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# Globally Stabilizing Robust Adaptive Voltage and Speed Regulator for Large-Scale Power Systems

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*Abstract*— **The paper presents a decentralized robust adaptive multivariable controller, with a guarantee of global stability, for large-scale uncertain power systems. The design is based on the reformulation of the conventional multi-machine power systems model into a new one with generators terminal voltages as state variables. This model while suitable for the application of modern control method, introduces a difficulty with regards to current design methods for large-scale systems. The interactions between generators treated as perturbations do not meet the common assumption on matching perturbations. A new robust adaptive method for a certain class of large-scale systems is therefore introduced. It does not require the matching condition, and can be seen as an extension of Shi and Sing's design method [1]. The main advantage of the proposed controller is that the asymptotic stability of the entire power system involves the mechanical and the electrical variables contrary to most methods encountered in the literature. Our solution consists of a nonlinear excitation and a turbine valve input canceling some nonlinearities of the model. An auxiliary control with linear and nonlinear components is used to stabilize the system. It compensates the unknown parameters in the model by continuously updating both the gain of the nonlinear components and parameters of the excitation. The adaptation algorithm for the gain uses sigma-modification whereas the one for the excitation parameters involves the projection method. Both algorithms are robust and prevent estimation drift. The computation of the linear part's matrix-gain involves the resolution of an algebraic Riccati equation and helps to solve the perturbation mismatching problem. A power system with nonlinear and dynamic loads is used to assess the proposed controller performance compared to the conventional AVR/PSS/GOVERNOR. The results show that transient performances are considerably improved after a severe contingency.**

## I. Introduction

The paper is about the design of a decentralized controller for large-scale uncertain multimachine power systems with a formal guarantee of global stability. The uncertainties may come from a topological or load variations (lines, load or generator tripping). The complexity of this problem comes from the fact that one aims to use local signals to compensate uncertain time-varying influences from remote sources while providing a proof of global stability. Several contributions have been reported on this important topic. In [2] a decentralized adaptive output feedback controller for large-scale power systems with unknown interconnections is presented. The interconnections are bounded by known polynomial functions and the outputs are rotors local angles. Reference [3] proposes a robust decentralized design scheme with unknown interconnections bounded by a general nonlinear function. Both solutions guarantee that the entire system is stable. Despite the relatively good results reported, generators terminal voltages are not included in the stability study. The multivariable characteristic of power systems is therefore not taken into account. The importance of the voltage profile in power system stabilization has been pointed out since the work in [4]. The general problem about the design of a robust adaptive decentralized voltage and speed (or power angle) regulator for uncertain large-scale power systems with formal proof of global stability is still an open problem.

The paper addresses this important issue in a multivariable framework, contrary to [12] in which a switching strategy is adopted to deal with the voltage regulation and transient stability problems simultaneously during the design of an adaptive control for a generator connected to an uncertain infinite bus. A complete power system model including the dynamics of terminal voltages and rotors angles is used for design. The two-axis model is used for the synchronous machine instead of the commonly used swing equation. Also the turbine dynamics are introduced resulting into a more realistic multivariable model with two inputs (excitation and turbine valve input) and two variables to be controlled (generator's terminal voltage and rotor speed). The solution extends the works in [5] and [6] to the case of power systems with unknown interconnection and aims to provide a guarantee of global stability.

Aggregating the power system components' dynamical models leads to dynamics equations, which are not suitable for modern control tools. Indeed, some variables such as stator currents are not state variables but appear in the model. A modeling procedure introduced by [6] is used in our solution. Its main characteristic and advantage is that generators terminal voltages are used as state variables and all the signals appearing in the model are either state

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variables or inputs. Design methods for uncertain large-scale system reported in the literature [1] [7] [8] are not appropriate for this multivariable model, however. The main problems when dealing with this model is that it is multivariable and the uncertainties do not satisfy the matching condition. Indeed, the approaches in [7] and [8] are for single input systems and the multivariable approach of [1] requires the matching condition for uncertain interconnections. The proposed solution extends therefore the method in [1] by removing the matching condition assumption and the works in [2] and [3] by taking into account generators terminal voltages in the stability study.

The design scheme consists of first reformulating the conventional multi-machine power systems model into a suitable one for modern control tools. Secondly, nonlinear excitation and valve input are derived to cancel some of the nonlinearities of the outputs dynamics. Next, linear and nonlinear auxiliary controls are used to stabilize the system. The linear stabilizing control has a fixed matrix-gain computed from the solution of an algebraic Riccati equation. The nonlinear stabilizing control has a time-varying gain continuously updated by an adaptation algorithm to compensate for the remote unknown interactions. An additional adaptation law is introduced to update the excitation and valve input parameters in order to compensate for local parametric uncertainties. The adaptation algorithms involve the sigma-modification for the estimated nonlinear part gain and the projection approach for the excitation parameters in order to prevent estimation drift.

The paper is organized as follows: The reformulation of multimachine power systems model is recalled in section 2. The design method for the decentralized multivariable robust adaptive voltage and speed regulator is exposed in Section 3. In Section 4, simulation results with a four-machine power system are shown. The paper ends with some conclusions in Section 5.

## II. Power Systems Model

We consider a power system with $n$ generators and $m$ buses. Each generator, illustrated at Fig. 1, consists of a synchronous machine powered by a static exciter and a steam turbine with its servo-valve. When generators terminal voltages are used as state variable, it is shown in [9] [13] that multimachine power systems can be represented by the following dynamics,

$$\dot{V}_{di} = \alpha_{1i} V_{di} + \alpha_{2i} V_{qi} + \rho_{1i} E_{fdi} + \Psi_{di}(\delta_i, \delta_k, V_{dk}, V_{qk}, \dot{V}_{dk}, \dot{V}_{qk}) \quad (1)$$

$$\dot{V}_{qi} = \beta_{1i} V_{di} + \beta_{2i} V_{qi} + \rho_{2i} E_{fdi} + \Psi_{qi}(\delta_i, \delta_k, V_{dk}, V_{qk}, \dot{V}_{dk}, \dot{V}_{qk}) \quad (2)$$

$$2H_i \dot{\omega}_i = T_{mi} - G_{ii}^f (V_{di}^2 + V_{qi}^2) + \Psi_{\omega i}(\delta_i, \delta_k, V_{dk}, V_{qk}) \quad (3)$$

$$T_{RHi} \dot{T}_{Mi} = -T_{Mi} + \left(1 - \frac{K_{HPi} T_{RHi}}{T_{CHi}}\right) P_{CHi} + \frac{K_{HPi} T_{RHi}}{T_{CHi}} P_{SVi} \quad (4)$$

$$T_{CHi} \dot{P}_{CHi} = -P_{CHi} + P_{SVi} \quad (5)$$

$$\dot{\delta}_i = (\omega_i - \omega_s)\omega_R \quad (6)$$

$$i = 1,\dots,n \quad k = 1,\dots,n \ ; k \neq i$$

$V_{di}$ and $V_{qi}$ represent the terminal voltage of the generator $i$ in the local d-q reference frame respectively. $E_{fdi}$ denotes the field voltage and is provided by the exciter. $T_{Mi}$, $P_{CHi}$ and $P_{SVi}$ are the mechanical torque, steam pressure and steam valve position respectively. $T_{RHi}$, $T_{CHi}$ and $K_{HPi}$ are the turbine and the servo parameters. $\omega_i$ represents the rotor speed and $\delta_i$ is the power angle expressed in the absolute reference frame. Without loss of generality, a thyristor type exciter is used and is modelled by a constant gain. This gain is included in the controller for simplicity.
$\Psi_{di}$, $\Psi_{qi}$ and $\Psi_{\omega i}$ represent the interconnections terms between generator $i$ and other generators of the grid. Details on the model parameters and the interactions terms are given in the appendix section.

*Remark 1:*
Since generators terminal voltages can experience abrupt variations particularly during short-circuits, they cannot be used a priori as state variables. Neglecting fast stator dynamics makes this possible however. Model (1-6) is based on this assumption and is valid therefore only after the discontinuity (i.e. for $t + t_0$ , $t > 0$) and not at $t_0$ the instant of the discontinuity. Generators terminal voltages when used as state variables are usually referred to as pseudo-state variables.

*Remark 2:*
Note that the interconnection terms $\Psi_{di}$ and $\Psi_{qi}$ depend on other generators terminal voltages derivatives. An equivalent form of dynamical model (1-6) in which interconnection terms do not depend on terminal voltages derivatives can be derived. The electrical part of the model has the following form,

$$\dot{V}_{di} = \alpha_{1i} V_{di} + \alpha_{2i} V_{qi} + \rho_{1i} E_{fdi} + \sum_{k=1}^{n} L_{dk}(\delta) E_{fdk} + \Omega_d(\delta, V_d, V_q) \quad (7)$$

$$\dot{V}_{qi} = \beta_{1i} V_{di} + \beta_{2i} V_{qi} + \rho_{2i} E_{fdi} + \sum_{k=1}^{n} L_{qk}(\delta) E_{fdk} + \Omega_q(\delta, V_d, V_q) \quad (8)$$

where

$\delta = [\delta_1 \quad \delta_2 \quad \cdots \quad \delta_n]^T$, $V_d = [V_{d1} \quad V_{d2} \quad \cdots \quad V_{dn}]^T$

$V_q = [V_{q1} \quad V_{q2} \quad \cdots \quad V_{qn}]^T$

The detailed computations to derive equations (7-8) and the expressions of $L_{dk}$, $L_{qk}$, $\Omega_d$ and $\Omega_q$ can be found in the appendix section.

## III. The Decentralized Adaptive Voltage and Speed Controller

In this section, decentralized multivariable voltage and speed controllers are proposed to stabilize a multimachine power system represented by dynamical equations (1-6). The parameters of this model depend on the loads characteristics and the network structure. They change therefore with the system operating condition, particularly after a load shedding, following a severe contingency.

Our objective is to propose a stabilizing controller even if local and interconnection parameters are unknown or are subject to change. Since the electrical dynamics strongly influence the rotor dynamical equation, our design strategy is to regulate simultaneously the generator terminal voltage and the speed. The good post-fault voltage profile resulting from this strategy will contribute to the improvement of the system's transient stability. The control inputs are the generator excitation $E_{fdi}$ and the turbine valve input $P_{SVi}$. Fig. 1 illustrates the control strategy and the main signals involved.

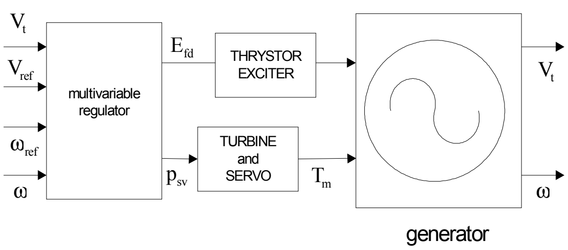


Fig. 1. Multivariable controller configuration

*A. Design Procedure*

First, let us derive the terminal voltage dynamical equation. Let $z_{1i}$ denote the difference between the terminal voltage $\sqrt{V_{di}^2(t)+V_{qi}^2(t)}$ and its steady state value $V_{ref}$.

$$z_{1i} = V_{di}^2(t)+V_{qi}^2(t)-V_{ref}^2 \tag{9}$$

Note that, the square of the terminal voltage is used, for simplicity. The dynamics of $z_1$ gives

$$\begin{aligned}\dot{z}_{1i} &= 2V_{di}\dot{V}_{di}+2V_{qi}\dot{V}_{qi}\\ &= 2(\alpha_{1i}V_{di}^2+\beta_{2i}V_{qi}^2+\alpha_{2i}V_{di}V_{qi}+\beta_{1i}V_{di}V_{qi}+V_{di}\Psi_{di0}+V_{qi}\Psi_{qi0})+\\ &\quad 2(\rho_{1i}V_{di}+\rho_{2i}V_{qi})E_{fdi}+2(\Psi_{di2}V_{di}+\Psi_{qi2}V_{qi})\end{aligned} \tag{10}$$

where

$$\Psi_{di2} = [\sum_{k=1}^{n} L_{dk}(\delta)E_{fdk}+\Omega_d(\delta,V_d,V_q)]-\Psi_{di}^0$$

$$\Psi_{qi2} = [\sum_{k=1}^{n} L_{qk}(\delta)E_{fdk}+\Omega_q(\delta,V_d,V_q)]-\Psi_{qi}^0$$

The parameters $\Psi_{di}^0$, $\Psi_{qi}^0$ and $\Psi_{\omega i}^0$ are the steady state values of interconnection terms $\Psi_{di}$, $\Psi_{qi}$ and $\Psi_{\omega i}$. $\Psi_{di2}$, $\Psi_{qi2}$ and $\Psi_{\omega i2}$ are defined as the difference between interconnections terms are theirs steady state values.

Since $\alpha_{1i}$, $\alpha_{2i}$, $\beta_{1i}$, $\beta_{2i}$, $\rho_{1i}$, $\rho_{2i}$ and the parameters of $\Psi_{di2}$, $\Psi_{qi2}$ and $\Psi_{\omega i2}$ are unknown, we introduce nominal operating condition parameters denoted $\alpha_{1iN}$, $\alpha_{2iN}$, $\beta_{1iN}$, $\beta_{2iN}$, $\rho_{1iN}$, $\rho_{2iN}$ in the voltage dynamical equation (10). The excitation is selected to partially linearize the voltage dynamics and its expression is,

$$E_{fdi} = \frac{v_{1i}-N}{2[(\rho_{1iN}+\Delta\hat{\rho}_{1i})V_{di}+(\rho_{2iN}+\Delta\hat{\rho}_{2i})V_{qi}]} \tag{11}$$

where

$$\begin{aligned}N = 2[&(\alpha_{1iN}+\Delta\hat{\alpha}_{1i})V_{di}^2+(\beta_{2iN}+\Delta\hat{\beta}_{2i})V_{qi}^2+(\alpha_{2iN}+\Delta\hat{\alpha}_{2i})V_{di}V_{qi}+\\ &(\beta_{1iN}+\Delta\hat{\beta}_{1i})V_{di}V_{qi}+V_{di}(\Psi_{di0N}+\Delta\Psi_{di0})+V_{qi}(\Psi_{qi0N}+\Delta\Psi_{qi0})]\end{aligned}$$

The parameters $\Delta\hat{\alpha}_{1i}$, $\Delta\hat{\alpha}_{2i}$, $\Delta\hat{\beta}_{1i}$, $\Delta\hat{\beta}_{2i}$, $\Delta\hat{\rho}_{1i}$, $\Delta\hat{\rho}_{2i}$, yet to be determined, denote the estimate of the difference between the unknown $\alpha_{1i}$, $\alpha_{2i}$, $\beta_{1i}$, $\beta_{2i}$, $\rho_{1i}$, $\rho_{2i}$ and the known parameters $\alpha_{1iN}$, $\alpha_{2iN}$, $\beta_{1iN}$, $\beta_{2iN}$, $\rho_{1iN}$, $\rho_{2iN}$.

Replacing (11) into (10) yields,

$$\begin{aligned}\dot{z}_1 &= \frac{d}{dt}(V_t^2(t)-V_{ref}^2) = v_{1i}+\theta_1V_{di}^2+\theta_2V_{di}V_{qi}+\theta_3V_{qi}^2+\theta_4V_{di}+\theta_5V_{qi}+\\ &\quad \theta_6V_{di}E_{fdi}+\theta_7V_{qi}E_{fdi}+2(\Psi_{di2}V_{di}+\Psi_{qi2}V_{qi})\\ &= v_{1i}+\Delta p_i^T\Phi_i+\Psi_{ivt}\end{aligned} \tag{12}$$

where

$\Delta p_i = \begin{bmatrix}\theta_{1i} & \theta_{2i} & \cdots & \theta_{7i}\end{bmatrix}$ is the difference between $p_i = \begin{bmatrix}\alpha_{1i}-\alpha_{1iN} & \alpha_{2i}-\alpha_{2iN} & \cdots & \rho_{2i}-\rho_{2iN}\end{bmatrix}$ and its estimate $\hat{p}_i = \begin{bmatrix}\Delta\hat{\alpha}_{1i} & \Delta\hat{\alpha}_{2i} & \cdots & \Delta\hat{\rho}_{2i}\end{bmatrix}$.

The interconnection term takes the following form

$$\Psi_{ivt} = 2(\Psi_{di2}V_{di}+\Psi_{qi2}V_{qi})$$

$$\Phi_i = \begin{bmatrix}V_{di}^2 & V_{di}V_{qi} & \cdots & V_{qi}E_{fdi}\end{bmatrix}^T$$

Introducing the nominal operating condition parameters into the rotor and the turbine dynamics gives

$$\dot{\omega}_i = \frac{1}{2H_i}(\Delta T_{mi}-G_{1IN}^f z_{1i})+\Psi_{i\omega a} \tag{13}$$

$$\Delta\dot{T}_{Mi} = -\frac{1}{T_{RHi}}\Delta T_{Mi}+(\frac{1}{T_{RHi}}-\frac{K_{HPi}T_{RHi}}{T_{RHi}T_{CHi}})\Delta P_{CHi}+\frac{K_{HPi}T_{RHi}}{T_{RHi}T_{CHi}}\Delta P_{SVi} \tag{14}$$

where

$\Delta X_i = X_i - X_{iN}$ is the difference between actual and nominal values of the turbine signals.

The valve input expression, shown below, is used to cancel the turbine original dynamics and impose new dynamics through the auxiliary input $v_{2i}$

$$\begin{aligned}P_{SVi} &= P_{SViN}+\frac{T_{CHi}}{K_{HPi}}\left\{\frac{1}{T_{RHi}}\Delta T_{Mi}-(\frac{1}{T_{RHi}}-\frac{K_{HPi}}{T_{CHi}})\Delta P_{CHi}+v_{2i}\right\}\\ &= \frac{T_{CHi}}{K_{HPi}}\left\{\frac{1}{T_{RHi}}T_{Mi}-(\frac{1}{T_{RHi}}-\frac{K_{HPi}}{T_{CHi}})P_{CHi}+v_{2i}\right\}\end{aligned} \tag{15}$$

To achieve zero steady state in voltage and speed, integrators are added into selected generators control loop. In the sequel, only those generators are considered. The extension of the method to a control loop without integrators is straightforward.

The system's dynamics in closed form can be rewritten as follows

$$\dot{x}_i = A_i x_i + B_i v_i + B_i \psi_{i1} + D_i \psi_{i2} \tag{16}$$

where

$$A_i = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & \dfrac{-G_{iiN}^f}{2H_i} & 0 & 0 & \dfrac{1}{2H_i} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad B_i = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \quad D_i = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$v_i = \begin{bmatrix} v_{1i} & v_{2i} \end{bmatrix}^T \quad \psi_{i1} = \begin{bmatrix} p_i^T \Phi_i & 0 \end{bmatrix}^T \quad \psi_{i2} = [\Psi_{ivt} \quad \Psi_{i\omega a}]^T$$

$$x_i = \begin{bmatrix} x_{1i} & x_{2i} & x_{3i} & x_{4i} & x_{5i} \end{bmatrix}^T$$

$$x_{1i} = \int z_1 d\tau,\ x_{2i} = z_1,\ x_{3i} = \int (\omega - \omega_{ref}) d\tau,\ x_{4i} = (\omega - \omega_{ref}),$$

$$x_{5i} = \Delta T_{mi}$$

*Remark 3*

The order of the resulting dynamics from the design process described above is less than the one of the original system. We can show that the zero dynamics of the two hidden dynamics are stable and do not affect the system stability [10]. They are not therefore considered in this study.

The sequel is devoted to the synthesis of all the generators stabilizing auxiliary inputs $v_{1i}$ and $v_{2i}$ which guarantee the stability of the entire power system. The system to be controlled consists of $n$ subsystems described by equation (16). The subsystems are interconnected through the terms $\Psi_{ivt}$ and $\Psi_{i\omega a}$ appearing in $\psi_i$. The main difficulties of this problem comes from the fact that the common assumption on the matching condition is not satisfied (since $B_i$ is different from $D_i$). Since generator excitations are bounded (i.e. $-E_{fd\,max} \le E_{fdi} \le E_{fd\,max}$) the dependency of $\Psi_{ivt}$ on $E_{fdi}$ is not problematic. The bounds on $E_{fdi}$ will be used to determine the bounds on $\Psi_{ivt}$. The main result of the paper is given by the following proposition and extends in a certain sense the result in [1] for a class of interconnections bounded by polynomials of order 2.

*Proposition*

The system described by (16) is ultimately bounded if the vector of decentralized auxiliary controls is defined as

$$v_i = K_i x_i - \hat{\rho}_i B_i^T P_i x_i (1 + \|x_i\|^2) \tag{17}$$

where the gain $K_i$ is given by

$$K_i = \frac{-1}{2\varepsilon_i} B_i^T P_i$$

and the matrix $P_i$ is a solution of the following Riccati equation

$$A_i^T P_i + P_i A_i - \frac{1}{\varepsilon_i} P_i B_i B_i^T P_i + \frac{1}{\mu_i} P_i D_i D_i^T P_i + \frac{1}{\mu_i} E^T E + \varepsilon_i Q_i = 0$$

The parameters $\varepsilon_i$ and $\mu_i$ are positive numbers. $Q_i$ is any positive definite matrix and $E = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$.

The gain $\hat{\rho}_i$ and the vector of parameters $\hat{p}_i$ are estimated using the following adaptation algorithms.

$$\dot{\hat{\rho}}_i = \gamma_i \left\| x_i^T P_i B_i \right\|^2 (1 + \|x_i\|^2)(1 + 2x_i^T P_i x_i) - \gamma_i \sigma_i \rho_i \tag{18}$$

$$\dot{\hat{p}}_i = \Gamma_i proj((1 + 2z_i^T P_i x_i)\Phi_i^T P_i x_i, \hat{p}_i) \tag{19}$$

$\gamma_i$ is a positive real number and $\Gamma_i$ is a positive definite matrix. ■

*Remark 4:*

The projection function $proj(\cdot,\cdot)$ is used to force the estimated parameters to remain within a predefined domain say, $\Omega_{\hat{p}_i} = \{ \hat{p}_i : \|\hat{p}_i\| \le p_{i\,max} \}$, and is defined as follow

$proj(y, \hat{p}) = y,$ if $f(\hat{p}) \le 0$

$proj(y, \hat{p}) = y,$ if $f(\hat{p}) \ge 0$ and $\dfrac{\partial f}{\partial \hat{p}} y \le 0$

$proj(y, \hat{p}) = [I - \dfrac{f(\hat{p})(\frac{\partial f}{\partial \hat{p}})^T (\frac{\partial f}{\partial \hat{p}})}{\left\| \frac{\partial f}{\partial \hat{p}} \right\|^2}] y,$ if $f(\hat{p}) > 0$

and $\dfrac{\partial f}{\partial \hat{p}} y > 0$

where $f(\hat{p}) = (\|\hat{p}\|^2 - \hat{p}_{max}) / (\varepsilon + 2\varepsilon \hat{p}_{max})$ and $\varepsilon$ is a positive real number. The projection function enjoys the following properties.

(P1) $\left\| proj(y, \hat{\theta}) \right\| \le \|y\|$

(P2) $\Delta\theta^T proj(y, \hat{\theta}) \ge \Delta\theta^T y$

The last term in (18) and the projection of (19) are introduced to prevent estimation drift due to parametric uncertainties and disturbances, which may appear in practice. Equations (18) and (19) are referred to as robust adaptation algorithms.

## B. *Proof of the proposition*

Let us consider the following Lyapunov function for the system consisting of n subsystems.

$$V = \sum_i^n [x_i^T P_i x_i + (x_i^T P_i x_i)^2 + \gamma_i^{-1} (\hat{\rho}_i - \rho_{0i})^2 + \Delta p_i^T \Gamma^{-1} \Delta p_i]$$

The parameter $\rho_{oi}$ is the unknown steady state value of the estimated parameter $\hat{\rho}_i$.

Let us derive the derivatives of each component of the Lyapunov function.

$$\frac{d}{dt}(x_i^T P_i x_i + (x_i^T P_i x_i)^2) = (1 + 2x_i^T P_i x_i) \frac{d}{dt}(x_i^T P_i x_i)$$

and

$$\frac{d}{dt}(x_i^T P_i x_i) = x_i^T(A_i^T P_i + P_i A_i)x_i + 2x_i^T P_i B_i v_i + 2x_i^T P_i B_i \psi_{i1} + 2x_i^T P_i D_i \psi_i$$
$$= x_i^T(\frac{1}{\varepsilon_i} P_i B_i R_i^{-1} B_i^T P_i - \frac{1}{\mu_i} P_i D_i D_i^T P_i - \frac{1}{\mu_i} E_i^T E_i - \varepsilon_i Q_i)x_i$$
$$+ 2x_i^T P_i B_i v_i + 2x_i^T P_i B_i \psi_{i1} + 2x_i^T P_i D_i \psi_{i2}$$

Completing the square using the following inequality

$$2x_i^T P_i D_i \psi_{i2} \le \frac{1}{\mu_i} x_i^T P_i D_i D_i^T P_i x_i + \mu_i \psi_{i2}^T \psi_{i2}$$

yields

$$\frac{d}{dt}(x_i^T P_i x_i) \le x_i^T(-\frac{1}{\mu_i} E_i^T E_i - \varepsilon_i Q_i)x_i + x_i^T P_i B_i \psi_{i1} + \mu_i \psi_{i2}^T \psi_{i2}$$
$$-2(\hat{\rho}_i - \rho_{0i} + \rho_{0i})(1 + \|x_i\|^2)$$

Also,

$$\frac{d}{dt}(\gamma_i^{-1}(\hat{\rho}_i - \rho_{0i})^2) = 2\gamma_i^{-1}(\hat{\rho}_i - \rho_{0i})\dot{\hat{\rho}}$$
$$= 2(\hat{\rho}_i - \rho_{0i})\{\|x_i^T P_i B_i\|^2 (1 + \|x_i\|^2)(1 + 2x_i^T P_i x_i) - \sigma_i(\hat{\rho}_i - \rho_{0i} + \rho_{0i}\}$$
$$= 2(\hat{\rho}_i - \rho_{0i})\|x_i^T P_i B_i\|^2 (1 + \|x_i\|^2)(1 + 2x_i^T P_i x_i) - 2\sigma_i(\hat{\rho}_i - \rho_{0i})^2 -$$
$$2\sigma_i(\hat{\rho}_i - \rho_{0i})\rho_{0i}$$

Again, completing the square with

$$-2\sigma_i(\hat{\rho}_i - \rho_{0i})\rho_{0i} \le \sigma_i(\hat{\rho}_i - \rho_{0i})^2 + \sigma_i \rho_{0i}^2$$

gives

$$\frac{d}{dt}(\gamma_i^{-1}(\hat{\rho}_i - \rho_{0i})^2) \le 2(\hat{\rho}_i - \rho_{0i})\|x_i^T P_i B_i\|^2 (1 + \|z_i\|^2)(1 + 2z_i^T P_i z_i)$$
$$-\sigma_i(\hat{\rho}_i - \rho_{0i})^2 + \sigma_i \rho_{0i}^2$$

The last Lyapunov function term derivative is

$$\frac{d}{dt}(\Delta p_i^T \Gamma^{-1} \Delta p_i) = -2\Delta p_i^T proj\{(1 + 2x_i^T P_i x_i)\Phi^T{}_i P_i x_i, \hat{p})$$
$$= -2\Delta p_i^T\{proj[(1 + 2x_i^T P_i x_i)\Phi^T{}_i P_i x_i, \hat{p}] -$$
$$(1 + 2x_i^T P_i x_i)\Phi^T{}_i P_i x_i + (1 + 2x_i^T P_i x_i)\Phi^T{}_i P_i x_i\}$$

Using property 2 yields

$$\frac{d}{dt}(\Delta p_i^T \Gamma^{-1} \Delta p_i) \le -2\Delta p_i^T(1 + 2x_i^T P_i x_i)\Phi^T{}_i P_i x_i$$

It is possible to show that, there exist positive constants $C_{oi}$, $C_{1ik}$ and $C_{2ik}$ such that

$$(\Psi_{\omega i}^r)^2 \le C_{0i} + \sum_{k=1}^{n} C_{1ik} x_{2k}^2$$

Also, positive constants $D_{1ik}$ and $D_{2ik}$, which depend on $E_{fd\,max}$, exist such that

$$(V_{di}\Psi_{di2} + V_{qi}\Psi_{qi2})^2 \le D_{0i} + \sum_{k=1}^{n} D_{1ik} x_{2k}^2$$

Therefore,

$$\psi_{i2}^T \psi_{i2} = 4(1 + (G_{iiN}^f)^2)(V_{di}\Psi_{di2} + V_{qi}\Psi_{qi2})^2 + (4H_i^2)^{-1}(\Psi_{\omega i}^r)^2$$
$$\le n_{0i} + \sum_{k=1}^{N}(m_{1ik} x_{2k}^2) \le n_{0i} + n_{1i}\sum_{i=1}^{N}\|x_i\|^2$$

where

$$n_{0i} = (4H_i^2)^{-1} C_{0i} + 4(1 + (G_{iiN}^f)^2) D_{0i}$$
$$m_{1ik} = 4(1 + (G_{iiN}^f)^2) D_{1ik} + (4H_i^2)^{-1} C_{1ik} \quad n_{1i} = \max_k\{m_{1ik}\}$$

The Lyapunov function derivative satisfies therefore,

$$\dot{V} \le -\sum_{i=1}^{n}\{(1 + 2x_i^T P_i x_i)x_i^T(\frac{1}{\mu_i} E_i^T E_i + \varepsilon_i Q_i)x_i - \sigma_i(\hat{\rho}_i - \rho_{0i})^2 + \sigma_i \rho_{0i}^2$$
$$-2\rho_{0i}(1 + \|x_i\|^2)(1 + 2x_i^T P_i x_i) + (1 + 2x_i^T P_i x_i)\mu_i(n_{0i} + n_{1i}\sum_{i=1}^{N}\|x_i\|^2)\}$$
$$\dot{V} \le -\sum_{i=1}^{n}\{L_{1i}\|x_i\|^2 + \sum_{i=1}^{N} L_{2i}\|x_i\|^4 + \sigma_i(\hat{\rho}_i - \rho_{0i})^2\} + \pi_0$$

where

$$\pi_0 = \sum_{i}^{N}(-2\rho_{0i} + \mu n_{oi} + \sigma_i \rho_{0i}^2)$$
$$L_{1i} = 2\rho_{0i}(\mu n_{0i} + \lambda_{min}(P_i)) - 5nn_{0i} - 2\mu n_{0i}\lambda_{max}(P_i) -$$
$$\lambda_{min}(\frac{1}{\mu_i} E^T E + \varepsilon_i Q_i)$$
$$L_{2i} = \lambda_{max}(P_i)\lambda_{min}(\frac{1}{\mu_i} E^T E + \varepsilon_i Q_i) + 4\rho_{oi}\lambda_{min}(P_i)$$

There exist constants $\upsilon$ and $\pi_0$ such that
$\dot{V}(\|x\|, (\hat{\rho} - \rho_0), \Delta\theta_i) \le -\upsilon V(\|x\|, (\hat{\rho}_i - \rho_{0i}), \Delta\theta_i) + \pi_0$.

We conclude that the system is ultimately bounded. It can be easily shown that the integrators in the voltage and speed control loops help ensure asymptotic stability. Indeed, it follows from the boundedness of the system that $\int z_{1i} d\tau$ and $z_{1i}$ are bounded. Therefore $\int_0^\infty z_{1i} d\tau$ exists and is finite. Since $\dot{z}_{1i}$ is also bounded, $z_{1i}$ is uniformly bounded. By Barbalat lemma, we conclude that $\|z_{1i}\|$ tends to zero as time goes to infinity. We follow the same scheme to show that $\|z_{2i}\|$ converges to zero.

The decentralized adaptive voltage and speed controller is now tested in a realistic multi generators power system containing static and dynamic nonlinear loads. The simulation file is built in the SimPowerSystems environment and complete models are used for power system components.

## IV. Simulation and Results

### A. System description

One test is used in this section to assess the proposed controller performance versus the conventional AVR/PSS and Governor system. The power system depicted at Fig. 2 is used for this purpose. The parameters of its components can be found in [11]. Loads characteristics are given by Table I. Note that Chr1 contains a nonlinear voltage dependent load whereas Chr3 main component is an induction motor.

### B. Controllers parameters

The parameters used to compute the controllers gains are $Q_i = I$, $\varepsilon_i = 0.1$ and $\mu_i = 0.75$. The resulting gain $K_i$ is

$$K_i = \begin{bmatrix} -14 & -7 & 0 & 0 & 0 \\ 0 & 0 & -41 & -24 & -4 \end{bmatrix}$$

The adaptation laws parameters $\sigma_i$, $\gamma_i$ and $\Gamma_i$ are $5$, $10^{-3}$ and $10^{-3} I$ respectively. The limits on the excitation and valve inputs are respectively $-11.5\ p.u. \le E_{fdi} \le 11.5\ p.u.$ and $0 \le p_{svi} \le 1.2\ p.u.$

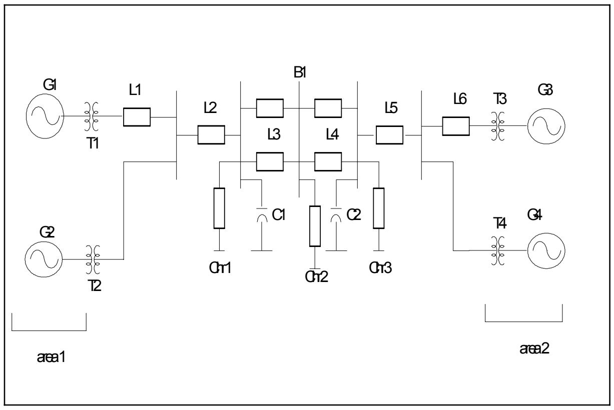


Fig. 2: A four-machine power system

### C. Test description and simulation results

A short-circuit of 12 cycles (about 200 ms) is applied at Bus B1. Fig.4 and 5 give the main signals of the system during the test. The proposed controller performances are compared to conventional regulators. It can be noted that very good transient performance is achieved with the multivariable adaptive controller. All the estimated gains and parameters remain bounded.

## V. Conclusion

A methodology for the design of a decentralized multivariable adaptive controller for uncertain large-scale power systems is exposed. The solution guarantees global stability of the entire system. Both the rotor speed and the terminal voltage of each generator are regulated. The design method extends previously exposed methods and requires the modeling of power systems into closed form suitable for modern control tools. The controller has a nonlinear linearizing part with time-varying parameters and an auxiliary stabilizing one. The auxiliary control consists of a linear component with a fixed matrix-gain computed after the resolution of a Riccati equation, and a nonlinear component with a time varying gain. The time-varying parameters and gain are continuously updated to accommodate the uncertainties of the power system. Simulation results assess that the system stability and transient performance are considerably improved as compared to conventional AVR/PSS/GOVERNOR regulator.

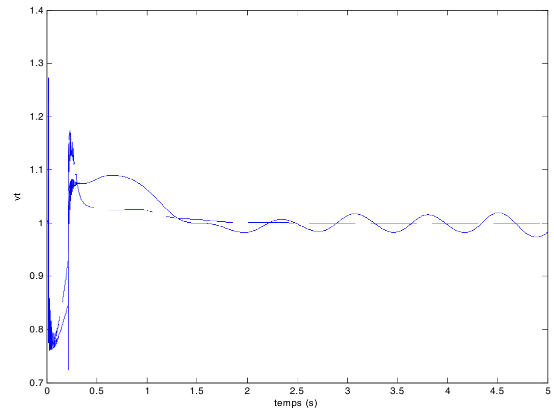


Fig. 3. G1 terminal Voltage profile
Dash line: adaptive controller, Solid line: Conventional Controller

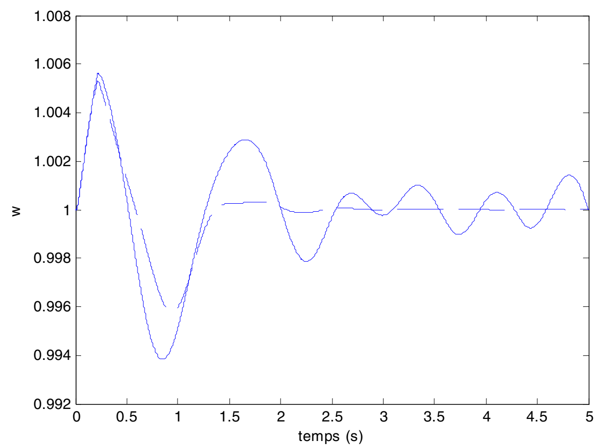


Fig. 4. G1 rotor speed
Dash line: adaptive controller, Solid line: Conventional Controller

## VI. Appendix

TABLE I:
NONLINEAR LOADS PARAMETERS

| | |
|---|---|
| Induction motor (in p.u.) $P = 7000\ MW$ $V = 25\ Kv \quad f = 60\ Hz$ | $R_s = 0.012 \quad Ll_s = 0.053$ $R_r^{'} = 0.04 \quad Ll_r^{'} = 0.053$ $L_M = 3 \quad H = 0.8$ |
| Voltage dependent load $V = 735\ Kv$ $P_0 = 500\ MW$ $Q_0 = 250\ MVar$ $V_0 = 0.994\ p.u.$ $V_{min} = 0.7\ p.u.$ $T_{p1} = T_{p2} = T_{q1} = T_{q2} = 0$ | $P = P_0 \left(\frac{V}{V_0}\right)^{n_p} \frac{1+T_{p1}s}{1+T_{p2}s}$ $Q = Q_0 \left(\frac{V}{V_0}\right)^{n_q} \frac{1+T_{q1}s}{1+T_{q2}s}$ If $V \ge V_{min}$ then $n_p = 1.3\ ,\ n_q = 2$ If $V < V_{min}$ then $n_p = n_q = 2$ |

*Expressions of the nonlinear terms* $\Psi_{di}$, $\Psi_{qi}$ *and* $\Psi_{\omega i}$.

$$\delta_{ik} = (\delta_k - \delta_i),\ \omega_{ik} = (\omega_k - \omega_i)$$

$$\Psi_{di} = \Psi_{1di} + \Psi_{2di}$$

$$\Psi_{1di} = \sum_{\substack{k=1\\k\neq i}}^{m} R_{ik}^{d}\{\cos(\delta_{ik} - \theta_{ik}^{d})V_{dk} - \sin(\delta_{ik} - \theta_{ik}^{d})V_{qk} +$$

$$\sum_{\substack{k=1\\k\neq i}}^{m} \omega_{ik} N_{ik}^{d} \sin(\delta_{ik} - \phi_{ik}^{d})V_{dk} - \sum_{\substack{k=1\\k\neq i}}^{m} \omega_{ik} N_{ik}^{d} \cos(\delta_{ik} - \phi_{ik}^{d})V_{qk} +$$

$$\Psi_{2di} = \sum_{\substack{k=1\\k\neq i}}^{m} N_{ik}^{d} \cos(\delta_{ik} - \phi_{ik}^{d})\dot{V}_{dk} - \sum_{\substack{k=1\\k\neq i}}^{m} N_{ik}^{d} \sin(\delta_{ik} - \phi_{ik}^{d})\dot{V}_{qk}$$

$$\Psi_{qi} = \Psi_{1qi} + \Psi_{2qi}$$

$$\Psi_{1qi} = \sum_{\substack{k=1\\k\neq i}}^{m} R_{ik}^{q}\{\cos(\delta_{ik} - \theta_{ik}^{q})V_{dk} - \sin(\delta_{ik} - \theta_{ik}^{q})V_{qk}\} +$$

$$\sum_{\substack{k=1\\k\neq i}}^{m} \omega_{ik} N_{ik}^{q} \sin(\delta_{ik} - \phi_{ik}^{q})V_{dk} - \sum_{\substack{k=1\\k\neq i}}^{m} \omega_{ik} N_{ik}^{q} \cos(\delta_{ik} - \phi_{ik}^{q})V_{qk}$$

$$\Psi_{2qi} = \sum_{\substack{k=1\\k\neq i}}^{m} N_{ik}^{q} \cos(\delta_{ik} - \phi_{ik}^{q})\dot{V}_{dk} - \sum_{\substack{k=1\\k\neq i}}^{m} N_{ik}^{q} \sin(\delta_{ik} - \phi_{ik}^{q})\dot{V}_{qk}$$

$$\psi_{\omega i} = V_{di}\{\sum_{\substack{k=1\\k\neq i}}^{m} M_{ik}\cos(\delta_{ik} + \varphi_{ik})V_{dk} - \sum_{\substack{k=1\\k\neq i}}^{m} M_{ik}\sin(\delta_{ik} + \varphi_{ik})V_{qk})\} +$$

$$V_{qi}\{\sum_{\substack{k=1\\k\neq i}}^{m} M_{ik}\sin(\delta_{ik} + \varphi_{ik})V_{dk} + \sum_{\substack{k=1\\k\neq i}}^{m} M_{ik}\cos(\delta_{ik} + \varphi_{ik})V_{qk})\}$$

*Steady state values of* $\Psi_{di}$, $\Psi_{qi}$ *and* $\Psi_{\omega i}$.

$$\Psi_{di}^{0} = \sum_{\substack{k=1\\k\neq i}}^{m} R_{ik}^{d}\sin(\delta_i^0 + \theta_{ik}^{d} - \theta_{ref}^{k})V_{ref}$$

$$\Psi_{qi}^{0} = \sum_{\substack{k=1\\k\neq i}}^{m} R_{ik}^{d}\cos(\delta_i^0 + \theta_{ik}^{d} - \theta_{ref}^{k})V_{ref}$$

$$\Psi_{\omega i}^{0} = \sum_{\substack{k=1\\k\neq i}}^{m} M_{ik}V_{ref}^{2}\cos(\theta_{ref}^{k} - \theta_{ref}^{i} + \varphi_{ik})$$

*Model parameters*

$$\alpha_{1i} = \frac{-\{\frac{x_{di}(G_{ii}^f)^2}{x'_{di}T'_{doi}} + \frac{x_{qi}(B_{ii}^f)^2}{x'_{qi}T'_{qoi}} + \frac{B_{ii}^f}{x'_{qi}T'_{qoi}}(\frac{x_{qi}}{x'_{di}} - 1)\}}{((G_{ii}^f)^2 + (B_{ii}^f)^2 + B_{ii}^f(\frac{1}{x'_{di}} - \frac{1}{x'_{qi}}))}$$

$$\alpha_{2i} = \frac{G_{ii}^f B_{ii}^f(\frac{x_{di}}{x'_{di}T'_{doi}} - \frac{x_{qi}}{x'_{qi}T'_{qoi}}) + \frac{G_{ii}^f}{x'_{di}T'_{doi}}(\frac{x_{di} - x'_{di}}{x'_{di}})}{(G_{ii}^f)^2 + (B_{ii}^f)^2 + B_{ii}^f(\frac{1}{x'_{di}} - \frac{1}{x'_{qi}})}$$

$$\alpha_{3ik} = \frac{-(\frac{x_{di}G_{ik}^f G_{ii}^f}{x'_{di}T'_{doi}} + \frac{x_{qi}B_{ik}^f B_{ii}^f}{x'_{qi}T'_{qoi}})}{((G_{ii}^f)^2 + (B_{ii}^f)^2 + B_{ii}^f(\frac{1}{x'_{di}} - \frac{1}{x'_{qi}}))}$$

$$\alpha_{4ik} = \frac{(\frac{x_{di}G_{ii}^f B_{ik}}{x'_{di}T'_{doi}} - \frac{x_{qi}G_{ik}^f B_{ii}}{x'_{qi}T'_{qoi}})}{((G_{ii}^f)^2 + (B_{ii}^f)^2 + B_{ii}^f(\frac{1}{x'_{di}} - \frac{1}{x'_{qi}}))}$$

$$\rho_{1i} = \frac{\frac{1}{T'_{doi}}\frac{G_{ii}^f}{x'_{di}}}{(G_{ii}^f)^2 + (B_{ii}^f)^2 + B_{ii}^f(\frac{1}{x'_{di}} - \frac{1}{x'_{qi}})}$$

$$\alpha_{5ik} = \frac{-(G_{ii}^f G_{ik}^f + B_{ii}^f B_{ik}^f)}{((G_{ii}^f)^2 + (B_{ii}^f)^2 + B_{ii}^f(\frac{1}{x'_{di}} - \frac{1}{x'_{qi}}))}$$

$$\alpha_{6ik} = \frac{(G_{ii}^f B_{ik}^f - B_{ii}^f G_{ik}^f)}{((G_{ii}^f)^2 + (B_{ii}^f)^2 + B_{ii}^f(\frac{1}{x'_{di}} - \frac{1}{x'_{qi}}))}$$

$$R_{ik}^{d} = \sqrt{\alpha_{3ik}^2 + \alpha_{4ik}^2} \quad \theta_{ik}^{d} = a\tan(\alpha_{4ik}/\alpha_{3ik})$$

$$N_{ik}^{d} = \sqrt{\alpha_{5ik}^2 + \alpha_{6ik}^2} \quad \phi_{ik}^{d} = a\tan(\alpha_{6ik}/\alpha_{5ik})$$

$$\beta_{1i} = \frac{(\frac{x_{di}G_{ii}^f}{x'_{di}T'_{doi}}(B_{ii}^f + \frac{1}{x'_{di}} - \frac{1}{x'_{qi}}) - \frac{G_{ii}^f}{x'_{qi}T'_{qoi}}(\frac{x_{qi}}{x'_{di}} + x_{qi}B_{ii}^f - 1))}{(G_{ii}^f)^2 + (B_{ii}^f)^2 + B_{ii}^f(\frac{1}{x'_{di}} - \frac{1}{x'_{qi}})}$$

$$\beta_{2i} = \frac{-(\frac{(B_{ii}^f + \frac{1}{x'_{di}} - \frac{1}{x'_{qi}})(x_{di}B_{ii}^f + \frac{x_{di} - x'_{di}}{x'_{di}})}{x'_{di}T'_{doi}} + \frac{(G_{ii}^f)^2}{x'_{qi}T'_{qoi}}x_{qi})}{(G_{ii}^f)^2 + (B_{ii}^f)^2 + B_{ii}^f(\frac{1}{x'_{di}} - \frac{1}{x'_{qi}})}$$

$$\beta_{3ik} = \frac{\frac{x_{di}G_{ik}^f}{x'_{di}T'_{doi}}(B_{ii}^f + \frac{1}{x'_{di}} - \frac{1}{x'_{qi}}) - \frac{x_{qi}G_{ii}^f B_{ik}^f}{x'_{qi}T'_{qoi}}}{(G_{ii}^f)^2 + (B_{ii}^f)^2 + B_{ii}^f(\frac{1}{x'_{di}} - \frac{1}{x'_{qi}})}$$

$$\beta_{4ik} = \frac{\frac{-(B_{ii}^f + \frac{1}{x'_{di}} - \frac{1}{x'_{qi}})x_{di}B_{ik}^f}{x'_{di}T'_{doi}} - \frac{x_{qi}G_{ii}^f G_{ik}^f}{x'_{qi}T'_{qoi}}}{(G_{ii}^f)^2 + (B_{ii}^f)^2 + B_{ii}^f(\frac{1}{x'_{di}} - \frac{1}{x'_{qi}})}$$

$$\beta_{5ik} = \frac{(G_{ii}^f B_{ik}^f - (B_{ii}^f + \frac{1}{x'_{di}} - \frac{1}{x'_{qi}})G_{ik}^f)}{((G_{ii}^f)^2 + (B_{ii}^f)^2 + B_{ii}^f(\frac{1}{x'_{di}} - \frac{1}{x'_{qi}}))}$$

$$\beta_{6ik} = \frac{(G_{ii}^f G_{ik}^f + (B_{ii}^f + \frac{1}{x'_{di}} - \frac{1}{x'_{qi}})B_{ik}^f)}{((G_{ii}^f)^2 + (B_{ii}^f)^2 + B_{ii}^f(\frac{1}{x'_{di}} - \frac{1}{x'_{qi}}))}$$

$$\rho_{2i} = \frac{\dfrac{-(B_{ii}^{f} + \dfrac{1}{x_{di}^{'}} - \dfrac{1}{x_{qi}^{'}})}{x_{di}^{'}T_{doi}^{'}}}{(G_{ii}^{f})^2 + (B_{ii}^{f})^2 + B_{ii}^{f}(\dfrac{1}{x_{di}^{'}} - \dfrac{1}{x_{qi}^{'}})}$$

$$N_{ik}^{q} = \sqrt{\beta_{5ik}^{2} + \beta_{6ik}^{2}} \quad \phi_{ik}^{q} = a\tan(\beta_{6ik} / \beta_{5ik})$$

$$M_{ik} = \sqrt{(G_{ik}^{f})^2 + (B_{ik}^{f})^2} \quad \varphi_{ik} = a\tan(B_{ik}^{f} / G_{ik}^{f})$$

*Derivation of equations* (2)

The electrical part of (1-6), when $\omega_{ik} \approx 0$, can be rewritten

$$\dot{x}_i = L_{ii}x_i + B_{ii}E_{fdi} + \sum_{k=1}^{n} L_{ik}x_k + \sum_{\substack{k=1 \\ k\neq i}}^{n} M_{ik}\dot{x}_k$$

$$L_{ii} = \begin{bmatrix} \alpha_{1i} & \alpha_{2i} \\ \beta_{1i} & \beta_{2i} \end{bmatrix},\ L_{ik} = \begin{bmatrix} R_{ik}^{d}\cos(\delta_{ik} - \theta_{ik}^{d}) & -R_{ik}^{d}\sin(\delta_{ik} - \theta_{ik}^{d}) \\ R_{ik}^{q}\cos(\delta_{ik} - \theta_{ik}^{q}) & -R_{ik}^{q}\sin(\delta_{ik} - \theta_{ik}^{q}) \end{bmatrix}$$

$$B_{ii} = \begin{bmatrix} \rho_{1i} \\ \rho_{21} \end{bmatrix} \quad M_{ik} = \begin{bmatrix} N_{ik}^{d}\cos(\delta_{ik} - \theta_{ik}^{d}) & -N_{ik}^{d}\sin(\delta_{ik} - \theta_{ik}^{d}) \\ N_{ik}^{q}\cos(\delta_{ik} - \theta_{ik}^{q}) & -N_{ik}^{q}\sin(\delta_{ik} - \theta_{ik}^{q}) \end{bmatrix}$$

Putting all the electrical part together gives

$$M\dot{x} = Lx + BE_{fdi}$$

$$M = \begin{bmatrix} I & M_{12} & \cdots & M_{1n} \\ M_{21} & I & \cdots & M_{2n} \\ & \cdots & \cdots & \\ M_{n1} & M_{n2} & \cdots & I \end{bmatrix},\ L = \begin{bmatrix} L_{11} & L_{12} & \cdots & L_{1n} \\ L_{21} & L_{22} & \cdots & L_{2n} \\ & \cdots & \cdots & \\ L_{n1} & L_{n2} & \cdots & L_{nn} \end{bmatrix}$$

$$B = diag(B_{ii})$$

Using the fact that $M = I + N$ for a certain matrix N and $L = L_1 + L_2$ with $L_1 = diag(L_{ii})$ and a certain matrix $L_2$, applying the formula $M^{-1} = I - M^{-1}N$ we get

$$\dot{x} = L_1x + BE_{fdi} + M^{-1}(L_2 - NL_1)x - M^{-1}NBE_{fdi}$$

Equation (2) and the expressions of the nonlinear terms are derived from the above equation.

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