Proceedings of the
44th IEEE Conference on Decision and Control, and
the European Control Conference 2005
Seville, Spain, December 12-15, 2005

Excellent control of flexible systems *via* control of the actuator-system interface

William J O'Connor

Abstract—**Many control strategies have been proposed for the common situation of an actuator attempting to control the end-point of a flexible mechanical system. It is here proposed that the best approach is to focus *not* on controlling the end point, *nor* on modeling/measuring the flexible system, but on the actuator-system interface, first to understand it, then to measure the two-way motion there, and finally to manage this motion. This leads to superb control. It is rapid, robust, energy efficient, unusually generic, and computationally light. It needs only modest sensing, only generic knowledge of the flexible system dynamics, and makes modest demands on the actuator dynamics. It can easily get the entire system to stop dead exactly at target, with slight vibration in transit.**

I. INTRODUCTION

THERE are many contexts, from space structures to disk drive heads, from medical mechanisms to long-arm manipulators, from cranes to robots, in which it is desired to achieve rapid and accurate position control of a load (or system end-point) by an actuator that is separated from the load by an intermediate flexible system. While all systems are to some extent flexible, issues related to flexibility become decisive as one tries to design lighter mechanisms, or systems that are more dynamically responsive, or softer, or more energy efficient, or simply long in one dimension

The system's actuator must then attempt to reconcile two, potentially conflicting, demands: position control and active vibration damping. Previous approaches, with differing degrees of success in specific cases, have included various classical and state feedback control techniques (often using simplified dynamic models); modal control (often considering a rigid-body, or zero frequency mode separately from vibration modes); sliding mode control; input command shaping; bang-bang control; optimal control; wave-based control; and control based on real-virtual system models [9].

Such methods either require accurate system models or they can offer only asymptotic approaches to target final states. Reference [1] observes that "to date *a general solution* to the control problem [of flexible structures] *has yet to be found*. One important reason is that computationally efficient (real-time) mathematical methods do not exist for solving the extremely complex sets of partial differential equations and incorporating the associated boundary conditions that most accurately model flexible structures." *(Emphasis added.)*

It is here contended that such a "general solution" is indeed possible now, one that applies to a wide range of flexible systems. It side-steps system sensing, modeling and identification issues by looking at the problem in a new way. Rather than treating the flexibility as a problem, it works with the flexibility to achieve system control in a natural way. It is here proposed that the key steps are a) understanding, b) measuring and then c) managing the interface between the actuator and the flexible system. These steps, in turn, can be achieved by the concept of two-way mechanical waves. The interface is seen as the wave gateway, controlled and managed by the actuator's motion.

Energy and momentum enter and leave the flexible system at the interface. They propagate in two directions within the system, from actuator to end-mass, and back again, albeit in ways that are faltering, complex, and highly dynamic. *Rest-to-rest motion corresponds to getting the energy and momentum into, and then out of, the system in just the right way to ensure that the entire system comes to rest at the target.* This view is one way of expressing the essence of the challenge. It is also revealing.

The actuator is the sole agent for all this. But the actuator interacts directly only with the part of the system dynamics to which it is directly connected: the interface. Its interaction with the rest of the system is indirect, and it is mediated by these local, interface dynamics. The two-way motion needs to be defined and measured only at the interface, where all the controlling is to be done, over time. In other words, only limited aspects of the system dynamics are relevant to the control problem, and these should be seen from the actuator's perspective. The actuator is simultaneously launching a wave into the system and responding to the returning wave. As will be seen, the returning wave component reveals to the controller all the system information it needs to achieve superb control, in a format that ideally suits this purpose.

The actuator here can have an independent sub-control system that takes its input from the higher controller. The nature of this sub-controller is not important. The directly controlled variable at the actuator can be motion or force at the interface, but not both: if one chooses one, the system determines the other. Similarly, the remotely controlled variable (at the end of the flexible system, and throughout) can be either motion or force, but again these cannot be

Manuscript received March 7, 2005. This work was funded in part by Enterprise Ireland Basic Research Grant, code SC/2001/319/.

W. J. O'Connor is with the Dept of Mechanical Engineering, University College Dublin, National University of Ireland, Belfield, Dublin 4, Ireland. (tel: +353-1-7161887; fax: +353-1-2830534; e-mail: william.oconnor@ucd.ie).

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specified independently of each other. Therefore, in single-actuator controlled flexible systems, there are four possible combinations: motion-motion, force-force, force-motion and motion-force. All arise in practice. The wave-interface control ideas apply to all four cases, but the focus here will be on motion-motion problems, with the actuator motion attempting to control the motion of the far end of the flexible system, assumed to be free (unconstrained). While, again, not essential to the main idea, it will also be assumed here that gravity effects, if present and relevant, are identical at the beginning and end of all manoeuvres.

Wave-based control of a broad range of flexible systems will be illustrated including distributed "second order", lumped "second order" and lumped "fourth order", where "order" here refers not to the system size (which is completely arbitrary) but to the corresponding partial or ordinary differential equations usually applied to such systems.

Wave-interface control is easier in practice than in theory and practice has often gone slightly ahead of theory in this work. While some explanations and insights will be given and (it is hoped) intuitions strengthened, the ultimate justification is here based less on theory and more on the demonstrable success of the approach. The strategy works, and it works very well, in exhaustive simulation tests, and, in some cases (e.g. gantry crane), on experimental rigs.

II. Understanding the Interface

A. Distributed systems

A simple first example of a flexible systems that is distributed (that is, continuous, rather than lumped) at the actuator-system interface is a gantry crane [4, 5]. The "actuator" is the overhead trolley, whose motion is the system "input", the "flexible system" is the hanging cable, flexing laterally under tension, and the "output" can be taken as the load motion. The "interface" is the connection between the cable and the trolley. See inset in Fig.4. The cable's lateral motion at the interface obeys a wave-like equation, with the trolley forming a movable boundary. The solution to the equation has two components, propagating in opposite directions, superposed. Provided a characterizing feature of the cable is known, at any instant these two component motions can be separately identified and measured at the interface by measuring two independent variables there, such as the lateral velocity and lateral force in the trolley-cable interface.

The cable wave boundary condition is that the superposed component motions at the boundary should be identical to the actuator (trolley) motion. The wave component traveling down the cable can be considered as launched by the actuator motion; the component propagating upwards as motion returning from the load towards the actuator, where it can be imagined to be absorbed from the flexible system, or reflected back into the system, or some combination of these. Thus, by measuring the flexible system interface, the actuator's motion can be resolved into two notional components, one part launching a wave (or motion) into the system, the other responding to a returning wave.

If the computer controlling the actuator's motion measures these components, it can then set the motion, for example, to launch an arbitrary wave into the system while simultaneously absorbing the returning wave.

These waves are transverse. The same arguments apply to flexible systems where the actuator and vibrational motions are longitudinal (along the axis of the flexible system) or rotational / torsional.

The case of beam-like structures with flexural stiffness under lateral bending is more complex. The biharmonic wave equation (fourth order in space) then applies, and four measurements are needed to resolve the motion, such as moment, rotation, shear force and lateral translation. The corresponding actuator may be capable of applying rotation, or lateral translation, or both. But despite the added complexity, if the local dynamic properties are known at the interface, by suitable measurements at the interface the actuator motion(s) can be resolved into notional launching and absorbing motions, in rotation and/or translation [7].

B. Lumped systems

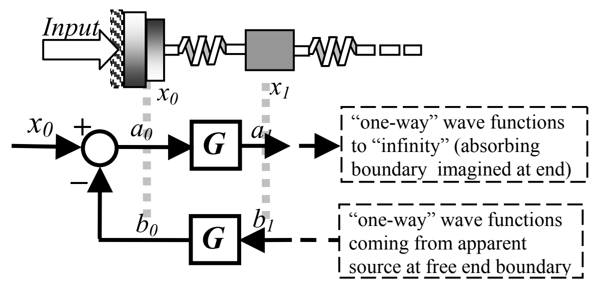

Fig.1 Notional wave model at actuator end. Upper and lower branches when superposed reproduce real system dynamics: $x_0=a_0+b_0$, $x_1=a_1+b_1$, etc. Also, by definition of G, $a_1=G(a_0)$, $b_0=G(b_1)$. Eqs. (4)&(5) follow.

In modeling robots, space structures, vehicles, etc, the inertia and stiffness are frequently considered as concentrated into separate elements. It can be shown [2, 8] that such a lumped flexible system with n inertial elements can be modeled by the superposition of two, "one-way" strings of n transfer functions, one modeling waves going out from the actuator, the other waves returning towards the actuator. The free end of the system can be modeled by a further single transfer function interconnecting the two systems (or by a simple reflection): the actuator end is modeled as a summing junction. The actuator end is of most interest here, and is shown in Fig.1 for the rectilinear mass-spring case, with only the first and last wave transfer function, G, of each branch shown.

Independently of the order of the system n, the component wave transfer functions are all of low order, but non-integer.

For example, for a uniform, rectilinear system of masses, m and springs of stiffness k, with $\omega_n=\sqrt{(k/m)}$

$$G(s) = 1 + \tfrac{1}{2}(s/\omega_n)^2 - \tfrac{1}{2}(s/\omega_n)\sqrt{(s/\omega_n)^2 + 4} \quad (1)$$

which is close to second order. The situation is more complex for a non-uniform system, but still each wave transfer functions is of similar form and dominated by the local dynamics, which in Fig.1 are the first spring and mass.

Fortunately an exact form of G is not needed. It can be modeled more than adequately for wave measurement purposes by simple computational analogues, such as the mass-spring-damper system inset in Fig.2, or the flexing arm inset into Fig.6. Two such "analogue computers" are needed, one for each "G" in the control diagrams (or for each G in equations (4-5) or (8-9)). These are "tuned" to the dynamics (inertia and stiffness) of the flexible system where it meets the actuator. They also have a damper, tuned to simulate approximately the effects, on the first mass, of an extension of the flexible system to infinity. The damping value is not critical.

III. MEASURING THE INTERFACE

In all cases, the actuator motion is to be separated into two notional motions, based on measurement of real quantities. For continuous systems where the flexible system at the actuator obeys the (second order) wave equation the actuator motion, x_0, is separated into the two notional components, a_0 and b_0:

$$a_0 = \tfrac{1}{2}\left(x_0 + \int f/Z\,dt\right), \quad (2)$$

$$b_0 = \tfrac{1}{2}\left(x_0 - \int f/Z\,dt\right), \quad (3)$$

where the measured quantities are x_0 itself and f, the actuator force. Z is the wave impedance. For the gantry crane case, f is the horizontal force applied by the trolley to the cable, and $Z=\sqrt{\rho T}$, with ρ the linear density and T the cable tension.

For lumped systems consisting of rectilinear mass-spring strings, the corresponding components of the actuator motion, x_0, are

$$a_0 = x_0 - G(x_1 - G(a_0)) \quad (4)$$

$$b_0 = G(x_1 - G(a_0)) \quad (5)$$

where x_1, the second measured quantity, is the position of the first mass, and G is a single-input-single-output operator, evaluated in practice in the time domain by the "analogues" as described. An even simpler scheme that fulfills all the control criteria with only marginal degradation in the resulting transient response is to set

$$a_0 = \tfrac{1}{2}\left(x_0 + \omega_n \int (x_0 - x_1)dt\right) \quad (6)$$

$$b_0 = \tfrac{1}{2}\left(x_0 - \omega_n \int (x_0 - x_1)dt\right) \quad (7)$$

For laterally-flexing, lumped systems with rotation, θ, and lateral translation, y, the two notional components, a and b, of the actuator motion are [7]

$$\begin{pmatrix}\theta_0\\ y_0\end{pmatrix}_a = \begin{pmatrix}\theta_0\\ y_0\end{pmatrix} - G\left[\begin{pmatrix}\theta_1\\ y_1\end{pmatrix} - G\begin{pmatrix}\theta_0\\ y_0\end{pmatrix}_a\right] \quad (8)$$

$$\begin{pmatrix}\theta_0\\ y_0\end{pmatrix}_b = G\left[\begin{pmatrix}\theta_1\\ y_1\end{pmatrix} - G\begin{pmatrix}\theta_0\\ y_0\end{pmatrix}_a\right] \quad (9)$$

where the subscript 0 refer to the actuator, the subscripts 1 refer to quantities at the first mass, and again the G are approximated by a pair of two-input, two output analogues.

IV. MANAGING THE INTERFACE

The control strategies involve a controlling computer (the "controller") requesting the actuator to move to achieve motion control of the flexible system. The "motion" input / output can be position, velocity or acceleration. The desired output may be to track a time-varying input, or to set the system at a specified target value. Again the wave-interface ideas apply to all these problems, but for brevity the focus below will be on the most common problem in flexible systems, namely, rest-to-rest manoeuvres to a target position of the flexible system end-point. Examples arise in robotics, disk drives, gantry cranes, and in slewing of space structures.

The controller calculates a motion to be requested from the actuator's sub-controller. The controller then measures the actuator's actual performance, x_0, and resolves it into the two notional components, a_0 and b_0. The control strategies involve setting the launch component, a_0, of the actuator's motion to follow a waveform decided by the controller while simultaneously adding an absorbing component, b_0.

A. Achieving steady state rapidly

Now if the launch component, a_0 is held steady (at a constant position, velocity or acceleration), then the effect of adding the absorb component, b_0, will be steadily to remove vibrations from the system, with b_0 reaching the same steady value as a_0 (position, velocity or acceleration). For example, if the launch waveform is set to a constant velocity (ramp displacement input), then the absorb waveform will quickly reach the same velocity, so that the total velocity of the actuator (sum of these waveforms) will become steady at twice the launch velocity, with zero system vibration. In other words, the entire system will be moving at the same velocity, as if rigid: a mixture of potential and kinetic energy becomes all kinetic.

If the launch waveform settles at a constant displacement, d, then the absorb waveform will also quickly settle at d, so that the entire system will come to rest at a displacement of $2d$.

How quickly the absorb waveform becomes equal to the steady launch waveform depends on the length and uniformity of the flexible system. For uniform systems it happens very quickly, typically in about one period of the lowest frequency cantilever (fixed-free) mode of the flexible system.

Why they become equal follows from the definitions of the notional components (2) to (9) and the active vibration damping effect inherent in the b_0 component of the actuator motion. For example, for rest-to-rest manoeuvres of systems using definitions (2) and (3), the initial and final momenta are zero, so that the force integrals must be zero. Therefore at the end

$$a_0 = b_0 = \tfrac{1}{2} x_0 = \text{constant} \tag{10}$$

Furthermore, inspection of (3) shows that b_0 adds "matched" viscous damping to absorb returning vibrations.

The other definitions in (4) to (9) for lumped systems have similar implications. In essence, the launch wave "pushes" the system a given amount, and in absorbing the return wave, the system then "pulls" the actuator the same amount again, gently, absorbingly. It can be shown that the steady state gain of all the wave transfer functions around the (implied) loop of Fig.1 is unity, so that, again, at rest $b_0=a_0=½x_0$.

Adding b_0 to the input, x_0, "cancels" the returning wave, or effectively "opens the feedback loop", absorbing energy, ensuring stability, as if the passive flexible system were extended to infinity with non-returning waves. It can be shown that opening of the loop of the complete wave model (only suggested in Fig.1) effectively cancels all system poles, because there are no poles in each component wave transfer functions (cf. e.g. (1)) or in their concatenation around the loop.

The attractive control features are very robust to "errors" in evaluating b_0, for example due to inaccuracies in Z, or in the transfer functions, G, or in their evaluation. Provided the main features of G are correct (unity steady-state gain, zero instantaneous response, with approximately the correct order, damping and characteristic frequency) then the control features are guaranteed.

B. *Direct position control of flexible system*

One solution for rest-to-rest position control is implicit in the above. To move the system a target distance, the controller simply sets the launch wave notional component to reach half the target distance and hold there, while simultaneously adding the second notional component which moves to absorb all returning waves. This guarantees that the system will quickly come to rest (zero vibration) exactly at target. The system's steady state error will be as small as that of the actuator (often negligible). Figure 2 shows this.

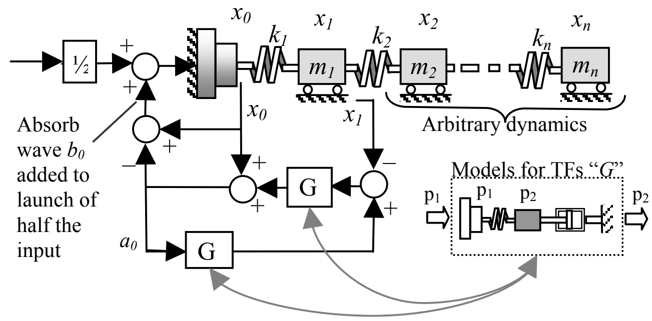

Fig.2. Wave control for tracking an arbitrary input, with (inset) analogue for G, whose input is "actuator" position p_1, output mass position p_2. Two of these are needed. a_0 and b_0 are determined following (4) & (5).

C. *Wave-echo control*

This method works extraordinarily well. But even better is possible. It is arbitrary how the launch waveform component reaches the (steady) half target value. This suggests the question: what waveform is best? There are many definitions of "best", but a waveform that caused the flexible system to come to rest as rapidly as possible is often highly desirable. For a given real actuator and flexible system, the best deceleration to rest at the end of the manoeuvre is probably a time reversal of the start-up from rest. Fortunately, the information needed to achieve this is available to the controller in the waveform absorbed from the system at the start-up. The strategy then is as follows.

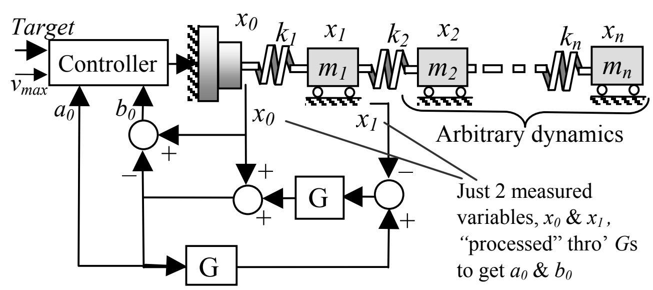

Fig.3. "Wave-echo" scheme implemented by "controller" computer when target position known beforehand.

To move the system a target displacement, the controlling computer initially sets the launch waveform for example to a ramp. Any ramp rate will work, but if the actuator is speed-limited to below v_{max}, then the launch ramp rate should be set to $½v_{max}$ so that for long manoeuvres the system will approach v_{max} at the half-way stage. To this launch ramp it adds the measured return wave, b_0, to produce the request input motion to the actuator. It also stores the absorb waveform values, $b_0(t)$, in memory. When the actuator position (the sum of these two waveforms) reaches half the target displacement, say at time T, the controller completes the launch waveform (which is not yet at its final ½-target value) using a time-reversed and inverted version of the absorb waveform, taken from memory, to settle at the ½-

target value. Symbolically, it sets $a_0(t) = ½target – b_0(2T\text{-}t)$. Thus, the "echo" absorbed from the system at the start-up is played back into the system, in reverse, to bring the launch wave to half the target. During this time, the absorbing action continues, so that the absorb waveform also ends up at half target, and the system comes to rest at the target.

But it does so wonderfully. Provided sufficient echo is available at the half-way, change-over point, the effect of the strategy is to cause the far end of the flexible system to stop dead, exactly at target. At this instant, the flexible system is straining to decelerate the end mass. The actuator, which is then short of the target, keeps moving in just the right way to let the system relax to rest, beginning with the end mass and ending at the actuator, all stopping exactly at target. Energy and momentum flow out of the system in the reverse order to their initial insertion. Total manoeuvre time is typically within 15% of time-optimal or input shaping control results.

V. SAMPLE RESULTS

Figure 4 shows the response of a gantry crane to this strategy. This is an example of a distributed flexible system, and it uses (2) and (3) to determine the launch and absorb components of the trolley's motion.

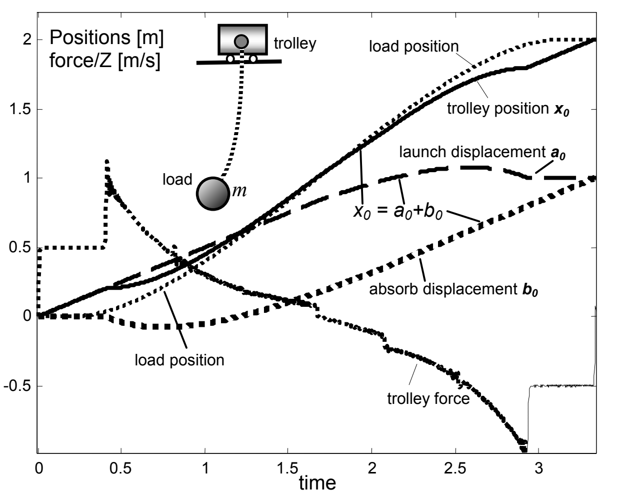

Fig.4. Load arrives and stops at target before trolley. Crane target distance 2m. Cable length=4m, ρ=0.1kg/m, m=2kg, T=23.54N, Z=1.534 Ns/m. Note time symmetry of load & trolley motions, due to a_0 and b_0.

Figure 5 is an sample response of a second order lumped system, of three equal masses and springs, with target distance of 1m, again using the "wave-echo" idea. The "Controller" in Fig.3 begins with launch-a_0 at ramp rate of $½v_{max}$. At x_0=½*target*, it completes the launch of a_0 to ½*target* by playing back b_0 recorded from start, inverted and time-reversed. Note the absence of vibration, and how the end mass arrives at the target well ahead of actuator, followed by a relaxation phase as the actuator comes to rest.

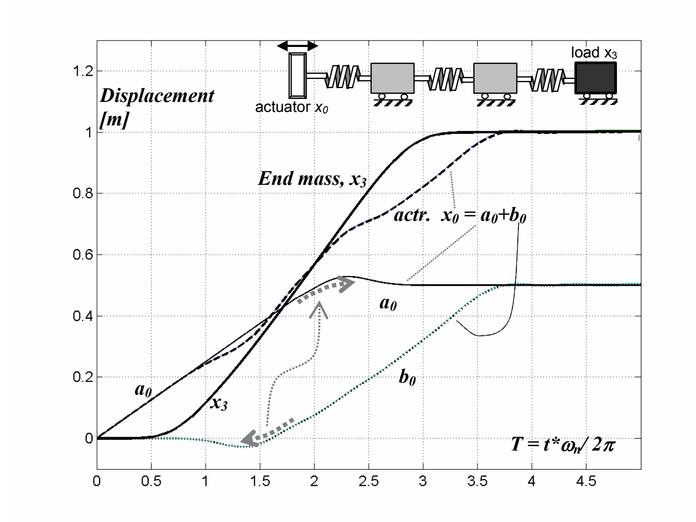

Fig. 5. Manoeuvre of uniform 3-mass-spring system through 1 m.

Figure 6 shows the control arrangement for a laterally flexing system of arbitrary length, with an actuator that can set an angle, θ_0, and/or lateral displacement, y_0. To avoid further clutter, only angles, θ, are shown in the subsystems for calculating G in the time domain, but y inputs and outputs can be added to the two subsystems as in the main system.

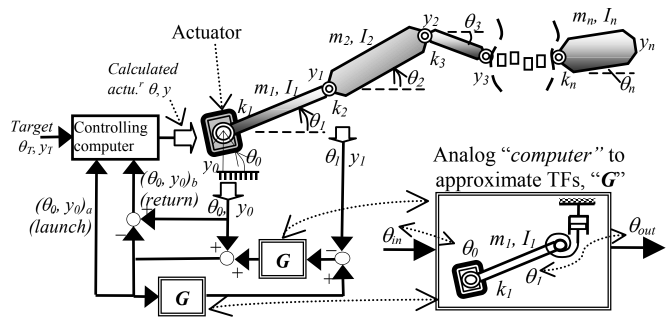

Fig.6. "Fourth order", laterally flexing system capable of both rotation and translation. To avoid further clutter only angular motion θ shown, although lateral translation, y, can be added to both main system and subsystems, G.

A sample response is shown in Fig.7, with the same echo feedback idea, from the half-way point, to complete the launch waveform to half the target value. The model here has 6 rotational inertial masses. This gets the last (6^{th}) inertial mass at the tip to move into line exactly, and then stop dead, without vibration and with minimal overshoot. It is difficult to imagine a physically possible better response, especially to be achieved by a single actuator at the other end. Furthermore, this entire manoeuvre is completed in a little over two periods of the fundamental cantilever vibration mode of the flexible system.

When lateral translation and rotation are combined simultaneously, the same ideas work, although some special care is needed. These issues are explored in [7].

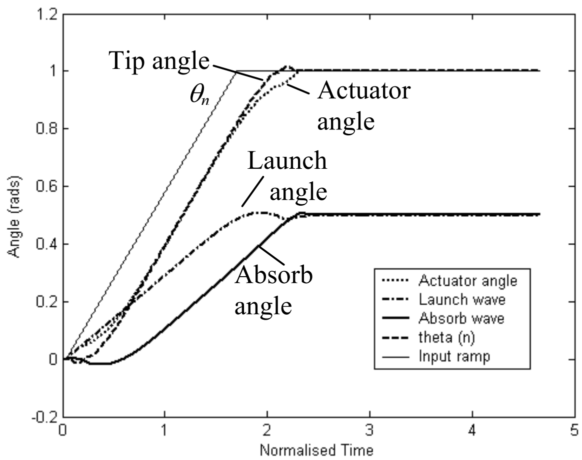

Fig. 7 Rotational manoeuvre of a flexing system, as in Fig.5, with 6 uniform inertial elements and revolute joint springs (n=6). The time is normalized to the period of the fundamental cantilever mode of the system.

In all of the above, even faster manoeuvre / transit times are possible simply by increasing the ramp rate (assuming the actuator can achieve this). The minor cost of doing so is to cause slightly increased vibration in transit, a small overshoot on arrival at target, followed by rapid settling exactly at target (zero steady-state error).

VI. Discussion and Conclusions

The strategies have been extensively tested and found to be highly robust to changes in the flexible system remote from the actuator, to actuator performance, to sensing errors, and to mismatches between the wave transfer functions, *G*, and the dynamics of the flexible system at the actuator.

Changes in the system dynamics remote from actuator require no changes in the control strategy. The strategy works independently of the order of the system (number of elements), whether the system has just one element or an arbitrarily large number of elements. The system itself can be uniform or non-uniform, entirely lumped, or partly lumped and partly distributed. The springs can have non-linear characteristics (extension hardening or softening), provided only that they return to the same relaxed state.

The control system needs no system model other than the wave transfer function model, *G*, tuned approximately to the dynamics of the first system element and spring. The returning (echo) waveform, b_0, reveals to the controller the entire system dynamics in just the form the controller needs to achieve perfect system deceleration to rest. In a sense, the system itself serves as the system model, which is therefore always accurate, up to date, and of the correct order. Or, to put it another way, all the required system identification is done in real time, as part of the controlled motion, with minimal computational overhead. This partly explains the control system's robustness to system changes. Other than the actuator's own motion, only one other sensed input is needed, and the second sensor supplying this information is located conveniently close to the actuator, where sensing is generally easiest and safest in practice.

In conclusion, no full or accurate system model is needed, nor system identification, nor modal analysis (whether with exact, or assumed modes, or truncated modes). There is no need for precise switching times, nor for ideal actuator behaviour. There is no separation of the motion into rigid-body and flexible modes, nor subsequent concerns about coupling between these two artificially separated motions. The chatter issues of sliding mode control do not arise, and actuator commands are smooth. Neither is there any fighting between position control and active vibration damping, these two functions having been seamlessly integrated into one motion. Modal control issues, such as mode spillover and uncontrolled modes, simply do not arise. The computational load is light, so real-time control is very feasible. The system is very robust to changes in the system dynamics and tolerant of non-ideal behaviour in the actuator.

Any one of these would constitute a major point of merit. To find them all on offer in one method seems almost too good to be true. Furthermore, there seem to be no obstacles to adapting it and extending it to even more complex situations.

Acknowledgment

This work was funded in part by Enterprise Ireland Basic Research Grant, code SC/2001/319/.

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