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Output Feedback Control for a Class of Uncertain MIMO Nonlinear Systems With Non-Symmetric Input Gain Matrix

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Abstract

In this paper, a new continuous output feedback control mechanism is developed for output tracking for a class of high-order multi-input nonlinear systems with an input gain matrix that is positive definite but non-symmetric. The controller yields semiglobal uniformly ultimately bounded (SGUUB) tracking while compensating for unstructured uncertainty in both the drift vector and the input matrix. First, a full-state feedback controller is designed based on limited assumptions on the structure of the system nonlinearities and the controller is proven to yield SGUUB tracking through a Lyapunov-based analysis. Then, an output feedback control design based on a high gain observer is proposed. A comprehensive stability analysis of the closed-loop system under output feedback is carried out and a recovery of the state feedback SGUUB result is demonstrated for the output feedback control system. Neural network estimation method is employed in both state and output feedback control design to feedforward compensate for the nonlinear system uncertainty.

1 Introduction

Over the years, a lot of progress has been reported in nonlinear systems – specifically, research has moved from analysis into systematic techniques for controller synthesis for classes of nonlinear systems. As reported in a survey paper by [10], many researchers either assume no uncertainty in the plant model or assume that the uncertainty is a product of unknown parameters with known nonlinearities – the latter case leading into adaptive control. For multi-input nonlinear systems that are representable in the parametric strict feedback form, Krstic *et. al.* [12] were able to formalize the adaptive backstepping design procedure; however, the gain matrix premultiplying the control is assumed to be known. In [11], a general procedure was presented for designing switching adaptive controllers for multi-input nonlinear systems which includes feedback linearizable systems, parametric-pure feedback systems, and systems with a control Lyapunov function that is linear in the parameters. Recently, Zhang *et. al.* [23] obtained global asymptotic convergence of the tracking error to the origin for the following subset of MIMO nonlinear systems

$$x^{(n)} = h(x, \dot{x}, \cdots, x^{(n-1)}, \theta_1) + G(x, \dot{x}, \cdots, x^{(n-2)}, \theta_2)u \tag{1}$$

where $(\cdot)^{(i)}$ denotes the i^{th} derivative with respect to time, the uncertain $\mathcal{C}^0$ functions $h(\cdot) \in \mathbb{R}^m$ and the uncertain positive-definite (p.d.) gain matrix $G(\cdot) \in \mathbb{R}^{m\times m}$ were assumed to be affine in the unknown constant parameter vectors θ_1 and θ_2. Other examples of adaptive results can be found in [6], [7], and many others (See references in [10]). Even where unstructured uncertainties are dealt with, classes of SISO systems make up a huge chunk of the total effort. For SISO nonlinear systems in the strict feedback form that are non-affine in the unknown parameters, a global uniform ultimate bounded result due to [19] employs a Nussbaum gain and a smooth parameter projection algorithm. In [5], Ding *et. al.* obtain an ultimately bounded output feedback result for uncertain SISO systems via a modification of the backstepping technique using a Nussbaum gain and a Lyapunov function that is flat in a specifiable region around the origin. It is to be noted that both these results are robust to disturbances and do not require any knowledge of the sign of the high-frequency gain.

As far as multi-input uncertain nonlinear systems are concerned, there are very few results available. In [21], a neural network-based state feedback adaptive controller was formulated for the following class of

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systems

$$x^{(n)} = h(x, \dot{x}, \cdots, x^{(n-1)}, \phi(t), t) + G(x, \dot{x}, \cdots, x^{(n-1)}, \phi(t), t)u \quad (2)$$

with the only restriction that $G(\cdot)$ be uniformly positive or negative definite. The proposed controller was shown to guarantee the semi-global convergence of the tracking error to a residual set. The drawback of the control strategy is that the estimation strategy utilized can lead to loss of controllability in which case the control input tends to zero.

In this paper, our goal is to design a continuous output feedback (OFB) tracking controller based on the limited assumptions that the nonlinearities are second-order differentiable and the input gain matrix is positive-definite but non-symmetric. The $\mathcal{C}^2$ restriction on nonlinearities is required in order to ensure that the Lyapunov analysis based synthesis procedure yields a continuous controller. The input gain matrix is assumed to be positive-definite (p.d.) because state controllability requires that the smallest singular value of that matrix be lowerbounded by a positive constant. However, we drop the requirement that the input gain matrix be symmetric since many practical nonlinear control systems do not possess such a property (See examples in [22, 17]). We decompose $G(\cdot)$ into the product of a symmetric p.d. matrix and a unity upper triangular matrix. The symmetric p.d. matrix is exploited in the Lyapunov based stability analysis while the unity upper triangular matrix allows for an algebraic loop free sequential synthesis of control signals $u_i(t)\ \forall\ i = 1, 2, ...m$. Next, we adapt a high-gain observer (HGO) result in [2] for full-state asymptotically stable systems to uniformly ultimately bounded systems in order to design a continuous OFB controller that drives the closed-loop trajectories for the tracking errors into an arbitrarily small residual set. In order to broaden the applicability of the approach, we introduce a modular feedforward scheme which is shown to be achievable via neural network compensation.

The rest of the paper is organized as follows. In Section 2, we present the general MIMO system and the input gain matrix decomposition. Section 3 proposes a full-state feedback (FSFB) controller for the general MIMO system. Then, an OFB control design of the same system with an HGO is presented in Section 4. In Section 5, the theoretical development of the paper is complemented with a numerical simulation for a two degrees-of-freedom (DOF) mechanical system.

2 Problem Formulation and Preliminaries

We consider a class of MIMO nonlinear systems having the form

$$x^{(n)} = h(\mathbf{x}) + G(\mathbf{x})u \quad (3)$$

where $x^{(i)}(t) \in \mathbb{R}^m$, $i = 0, 1, ..., n-1$ are the system states, $\mathbf{x} = [x^T\ \dot{x}^T\ ...\ (x^{(n-1)})^T]^T \in \mathbb{R}^{mn}$, $u(t) \in \mathbb{R}^m$ represents the control input, and $h(\mathbf{x}) \in \mathbb{R}^m$ and $G(\mathbf{x}) \in \mathbb{R}^{m\times m}$ are uncertain $\mathcal{C}^2$ nonlinearities. We assume $G(\mathbf{x})$ is positive definite but non-symmetric.

The following matrix decomposition will be a key factor in the proposed control design.

Lemma 1 *Any positive-definite, non-symmetric matrix $P \in \mathbb{R}^{m\times m}$ can be decomposed as*

$$P = ST \quad (4)$$

where $S \in \mathbb{R}^{m\times m}$ is positive definite and symmetric, and $T \in \mathbb{R}^{m\times m}$ is an unity upper triangular matrix.

Proof: We can use the fact that all leading principal minors of a real, positive definite matrix are positive ([15], Theorem 5.10) along with Lemma 1 given in [3]. ■

Invoking Lemma 1, (3) can be rewritten as

$$M(\mathbf{x})x^{(n)} = \varphi(\mathbf{x}) + T(\mathbf{x})u \quad (5)$$

where $M(\mathbf{x}) = S^{-1}(\mathbf{x}) \in \mathbb{R}^{m\times m}$ is symmetric and positive definite, $\varphi(\mathbf{x}) = S^{-1}(\mathbf{x})h(\mathbf{x}) \in \mathbb{R}^m$, and $S(\mathbf{x}), T(\mathbf{x})$ are defined as in Lemma 1. We assume that the matrix $M(\cdot)$ is bounded by

$$\underline{m}\left\|\xi\right\|^2 \leqslant \xi^T M(\cdot)\xi \leqslant \bar{m}(\cdot)\left\|\xi\right\|^2 \quad \forall \xi \in \mathbb{R}^m \quad (6)$$

where $\underline{m} \in \mathbb{R}$ denotes a positive constant, and $\bar{m}(\cdot) \in \mathbb{R}$ denotes a positive, non-decreasing function. We now differentiate (5) to obtain

$$M(\mathbf{x})x^{(n+1)} = f(\mathbf{x}, x^{(n)}) + T(\mathbf{x})\dot{u} \quad (7)$$

where the uncertain nonlinearity $f(\mathbf{x}, x^{(n)})$ is given by

$$f(\mathbf{x}, x^{(n)}) = \dot{\varphi}(\mathbf{x}) - \dot{M}(\mathbf{x})x^{(n)} + \dot{T}(\mathbf{x})G^{-1}(\mathbf{x})\left(x^{(n)} - \varphi(\mathbf{x})\right). \quad (8)$$

The $(n+1)^{th}$ order system in (7) will be used as a basis for the subsequent control design and stability analysis.

Let $x_d(t) \in \mathbb{R}^m$ be a $\mathcal{C}^{n+2}$ reference trajectory such that

$$x_d^{(i)}(t) \in \mathcal{L}_\infty, \quad i = 0, 1, ..., n+2 \quad (9)$$

and $\mathbf{x}_d = [x_d^T \ \dot{x}_d^T \ ... \ (x_d^{(n-1)})^T]^T \in \mathbb{R}^{mn}$. The output tracking error $e_1(t) \in \mathbb{R}^m$ is defined as follows

$$e_1 = x_d - x. \tag{10}$$

The control objective is to ensure that $e_1(t) \to 0$ as $t \to \infty$ as well as guarantee the boundedness of all closed-loop signals. To simplify the subsequent control design, we introduce the following auxiliary error signals $e_i(t) \in \mathbb{R}^m$, $i = 2, 3, ..., (n+1)$

$$\begin{aligned} e_2 &= \dot{e}_1 + e_1 \\ e_3 &= \dot{e}_2 + e_2 + e_1 \\ e_4 &= \dot{e}_3 + e_3 + e_2 \\ &\vdots \\ e_{n+1} &= \dot{e}_n + e_n + e_{n-1}. \end{aligned} \tag{11}$$

Following [20], it is easy to obtain

$$e_i(t) = \sum_{j=0}^{i-1} a_{ij} e_1^{(j)}(t) \qquad \forall \ i = 2, 3, ..., (n+1) \tag{12}$$

where the known constant coefficients a_{ij} are generated via a Fibonacci number series [20].

3 State Feedback Control

3.1 Control Law

In this section, we assume full state feedback, *i.e.*, $x^{(i)}(t)$, $i = 0, ..., n-1$ in (3) are measurable. We propose the following state feedback control law

$$u = \int_{t_0}^{t} (K + I_m)(e_{n+1}(\tau) + e_n(\tau))\, d\tau + \int_{t_0}^{t} \hat{f}(\tau) d\tau \tag{13}$$

where $K \in \mathbb{R}^{m \times m}$ is a positive-definite, diagonal matrix, I_m is the $m \times m$ identity matrix, and $\hat{f}(t) \in \mathbb{R}^m$ denotes a yet to be designed feedforward component included to compensate for the term $f(\mathbf{x}, x^{(n)})$ in (7). We assume $\hat{f}(t)$ is designed such that $\hat{f}(t) \in \mathcal{L}_\infty$. Note that $e_n(t)$ and $\int e_{n+1}(t)dt$ in (13) are measurable since they are a function of the system states and the reference trajectory. Note that $\int e_{n+1}(t)dt$ is measurable via integrating the right hand side of the last definition of (11).

3.2 Error System Development

We define the error signal $r(t)$ as follows

$$r = e_{n+1} + e_n. \tag{14}$$

After taking the time derivative of (14), multiplying both sides by $M(\mathbf{x})$, and then substituting from the derivative of (12) for $i = n+1$, we have

$$M\dot{r} = M \sum_{j=0}^{n} a_{n+1j} e_1^{(j+1)} + M\dot{e}_n. \tag{15}$$

After applying (10) to (15), we have

$$\begin{aligned} M\dot{r} &= M x_d^{(n+1)} - M x^{(n+1)} \\ &\quad + M\left(\sum_{j=0}^{n-1} a_{n+1j} e_1^{(j+1)} + \dot{e}_n\right) \\ &= M\left(x_d^{(n+1)} + \sum_{j=0}^{n-1} a_{n+1j} e_1^{(j+1)} + \dot{e}_n\right) \\ &\quad - f - T\dot{u} \end{aligned} \tag{16}$$

upon the use of (7). We now arrange (16) into the following form

$$M\dot{r} = -\frac{1}{2}\dot{M}r - e_{n+1} - \bar{T}\dot{u} - \dot{u} + N \tag{17}$$

where $N(\mathbf{x}, x^{(n)}, t)$ is defined as follows

$$\begin{aligned} N = \ & M\left(x_d^{(n+1)} + \sum_{j=0}^{n-1} a_{n+1j} e_1^{(j+1)} + \dot{e}_n\right) - f \\ & + \tfrac{1}{2}\dot{M}r + e_{n+1} \end{aligned} \tag{18}$$

and $\bar{T}(\mathbf{x}) \triangleq T(\mathbf{x}) - I_m$. Note that $\bar{T}(\mathbf{x})$ is a strictly upper triangular matrix. We now define

$$N_d(t) = N(\mathbf{x}_d, x_d^{(n)}, x_d^{(n+1)}), \tag{19}$$

and rewrite (17) as

$$M\dot{r} = -\frac{1}{2}\dot{M}r - e_{n+1} - \bar{T}\dot{u} - \dot{u} + N_d + \tilde{N} \tag{20}$$

where $\tilde{N} \triangleq N - N_d$. Note that $N_d(t), \dot{N}_d(t) \in \mathcal{L}_\infty$ due to (9) and the $\mathcal{C}^2$ condition on $G(\mathbf{x})$ and $h(\mathbf{x})$. Since N defined in (18) is $\mathcal{C}^1$, it can be shown that $\tilde{N}$ can be upper bounded as follows

$$\left\| \tilde{N} \right\| \leq \rho_N(\|z\|)\, \|z\| \tag{21}$$

where

$$z \triangleq [\ e_1^T \quad e_2^T \quad ... \quad e_n^T \quad r^T \ ]^T \tag{22}$$

and $\rho_N(\cdot) : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is a globally invertible, nondecreasing function. By employing the time derivative of the control input vector of (13), the vector $\bar{T}\dot{u}$ in (20) can be written as

$$\bar{T}\dot{u} = \begin{bmatrix} \sum_{j=2}^{m} T_{1j}\dot{u}_j \\ \sum_{j=3}^{m} T_{2j}\dot{u}_j \\ \vdots \\ T_{(m-1)m}\dot{u}_m \\ 0 \end{bmatrix} = \begin{bmatrix} \Lambda + \Phi_d \\ 0 \end{bmatrix}, \tag{23}$$

where the auxiliary signals are defined as: $\Lambda \triangleq [\Lambda_1 \ \Lambda_2 \cdots \Lambda_{m-1}]^T$, $\Phi_d \triangleq [\Phi_{d1} \ \Phi_{d2} \cdots \Phi_{d(m-1)}]^T$, and the individual elements are defined as

$$\Lambda_i = \sum_{j=i+1}^{m} \left[\tilde{T}_{ij}(K_j+1)r_j + \tilde{T}_{ij}\hat{f}_j + T_{ij}(\mathbf{x}_d)(K_j+1)r_j\right] \quad (24)$$

$$\Phi_{di} = \sum_{j=i+1}^{m} T_{ij}(\mathbf{x}_d)\hat{f}_j \ . \quad (25)$$

In (24) above, $\tilde{T}_{ij} \triangleq T_{ij}(\mathbf{x}) - T_{ij}(\mathbf{x}_d)$ while the subscript j (ij) denotes the j^{th} (ij^{th}) element of the corresponding vector (matrix). It can be shown that Λ_i is upper bounded as follows

$$\|\Lambda_i\| \le \rho_{\Lambda i}(\|z\|)\,\|z\| \quad (26)$$

where $z(t)$ was defined in (22), and $\rho_{\Lambda i}(\cdot) : \mathbb{R}_{\ge 0} \to \mathbb{R}_{\ge 0}$ is a globally invertible, nondecreasing function containing only the diagonal elements $i+1$ to $m-1$ of K due to the strictly upper triangular nature of $\bar{T}(\mathbf{x})$. After taking the derivative of (13) and substituting into (20), one can arrive at the following closed-loop dynamics for $r(t)$

$$M\dot{r} = -\frac{1}{2}\dot{M}r - e_{n+1} - (K + I_m)r + \Pi + \Psi_d \quad (27)$$

where

$$\Pi = \tilde{N} - \begin{bmatrix} \Lambda \\ 0 \end{bmatrix}, \quad \Psi_d = N_d - \begin{bmatrix} \Phi_d \\ 0 \end{bmatrix} - \hat{f}. \quad (28)$$

It is not difficult to show that $\Psi_d(t), \dot{\Psi}_d(t), T(\mathbf{x}_d) \in \mathcal{L}_\infty$. Also note that $\Psi_d(t)$ is not a function of any control gains. The stability of the closed-loop system is stated by the following theorem:

Theorem 2 *Provided the elements of K are selected sufficiently large, the control law of (13) ensures that: (a) all closed-loop signals stay bounded for all time, and (b) tracking is locally uniformly ultimately bounded in the sense that*

$$\|e_i(t)\| \leqslant \epsilon, \quad i = 1, \cdots, n+1 \quad \forall\, t \in [t_0 + T, \infty) \quad (29)$$

where ϵ, T are some positive constants. (See Appendix A for proof)

4 Output Feedback Control Law

When $x(t)$ is the output of the system and is the only measurable state, we can only measure $e_1(t)$ given $x(t)$ and $x_d(t)$. Motivated by the result in [2], an estimate $\hat{e}(t) = \begin{bmatrix} \hat{e}_1(t) & \hat{e}_2(t) & \cdots & \hat{e}_{n+1} \end{bmatrix}^T \in \mathbb{R}^{m(n+1)}$ for the auxiliary error signals $e_i(t) \in \mathbb{R}^m \ \forall\, i = 1, 2, \cdots, n+1$ can be obtained via the following HGO

$$\begin{aligned}
\dot{\hat{e}}_1 &= \hat{e}_2 - \hat{e}_1 + \frac{\alpha_1}{\varepsilon}(e_1 - \hat{e}_1) \\
\dot{\hat{e}}_2 &= \hat{e}_3 - \hat{e}_2 - \hat{e}_1 + \frac{\alpha_2}{\varepsilon^2}(e_1 - \hat{e}_1) \\
&\vdots \\
\dot{\hat{e}}_n &= \hat{e}_{n+1} - \hat{e}_n - \hat{e}_{n-1} + \frac{\alpha_n}{\varepsilon^n}(e_1 - \hat{e}_1) \\
\dot{\hat{e}}_{n+1} &= \frac{\alpha_{n+1}}{\varepsilon^{n+1}}(e_1 - \hat{e}_1)
\end{aligned} \quad (30)$$

where $\alpha_i \in \mathbb{R}^{m\times m} \ \forall\, i = 1, 2, \cdots, n+1$ are yet to be designed gain matrices, and ε is a positive scalar. To make for facile analysis in the singularly perturbed form, we further define scaled observer errors $\eta_i(t) \in \mathbb{R}^m \ \forall\, i = 1, 2, \cdots, n+1$ as follows

$$\eta_i(t) = \frac{1}{\varepsilon^{n+1-i}}(e_i - \hat{e}_i). \quad (31)$$

After differentiating (31) and taking advantage of the definition of (11) and the design of (30), we obtain the following dynamic observer error system:

$$\begin{aligned}
\varepsilon\dot{\eta}_1 &= -\alpha_1\eta_1 + \eta_2 - \varepsilon\eta_1 \\
\varepsilon\dot{\eta}_i &= -\alpha_i\eta_1 + \eta_{i+1} - \varepsilon^2\eta_{i-1} - \varepsilon\eta_i \\
\varepsilon\dot{\eta}_{n+1} &= -\alpha_{n+1}\eta_1 + \varepsilon\dot{e}_{n+1}
\end{aligned} \quad (32)$$

where $i = 1, 2, \cdots, n$. By defining $\eta(t) = \begin{bmatrix} \eta_1^T(t) & \eta_2^T(t) & \cdots & \eta_{n+1}^T(t) \end{bmatrix}^T \in \mathbb{R}^{m(n+1)}$, we can write a more compact form of (32) as follows

$$\varepsilon\dot{\eta}(t) \ = \ A_0\eta(t) + \varepsilon g \quad (33)$$

where

$$g = -\begin{bmatrix} \eta_1 & \varepsilon\eta_1 + \eta_2 & \cdots & \varepsilon\eta_{n-1} + \eta_n & -\dot{e}_{n+1} \end{bmatrix}^T.$$

and $\alpha_i \ \forall\, i = 1, 2, \cdots, n+1$ are chosen such that

$$A_0 \triangleq \begin{bmatrix} \alpha_1 & I_m & 0_m & 0_m \\ \cdot & \cdot & \cdot & \cdot \\ \alpha_n & 0_m & 0_m & I_m \\ \alpha_{n+1} & 0_m & 0_m & 0_m \end{bmatrix}$$

is Hurwitz[1]. The boundary-layer system $\frac{d\eta(\tau)}{d\tau} = A_0\eta(\tau)$ (obtained by applying a change of variable $\tau = t/\varepsilon$ and then setting $\varepsilon = 0$) induces a Lyapunov function $W(\eta) = \eta^T P_0 \eta$ that satisfies the following properties

$$\begin{cases} \lambda_{\min}(P_0)\|\eta\|^2 \le W(\eta) \le \lambda_{\max}(P_0)\|\eta\|^2, \\ \dot{W} = \frac{\partial W}{\partial \eta}\dot{\eta} \le -\lambda_w\|\eta\|^2, \quad 0 < \lambda_w \le 1, \\ \left\|\frac{\partial W}{\partial \eta}\right\| \le 2\|P_0\|\|\eta\|, \quad \|P_0\| \triangleq \lambda_{\max}(P_0). \end{cases} \quad (34)$$

[1] The notation I_x and 0_x denotes, respectively, $x \times x$ identity and null matrices for any positive integer x.

In the above equation, $P_0 \in \mathbb{R}^{m(n+1)\times m(n+1)}$ is a p.d. matrix that satisfies $P_0A_0 + A_0^TP_0 = -I_{m(n+1)}$. From (34), it is clear that $\eta(t) = 0$ is a globally exponentially stable equilibrium of the boundary-layer system.

From (32), it is clear that the solution of $\eta(t)$ contains terms like $\frac{1}{\varepsilon}e^{-\omega t/\varepsilon}$ for some $\omega > 0$ (*i.e.*, the amplitude of $\eta(t)$ is $O(\frac{1}{\varepsilon})$). This so-called peaking phenomenon can possibly drive an output feedback controller (that is based on a naive application of the separation principle) out of its region of attraction, thereby causing instability. To suppress the amplitude peaking of $\eta(t)$, the full-state control design of (13) is modified to an output feedback saturated control as follows [2]

$$\begin{aligned} u(t) = \; & \int_{t_0}^{t} sat\{(K+I_m)(\hat{e}_{n+1}(\tau)+\hat{e}_n(\tau))\}d\tau \\ & + \int_{t_0}^{t} \hat{f}(\tau)d\tau. \end{aligned} \tag{35}$$

where the time derivative of (35) is saturated in the variables $\hat{e}_n(t)$ and $\hat{e}_{n+1}(t)$ outside a compact set $\mathcal{D}_c \triangleq \{z(t) \in \mathbb{R}^{m(n+1)} \mid V(t) \leq c\}$ of the region of attraction $\mathcal{D}_z$ (See Appendix A) for the full-state feedback system. After combining (20), (23), (28), (33) and (35), we obtain the following closed-loop error dynamics for (3)

$$\begin{aligned} \dot{z}(t) &= \hat{\Phi}(z(t),\eta(t)), \\ \varepsilon\dot{\eta}(t) &= A_0\eta(t) + \varepsilon\hat{g}(z(t),\eta(t)) \end{aligned} \tag{36}$$

where $\hat{\Phi}(z(t),\eta(t))$ and $\hat{g}(z(t),\eta(t))$ are defined as follows

$$\begin{aligned} \hat{\Phi}(\cdot) = \; & \left[\begin{array}{ccc} e_2 - e_1 & e_3 - e_2 - e_1 & \cdots \\ \cdots & e_{n+1} - e_n - e_{n-1} & \phi(z,\eta) \end{array}\right]^T \\ \hat{g}(\cdot) = \; & -\left[\begin{array}{cccc} \eta_1 & \varepsilon\eta_1 + \eta_2 & \cdots & \varepsilon\eta_{n-1} + \eta_n \\ & -\phi(z,\eta) + e_{n+1} - e_n - e_{n-1} & & \end{array}\right]^T \\ \phi(\cdot) \triangleq \; & M^{-1}\Big[-\tfrac{1}{2}\dot{M}r - e_{n+1} - \hat{f} \\ & -sat\{(K+I_m)(\hat{e}_{n+1}+\hat{e}_n)\} \\ & -\bar{T}(sat\{(K+I_m)(\hat{e}_{n+1}+\hat{e}_n)\} + \hat{f}) \\ & +N_d + \tilde{N}\Big]. \end{aligned} \tag{37}$$

From the global boundedness of $\hat{f}(t)$, the global saturation of the estimated states, and the boundedness of $z(t)$ inside $\mathcal{D}_c$, it is easy to see that $\left\|\hat{\Phi}(z(t),\eta(t))\right\| \leq k_1 \;\forall\; z(t) \in \mathcal{D}_c$ and $\forall\; \eta(t) \in \mathbb{R}^{m(n+1)}$. Here, k_1 is a positive constant independent of ε.

4.1 OFB Stability Results

To directly construct a Lyapunov function to prove the stability of (36) is non-trivial owing to the augmented set of dynamics as well as saturation introduced in the OFB design. Instead, the proof is split into multiple steps (as similarly done in [2]) to reduce the complexity at each step. First, we prove the existence of a positively invariant set for the solutions of (36). In the second step, we regain the boundedness of solutions of (36) provided the trajectory $(z(t), \hat{e}(t))$ starts inside a compact subset of $\mathcal{D}_z \times \mathbb{R}^{m(n+1)}$. We are then able to show that the HGO constant ε can be chosen small enough to ensure that any trajectory $(z(t), \hat{e}(t))$ starting in the aforementioned compact subset results in $\eta(t)$ entering the invariant set (from step 1) before $z(t)$ can escape. In the third step, we recover semiglobal ultimate boundedness for (36).

We define $\mathcal{Z}$ to be any compact set in the interior of $\mathcal{D}_z$ such that $\mathcal{Z} \subset \mathcal{D}_c \subset \mathcal{D}_z$. We further define $\mathcal{H}$ to be any compact set in the interior of $\mathbb{R}^{m(n+1)}$. Let $\mathcal{D}_\varepsilon \triangleq \{\eta(t) \in \mathbb{R}^{m(n+1)} \mid W(\eta(t)) \leq \varrho\varepsilon^2\}$ be a compact set where $W(t)$ was defined in (34), ϱ is a positive constant that is yet to be selected, while ε is the HGO constant. In our analysis of the stability of (36), we will consider trajectories for closed-loop solutions $(z(t), \hat{e}(t))$ that start inside $\mathcal{Z} \times \mathcal{H}$. With these definitions, we present the stability analysis for the system of (36) via the following theorems:

Theorem 3 ***(Invariant Set Theorem)*** *Given* $\Sigma \triangleq \mathcal{D}_c \times \mathcal{D}_\varepsilon$*, there exists an* $\bar{\varepsilon}_1 > 0$ *such that* $\forall\varepsilon \in (0, \bar{\varepsilon}_1]$*,* Σ *is a positively invariant set for the trajectory* $(z(t), \eta(t))$*. (See Appendix B for proof)*

Theorem 4 ***(Boundedness Theorem)*** *There exists an* $\bar{\varepsilon}_2 \leq \bar{\varepsilon}_1$ *such that* $\forall\varepsilon \in (0, \bar{\varepsilon}_2]$*, any trajectory* $(z(t), \hat{e}(t))$ *that starts inside* $\mathcal{Z} \times \mathcal{H}$ *is bounded for all time. (See Appendix D for proof)*

Theorem 5 ***(Ultimately Boundedness Theorem)*** *Given any solution* $(z(t), \hat{e}(t))$ *that starts in* $\mathcal{Z} \times \mathcal{H}$ *and given any small* $\delta > \sqrt{2\lambda_3^{-1}\nu_0}$*, there exists an* $0 < \bar{\varepsilon}_3(\delta) \leq \bar{\varepsilon}_2$ *and a* $T_1(\delta) > 0$ *such that* $\|z(t)\| \leq \delta$ *and* $\|\hat{e}(t)\| \leq 2\delta \;\forall\; t \geq T_1$ *and* $\forall\; \varepsilon \in (0, \bar{\varepsilon}_3]$*. (See Appendix E for proof)*

5 Simulation

See Appendix F.

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Appendices available upon request.