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On Scalar LQG Control with Communication Cost

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Abstract— **We study the LQG control of scalar systems under communication constraints by naturally extending the LQG cost to include a quadratic penalty for communication. We show that the resulting optimization problem is quasiconvex in the communications parameter so that it can be solved in a computationally efficient manner.**

I. Introduction

The issue of control under communication constraints has recently been an active area of research. While most traditional analysis of control systems assume that the observation is readily available in uncorrupted form, the situation in real engineering systems is often quite different [4]. With networked systems becoming more ubiquitous, there is a pressing need to understand the impact of system performance due to various communication schemes and/or constraints.

Most of the recent works in this area have adopted, more or less, an information theoretic point of view by studying the performance/rate tradeoff. For example, [9], [10], [26], [27], [7], [14], [15], [28], and [20] studied the stabilization of linear systems with quantized state feedback. In addition to rate constraints many researchers have also looked at system performance under various noisy channel models (see, for example, [13], [8], and [19]), and much progress has been made in addressing the fundamental limits of performance for feedback systems, in the presence of a communication channel (see, for example, [22], [17], [11], and [16]).

In the spirit of the traditional linear quadratic Gaussian (LQG) framework, we would like approach the problem of controlling a system under communication constraints from an optimization perspective. Building on the classic LQG framework is not a new concept: Borkar and Mitter [4] examined the LQG problem with finite alphabet codewords being transmitted to the controller with ensuing delay and distortion; Tatikonda et al. [21] [23] examined LQG performance in the presence of a noisy analog feedback channel using sequential rate distortion; Gupta et al. [12] studied LQG control across a packet dropping link. Instead of explicitly introducing communication constraints such as a power-constrained Gaussian channel, an erasure channel, or a rate-constrained channel, we would like the communication constraint to be reflected in the cost formulation. That is, the aim of this work is to naturally extend the traditional LQG problem to account for communication constraints by adding a quadratic communication cost into the LQG problem formulation. We show that the resulting optimization problem is quasiconvex in C, the communications parameter. This allows us to easily compute the optimal solution.

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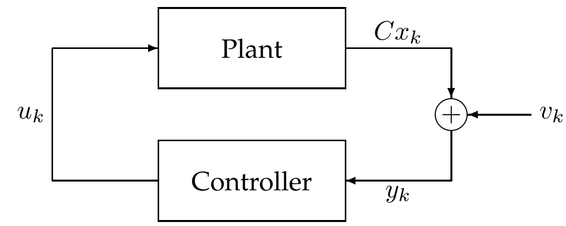

Fig. 1. System

This paper is organized as follows. In Section II, we formulate the problem. In Section III, we review results from traditional LQG control theory. In section IV, we examine the LQG problem with the new cost constraint and show that the solution can be obtained efficiently by proving that the resulting optimization is quasiconvex.

II. Problem Statement

Here, we describe the different components of our problem.

A. Plant

Consider the canonical scalar discrete-time, stochastic, linear system:

$$\begin{cases} x_{k+1} = Ax_k + Bu_k + w_k, & k \geq 0 \\ y_k = Cx_k + v_k \end{cases} \tag{1}$$

where $A \in \mathbb{R}$, $B \in \mathbb{R}$ are known nonzero values, $\{x_k \in \mathbb{R}\}$ is the state sequence, $\{u_k \in \mathbb{R}\}$ is the control sequence, and $\{y_k \in \mathbb{R}\}$ is the observed system output, and $C \in \mathbb{R}_{++}$ is the observation gain.[1] Disturbances $\{w_k \in \mathbb{R}\}$ and $\{v_k \in \mathbb{R}\}$ are uncorrelated white Gaussian noise processes with zero mean and covariances $W \in \mathbb{R}_{++}$ and $V \in \mathbb{R}_{++}$, respectively. The initial condition, x_0, is zero mean, has covariance $\Pi_0 \in \mathbb{R}_{++}$, and is uncorrelated with the processes $\{w_k\}$

[1] $\mathbb{R}_{++}$ denotes the strictly positive reals.

and $\{v_k\}$. In symbols,

$$E\begin{bmatrix} w_i \\ v_i \\ x_0 \end{bmatrix}\begin{bmatrix} w_j & v_j & x_0 & 1 \end{bmatrix} = \begin{bmatrix} W\delta_{ij} & 0 & 0 & 0 \\ 0 & V\delta_{ij} & 0 & 0 \\ 0 & 0 & \Pi_0 & 0 \end{bmatrix},$$

where δ_{ij} is the Kronecker delta function (identically zero except when $i = j$, where $\delta_{ii} = 1$).

B. Channel

The channel between the plant and the controller is an additive white Gaussian noise (AWGN) channel. Notice that we do not place an explicit power constraint on this channel. In the spirit of LQG, the communication constraint is factored into the cost function, this will be made apparent shortly.

C. Controller

At any instance in time, the controller has access to past and present observations, as well as all past control signals. That is, the control input is of the form

$$u_k = \mu_k(y_0, y_1, \ldots, y_k, u_0, u_1, \ldots, u_{k-1}),$$

where μ_k is a function of past and present observations and past controls. Fig. 1 shows the interconnection of the various components.

D. Quadratic Performance Objective

Our goal is to design the optimal control law, $\{u_k\}$, and the optimal measurement gain, C, to achieve the minimal average cost

$$J^* \triangleq \min_{C,\{u_k\}} \lim_{n\to\infty} E\left[\frac{1}{n}\sum_{k=0}^{n-1}\left(Qx_k^2 + Ru_k^2 + SC^2\right)\right], \quad (2)$$

where $Q \in \mathbb{R}_{++}$, $R \in \mathbb{R}_{++}$, and $S \in \mathbb{R}_{++}$. This is a natural extension of the LQG cost objective. By allowing C to become a variable parameter, we gain an additional degree of freedom in the design process: the control engineer now gets to design both the control law and the amplifier used by the plant to transmit the state vector. A quadratic cost is then placed on the gain of the amplifier. One can think of this new cost as the charge on the power-gain of the amplifier.

III. Classic LQG Problem

It is instructive to review the results of traditional LQG first before continuing onto our problem. As these results are standard and well known, we will state them without giving the details of the proof.

***Lemma** 1 (LQG):* For a fixed $C \in \mathbb{R}_{++}$, given the discrete-time stochastic linear system (1) and the average cost

$$J_{\mathrm{LQG}} = \lim_{n\to\infty} E\left[\frac{1}{n}\sum_{k=0}^{n-1}\left(Qx_k^2 + Ru_k^2\right)\right],$$

the optimal performance is:

$$\begin{aligned} J^*_{\mathrm{LQG}} &\triangleq \min_{\{u_k\}} J_{\mathrm{LQG}} \\ &= PW + \left(A^2P - P + Q\right)\bar{\Sigma}, \end{aligned} \quad (3)$$

where P and $\bar{\Sigma}$ satisfy

$$\begin{aligned} P &= A^2P - (APB)^2(B^2P+R)^{-1} + Q, && (4) \\ \bar{\Sigma} &= \Sigma - (\Sigma C)^2(C^2\Sigma + V)^{-1}, \\ \Sigma &= A^2\Sigma - (A\Sigma C)^2(C^2\Sigma+V)^{-1} + W. && (5) \end{aligned}$$

and $P > 0$, $\Sigma > 0$. That is, P and Σ are the positive roots to the quadratic equations (4) and (5), respectively.
Some of the key ideas from LQG theory can be summarize as follows:

- At each instance in time, the optimal control is linear in the current state estimate. That is, $u_k = L_k\hat{x}_k$ where $\hat{x}_k$ is the optimal estimate of the current state. In the steady state, $u_k = L\hat{x}_k$ where $L = -(BPA)(B^2P+R)^{-1}$.
- (Certainty Equivalence) The control gain, L, is independent of the statistics of the problem. That is, it remains the same if we were solving the deterministic LQR problem.
- (Separation) The optimal solution to this problem is obtained by independently solving the estimation problem (assuming no control) and the control problem assuming perfect information.

The details are omitted as they are readily available in standard textbooks on optimal control theory; for example, see §5.2 of [2], §4.5 in [3], or §5.3-4 [18].

The terms in (3) can be interpreted as the cost associated with perfect information, PW, and an additional cost due to estimation error, $(A^2P - P + Q)\bar{\Sigma}$. We note that equations (4) and (5) are the scalar algebraic Riccati equations (ARE) associated with the linear quadratic regulator (LQR) problem and the linear estimation problem (*à la* Kalman filtering), respectively. Conditions for their solutions are well known and we state it as a lemma.

***Lemma** 2 (ARE):* If $Q > 0$ and $B \neq 0$, then there exists a $P > 0$ that is the unique positive solution to the ARE (4). Similarly, if $V > 0$ and $C \neq 0$ then there exists a $\Sigma > 0$ that is the unique positive solution to the ARE (5).

Proof: The idea is that in order for the LQG problem to be well-posed, we must be able to control the system ($B \neq 0$) and observe the system ($C \neq 0$). Furthermore, to avoid degeneracies, the cost must account for the state ($Q > 0$) and there should be measurement noise ($V > 0$). For details, see Proposition 4.4.1 in [2]. ∎

It is evident that in order to solve the LQG problem, we must solve ARE's (4) and (5). Fortunately, in the scalar case, the ARE is simply a quadratic equation and we can solve for the solution in closed form. We note that for general higher-dimensional systems, there are no known closed form solutions to matrix ARE's. However, they be solved numerically using semidefinite programming (SDP) (see [24], [5], or [29]). In order to gain insight into our problem which can be easily obscured by the many difficulties associated with vector systems, we examine only the scalar case in this paper. More discussion on general vector systems can be found Section V.

IV. Communication Constrained LQG

Given the results in the previous section, it is straight forward to see that in our formulation, the separation principle between estimation and control continue to hold (see [25], [1], or §5.2 in [2]). The optimal cost, (2), becomes

$$J^* = \min_C \left\{ \min_{\{u_k\}} \lim_{n\to\infty} E\left[\frac{1}{n}\sum_{k=0}^{n-1}\left(Qx_k^2 + Ru_k^2 + SC^2\right)\right]\right\}$$

and we have the following:

***Theorem** 3:* Given the linear system (1) and a fixed $C \in \mathbb{R}_{++}$,

$$\begin{aligned} J(C) &\triangleq \min_{\{u_k\}} \lim_{n\to\infty} E\left[\frac{1}{n}\sum_{k=0}^{n-1}\left(Qx_k^2 + Ru_k^2 + SC^2\right)\right] \\ &= PW + C^2S + \left(A^2P - P + Q\right)\bar{\Sigma}, \qquad (6) \end{aligned}$$

where P and $\bar{\Sigma}$ are defined in (4) and (5), respectively.

The terms in (6) can be roughly interpreted as the cost associated with perfect information, PW; additional cost due to estimation error, $(A^2P - P + Q)\bar{\Sigma}$; and cost due to communication, C^2S. We note that the last two terms are related as the choice of measurement gain, C, directly influence the quality of estimation.

We first examine the implications of the ARE's (4) and (5). From (4), we can write

$$A^2P - P + Q = \frac{(APB)^2}{B^2P + R},$$

which implies

$$A^2P - P + Q > 0. \qquad (7)$$

Similarly, from (5), we write

$$\Sigma = \frac{A^2V\Sigma}{C^2\Sigma + V} + W,$$

which implies

$$\begin{aligned} \frac{A^2V\Sigma}{C^2\Sigma + V} < \Sigma \quad &\Rightarrow \quad A^2\frac{V}{C^2\Sigma + V} < 1 \qquad (8) \\ &\Rightarrow \quad A^2\left(\frac{V}{C^2\Sigma + V}\right)^2 < 1 \\ &\Rightarrow \quad (AV)^2 < \left(C^2\Sigma + V\right)^2. \qquad (9) \end{aligned}$$

To make our presentation more concise, we need to define a few more functions:

***Lemma** 4:* Let

$$\begin{aligned} h(x) \quad \triangleq \quad & 3\,x^4 + 8\,x^3V + 2\,x^2V^2(A^2+3) \\ & -4\,xV^3A^2(A^2-1) - V^4(A^2-1)^2, \end{aligned}$$

then for $x > \max\{0, V(A^2-1)\}$, $h''(x) > 0$ and $h'(x) \geq 0$. [2]

Proof:

$$\begin{aligned} h'(x) &= 12x^3 + 24x^2V + 4xV^2(A^2+3) \\ &\quad -4V^3A^2(A^2-1) \\ h''(x) &= 36x^2 + 48xV + 4V^2(A^2+3) \end{aligned}$$

It is clear that for $x > 0$, $h''(x) > 0$.

If $|A| < 1$, then $h'(0) = 4V^3A^2(1-A^2) > 0$. If $|A| \geq 1$, then $h'(V(A^2-1)) = 12V^3A^4(A^2-1) \geq 0$. Thus, $h'(x) \geq 0$ for $x > \max\{0, V(A^2-1)\}$. ■

We are now ready to examine the convexity of cost function (6). Let

$$g(C) \triangleq \frac{2V(A^2P - P + Q)\Sigma^2}{\left((V + C^2\Sigma)^2 - (AV)^2\right)^3},$$

then the second derivative of $J(C)$ with respect to C can be written as

$$J''(C) \quad = \quad 2\,S + g(C)h(C^2\Sigma). \qquad (10)$$

***Theorem** 5:* When the system is unstable, $|A| \geq 1$, the cost $J(C)$ is convex in $C > 0$, i.e., $J''(C) \geq 0$ for $C > 0$.

Proof: The first term in $J''(C)$, $2S$, is always positive.

By (7) and (9), we see that $g(C) > 0$.

From (8), we know that

$$C^2\Sigma > V(A^2-1).$$

Together with Lemma 4, we have

$$\begin{aligned} h(C^2\Sigma) &\geq h(V(A^2-1)) \\ &= 3A^4(A^2-1)^2V^4 > 0. \end{aligned}$$

Therefore, $J''(C) > 0$ and $J(C)$ is convex in $C > 0$. ■

Unfortunately, this convexity doesn't hold when the system is stable. When the system is stable, $|A| < 1$ so $h(0) < 0$. Since S can be chosen arbitrarily small, there may exist some small $C > 0$ such that $J''(C) < 0$. Therefore, J is not always convex in C.

Although convexity of J under all circumstances would have been nice, we can prove something that's almost as good: quasiconvexity.

***Definition** 6:* A function $\beta : \mathbb{R} \to \mathbb{R}$ is said to be *quasiconvex* (or *unimodal*) if its domain and all its sublevel sets

$$S_\alpha = \{x \in \mathbf{dom}\,\beta \,|\, \beta(x) \leq \alpha\},$$

for $\alpha \in \mathbb{R}$, are convex.

[2]The symbol $'$ denotes derivative where the differentiation variable should be clear from the context.

We will show that J is quasiconvex in $C > 0$ when $|A| < 1$ by proving that $J(C)$ exhibits the following property.

***Lemma** 7:* Suppose $\beta : \mathbb{R} \rightarrow \mathbb{R}$ is twice differentiable. The function β is *quasiconvex* if and only if for all $x \in \mathbf{dom}\,\beta$,

$$\beta'(x) = 0 \Rightarrow \beta''(x) \geq 0,$$

i.e. at any point with zero slope, the second derivative is nonnegative.

Proof: See §3.4 of [6]. ∎

Before showing J is quasiconvex, we need to examine the relationship of Σ and $C^2\Sigma$ with respect to C. From (5), Σ can be seen as a function, $\Sigma(C)$, of C. Similarly, $C^2\Sigma$ can be written as a function of C.

***Lemma** 8:* Let

$$\begin{aligned} f(C) &= C^2\Sigma \\ &\triangleq \frac{1}{2}(C^2W - V + A^2V) + \\ &\quad \frac{1}{2}\sqrt{C^4W^2 + 2(1+A^2)C^2WV + (A^2-1)^2V^2} \end{aligned}$$

Then for $C > 0$, we have

1) $\Sigma(C) > 0$ and $\Sigma'(C) < 0$;
2) $f(C) > 0$ and $f'(C) > 0$.

Proof: Clearly, $\Sigma(C) > 0$ and $f(C) > 0$. Notice that all coefficients of powers of C in $f(C)$ are positive, so f is increasing in C. That is, $f'(C) > 0$ for $C > 0$. By equation (5) and the definition of f, we can write

$$\Sigma(C) = W\left(1 + \frac{A^2V}{f(C) + V(1-A^2)}\right)$$

By relation (8), for all $C > 0$,

$$f(C) + V(1-A^2) > 0,$$

so $\Sigma'(C) < 0$. This is intuitively satisfying as Σ is the prediction error covariance and should decrease as we increase signal gain (effectively increasing the signal-to-noise ratio). ∎

***Lemma** 9:* For $|A| < 1$, there exists some positive C_1 where $J''(C) > 0$ for $C \geq C_1$ and $J''(C)$ is strictly increasing in C for $0 < C < C_1$.

Proof: Recall the expression for J'' in equation (10). By Lemma 4 and

$$h(\max\{0, V(A^2-1)\}) = h(0) < 0,$$

we know there exists a unique $x_1 > 0$ where $h(x_1) = 0$. When $|A| < 1$, $f(0) = 0$. By Lemma 8 and noting that $f(C) \rightarrow \infty$ as $C \rightarrow \infty$, we know that there exists a unique $C_1 > 0$ where $f(C_1) = x_1$. Hence, for all $C \geq C_1$, $J''(C) > 0$ because $h\circ f\,(C) \geq 0$.[3]

Now consider the interval $0 < C < C_1$. In this interval, $h\circ f\,(C) < 0$; by Lemma 8 and Lemma 4,

$$\frac{\partial}{\partial C}\Big[h\circ f\,(C)\Big] \geq 0.$$

We have already argued in Theorem 5 that $g(C) > 0$. Now, from Lemma 8, $\Sigma' < 0$ and $f' > 0$, so the numerator of $g(C)$ is decreasing in C while the denominator is increasing in C, thus g is decreasing in C: $g'(C) < 0$. Therefore, $g(C)h(f(c))$, and hence $J''(C)$, is strictly increasing in C for $0 < C < C_1$. See Figure 2(a) for illustration. ∎

The following is a direct result of Lemma 9.

***Corollary** 10:* When $|A| < 1$, if J is not convex, then there exists a unique $0 < C_0 < C_1$ where

$$J''(C)\begin{cases} < 0 & \text{if } C < C_0, \\ = 0 & \text{if } C = C_0, \\ > 0 & \text{if } C > C_0. \end{cases}$$

See Figure 2(a) for illustration.

This corollary tells us that the plot of $J'(C)$ only has one "dip". See Figure 2(b) for illustration.

***Theorem** 11:* When $|A| < 1$, if $J(C)$ is not convex in $C > 0$, it is quasiconvex in $C > 0$.

Proof: Assume $J(C)$ is not convex in $C > 0$. Consider the expression for $J'(C)$:

$$J'(C) = \frac{2C\left(V^2S(1-A^2)) + C^4\Sigma^2S + V\Sigma(\cdots)\right)}{(1-A^2)V^2 + 2C^2V\Sigma + C^4\Sigma^2}$$

Clearly, $J'(0) = 0$ so that means there exists a unique $C^* > C_0$ where $J'(C^*) = 0$. But $C^* > C_0$ means that $J''(C^*) > 0$ and quasiconvexity follows from Lemma 7. See Figure 2(c) for illustration. ∎

Finally, we summarize our results in the following theorem:

***Theorem** 12:* Given A, $B \neq 0$, $Q > 0$, $R > 0$, $S > 0$, and the scalar discrete-time linear system

$$\left\{ \begin{array}{rcl} x_{k+1} &=& Ax_k + Bu_k + w_k, \quad k \geq 0 \\ y_k &=& Cx_k + v_k \end{array} \right.$$

whose control input is of the form

$$u_k = \mu_k(y_0, y_1, \ldots, y_k, u_0, u_1, \ldots, u_{k-1}).$$

For $C > 0$, the cost function

$$J(C) = \min_{\{u_k\}}\left\{\lim_{n\rightarrow\infty} E\left[\frac{1}{n}\sum_{k=0}^{n-1}\left(Qx_k^2 + Ru_k^2 + SC^2\right)\right]\right\}$$

is *convex* when $|A| \geq 1$ and *quasiconvex* when $|A| < 1$.

The convexity of J allows us to summarize the optimal solution to our communication constrained LQG problem in the following table.

System	Convexity	Optimal C
$\|A\| \geq 1$	Convex	$C > 0$ such that $J'(C) = 0$
$\|A\| < 1$	Convex	$C = 0$ with $J^* = \frac{QW}{1-A^2}$
$\|A\| < 1$	Quasiconvex[4]	$C > 0$ such that $J'(C) = 0$

[3] Here, $h\circ f\,(C) = h(f(C))$

[4] This is the case where J is NOT convex but only quasiconvex

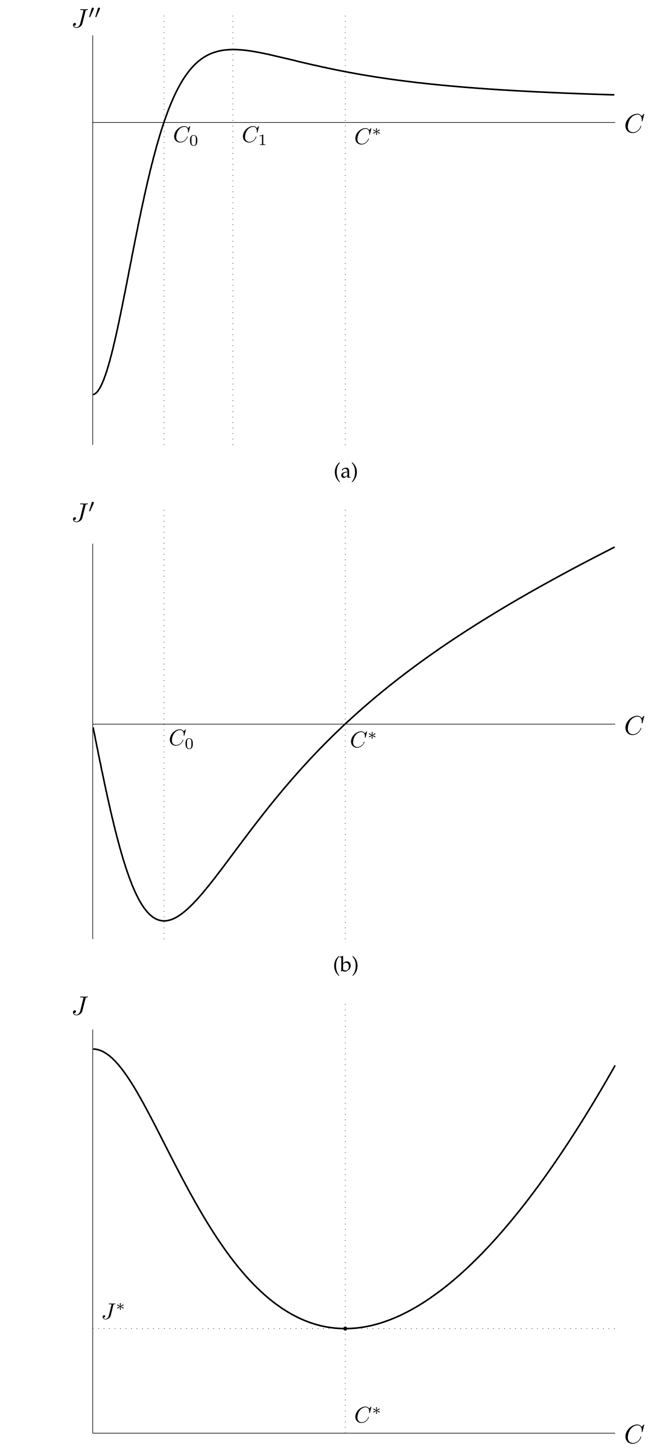

Fig. 2. Plots of (a) J'', (b) J', and (c) J for a stable system whose cost is quasiconvex in C. The system parameters are: $A = 0.8, B = 10, Q = R = S = W = V = 1$.

V. Conclusion and Future Work

In this paper, we formulated an LQG problem with a quadratic communication cost. We showed that the optimization problem is quasiconvex in C and if the system is unstable, the problem is convex. That is, it can be solved in a computationally efficient manner. Instead of viewing the problem from an information-theoretic rate perspective, we examined it from an energy perspective, consistent with traditional LQG. We studied the scalar case in this paper to gain basic insight into this new performance criterion.

The natural extension is to generalize to higher dimensional systems. As we pointed out earlier, for higher dimensional systems, (4) and (5) become matrix ARE's. There are no known closed form solutions although we can solve them efficiently using SDP by formulating the problem as a linear matrix inequality (LMI), see [5], [29], or [24] for details. We are currently exploring the problem for general vector systems.

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