Perception-based navigation through weak chaos control

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Abstract— In this paper a new navigation control strategy based on neurophysiological studies on perception is introduced. The underlying idea is that perception can be represented by chaotic attractors whose dynamical evolution depends on sensorial stimuli. To achieve this goal sensors equipped on a robot are associated with reference trajectories controlling the chaotic system (a multiscroll system) with a new technique based on Pyragas control, but extended to multi-reference signals. This technique is called *weak chaos control*, since applied to the multiscroll system allows us to obtain periodic orbits never shown by the uncontrolled system. This represents a key issue in the representation of perception under the concurrent control of different sensorial stimuli. The approach has been successfully applied to a roving robot model.

I. INTRODUCTION

The problem faced in this paper is to control the navigation of a robot endowed with a (potentially high) number of sensors. A key issue is to realize this control in real-time and on a working robot prototype, so the methodology has to be easily realized. The main inspiration comes from living systems: even if a huge number of sensors concurrently sends to the higher regions in the brain a lot of signals, complex decision making takes place in real-time. Indeed, till nowadays it is not completely clear how cognitive brain functions allow to take a decision in connection with the external environment: this complex topic involves many different disciplines, such as neurosciences, psychology, behavioral biology, bio-robotics and computer sciences. With the term *cognition* we intend a lot of mental processes like attention, planning, reasoning, understanding language and others. Different levels of abstraction are commonly used to try to model cognitive phenomena, starting from the simplest stimulus-reaction approach, to arrive to face, at a high level, with the so-called action-perception cycle [1]. This cycle includes a *central processing unit* whose role is to modify the environment through effectors and to take external information through sensors.

The action-oriented perception is a new interpretation of cognitive brain functions and it is very far from the traditional one which is based on the concept of thinking [2]. According to the action-oriented perception concept, competencies result from evolutionary adaptation; moreover information arises from coupling the organism to the environment. So new artificial perceptive schemes should directly consider the robot embodiment within the environment and concurrently process input sensorial stimuli to choose the suitable action. This paradigm differs from simple stimulus-reaction approaches since it recognizes the existence of internal (mental) representation [2]-[5]. Cortex processes information coming from objects identified in the environment through spike trains from receptors by enrolling dedicated neural assemblies. These are nonlinear dynamical coupled systems whose collective dynamics constitutes the mental representation of the stimulus [5]. Freeman in his works [3]-[5] conceives the hypothesis that cerebral activity can be represented by a chaotic dynamics. He attained this result by different experiments on rabbits which inhaled in a pre-programmed way several smells. Through the electroencephalogram (EEG), Freeman evaluated the action potentials in the olfactory bulb and he noticed that the potential waves showed a typical chaotic behavior. So he came to the conclusion that an internal mental representation (cerebral pattern) of a stimulus is the result of a chaotic dynamics in the sensory cortex in cooperation with the limbic system that implements the supporting processes of intention and attention [3].

In this paper we give a formal representation of this idea through a new technique for chaos control. This allows to face with the navigation control problem using the bioinspired paradigm of brain pattern formation. According to [5] perceived stimuli are converted from brain in a particular cerebral pattern characterized by a chaotic behavior. So we used a new chaos control approach to realize a real-time control technique for the navigation in a robot.

The control technique presented in this paper can be considered as belonging to the class of feedback methods. It presents both differences and analogies with the Pyragas approach [8], [9]. Our control technique is an adaptive control method like the Pyragas approach, because it uses control inputs that depend on the error between reference and actual output dynamics. These are modulated by an adjustable parameter K > 0. The main difference with respect to the Pyragas method is that in our technique it is possible to have several external contributes represented by as many reference signals as desired, that contribute to control the system. This approach is new, because so far chaos control techniques require one reference dynamics to control the chaotic system. So, if we think to consider as reference dynamics the sensorial stimuli perceived by sensors (which can be more than one), it is possible to obtain several configurations controlling the system depending on which are the currently active stimuli and their levels of activation.

This new chaos control technique, applied to a recently introduced chaotic system, allows to obtain also periodic orbits never shown by the uncontrolled system. This is the reason why this technique is called *weak chaos control*. This

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is a key issue to represent the concept of *percept* used for perception-based navigation control. All sensor signals will be mapped as different potential reference limit sets for the chaotic system (the controlled system), and therefore they will be mapped in the system phase plane. This allows to associate to each sensor a limit set topographically placed in the phase plane, in a way that reflects the topographic position of the sensor in the robot (front, left, back,...). This technique therefore allows to map into the phase plane of the slave system the real embodiment of the robot, referred to the sensor position, allowing for an efficient implementation of the navigation control method, based on the trajectory control directly in the phase plane.

In literature, among the methods for perception-based robot navigation control, one is based on a composition of genetic algorithms and fuzzy controller. Sensory networks are used to represent a perception mechanism, and a mobile robot uses a collision avoidance behavior to change the attention range according to environment [10]. Another technique is based on Turing patterns realized by Cellular Nonlinear Networks (CNNs) [11].

The new robot navigation technique proposed in this paper introduces the use of a chaotic system able to be controlled in real-time towards less complex orbits, like periodic orbits, considered as *perceptive orbits*. These are subject to realtime modifications that represent the corresponding modifications in the environment. The first experimental results are reported in the paper. They are encouraging and open the way to an analytical approach to optimize the system performance. Moreover, both the system used and the control methodology introduced can be easily implemented in a low-level hardware structure for the realization of real-time control in a robot prototype.

II. THE MULTISCROLL SYSTEM

In this section the chaotic circuit used as perceptive system is introduced. Since the perceptive system should be able to deal with a great number of sensorial stimuli and represent them, a chaotic system able to generate multiscrolls recently introduced in [12] has been adopted. This multiscroll system can be viewed as a generalization of the Chua's double scroll attractor, subsequently represented through saturated piecewise linear functions and of other circuits able to generate *n*-scrolls [14]. It is able to generate one-dimensional (1-D) *n*-scroll, two-dimensional (2-D) $n \times m$ -grid scroll or threedimensional (3-D) $n \times m \times l$ -grid scroll chaotic attractors by using saturated function series. In this paper a 2-D multiscroll system has been chosen. It is described by the following differential equations [12]:

$$\begin{cases} \dot{x}_1 = x_2 - \frac{d_2}{b} f(x_2; k_2; h_2; p_2, q_2) \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = -ax_1 - bx_2 - cx_3 + d_1 f(x_1; k_1; h_1; p_1, q_1) \\ + d_2 f(x_2; k_2; h_2; p_2, q_2) \end{cases}$$
(1)

where the following so-called saturated function series f(x;k;h;p,q) has been used:

$$f(x;k;h;p;q) = \sum_{i=-p}^{q} f_i(x;k;h)$$
(2)

where k > 0 is the slope of the saturated function, h > 2 is called *saturated delay time*, *p* and *q* are positive integers, and

$$f_i(x;k;h) = \begin{cases} 2k & \text{if } x > ih+1, \\ k(x-ih)+k & \text{if } |x-ih| \le 1, \\ 0 & \text{if } x < ih-1 \end{cases}$$
$$f_{-i}(x;k;h) = \begin{cases} 0 & \text{if } x > -ih+1, \\ k(x+ih)-k & \text{if } |x+ih| \le 1, \\ -2k & \text{if } x < -ih-1 \end{cases}$$

System (1) can generate a grid of $(p_1 + q_1 + 2) * (p_2 + q_2 + 2)$ scroll attractors. Parameters p_1 (p_2) and q_1 (q_2) control the number of scroll attractors in the positive and negative direction of the variable x_1 (x_2) , respectively. In order to obtain a 2-D grid of scroll attractors, the guidelines discussed in [12] have been taken into account.

The parameters used in the following $(a = b = c = d_1 = d_2 = 0.7, k_1 = k_2 = 50, h_1 = h_2 = 100, p_1 = p_2 = 1, q_1 = q_2 = 2)$ have been chosen taking into account these guidelines to generate a 2-D 5 × 5 grid of scroll attractors. An example of the chaotic attractor of system (1) is given in Fig. 1.

III. CONTROL OF THE MULTISCROLL SYSTEM

In our approach the perceptive system is represented by the multiscroll attractor of equations (1), while sensorial stimuli are represented by dynamical systems with periodic behavior. Since one of the main characteristics of perceptive systems is that sensorial stimuli strongly influence the spatial-temporal dynamics of the internal state, a suitable scheme to control the chaotic behavior of the multiscroll system on the basis of sensorial stimuli should be adopted.

Chaos control refers to a process wherein a tiny perturbation is applied to a chaotic system in order to realize a desirable behavior (e.g. chaotic, periodic, etc.). Several techniques have been developed for the control of chaos [6].



Fig. 1. Projection of the 5x5 grid of scroll attractors in the plane x_1 - x_2 .

A commonly used chaos control strategy is to select the desirable behavior among the unstable periodic orbits embedded in the system. The stabilization of one of the unstable periodic orbits can be obtained by applying a small perturbation when the chaotic orbit passes close to the desired behavior.

In general the strategies for the chaos control can be divided in two classes: closed loop and open loop. The first class includes those methods which select the perturbation based upon a knowledge of the state variables, and oriented to control a prescribed dynamics. The two most important feedback methods are the Ott-Grebogi-York (OGY) [6], [7] approach and the Pyragas technique [8]. The second class includes those strategies which consider the effect of external perturbations on the evolution of the system.

In the OGY's method, the changes of the parameter are discrete in time since this method is based on the use of Poincaré maps. In view of our application, a continuous-time technique like the Pyragas's method is more suitable.

In this method [8],[9] the following model is taken into account

$$\frac{dy}{dt} = P(y,x) + F(t), \qquad \frac{dx}{dt} = Q(y,x)$$
(3)

where y is the output of the system (i.e. a subset of the state variables) and the vector x describes the remaining state variables of the system. F(t) is the additive feedback perturbation which forces the chaotic system to follow the desired dynamics. Pyragas [8],[9] introduced two different methods of permanent control in the form of feedback. In the first method F(t) assumes the following form:

$$F(t) = K[\hat{y}(t) - y(t)] \tag{4}$$

where \hat{y} represents the external input (i.e. the desired dynamics), and *K* represents a vector of experimental adjustable weights (adaptive control). In the second method the idea consists in substituting the external signal \hat{y} in (4) with the delayed output signal $y(t - \tau)$:

$$F(t) = K[y(t - \tau) - y(t)]$$
(5)

where τ is a delay time. This feedback performs the function of self-control.

IV. CONTROL SCHEME

In our case a strategy based on equations (4) has been applied. The desired dynamics is provided by the periodic behavior associated with the sensorial stimuli. Since more than one stimulus can be present at the same time, the Pyragas method has been generalized to account for more than one external forcing.

Hence, the equations of the controlled multiscroll system can be written as follows:

$$\begin{cases} \dot{x}_1 = x_2 - \frac{d_2}{b} f(x_2; k_2; p_2, q_2) + \sum_i K_{x_{1i}}(x_{1m_i} - x_1) \\ \dot{x}_2 = x_3 + \sum_i K_{x_{2i}}(x_{2m_i} - x_2) \\ \dot{x}_3 = -ax_1 - bx_2 - cx_3 + d_1 f(x_1; k_1; h_1; p_1, q_1) \\ + d_2 f(x_2; k_2; h_2; p_2, q_2) \end{cases}$$
(6)

where x_{1m_i} , x_{2m_i} are the state variables of the reference circuits that will be described in details above and $K_{x_{1i}}$, $K_{x_{2i}}$ represent the control gains. It can be noticed that the control acts only on the state variables x_1 and x_2 . The complete control scheme is showed in Fig. 2.

Each reference trajectory x_{1m_i} , x_{2m_i} is a periodic trajectory generated by a multiscroll circuit associated with a sensor. When this sensor is not active, the reference circuit is not taken into account. When the sensor is active, the reference circuit is used to generate the feedback perturbation for the controlled multiscroll system.

The parameters of the multiscroll system for the reference trajectories are chosen according to:

$$a = b = c = d_1 = d_2 = 1, k_1 = k_2 = 50$$

$$h_1 = h_2 = 100, p_1 = p_2 = 1, q_1 = q_2 = 2$$
(7)

Let us recall that each of the scrolls surrounds a saddle point of index 2 [12]. When parameters (7) are adopted, instead of saddle points of index 2 these equilibrium points are centers (in fact, the linearized system has a pair of eigenvalues with null imaginary part). Thus, sinusoidal signals can be generated depending on the initial conditions of the reference system.

For this reason, a generic reference trajectory can be written as follows:

$$x_{1m_i}(t) = A_{x_1} sin(\frac{2\pi t}{T_{x_1}} - \varphi_{x_1}) + x_1^*$$
(8)

$$x_{2m_i}(t) = A_{x_2} sin(\frac{2\pi t}{T_{x_2}} - \varphi_{x_2}) + x_2^*$$
(9)

where (x_1^*, x_2^*) indicate the coordinates of the center point on the plane $x_1 - x_2$.



Fig. 2. Control scheme referred to 3 reference systems $(e_{x1} = \sum_i K_{x_{1i}}(x_{1m_i} - x_1), e_{x2} = \sum_i K_{x_{2i}}(x_{2m_i} - x_2)).$

A. Choice of the control gains

A range for the control gains such that the controlled system shows a limit cycle behavior can be defined numerically. In particular, considering a single reference signal, numerical considerations show the existence of a control gain range defined in the following way:

$$K_{x_1}, K_{x_2} \ge K_{min} \tag{10}$$

where $K_{min} = 0.6$.

Below K_{min} the system presents a chaotic behavior, while if control gains are above K_{min} a cyclic behavior occurs. Even if it should be possible to exploit also chaotic dynamics, in this work we used information related only to control actions able to give rise to a limit cycle dynamics. Furthermore, we have chosen to consider equal control gain values for variables x_1 and x_2 of the multiscroll system.

In particular, as concerns the control by using a single reference dynamics, for low values of K_{x_1} and K_{x_2} (as shown in Fig. 3(a)) the control of the multiscroll attractor has a residual error, however for the purpose of navigation control this weak condition is still acceptable. For higher values of the of K_{x_1} and K_{x_2} (as shown in Fig. 3(b)), the steady error is almost zero.

It is important to outline that the system, for control gains defined by condition (10), converges to a limit cycle for any initial condition.

One of the most interesting aspects of this technique, applied to multiscroll system into consideration, takes place when there are more than one external reference. For example, consider the case in which there are two concurrently active sensors, and so there are two reference signals in the multiscroll phase plane. If the control gains of the two reference systems are not equal, extensive simulations



Fig. 3. Behavior of the multiscroll system controlled by a reference cycle. (a) Low control gains: $K_{x1} = K_{x2} = 0.7$; (b) High control gains: $K_{x1} = K_{x2} = 1.8$.



Fig. 4. Limit cycle obtained when the multiscroll system is controlled by two sensorial stimuli. (a) Control gains: $K_{1x_1} = K_{1x_2} = K_{2x_1} = K_{2x_2} = 0.8$; (b) Control gains: $K_{1x_1} = K_{1x_2} = 2$, $K_{2x_1} = K_{2x_2} = 0.6$.



Fig. 5. An example of the evolution of the multiscroll system when controlled by reference dynamics associated with sensors. (a) When there is no input sensorial stimulus, the system has chaotic behavior; (b) when a single stimulus is perceived, the system converges to the reference cycle; (c) the system persists in the limit cycle behavior until the sensorial stimulus is no more perceived; (d) when the stimulus ends, the system behaves chaotically.

indicate that the resulting controlled limit cycle will be placed, in the phase plane, near the reference cycle with the higher control gain. If the two reference dynamics have the same control gains, the resulting cycle will be placed exactly at halfway between them. These results are shown in Fig. 4.

Using the control technique described in this section, it is possible to implement the idea underlying Freeman's neurophysiological theories, considering the sensory cortex dynamics as wandering in a chaotic attractor, represented by the multiscroll system, controlled by incoming sensory stimuli. Of course the dynamics of the cerebral pattern is a consequence of the perceived stimuli. If no stimulus is perceived, the multiscroll presents a chaotic behavior. When stimuli are perceived it converges to a limit cycle that constitutes a representation of the concurrent activation of the sensorial stimuli. When stimuli are no longer active, the multiscroll returns to its default chaotic dynamics. An example of this process is shown in Fig. 5.

V. ROBOT NAVIGATION

In the previous section it has been shown that how using the chaos control technique described above the multiscroll attractor can be enslaved to follow one or more reference trajectories, which represent the sensorial stimuli of our perceptive system. In this section we apply this perceptive system to control navigation of a roving robot.

The navigation problem is applied to a robot placed in an unknown environment which must avoid obstacles and reach targets. This is addressed by associating the perception of an obstacle with a stimulus and associating it with a representation (pattern) in accordance with Freeman's theories [3]-[5]. Finally, an action is associated to the emerging pattern depending from stimuli. This entails that a reference cycle will appear in the phase plane and, by suitable control gains, the multiscroll system will converge to a cycle. To achieve this goal, a chaotic dynamics is used to represent the evolution of the perceptive system which drives the choices of the robot. The stabilization of chaotic trajectories on a cycle is led by stimuli that cause the formation of an internal representation (cerebral pattern) and are represented through natural cycles of reference systems. The behavior of the controlled system is finally associated to an action, in terms of speed and rotation angle, that represents the robot response to the stimuli perceived from the environment.

An essential point of the approach is that, among the 5×5 possible reference cycles which can be used, only a subset of them has been effectively taken into account and that their distribution in the phase plane $x_1 - x_2$ reflects the topological distribution of robot sensors. This is shown in Fig. 6, which refers to four cycles connected to front, back, right and left distance sensors. Moreover, in this work we have chosen to link control gains with the intensity of perceived sensorial stimuli. In details, once defined the number of sensors used, the interval $[d_{min}, d_{max}]$ (where d_{min} and d_{max} represent, in the case of distance sensors, the min and max distance values in the sensor scale) is scaled in an interval $[K_{min}, K_{max}]$ where $K_{min} = 0.6$ and K_{max} has been arbitrarily fixed to 20.

The technique, based on placing reference cycles in the phase plane in accordance with the distribution of sensors on the robot, permits to implement a great number of sensors. In our tests only distance and target sensors have been used, although other sensors can be used.

Distance sensors have a visibility range that describes the area where the robot is able to detect static and dynamic obstacles, while target sensors return the target sighting angle with respect to the frontal axis of the robot.

As concerns the action (in terms of absolute value of speed and heading) performed by the robot, this depends on the multiscroll behavior. In particular, the action performed by the robot is strongly related to the position of the cycle of the controlled system. When no stimulus is perceived the system evolves in chaotic behavior and the action of the robot is to explore the environment. In this case the robot moves with constant speed and without modifying its orientation.

Instead, when the multiscroll system is controlled by external stimuli and converges to a limit cycle (i.e. a periodic pattern), the action to be executed is chosen according to the relative position of the cycle in the phase plane. A vector



Fig. 6. Four reference cycles representing four sensors of the robot. Cycles can be generated by system (1) with parameters: $a = b = c = d_1 = d_2 = 1$, $k_1 = k_2 = 50$, $h_1 = h_2 = 100$, $p_1 = p_2 = 1$, $q_1 = q_2 = 2$ and the following initial conditions: ref. 1, $\mathbf{x}_0 = (40, 240, -30)^T$; ref. 2, $\mathbf{x}_0 = (240, 40, -30)^T$; ref. 3, $\mathbf{x}_0 = (40, -160, -30)^T$; ref. 4, $\mathbf{x}_0 = (-120, 40, -30)^T$. Equivalently, equations (8) and (9) can be used.



Fig. 7. SPICE simulation of a 2×2 grid of scroll attractors.

pointing to the center of the limit cycle of the controlled multiscroll attractor is defined; predefined actions are chosen on the basis of module and orientation of this vector.

A similar strategy has been adopted for the target. When a target is in the range of robot visibility, it is considered as an obstacle located in a position symmetric with respect to the motion direction. This obstacle is associated with a reference cycle which controls the multiscroll attractor with a low gain, so that avoiding obstacles has priority over reaching the target. In this way the generated reference cycle has the task to weakly suggest a rotation towards the target, since it was considered more important to avoid close obstacles than to reach targets.

VI. SIMULATION RESULTS

This section shows simulation results related to circuit implementation and validation of the navigation control strategy.

First, in order to show the feasibility of the methodology, a circuit implementation of the multiscroll system has been taken into account, following the guidelines discussed in [16]. SPICE simulations of a 2×2 multiscroll system are shown in Fig. 7.

To test the performance of the control strategy, a software tool for mobile robot simulations has been developed. A robot model navigating in an environment with obstacles and targets has been simulated. Four targets placed as in Fig. 8 have been considered. The circle around the target represents the range in which the target can be sensed. Obstacles are represented by walls and by the black rectangle in the right corner of the environment. The robot has to navigate in this environment and reach the target, while avoiding obstacles. When a target is found, this is disabled, so that the robot can navigate toward the other targets.

Fig. 8 shows an example of the trajectory followed by the robot. As it can be noticed the robot is able to successfully perform the navigation task. Another test concerns the capabilities of the robot to explore the whole environment. To test this behavior, a long simulation run has been carried out. The result is shown in Fig. 8(b), which demonstrates that the robot successfully explores the whole environment. Movies of these simulations are available in the web page [17].



Fig. 8. (a) Trajectory followed by the robot in the initial phase of the simulation; the circle indicates the starting point. Targets are indicated by small squares surrounded by circles representing the range in which the target is visible. (b) Trajectories followed by the robot at the end of the simulation: the whole environment has been explored by the robot.

VII. CONCLUSIONS

In this work a new navigation control strategy has been introduced for robots moving in unknown environments. The main element of this real-time control is the use of a multiscroll chaotic system suitably controlled according to the perceived sensorial stimuli. To achieve this goal every sensor on the robot is represented with a reference cycle in the multiscroll phase plane, which is, whenever the sensor is active, a reference system able to drive the multiscroll system to converge to a limit cycle.

In this context a key role is assumed by the control gains which are related to the sensorial information. Suitable control gains and their lower and upper bounds were experimentally designed. The design has been successfully tested in several simulations where obstacles were able to stimulate up to eight sensors simultaneously. After a cycle is formed on the multiscroll, the cycle center position vector is determined with respect to phase plane origin: the modulus and phase determine the robot action for the next step. Moreover, reference trajectories used in this work are obtained through simple oscillators and not with multiscroll chaotic systems. This represents an advantage for a hardware implementation.

It is to be outlined that the introduced navigation control strategy has some key characteristics that make it very simple from the implementation point of view. In fact the multiscroll system is a chaotic system that is represented only by three equations. Moreover the saturated functions, representing the nonlinearities, can be realized very simply and with lowcost components, like operational amplifiers. The measures coming from each sensor, in the case treated in this paper, are used to modulate an associated oscillator, so also the realization of the sensor interface is very simple. Finally the control action is based on simple rules that can be easily implemented for real-time control.

The paper reports the experimental results. They are encouraging and open the way to an analytical approach to optimize the system performance. Currently some efforts are being paid in order to improve the strategy, and in particular to add flexibility to the action selection stage. In fact, up to now the methodology reflects the so-called "fixed action" strategy. An improvement will be gained by letting the action selection stage be the result of an unsupervised learning algorithm, so as to adaptively react to environment stimuli and reach a deliberative behavior. Moreover the reference cycle shapes could reflect direct sensory data, and control gains could be selected considering also sensor sensitivity, so as to represent the concept of attention.

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