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H_∞ Dynamic observer design with application in fault diagnosis

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***Abstract*— Most observer-based methods applied in *fault detection and diagnosis* (FDD) schemes use the classical two-degrees of freedom observer structure in which a *constant matrix* is used to stabilize the error dynamics while a *post filter* helps to achieve some desired properties for the residual signal. In this paper, we consider the use of a more general framework which is the dynamic observer structure in which an observer gain is seen as a filter designed so that the error dynamics has some desirable frequency domain characteristics. This structure offers extra degrees of freedom and we show how it can be used for the sensor faults diagnosis problem achieving detection and estimation at the same time. The use of weightings to transform this problem into a standard H_∞ problem is also demonstrated.**

I. Introduction

Model-based approaches have been a useful tool to solve the *fault diagnosis problem* specially for LTI systems. They are represented by the two-stage structure which is now widely accepted by the fault diagnosis community [2] and that consists of the following: (i) A residual generation module that generates a fault indicating signal (residual) using the available input/output information, (ii) A decision making phase where the residuals are examined to determine if a fault has occurred.

The observer-based approach, in which an observer plays the role of the residual generation module, is one of the most famous techniques used for residual generation. Many standard observer-based techniques exist in the literature providing different solutions to both the theoretical and practical aspects of the problem (see [5], [7] for good surveys). The basic idea behind this approach is to estimate the outputs of the system from the measurements by using either *Luenberger* observers in a deterministic framework [1] or *Kalman* filters in a stochastic framework [13]. The weighted output estimation error is then used as the residual in this case. Different aspects of the fault diagnosis problem have been considered by using this methodology. Beard used this idea to develop existence conditions for directional residuals (residuals that achieve fault isolation) [1]. Fault isolation has also been considered by using the dedicated observer scheme [11], where a bank of observers is used to differentiate between different faults. The problem of robustness to disturbances and uncertainties has also seen much attention and different successful techniques have been applied such as the *Unknown Input Observer* (UIO) [3] and the *eigen structure assignment* approach [10]. Optimization techniques (specially H_∞) have also been widely used in fault detection to minimize the disturbance effect and maximize the fault effect when complete decoupling is impossible [2], [7], [4], [8]. In all of these works, the residual generator can be parameterized by the same two-degrees of freedom structure, in which a *constant observer gain* and a *post filter* achieve different specifications of the fault diagnosis problem. In this paper, we consider a more general framework, making use of the dynamic structure in [9] where an observer gain is seen as a filter designed so that the error dynamics has some desirable frequency domain characteristics. We apply this structure for the sensor faults estimation problem where the objective of estimating the faults magnitudes is considered (in addition to detection and isolation). We show that, unlike the classical structure, this objective is achievable by minimizing the estimation error in a narrow frequency band. Different frequency patterns are also considered and the use of weightings to model the problem as a standard H_∞ problem is illustrated. The introduced techniques are demonstrated on a model of the $PROCON^{TM}$ *level/temperature* process training system.

II. Preliminaries and notation

The linear fault detection and diagnosis (FDD) problem considers the general class of LTI-MIMO systems affected by faults that can be modeled as follows:

$$\dot{x}(t) = Ax(t) + Bu(t) + R_1 f(t) \tag{1}$$

$$y(t) = Cx(t) + Du(t) + R_2 f(t) \tag{2}$$

where $x(t) \in \mathcal{R}^n$, $u(t) \in \mathcal{R}^m$, $y(t) \in \mathcal{R}^p$ and $f(t) \in \mathcal{R}^s$, and where the matrices A, B, C, D, R_1 and R_2 are known matrices of appropriate dimensions. Here $f(t)$ is the fault vector, and can represent the different types of system faults (i.e, sensor, actuator and component faults).

As mentioned in section I, the most famous technique used for residual generation is the observer-based approach that uses the following *Luenberger* observer structure:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L\,(y(t) - \hat{y}(t)) \tag{3}$$

$$\hat{y}(t) = C\hat{x}(t) + Du(t) \tag{4}$$

in addition to a weighting $Q(s)$ to generate the residual as:

$$r = Q(s)(y - \hat{y}); \qquad r(t) \in \mathcal{R}^q \tag{5}$$

The residual obtained from (5) is therefore the weighted output estimation error of the observer, and the residual generator (3)-(5) has two degrees of freedom, namely, the constant observer gain L and the post filter $Q(s)$. This freedom can be used to achieve different specifications of the FDD problem. The following definitions are widely accepted by the FDD community and are related to the different tasks of a residual generator [2]: (note that in these definitions the transient period of the residuals is not considered)

Definition 1: (Fault detection) The residual generator achieves fault detection if the following condition is satisfied:

$$r_i(t) = 0\ ;\quad \text{for } i = 1, \cdots, q\ ;\ \forall t$$
$$\iff \qquad f_i(t) = 0\ ;\quad \text{for } i = 1, \cdots, s\ ;\ \forall t$$

Definition 2: (Fault isolation) The residual generator achieves fault isolation if the residual has the same dimension as $f(t)$ (i.e, $q = s$) and if the following condition is satisfied:

$$(r_i(t) = 0\ ;\quad \forall t \iff f_i(t) = 0\ ;\quad \forall t)\ ;\ \text{for } i = 1, \cdots, s$$

Definition 3: (Fault identification) The residual generator achieves fault identification if the residual has the same dimension as $f(t)$ and if the following condition is satisfied:

$$(r_i(t) = f_i(t)\ ;\quad \forall t)\ ;\ \text{for } i = 1, \cdots, s$$

According to the previous definitions, in fault detection a binary decision could be made either that a fault occurred or not, while in fault isolation the location of the fault is determined and in fault identification the size of the fault is estimated. The relative importance of the three tasks is subjective and depends on the application, however it is important to note that fault identification implies isolation and that fault isolation implies detection (but not the opposite). Necessary and sufficient conditions for fault detection and isolation have been developed in [6]. For the sensor faults diagnosis problem (which is our focus in this paper), the system (1)-(2) is the special case where $R_1 = 0$, $R_2 = I_p$ and $f(t) = f_s(t) \in \mathcal{R}^p$. Using the classical residual generator in (3)-(5), the observer error dynamics is given from (6)-(7) (where $e = x - \hat{x}$, $\tilde{y} = y - \hat{y}$).

$$\dot{e}(t) = (A - LC)e(t) - Lf_s(t) \tag{6}$$
$$\tilde{y}(t) = Ce(t) + f_s(t) \tag{7}$$

The fault vector f_s has direct effect on the output estimation error $\tilde{y}$, and hence on the residual. Therefore *sensor fault detection* according to definition 1 is achievable by this structure [2]. *Fault isolation* can also be achieved by using the dedicated observer scheme, where a bank of observers (3)-(4) is used to differentiate between different faults. However, for this approach, the number of sensor faults need to be known a priori, and also restrictive observability conditions need to be satisfied [11]. In this paper we consider the multiple sensor faults identification problem using a novel approach. Our methodolgy is based on the extension of the *Luenberger* structure in (3)-(4) to a more general dynamic framework. We tackle the case when f_s are in a narrow frequency band by showing that the sensor fault identification problem is equivalent to an output zeroing problem which is solvable only with a dynamic observer. We further consider the cases of low and high frequency ranges showing that the problem can be modeled as a weighted H_∞ problem. The following definitions and notation will be used throughout the paper:

Definition 4: ($\mathcal{L}_2$ space) The space $\mathcal{L}_2$ consists of all $\mathcal{L}$ebesque measurable functions $u : \mathcal{R}^+ \to \mathcal{R}^q$, having a finite $\mathcal{L}_2$ norm $\|u\|_{\mathcal{L}_2}$, where $\|u\|_{\mathcal{L}_2} \triangleq \sqrt{\int_0^\infty \| u(t) \|^2\ dt}$, with $\|u(t)\|$ as the *Euclidean* norm of the vector $u(t)$.

For a system $H : \mathcal{L}_2 \to \mathcal{L}_2$, we will represent by $\gamma(H)$ the $\mathcal{L}_2$ gain of H defined by $\gamma(H) = \sup_u \frac{\|Hu\|_{\mathcal{L}_2}}{\|u\|_{\mathcal{L}_2}}$. It is well known that, for a linear system $H : \mathcal{L}_2 \to \mathcal{L}_2$ (with a transfer matrix $\hat{H}(s)$), $\gamma(H)$ is equivalent to the H-infinity norm of $\hat{H}(s)$ defined as follows: $\gamma(H) \equiv \| \hat{H}(s) \|_\infty \triangleq \sup_{\omega \in \mathcal{R}} \sigma\text{max}(\hat{H}(j\omega))$, where $\sigma_{\max}$ represents the maximum singular value of $\hat{H}(\jmath\omega)$. The matrices I_n, 0_n and 0_{nm} represent the identity matrix of order n, the zero square matrix of order n and the zero n by m matrix respectively. $Diag_r$(a) represents the diagonal square matrix of order r with $\begin{bmatrix} a & a & \cdots & a \end{bmatrix}_{1\times r}$ as its diagonal vector, while $diag(a_1,\ a_2, \cdots,\ a_r)$ represents the diagonal square matrix of order r with $\begin{bmatrix} a_1 & a_2 & \cdots & a_r \end{bmatrix}$ as its diagonal vector. The symbol $\hat{T}_{yu}$ represents the transfer matrix from input u to output y. The partitioned matrix $G = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$ (when used as an operator from u to y, i.e, $y = Gu$) represents the state space representation ($\dot{\xi} = A\xi + Bu,\ y = C\xi + Du$), and in that case the transfer matrix is $\hat{G}(s) = C(sI - A)^{-1}B + D$. We will also make use of the following property on the rank of $\hat{G}(s)$ [12] (if s is not an eigenvalue of A and where n is the dimension of the matrix A):

$$\text{rank} \begin{bmatrix} A - sI & B \\ C & D \end{bmatrix} = n + \text{rank}\left(\hat{G}(s)\right) \tag{8}$$

III. NARROW FREQUENCY BAND SENSOR FAULTS DIAGNOSIS

In almost all observer-based FDD designs, maximizing the faults effect on the observer's estimation error is considered as an optimal objective. However, for the sensor faults case (as shown in (6)-(7)) the opposite is true. By minimizing e, the output estimation error $\tilde{y}$ converges to f_s which guarantees fault identification in this case. In this section, we consider the solution of this design problem (when f_s is in a narrow frequency band around a nominal frequency ω_o) by using a dynamic observer structure, showing that the problem is not tractable for the static gain structure in (3)-(4).

A. Dynamic generalization of the classical structure

Throughout this paper, following the approach in [9], we will make use of dynamical observers of the form:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + \eta(t) \tag{9}$$
$$\hat{y}(t) = C\hat{x}(t) + Du(t) \tag{10}$$

where $\eta(t)$ is obtained by applying a dynamical compensator on the output estimation error $(y - \hat{y})$, i.e $\eta(t)$ is given from

$$\dot{\xi} = A_L\xi + B_L(y - \hat{y}) \tag{11}$$
$$\eta = C_L\xi + D_L(y - \hat{y}). \tag{12}$$

We will also write

$$K = \left[\begin{array}{c|c} A_L & B_L \\ \hline C_L & D_L \end{array}\right] \tag{13}$$

to represent the compensator in (11)-(12). It is straightforward to see that this observer structure reduces to the usual observer in (3)-(4) in the special case where the gain K is the constant gain given by $K = \left[\begin{array}{c|c} 0_n & 0_{np} \\ \hline 0_n & L \end{array}\right]$. The additional dynamics provided by this observer brings additional degrees of freedom in the design, something that will be exploited in the minimization of the sensor faults effect. First, note that the observer error dynamics in (6) is now given by ($\dot{e} = \; Ae - \eta$) which can also be represented by the following so-called standard form:

$$\dot{z} = \begin{bmatrix} A \end{bmatrix} z + \begin{bmatrix} 0_{np} & -I_n \end{bmatrix} \begin{bmatrix} \omega \\ \nu \end{bmatrix} \tag{14}$$

$$\begin{bmatrix} \zeta \\ \varphi \end{bmatrix} = \begin{bmatrix} I_n \\ C \end{bmatrix} z + \begin{bmatrix} 0_{np} & 0_n \\ I_p & 0_{pn} \end{bmatrix} \begin{bmatrix} \omega \\ \nu \end{bmatrix} \tag{15}$$

where

$$\begin{aligned} \omega &= f_s \;, \quad \nu = \; \eta = K(y - \hat{y}) \\ \zeta &= e = x - \hat{x} \;, \quad \varphi = \; y - \hat{y} \end{aligned} \tag{16}$$

which can also be represented by Fig. 1 where the plant G has the state space representation in (17) with the matrices in (14)-(15) and where the controller K is given in (13).

$$\hat{G}(s) = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array} \right] \tag{17}$$

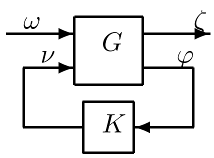
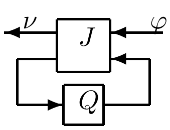


Fig. 1. Standard setup.

All possible observer gains for the observer (9)-(13) (including the static case (3)-(4)) can then be parameterized by the set of all stabilizing controllers for the setup in Fig. 1. This is a standard result (see [12]) and, for the problem considered in this paper, it can be represented by the following theorem (as special case of Theorem 11.4 in [12]):

Theorem 1: Let F and L be such that $A+LC$ and $A-F$ are stable; then all possible observer gains K for the observer (9)-(13) can be parameterized as the transfer matrix from φ to ν in Fig. 2 with any $\hat{Q}(s) \in RH_\infty$.



$$; \; \hat{J}(s) = \left[\begin{array}{c|cc} A - F + LC & -L & -I_n \\ \hline F & 0_{np} & I_n \\ -C & I_p & 0_{pn} \end{array} \right]$$

Fig. 2. Parametrization of all observer gains.

B. State and sensor faults estimation

As mentioned earlier, our objective is to minimize (in some sense) the effect of sensor faults (in a narrow frequency band around a nominal frequency ω_o) on the state estimation error in order to achieve sensor faults estimation. Towards that goal, we will denote $\mathcal{G}$ as the set of all scalar continuous functions $g(\omega)$ which are symmetric around ω_o, and $F_s(j\omega)$ as the fourier transform of $f_s(t)$. We will then define an optimal observer gain in $\mathcal{L}_2$ sense as follows:

Definition 5: (Optimal observer gain) An observer gain is said to be optimal with respect to the nominal frequency ω_o if the following property is satisfied for the estimation error $e(t)$ resulting from the sensor faults vector $f_s(t)$:

"$\forall \; \epsilon \; > \; 0$ and $\forall \; g(\omega) \; \in \; \mathcal{G}$, $\exists \; \Delta\omega \; > \; 0$ such that $F_s(j\omega)$ in (18) $\implies \; \| \, e \, \|_{\mathcal{L}_2} \leq \; \epsilon$" .

$$|F_s(j\omega)| \; = \; \begin{cases} g(\omega) \; ; & |\omega - \omega_o| < \Delta\omega \\ 0 \; ; & \text{otherwise} \end{cases} \tag{18}$$

Equation (18) means that the frequency pattern for $f_s(t)$ is confined to the region $[\omega_o - \Delta\omega, \omega_o + \Delta\omega]$. It is then clear that an optimal observer gain is one that satisfies $\hat{T}_{ef_s}(j\omega_o) = 0$. The following lemma shows that a static observer gain can never be an optimal observer gain.

Lemma 1: A static observer gain (such as the constant matrix L in (3)-(4)) can never be an optimal observer gain according to Definition 5.

Proof: The proof follows by noting that the transfer matrix from f_s to e (as seen in (6)) is $\hat{T}_{ef_s}(s) \; = \; \left[\begin{array}{c|c} A - LC & -L \\ \hline I_n & 0_{np} \end{array} \right]$. And since the gain L is chosen to stabilize $(A - LC)$, then $(\forall \omega_o)$ $j\omega_o$ is not an eigenvalue of $(A - LC)$. Therefore, by using (8), we have rank $\left(\hat{T}_{ef_s}(j\omega_o)\right)$ = rank $\left[\begin{array}{cc} A - LC - j\omega_o I_n & -L \\ I_n & 0_{np} \end{array} \right] - n$. But rank $\left[\begin{array}{cc} A - LC - j\omega_o I_n & -L \\ I_n & 0_{np} \end{array} \right]$ = rank $\left[\begin{array}{cc} L & 0_n \\ 0_{np} & I_n \end{array} \right]$ = $n+$rank(L). Therefore, rank $\left(\hat{T}_{ef_s}(j\omega_o)\right) \neq 0$ unless $L = 0$. This implies that no gain L can satisfy $\hat{T}_{ef_s}(j\omega_o) = 0$, and therefore a static observer gain can never be an optimal gain according to Definition 5. $\triangle$

We now consider the case of the dynamic observer (9)-(13). As a result of the gain parametrization presented in theorem 1, the transfer matrix from f_s to e, achievable by an internally stabilizing gain K, is equal to the Linear Fractional Transformation (LFT) between T and Q as follows [12]:

$$\hat{T}_{ef_s}(s) \equiv LFT(T, Q) = \hat{T}_{11}(s) + \hat{T}_{12}(s)\hat{Q}(s)\hat{T}_{21}(s) \tag{19}$$

where $\hat{Q}(s) \in RH_\infty$ and where T is given from

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \left[\begin{array}{cc|cc} A - F & F & 0_{np} & -I_n \\ 0_n & A + LC & L & 0_n \\ \hline I_n & 0_n & 0_{np} & 0_n \\ 0_{pn} & C & I_p & 0_{pn} \end{array} \right] \tag{20}$$

We will denote $\hat{T}_{11}(s)$, $\hat{T}_{12}(s)$ and $\hat{T}_{21}(s)$ by $\hat{T}_1(s)$, $\hat{T}_2(s)$ and $\hat{T}_3(s)$ respectively. The following lemma presents a result on the invertibility of the transfer matrices $\hat{T}_2(s)$ and $\hat{T}_3(s)$ at a frequency ω_o (i.e, at $s = j\omega_o$).

Lemma 2: The $(n \times n)$ and $(p \times p)$ matrices $\hat{T}_2(j\omega_o)$ and $\hat{T}_3(j\omega_o)$ are invertible if $j\omega_o$ is not an eigenvalue of A.

Proof: By (20), $\hat{T}_2(s) = \left[\begin{array}{cc|c} A - F & F & -I_n \\ 0_n & A + LC & 0_n \\ \hline I_n & 0_n & 0_n \end{array} \right]$ $= \left[\begin{array}{c|c} A - F & -I_n \\ \hline I_n & 0_n \end{array} \right]$. Similarly, $\hat{T}_3(s) = \left[\begin{array}{c|c} A + LC & L \\ \hline C & I_p \end{array} \right]$.

Therefore, using the rank property in (8):

i) $\text{rank}\left(\hat{T}_2(j\omega_o)\right) = \text{rank}\begin{bmatrix} A-F-j\omega_o I_n & -I_n \\ I_n & 0_n \end{bmatrix} - n$

ii) $\text{rank}\left(\hat{T}_3(j\omega_o)\right) = \text{rank}\begin{bmatrix} A+LC-j\omega_o I_n & L \\ C & I_p \end{bmatrix} - n$

But $\text{rank}\begin{bmatrix} A-F-j\omega_o I_n & -I_n \\ I_n & 0_n \end{bmatrix} = 2n, \ \forall\omega_o$. Also, $\text{rank}\begin{bmatrix} A+LC-j\omega_o I_n & L \\ C & I_p \end{bmatrix} = \text{rank}\begin{bmatrix} A-j\omega_o I_n & L \\ 0_{pn} & I_p \end{bmatrix} = n+p$; if $j\omega_o$ is not an eigenvalue of A. Therefore, $\text{rank}\left(\hat{T}_2(j\omega_o)\right) = n$ and $\text{rank}\left(\hat{T}_3(j\omega_o)\right) = p$ (full ranks) if $j\omega_o$ is not eigenvalue of A, and the proof is completed. $\triangle$

Based on the results in lemma 2, it can be proven that, for $\hat{T}_{ef_s}(s)$ in (19), $\exists$ a transfer matrix $\hat{Q}(s) \in RH_\infty$ that satisfies $\hat{T}_{ef_s}(j\omega_o) = 0$ (see Appendix I for details about computing $\hat{Q}(s)$). Therefore, for the dynamic observer in (9)-(13), an optimal gain (in the sense of Definition 5) can be found (unlike the static case). This shows the advantage of using the dynamic observer in this case. To summarize, we will define an optimal residual generator as follows:

Definition 6: (Optim. residual for narrow frequency band) The observer (9)-(13) along with $r = y - \hat{y}$ is an optimal residual generator for the sensor faults identification problem (with faults in a narrow frequency band around ω_o) if the gain K is chosen as the Linear Fractional Transformation $LFT(J,Q)$ in Fig. 2 where $\hat{Q}(s) \in RH_\infty$ solves the problem $\hat{T}_{ef_s}(j\omega_o) = 0$ for $\hat{T}_{ef_s}(s)$ in (19).

Remarks

- According to this definition, an optimal residual generator guarantees sensor faults estimation and at the same time state estimation. An advantage of having *state* estimation is the possibility to use the observer in fault tolerant output feedback control.
- From the special cases of interest is the case of sensor bias, where this approach can be used to get an *exact* estimation of all biases at the same time. A sufficient condition is that A has no eigenvalues at the origin.

IV. H_∞ SENSOR FAULTS DIAGNOSIS

We here consider two different cases: the low frequency range and the high frequency range. For the first case, we assume the system to be affected by sensor faults of low frequencies determined by a cutoff frequency ω_l, i.e the frequency pattern for $f_s(t)$ is confined to the region $[0, \omega_l]$. In the high frequency case, we assume these frequencies to be confined to the region $[\omega_h, \infty)$. Since, the error dynamics can be represented by Fig. 1 with the plant G in (17) having the matrices defined in (14)-(15) and with the controller K in (13), then these two cases can be solved by adding weightings to the setup in Fig. 1 that emphasize the frequency range under consideration, and by solving these problems as weighted H_∞ problems. However, before introducing weightings, it is important to note that the standard form in (14)-(15) does not satisfy all of the regularity assumptions in the H_∞ framework, and hence observer synthesis can not be carried out directly using the standard H_∞ solution. Fortunately, regularization can be done by extending the external output ζ in Fig. 1 to include the "scaled" vector $\beta\nu$; with $\beta > 0$. It can be seen that the standard form in (14)-(15) has now the following form:

$$\dot{z} = \begin{bmatrix} A \end{bmatrix} z + \begin{bmatrix} 0_{np} & -I_n \end{bmatrix} \begin{bmatrix} \omega \\ \nu \end{bmatrix} \tag{21}$$

$$\begin{bmatrix} \begin{bmatrix} e \\ \beta\nu \end{bmatrix} \\ \varphi \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} I_n \\ 0_n \end{bmatrix} \\ C \end{bmatrix} z + \begin{bmatrix} \begin{bmatrix} 0_{np} \\ 0_{np} \end{bmatrix} & \begin{bmatrix} 0_n \\ \beta I_n \end{bmatrix} \\ I_p & 0_{pn} \end{bmatrix} \begin{bmatrix} \omega \\ \nu \end{bmatrix} \tag{22}$$

which can also be represented by the standard setup in Fig. 1 with the same variables in (16) , except for redefining the matrices of $\hat{G}(s)$ in (17) and defining ζ as: $\zeta \triangleq \begin{bmatrix} e & \beta\nu \end{bmatrix}^T$. All the regularity assumptions below [12] are now satisfied iff A has no eigenvalues on the imaginary axis:

1) (A,B_2) stabilizable: satisfied for any matrix A. (C_2,A) detectable: satisfied, since (A, C) is detectable.
2) $D_{21}D_{21}^T$ and $D_{12}^T D_{12}$ are nonsingular.
3) $\text{rank}\begin{bmatrix} A-j\omega I & B_2 \\ C_1 & D_{12} \end{bmatrix} = 2n =$ full column rank $\forall\omega$. $\text{rank}\begin{bmatrix} A-j\omega I & B_1 \\ C_2 & D_{21} \end{bmatrix} = n+p =$ full row rank; iff $j\omega$ is not an eigenvalue of A.
4) $D_{22} = 0$.

The following lemma demonstrates a certain equivalence relationships between the standard form in (14)-(15) and the regularized one in (21)-(22) (proof is omitted).

Lemma 3: Let R_1 be the setup in Fig. 1 associated with (14)-(15), R_2 be the one associated with (21)-(22) and consider a stabilizing controller K for both setups. Then $\| \hat{R}_1 \|_\infty < \gamma$ if and only if $\exists\ \beta > 0$ such that $\| \hat{R}_2 \|_\infty < \gamma$.

A. The low frequency range case

We now consider faults of low frequencies determined by a cutoff frequency ω_l. The weighting $\hat{w}_l(s) = \frac{as+b}{s}$, [12], emphasizes this low frequency range with "b" selected as ω_l and "a" as an arbitrary small number for the magnitude of $\hat{w}_l(j\omega)$ as $\omega \to \infty$. Therefore, with a diagonal transfer matrix $\hat{W}(s)$ that consists of these weightings, the problem in Fig. 1 can be modified to the weighted version in Fig. 3.

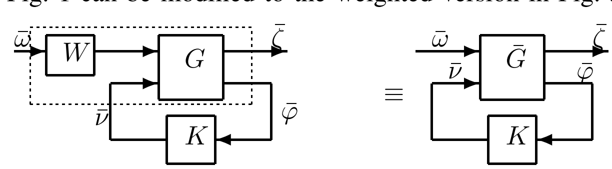


Fig. 3. Weighted standard setup.

It can be seen that $\bar{G}$ is given by:

$$\hat{\bar{G}}(s) = \left[\begin{array}{c|c|c} \begin{bmatrix} A_w & 0_{pn} \\ 0_{np} & A \end{bmatrix} & \begin{bmatrix} I_p \\ 0_{np} \end{bmatrix} & \begin{bmatrix} 0_{pn} \\ -I_n \end{bmatrix} \\ \hline \begin{bmatrix} 0_{np} & I_n \\ 0_{np} & 0_n \\ C_w & C \end{bmatrix} & \begin{bmatrix} 0_{np} \\ 0_{np} \\ D_w \end{bmatrix} & \begin{bmatrix} 0_n \\ \beta I_n \\ 0_{pn} \end{bmatrix} \end{array}\right] \tag{23}$$

where $A_w = 0_p$, $C_w = diag_p(b)$ and $D_w = diag_p(a)$. However, this standard form violates assumptions 1 and 3 of the regularity assumptions summarized earlier. Therefore, we introduce the modified weighting $\hat{w}_{lmod}(s) = \frac{as+b}{s+c}$; with arbitrary small positive "c". With this modification, the augmented plant $\bar{G}$ is the same as (23) except for A_w which is now given by the stable matrix $diag_p(-c)$ and C_w given by $diag_p(b-ac)$. Similar to the non weighted case, all the regularity assumptions are satisfied iff A has no eigenvalues on the imaginary axis. We define the regular H_∞ problem associated with the low frequency range as follows:

Definition 7: (Low freq. H_∞) Given $\beta > 0$, find $\mathcal{S}$, the set of admissible controllers K satisfying $\| \hat{T}_{\bar{\zeta}\bar{\omega}} \|_\infty < \gamma$ for the setup in Fig. 3 where $\bar{G}$ has the representation (23) with $A_w = diag_p(-c)$, $C_w = diag_p(b-ac)$ and $D_w = diag_p(a)$.

Based on the previous results, we now present the main result of this section in the form of the following definition for an optimal residual generator in $\mathcal{L}_2$ sense:

Definition 8: (Optimal residual for low frequencies) An observer (9)-(13) along with $r = y - \hat{y}$ is an optimal residual generator for the sensor faults identification problem (with faults of low frequencies below ω_l) if the dynamic gain $K \in \mathcal{S}^*$ (the set of controllers solving the H_∞ optimal control problem in Definition 7 with the minimum possible γ).

Comments

- A residual generator that is optimal in the sense of Definition 8 can be found using an iterative binary search algorithm over β to achieve the minimum possible γ.
- The constants a and c should be selected as arbitrary small positive numbers, while b must approximately be equal to ω_l (the cutoff frequency). Different weightings could also be used for the different sensor channels.

B. The high frequency range case

The SISO weighting $\hat{w}_{hmod}(s) = \frac{s+(a\times b)}{\epsilon s+b}$ [12], could be selected to emphasize the high frequency range $[w_h, \infty)$ with "b" selected as w_h and, "a" and "ϵ" > 0 as arbitrary small numbers. Similar to the low frequency range, a regular H_∞ problem related to this case can be defined. Also, an optimal residual generator can be defined in a similar way to Definition 8 (details are omitted due to similarity).

V. SIMULATION RESULTS

The PROCON Level/Flow/Temperature Process Control System (Fig. 4) includes two rigs which can be connected to achieve simultaneous level and temperature control.

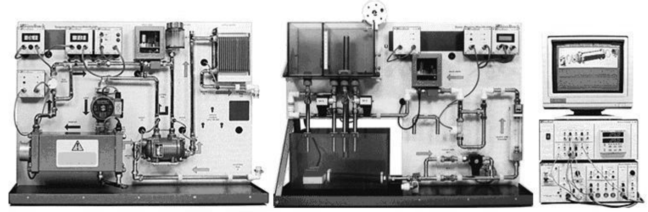


Fig. 4. The Level/Flow/Temperature Process Control System.

In the simulations, we consider the configuration obtained by connecting the two modules in cascade as shown in Fig. 5. In this case, there are two water circuits, namely, the hot water circuit and the cold water circuit. The water of both circuits flows into a heat exchanger where the heat energy can be transferred from the hot water flow into the cold water flow. The hot water temperature is controlled manually by the on-off switch of the heater, while the flow rates of both circuits can be controlled through the two servo valves connected to the computer. A level sensor is used to measure the level of the cold water in the main upper tank, while the temperature (at exactly one position) can be measured through the transmitter. It is important to note that there are 5 available positions for temperature measurement: T_1 (T_2) for the hot water input flow to (output flow from) the heat exchanger, T_3 (T_4) for the cold water input (output) flow, and T_5 for the cold water output flow from the cooling radiator. In this experiment, our objective is to control the water level and the temperature of the hot water circuit by controlling the flow rates of the valves. According to this configuration, the process has two inputs (the cold water and hot water servo valves) and two outputs (the level of the water in the upper tank and the temperature T_2). The inputs will be denoted u_1 and u_2 respectively and they both have the same operating range of 0 to 4 litres/min. The operating ranges for the outputs y_1 and y_2 are (0, 14 cm) and (0, 100 Celsius) respectively. The heater set point (i.e, T_1) is chosen as 80 Celsius, while the cold water in the reservoir is at the room's temperature (i.e, $T_3 \approxeq 23$ Celsius).

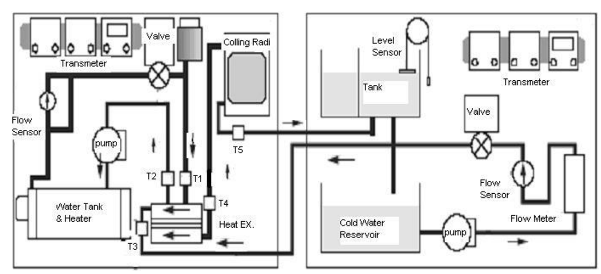


Fig. 5. Structure of the connected rigs.

Identification experiments are conducted, and for the operating point ($u_1 = 2.8$, $u_2 = 0.8$, $y_1 = 6.35$ and $y_2 = 35$) a 5^{th} order model of the form ($\dot{x} = Ax + Bu;\ y = Cx + Du$) is identified (see Appendix II for the system matrices). This model is used to demonstrate the proposed schemes. The system is first controlled as seen in Fig. 6 .

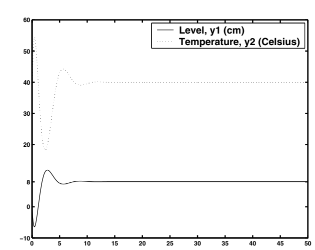


Fig. 6. Actual system outputs for the controlled process.

Case study 1: The system is assumed to be affected by sensor biases. This is the special case where $\omega_o = 0$ for the problem in section III-B, and since A has no eigenvalues on the origin, an optimal observer gain can be designed. This gain K, in our case, is the LFT in Fig. 2 with $\hat{Q}(s) = \hat{Q}(0)$ (computed using Appendix I) as follows:

$$\hat{Q}(0) = \begin{bmatrix} 104.96 & -75.24 & -6.86 & -74.84 & 36.31 \\ -116.58 & -356.17 & 783.55 & -694.06 & 112.44 \end{bmatrix}^T$$

Using this gain with initial conditions as $\begin{bmatrix} 0 & 0 & 0.1 & 0 & 0.005 \end{bmatrix}$, two biases changing with time are simultaneously estimated as seen in Fig. 7. The state

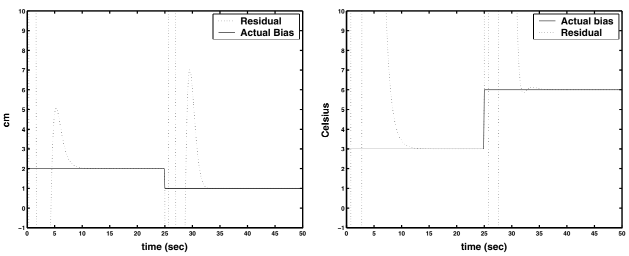


Fig. 7. (a) Bias estimation for y_1 (b) Bias estimation for y_2 .

and output estimation errors also converge to 0 in this case.
Case study 2: We consider the case of low frequency sensor faults (in the range $[0,\ 5$ rad/sec$]$). Using the design in section IV-A (and with $a = 0.01,\ b = 5$ and $c = 0.001$), the optimal observer gain is obtained by solving the H_∞ problem in Definition 7 using the command *hinfsyn* in MATLAB, with minimum γ as 0.1 and with $\beta = 1$. Using this observer for fault diagnosis, a correct estimation of the low frequency sensor faults is shown in Fig. 8.

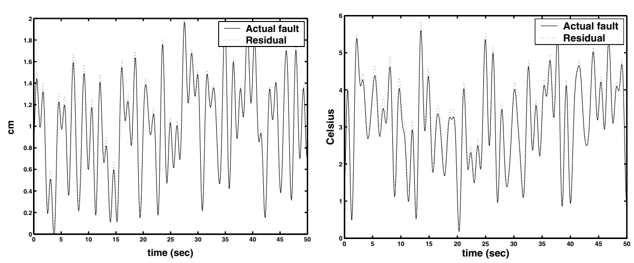


Fig. 8. (a) Fault estimation for y_1 (b) Fault estimation for y_2 .

VI. CONCLUSION

We considered the use of a dynamic observer structure for the sensor faults diagnosis problem. This structure offers extra degrees of freedom over the classical Luenberger structure and we showed how it can be used for the sensor faults and state estimations problems. For the narrow frequency band case, the problem was shown to be equivalent to an output zeroing problem for which a dynamic gain is necessary. The use of appropriate weightings to transform this problem into a standard H_∞ control problem was also demonstrated.

APPENDIX I
ALGORITHM FOR $\hat{Q}(s)$ COMPUTATION

If $j\omega_o$ is not an eigenvalue of A, then (from Lemma 2) the matrices $\hat{T}_2(j\omega_o)$ and $\hat{T}_3(j\omega_o)$ are invertible, and the matrix equation $\hat{T}_{ef_s}(j\omega_o) = 0$ can be solved for $\hat{Q}(j\omega_o)$ as follows:

$$\hat{Q}(j\omega_o) = -\hat{T}_2^{-1}(j\omega_o)\ \hat{T}_1(j\omega_o)\ \hat{T}_3^{-1}(j\omega_o) = \hat{Q}_{re} + j\hat{Q}_{im}$$

where $\hat{Q}_{re}$ and $\hat{Q}_{im}$ are $n \times p$ matrices that represent the real and imaginary parts respectively.

Let $\hat{Q}(s) = \left[\begin{array}{c|c} A_q & B_q \\ \hline C_q & D_q \end{array}\right]$; where $A_q \in \mathcal{R}^{\ell\times\ell}$, $B_q \in \mathcal{R}^{\ell\times p}$, $C_q \in \mathcal{R}^{n\times\ell}$ and $D_q \in \mathcal{R}^{n\times p}$ and where ℓ is the order of $\hat{Q}(s)$. Then computing $\hat{Q}(s) \in RH_\infty$ reduces to solving:

$$C_q\ (j\omega_o I_\ell - A_q)^{-1}\ B_q + D_q = \hat{Q}_{re} + j\hat{Q}_{im} \qquad (24)$$

for a stable A_q with a suitable order ℓ, and for B_q, C_q and D_q. By choosing $\ell = n$, $C_q = I_n$ and $A_q = -I_\ell$, the problem in (24) then reduces to solving the equations:

$$\frac{1}{1+\omega_o^2}\ B_q^{(ij)} + D_q^{(ij)} = \hat{Q}_{re}^{(ij)};\ i = 1,\cdots,n;\ j = 1,\cdots,p$$

$$\frac{-\omega_o}{1+\omega_o^2}\ B_q^{(ij)} = \hat{Q}_{im}^{(ij)};\ i = 1,\cdots,n;\ j = 1,\cdots,p$$

where $B_q^{(ij)}$ and $D_q^{(ij)}$ are the elements in row i and column j of the $n \times p$ matrices B_q and D_q respectively.

APPENDIX II
SYSTEM MATRICES

$$A = \begin{bmatrix} -0.0084 & -0.0012 & 0.0155 & 0.0280 & 0.0017 \\ -0.0046 & -0.0352 & -0.0227 & 0.0150 & 0.0082 \\ -0.0825 & -0.0122 & -0.0773 & 0.0661 & 0.3209 \\ -0.2105 & 0.0336 & -0.0929 & -0.3418 & -0.1551 \\ 0.0388 & -0.0754 & -0.1532 & 0.0126 & -0.1602 \end{bmatrix}$$

$$C = \begin{bmatrix} -6.7795 & -0.7974 & 0.0766 & 0.1585 & 0.0444 \\ -0.1862 & 19.2450 & -0.4087 & -0.0602 & -0.3102 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & -0.12 & -0.28 & 0.06 \\ 0 & 0.01 & 0.28 & -0.12 & -0.24 \end{bmatrix}^T, D = \begin{bmatrix} 0 & 0.01 \\ 0 & -0.04 \end{bmatrix}$$

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