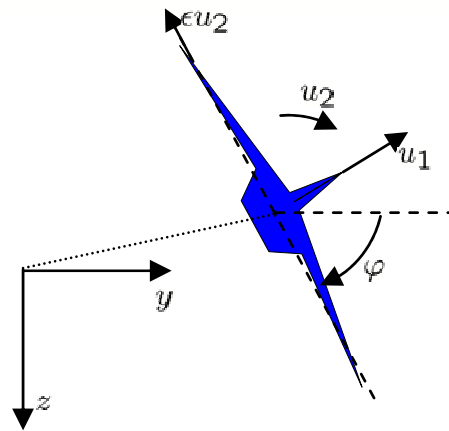


Fig. 1. PVTOL aircraft.

A. The decoupled PVTOL model

If in system (1) we pose $\epsilon = 0$, we obtain the equation of a simplified model with no coupling between rolling moment and lateral force. This is a desired condition from a pilot perspective, because it allows to tackle two different control tasks with decoupled inputs, i.e. thrust for the altitude control and roll moment for the direction.

The simplified system can be exactly linearized (with no zero dynamics) introducing u_1 and \dot{u}_1 as new states and

then using \ddot{u}_1 and u_2 as control inputs (dynamic extension). Differentiating (1) (with $\epsilon = 0$) twice it follows:

$$\begin{aligned} \begin{bmatrix} y^{(4)} \\ z^{(4)} \end{bmatrix} &= \begin{bmatrix} -u_1 \dot{\varphi}^2 \sin \varphi + 2\dot{u}_1 \dot{\varphi} \cos \varphi \\ u_1 \dot{\varphi}^2 \cos \varphi + 2\dot{u}_1 \dot{\varphi} \sin \varphi \end{bmatrix} + \\ &+ \begin{bmatrix} \sin \varphi & u_1 \cos \varphi \\ -\cos \varphi & u_1 \sin \varphi \end{bmatrix} \begin{bmatrix} \ddot{u}_1 \\ u_2 \end{bmatrix} \quad (2) \\ &= \alpha(u_1, \dot{u}_1, \varphi, \dot{\varphi}) + \beta(u_1, \varphi) [\ddot{u}_1 \ u_2]^T, \end{aligned}$$

where $\beta(u_1, \varphi)$ is invertible if u_1 is not zero. This condition corresponds to the intuitive idea that if no thrust force is applied, only the orientation of the PVTOL may be affected. Choosing

$$[\ddot{u}_1 \ u_2]^T = \beta(u_1, \varphi)^{-1} (-\alpha(u_1, \dot{u}_1, \varphi, \dot{\varphi}) + [v_1 \ v_2]^T) \quad (3)$$

the system is completely linearized

$$\begin{bmatrix} y^{(4)} \\ z^{(4)} \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}. \quad (4)$$

This implies that we can parameterize the trajectory manifold of the system by means of $y(t)$, $z(t)$ and their first four derivatives. It turns out that, in order to have bounded state trajectories, $y(t)$ and $z(t)$ must be C^4 (or at least have absolute continuous third derivatives). Moreover those trajectories may be tracked by using a linear feedback for v_1 and v_2 .

B. Input-Output linearization of the PVTOL model: the Driven Pushed Pendulum

As in the decoupled case we may input-output linearize the system differentiating the outputs y and z until at least one input appears. Differentiating twice (1), however, both inputs appear. Hence the system has a relative degree $r=[2 \ 2]$. This implies that there exist unobservable internal dynamics of dimension two. Hence if we pose

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \sin \varphi & -\cos \varphi \\ -\frac{1}{\epsilon} \cos \varphi & -\frac{1}{\epsilon} \sin \varphi \end{bmatrix} \left(\begin{bmatrix} 0 \\ -g \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \right) \quad (5)$$

the dynamics of the system become

$$\begin{aligned} \ddot{y} &= v_1 \\ \ddot{z} &= v_2 \\ \ddot{\varphi} &= \frac{1}{\epsilon}(g - v_2) \sin \varphi - \frac{1}{\epsilon} v_1 \cos \varphi. \end{aligned} \quad (6)$$

As expected, the output dynamics are linear, however unobservable internal dynamics are also present. If the inputs and outputs are constrained to be zero $y(0) = 0$, $z(0) = 0$, $\dot{y}(0) = 0$ and $\dot{z}(0) = 0$ and posing $v_1(\cdot) = 0$ and $v_2(\cdot) = 0$, the zero dynamics of the system are given by

$$\ddot{\varphi} = \frac{g}{\epsilon} \sin \varphi. \quad (7)$$

which, for $\epsilon > 0$, describe the motion of an undamped pendulum with an unstable equilibrium point in $(\varphi, \dot{\varphi}) = (0, 0)$ and a stable (but not asymptotically stable) equilibrium point in $(\varphi, \dot{\varphi}) = (\pi, 0)$. The phase portrait of (7) is shown in fig. 2.

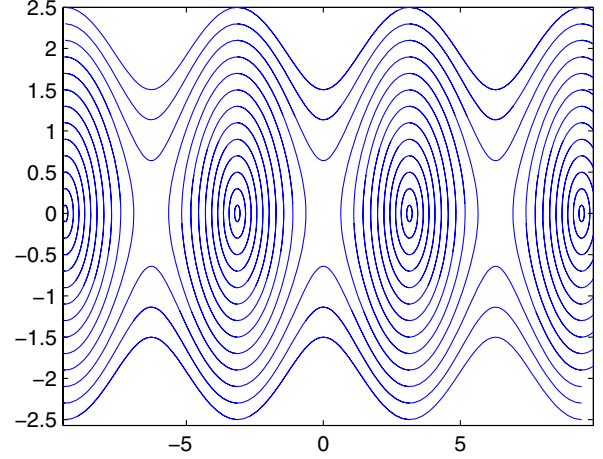


Fig. 2. Phase portrait of the zero dynamics for $\epsilon = 1$ and $g = 1$.

Remark 1: The control $[u_1 \ u_2]^T$ in (5) is well defined as long as the coupling coefficient ϵ is not zero. Moreover a strong coupling allows to bound the control effort. For this reason coupling is beneficial and ϵ should not be too small.

Remark 2: The Input-Output linearized system (6) is a DPP driven by the y and z accelerations.

Remark 3: If ϵ is not zero the system is not flat w.r.t. the y and z outputs anymore. In [11] the system was shown to be flat if the coordinates of the *Huygens center of oscillation* are chosen as outputs. These outputs however are not convenient to parameterize trajectories.

III. PROBLEM STATEMENT

The objective is to explore the trajectory manifold of the PVTOL using the y and z trajectories of the center of gravity as “parameters”. In other words we can summarize our objective as follows:

Problem: Given $y_d(\cdot)$ and $z_d(\cdot)$ “sufficiently smooth” trajectories of the center of gravity we want to find a “bounded” roll trajectory $\varphi(\cdot)$ satisfying

$$\ddot{\varphi} = \frac{1}{\epsilon}(g - \ddot{z}_d) \sin \varphi - \frac{1}{\epsilon} \ddot{y}_d \cos \varphi. \quad (8)$$

We have to formalize what we mean for “sufficiently smooth” and “bounded” trajectories.

First of all we assume that $y_d(\cdot)$ and $z_d(\cdot)$ are C^2 trajectories. Moreover if we define

$$a^2(t) = \ddot{y}_d^2 + (\ddot{z}_d - g)^2, \quad (9)$$

we assume

$$a_{min} \leq a(t) \leq a_{max} \quad (10)$$

Now we rewrite the roll dynamics (8) in the form

$$\ddot{\varphi} = \frac{1}{\epsilon} a(t) \sin(\varphi - \varphi_{qs}(t)). \quad (11)$$

where $\varphi_{qs}(t)$ is the *quasi-static* roll angle and satisfies

$$\tan \varphi_{qs}(t) = \frac{\ddot{y}_a(t)}{(g - \ddot{z}_a(t))}. \quad (12)$$

In fig. 3 the space of the admissible accelerations is shown with the definition of the *quasi-static* roll angle. Observe that the *quasi static* angle is just the angle for which the pendulum, or the PVTOL, is aligned with the direction of the resultant acceleration so that at that instant the rolling moment is zero.

Hence, the task is to find a bounded roll trajectory in the sense that the difference

$$\theta(t) = \varphi(t) - \varphi_{qs}(t) \quad (13)$$

is bounded.

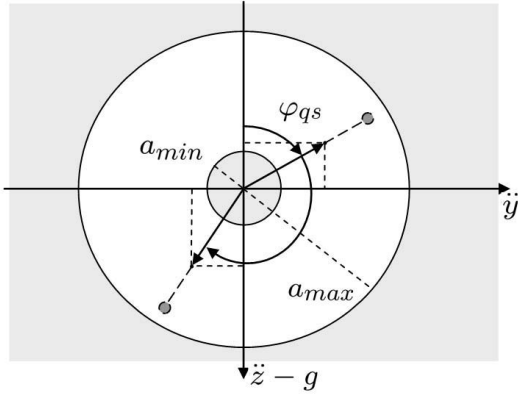


Fig. 3. Acceleration configuration and quasi-static roll angle

IV. DICHOTOMY AND EXISTENCE OF THE BOUNDED TRAJECTORY

The roll dynamics in (11) has the same form as the roll dynamics of a motorcycle (rewritten in terms of the quasi-static angle) studied in [3] and [5]. In this case the coupling coefficient ϵ plays the role of the height h of the motorcycle, while the acceleration $a(t)$ and the *quasi-static* angle $\varphi_{qs}(t)$ are defined in a more general way. In fact, differently from the motorcycle, in the PVTOL both the vertical and the lateral accelerations can be driven.

Using the same procedure as in [5], under the assumptions introduced above, we can prove that there exists a bounded roll trajectory.

We proceed as in [3] and [5] and rewrite the roll dynamics in terms of the error from the *quasi-static* angle as:

$$\ddot{\theta} = \frac{1}{\epsilon} a(t) \sin(\theta) + \ddot{\varphi}_{qs}(t), \quad (14)$$

which may be written in terms of the linearization around the equilibrium trajectory $\theta(\cdot) = 0$ as

$$\begin{aligned} \ddot{\theta} &= \frac{1}{\epsilon} a(t) \theta - \frac{1}{\epsilon} a(t) (\theta - \sin \theta) + \ddot{\varphi}_{qs}(t) \\ &= \alpha^2(t) \theta - \alpha^2(t) (\theta - \sin \theta) + \ddot{\varphi}_{qs}(t) / \alpha^2 \end{aligned} \quad (15)$$

If we consider the linear time varying system driven by a bounded external input

$$\ddot{\gamma} = \alpha^2(t) \gamma - \alpha^2(t) \mu(t) \quad (16)$$

it can be proven [3] that the undriven system admits an exponential dichotomy and, hence, working in a noncausal fashion, that for any bounded input $\mu(\cdot)$ a bounded solution $\gamma(\cdot)$ exists. The linear map $\mu(\cdot) \rightarrow \gamma(\cdot)$ is denoted by \mathcal{A} . Observe that, apparently, it seems from (16) that $\ddot{\varphi}_{qs}(t)$ should be well defined and then $a(t)$ should be at least \mathcal{C}^2 as assumed in [3]. However in [5] it is shown that this is not necessary. In fact, defining the nonlinear operator \mathcal{N}

$$\theta \rightarrow \mathcal{A} [\theta - \sin \theta + \ddot{\varphi}_{qs}(t) / \alpha^2(\cdot)] =: \mathcal{N} [\theta(\cdot)], \quad (17)$$

the following propositions hold:

Proposition 1: A bounded curve $\theta(\cdot)$ is a solution of (16) if and only if it is a fixed point of \mathcal{N} , i.e.,

$$\theta(\cdot) = \mathcal{N} [\theta(\cdot)].$$

Proposition 2: $\mathcal{A} [\ddot{\varphi}(\cdot) / \alpha^2] = \mathcal{A} [\varphi(\cdot)] - \varphi(\cdot)$ for all bounded $\varphi(\cdot)$.

Hence, defining the curve

$$\eta = \mathcal{A} [\varphi_{qs}(\cdot)] - \varphi_{qs}(\cdot) = \mathcal{A} [\ddot{\varphi}_{qs}(\cdot) / \alpha^2(\cdot)],$$

with $\bar{\eta} = \|\eta\|$, the following theorem may be proven:

Theorem 1: If

$$\bar{\eta} = \|\mathcal{A} [\ddot{\varphi}_{qs}(\cdot) / \alpha^2(\cdot)]\| < 1$$

then there is a $\delta < \pi/2$ such that \mathcal{N} is a contraction on the invariant set \bar{B}_δ . The unique fixed point $\theta(\cdot)$ of \mathcal{N} in \bar{B}_δ is a bounded trajectory of (14) so that $\varphi(\cdot) = \varphi_{qs}(\cdot) + \theta(\cdot)$ is a bounded trajectory of (8).

We refer the reader to [5] for the proofs and for a detailed treatment.

Remark 1: In [11] the authors determined a bounded roll trajectory under the assumption that $\|\ddot{y}(\cdot)\|_1 + \|\ddot{y}(\cdot)\|_\infty$ was not too large. The result illustrated here is much stronger, in fact we only ask $a(t)$ to be bounded.

V. NONLINEAR INVERSION USING PROJECTION

In order to compute a bounded roll trajectory on a finite interval $[0 T]$, we use an optimization based on a projection operator introduced by Hauser et al. in [7] and [8]. This technique was already used in [4] for the motorcycle example.

The method consists of embedding the original system (8), or equivalently (11), in the driven system

$$\ddot{\varphi} = \frac{1}{\epsilon} a(t) \sin(\varphi - \varphi_{qs}(t)) + u_{ext}, \quad (18)$$

where u_{ext} is a fictitious input. As in [4], u_{ext} is used to drive the system along any desired feasible trajectory.

If the accelerations of the center of gravity vary slowly, we can imagine the roll trajectory to be close to the *quasi-static* one. Hence, we can use the *quasi-static* as an initial guess to find the real trajectory. If we rewrite (18) in state space form as

$$\dot{x} = f(t, x, u_{ext}) \quad (19)$$

where $x = (\varphi, \dot{\varphi})$ and $x_{qs} = (\varphi_{qs}, \dot{\varphi}_{qs})$, we may pose the following optimization problem:

$$\begin{aligned} & \text{minimize} \\ & h(x, u) = \frac{1}{2} \int_0^T \|x(\tau) - x_{qs}(\tau)\|_Q^2 + \rho |u_{ext}(\tau)|^2 d\tau \\ & \quad + \frac{1}{2} \|x(T) - x_{qs}(T)\|_P^2 \\ & \text{subject to } \dot{x} = f(t, x, u_{ext}). \end{aligned} \quad (20)$$

where Q , ρ and P are weighting parameters.

Using a high weight ρ for the input, we may obtain a trajectory which is close to the bounded roll trajectory we are looking for. Observe that we do not impose any constraint on the initial condition, on the contrary we optimize on it to further reduce the control effort. We use a projection operator approach, introduced by Hauser in [7], to solve this constrained optimization problem.

Let the couple $\eta = (x(t), u(t))$ be a trajectory of the system (19) and denote by \mathcal{T} the manifold of all the trajectories. Given a bounded curve $\xi = (\alpha(t), \mu(t))$, the linear feedback

$$u(t) = \mu(t) + K(t)(\alpha(t) - x(t)) \quad (21)$$

may be used to track the $\xi(\cdot)$ trajectory. If $K(t)$ is asymptotically stabilizing, the nonlinear feedback system (19), (21) defines, *de-facto*, a continuous nonlinear *Projection Operator*

$$\mathcal{P} : \xi = (\alpha, \mu) \rightarrow \eta = (x, u) \quad (22)$$

with $\mathcal{P} = \mathcal{P} \circ \mathcal{P}$. Note that $\xi \in \mathcal{T}$ if and only if $\xi = \mathcal{P}(\xi)$. The constrained optimization problem (20) is then equivalent to the unconstrained problem:

$$\min_{\xi \in \mathcal{U}} g(\xi) = h(\mathcal{P}(\xi))$$

where $\mathcal{P}(\mathcal{U}) \subset \mathcal{U} \subset \text{dom}\mathcal{P}$.

In [8], Hauser and Meyer proved that, given $\xi \in \mathcal{T}$ there exists $\epsilon > 0$ such that each trajectory $\eta \in \mathcal{T}$, with $\|\eta - \xi\| < \epsilon$, satisfies

$$\eta = \mathcal{P}(\xi + \zeta)$$

where $\zeta \in T_\xi \mathcal{T}$ and is unique. Moreover $\zeta \rightarrow \mathcal{DP} \cdot \zeta$ is the projection operator, obtained by linearizing (19) about ξ , and $\zeta \in T_\xi \mathcal{T}$ if and only if $\zeta = \mathcal{DP}(\zeta)$, i.e. if and only if ζ is a trajectory of the linearized system.

Using these results, a Newton method may be used to find a descent direction, i.e. a trajectory $\zeta \in T_\xi \mathcal{T}$. Hence, the corresponding trajectory of the system is obtained by means of projection.

The Newton method based on the Projection Operator may be summarized as follows:

Given an initial trajectory $\xi_0 \in \mathcal{T}$

- Solve the time varying LQR problem

$$\min_{\zeta=(z,v)} Dg(\xi_i) \cdot \zeta + \frac{1}{2} D^2 g(\xi_i)(\zeta, \zeta)$$

subject to $\dot{z}(t) = A(\xi_i(t))z(t) + B(\xi_i(t))v(t)$;

- Step size $\gamma_i = \arg \min_{\gamma \in (0,1]} g(\xi + \gamma \zeta_i)$;
- Project $\xi_{i+1} = \mathcal{P}(\xi_i + \gamma_i \zeta_i)$.

The so computed optimal roll trajectory may be used to track an assigned $(y(\cdot), z(\cdot))$ trajectory while maintaining a bounded roll angle. This can be accomplished by linearizing the (input-output linearized) dynamics (6) about the approximate trajectory. Hence the resulting control action is given by:

$$u = u_d(t) + K(t)(x_d(t) - x(t)) \quad (23)$$

where $K(\cdot)$ is the linear quadratic regulator gain obtained by solving the Riccati equation for the (time varying) linearized system.

VI. SIMULATIONS

We show two examples of maneuvers. We used a value of 0.3 for ϵ and we normalized gravity to one. We performed a circular trajectory in the $y - z$ plane, i.e. we asked the PVTOL to track the trajectories $y_d(t) = -z_{max}/\omega_0^2 \sin(\omega_0 t)$ and $z_d(t) = -z_{max}/\omega_0^2 (1 - \cos(\omega_0 t))$ where $\omega_0 = 2\pi \cdot 1.5/10$. The vertical acceleration was set to different values in the two simulations. In particular, in the first case we chose $|z_{max}| = 0.9$, so that the resulting vertical acceleration $g - \ddot{z}_d(t)$ remains always positive. As shown in fig. 4 the maneuver results in an upright roll trajectory. The red dashed lines represent the resulting accelerations. Observe that the quasi static model (green) is aligned at each instant along the acceleration direction.

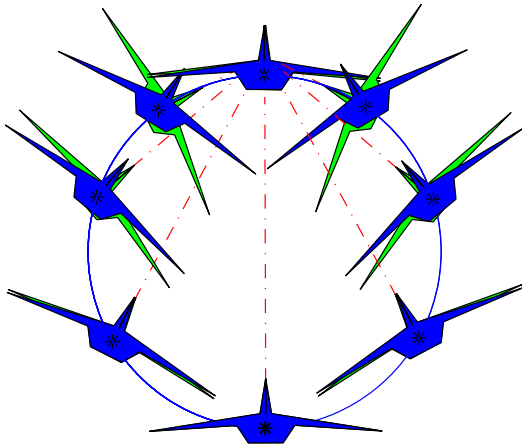


Fig. 4. Upright trajectory: optimal (blue) and quasi-static (green)

In fig. 5 and fig. 6 the roll and roll rate optimal trajectories are compared with the quasi-static ones. Note that the optimal trajectories display a degree of anticipation and are smoother than the quasi-static due to the filtering action of the dynamics.

In the second simulation we set $|z_{max}|$ to 1.2. In this way the global vertical acceleration $g - \ddot{z}_d(t)$ changes sign. An upright roll trajectory is no more possible and a barrel roll occurs fig. 7.

In fig. 8 and in fig. 9 the optimal and quasi static roll and roll rate trajectories are compared as in the previous case.

VII. CONCLUSIONS

We have described a technique to explore the manifold of the trajectories of the PVTOL aircraft. As in the differentially flat decoupled model, we are able to parameterize

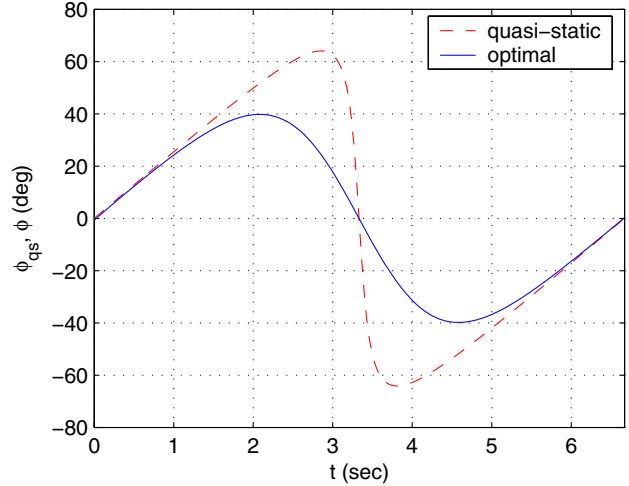


Fig. 5. Quasi-static and optimal roll trajectories (upright trajectory)

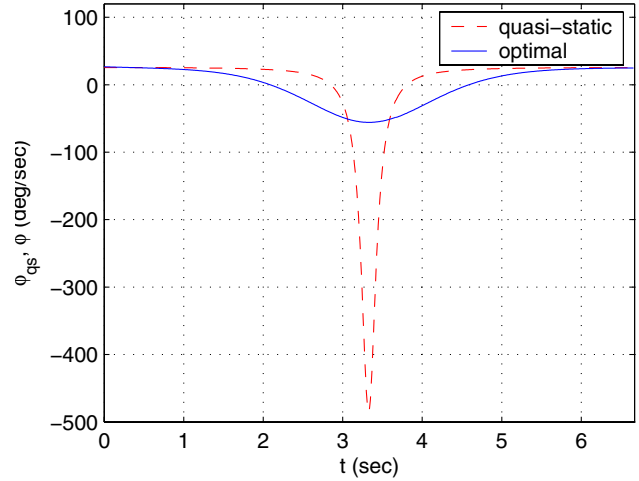


Fig. 6. Quasi-static and optimal roll rate trajectories (upright trajectory)

the manifold using the trajectories of the center of gravity of the PVTOL as "parameters", however, unlike in the decoupled case, they have to be only C^2 (and not C^4). Using results proven in [5] we have shown that, given any bounded C^2 trajectories for the center of gravity, a bounded roll trajectory exists. We found an approximation of such trajectory on an interval $[0 T]$ by solving an optimal control problem obtained by embedding the roll dynamics into a controlled system. The nonlinear optimal control problem was solved by using a nonlinear *projection operator*.

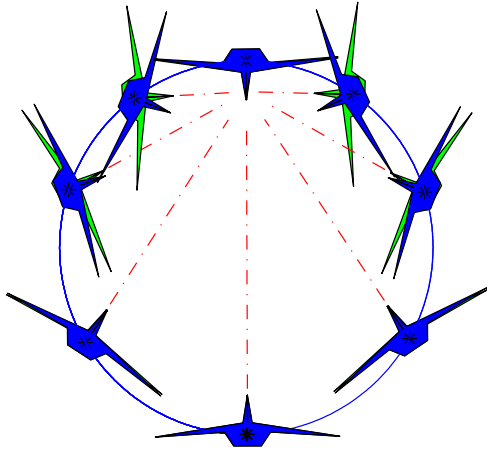


Fig. 7. Barrel roll trajectory: optimal (blue) and quasi-static (green)

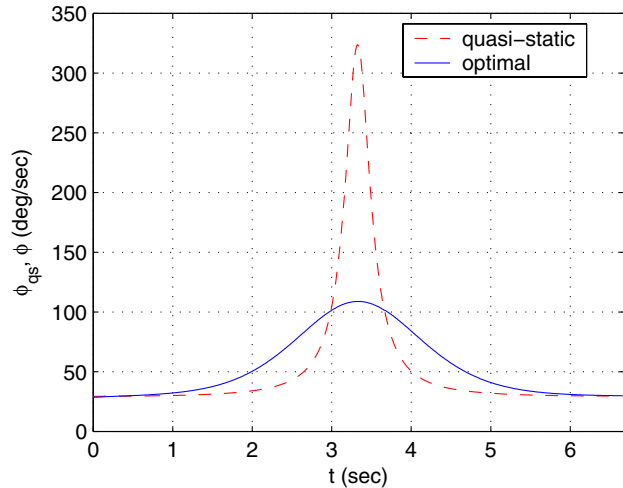


Fig. 9. Quasi-static and optimal roll rate trajectories (barrel roll trajectory)

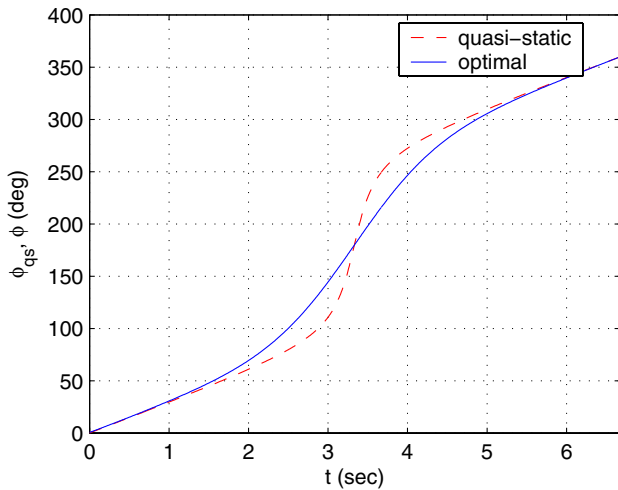


Fig. 8. Quasi-static and optimal roll trajectories (barrel roll trajectory)

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