Proceedings of the
44th IEEE Conference on Decision and Control, and
the European Control Conference 2005
Seville, Spain, December 12-15, 2005

Approximate Control of Formations of Multiagent systems

Grigoris Lionis and Kostas J. Kyriakopoulos

Abstract— **In this paper we are investigating how the local controllability properties of each agent interplays with the motion feasibility of the complete multiagent system, studying systems that are either small time locally controllable (STLC), or locally controllable but in bounded time. We present an approximation strategy, which allows us to plan the motion of the system using fully actuated holonomic vehicles, and to incorporate the constraints on the motion of the agents in a latter step, satisfying the formation constraints arbitrarily well.We apply this approximation strategy to tracking both static trajectories or arbitrary moving references.**

I. INTRODUCTION

During the past few years robotics, following the human social paradigm, has increasingly adopted cooperative schemes of several robots engaged in complicated tasks. Enhanced robustness and better adaptation to changing environments are two additional benefits of the multi-robotic approach. The possibility of distributing a task to many simple agents reduces cost, increases usefulness and in some very interesting cases multi agent cooperation is the *only feasible* solution to a number of problems.

In this context, multi agent robotic systems have been studied extensively. The problem of N agents moving independently on a plane has been studied both for decentralized ([2], [8]) and centralized ([7], [9]) setups. Flocking behavior of multiagent systems, i.e. movement of all agents with the same velocity, has been studied in ([15],[11]),while in ([12] the authors propose a method for creating a formation Lyapunov function, given independent agent Lyapunov functions. A team of robots following simple rules and how these rules lead to uniform spreading of the agents is the problem studied in [14] and in [1] the authors study the problem of controlling a group of robots and propose ways to decompose the control of the position and configuration of the group, while an information exchange algorithm which leads to establishing a common shared trajectory is proposed in [4] . In [13], the authors study the problem of establishing the feasibility of a specified formation,and for the feasible formations the problem of abstracting the formation onto a lower dimensional control space.Their work is focused in the interplay between kinematic and formation constraints. In [16] the authors study the feasibility problem, focusing on sensing and communication issues, and derive conditions under which formation stabilization is possible.

This work was partially supported by the EU project IST-IP-507006 ISWARM (Intelligent Small World Autonomous Robots for Micro-Manipulation) and by the Greek State Scholarships Foundation (IKY).

G. Lionis is a PhD Student in the Department of Mechanical Engineering, Technical University of Athens, Greece, glion@mail.ntua.gr

K. Kyriakopoulos is with Faculty of Mechanical Engineering, Technical University of Athens, Greece, kkyria@mail.ntua.gr

In this paper, we are studying a certain class of the multi agent cooperation problem and in particular the formation problem, and how we can solve it approximately. The formation is defined as a number of constraints that have to be satisfied during the motion of the system, plus a number of output variables that have to be controlled to specific values.Our approach lies on providing for this problem, control strategies that approximate the formation constraints arbitrary well. In most practical cases an approximation of the constraints is adequate for the execution of the given task. Moreover, our control strategy has a number of advantages.

- Relies on general controllability properties (local controllability),and can thus be used for a very large class of agents
- Satisfies the constraints with arbitrary precision, for static control tasks, while for dynamic control tasks, provides bounds on the velocity of the input
- Can be used with a heterogeneous multiagent system
- Decomposes the formation planning problem with the low-level agent actuation, providing very small computational load and can be thus used in systems with limited capabilities,eg. micro-robots

II. PROBLEM FORMULATION

We are interested in checking the feasibility motion of formation of agents.In particular,we are interested in checking the formation capabilities of a set of n agents, each satisfying

$$\dot{q}_i = f_i(q_i, u_i) \tag{1}$$

where the state of each agent is denoted by $q_i \in M_i$,where M_i is a metric space, and let $\mathcal{U}_i$ denote the actuation space for agent i. The formation directive is described by a number of constraints that the states of the agents have to satisfy

$$H_j(q) = 0 \; j\epsilon\{1, ..., k\} \tag{2}$$

where $q = [q_1^T \; ...q_n^T]^T$ is the state of the complete system, and we stack all the constraint functions in $H(q) = [H_1(q), H_2(q), ...]^T$.We will use subscripts to refer to projection of q onto a subsystem, and we will use superscripts to number specific points of q. We denote as $M = M_1 \times M_2 \times ...M_n$, the state space, and as $\mathcal{U} = \mathcal{U}_1 \times ..\mathcal{U}_n$ the actuation space for the composite system. We denote as

$$p = G(q) \tag{3}$$

the set of output variables. The objective is to control p from $p(0)$ to p_f. We can thus define a multiagent system as

$$\Sigma = (M, \mathcal{U}, \{f_i\}, H, G) \tag{4}$$

and we can define the problem as

0-7803-9568-9/05/$20.00 ©2005 IEEE

- Determine if Σ has any motion capabilities, i.e. if $\exists u \in \mathcal{U}$ s.t. $q(t)$ satisfies eq. (2)
- Determine if there is a admissible control signal, steering p to p_f

Our main attention will be given to a large class of interesting locomotion systems described by driftless affine control systems, where the dynamics take the form

$$\dot{q}_i = f(q_i)u_i \tag{5}$$

III. Basic Definitions

We denote as u_i an actuation vector , i.e. $u_i \in \mathcal{U}_i$ and as $\bar{u}_i^T$ an actuation signal of length T i.e. $\bar{u}_i^T : [0,T] \to \mathcal{U}_i$, while by $\bar{u}_i^T(t) \in \mathcal{U}_i$ we denote the actuation vector at time $t \leq T$.

We will write as $\Phi(q_i^0, \bar{u}_i^T, T_1)$ the solution of (1), at time T_1, starting from $q_i(0) = q_i^0$ and having as input $\bar{u}_i^T$.

$B(q_i, \epsilon)$denotes a ball of radius ϵ around q_i. We denote as $R(q_i(t_1), \mathcal{U}_i, t_2)$ the (time) reachable set of (1) from $q_i(t_1)$, i.e.

$$R(q_i(t_1), \mathcal{U}_i, t_2) = \{q_i \in M_i | \exists \bar{u}_i, t \in [t_1, t_2] \Phi(q_i(t_1), \bar{u}_i, t) = q_i\}$$

. (Where obviously the actuation signal $\bar{u}_i$ is defined as $\bar{u}_i : [t_1, t_2] \to \mathcal{U}_i$)

We denote as

$$M_H = \{q \in M | H(q) = 0\}$$

Definition 1: System Σ will be called consistent iff

$$\exists q^f \in M_H \ s.t. G(q^f) = p_f$$

and moreover q^f, q^0 belong in the same connected subset of M_H.

Definition 2: System Σ will be called *motion feasible in* M if $\forall q \in M_H \ \exists \ t_I > 0$ and a function $\bar{u}^{t_I} : (0, t_I) \to \mathcal{U}$ s.t. $H(\Phi(q, \bar{u}^{t_I}, t)) = 0, \forall t < t_I$

Definition 3: System Σ will be called *exactly steerable* in time T iff

$$\forall q^1, q^2 \in M_H, \exists \bar{u}^T \to \mathcal{U}$$

s.t.

$$\Phi_i(q_i^1, \bar{u}_i^T, T) = q_i^2$$

and moreover

$$\Phi_i(q_i^1, \bar{u}_i^T, t) \in M_H, \forall t < T$$

Definition 4: System Σ will be called *approximately* ϵ_r *steerable* iff

$$\forall q^1, q^2 \in M_H, \exists \bar{u}^T : [0,T] \to \mathcal{U}$$

s.t.

$$\Phi_i(q_i^1, \bar{u}_i^T, T) = q_i^2$$

and moreover

$$||H(\Phi_i(q_i^1, \bar{u}_i^T, t))|| \leq \epsilon_r, \forall t < T$$

A *exactly steerable system* is obviously *motion feasible*, while the converse is not true. Moreover, an *approximately steerable* may fail to be *motion feasible*.

IV. Analysis

A. Smooth Systems

We begin our analysis by deriving feasibility conditions for formations of smooth agents, following a similar line of thought as in [13]. When the state equation of each agent is (5), and f is a smooth function (i.e. $f_i \in C^\infty$) the problem is relatively easy. Since $H_j(q) = 0$ we have that $\frac{dH_j}{dt} = 0$, and if we assume that the constraints are time-invariant we have that $\sum_i \frac{\partial H_j}{\partial q_i} \dot{q}_i = 0$ or, in matrix notation we have that

$$A\dot{q} = 0 \tag{6}$$

with $A_{ij} = \frac{\partial H_i}{\partial q_j}$ .and $\dot{q}$ in matrix form as

$$\dot{q} = F(q)u \tag{7}$$

, with $F = diag(f_i(q))$.If we substitute (7) into (6) we get $A(q)F(q)u = 0$ or

$$P(q)u = 0 \tag{8}$$

with $P(q) = A(q)F(q)$. In other words, in this case, the admissible controls must lie within the null space of $P(q)$ so that the system moves satisfying the constraints.

Assuming that the null space of $P(q)$ is of constant dimension d, then (8) is equivalent with

$$u = \Lambda(q)u^* \tag{9}$$

$\Lambda(q)$ corresponds to a basis for the null space of $P(q)$ and the system satisfies the constraints for any u^*. System Σ is *motion feasible* iff $\forall q \in M : \Lambda(q) \neq 0$.

$H(q) = 0$ defines a surface at which q evolves. For simplicity we assume that

$$H(q) = 0 \Leftrightarrow q = C(q^*) \tag{10}$$

with q^* defining coordinates on the zero H surface. We can also assume, that

$$\forall q \in M_H, q^* = D(q) \tag{11}$$

i.e. that every state that complies with the constraints is mapped to a single point at the zero constraint surface. We may write the "objective" function as $G(q) = G^*(q^*)$. If the objective function cannot be written in q^* coordinates, this means that the nature of the objective function imposes another constraint of the motion of the vehicle, not found in $H(q) = 0$. In this case, the formation functions should be augmented with the further constraint arising from the objective function, so that the objective function can be written as a function of q^*. We assume that Σ is consistent, therefore, the specification $\lim_{t \to t_{final}} p = p_f$ is equivalent with $\lim_{t \to t_{final}} q^* \in \mathcal{A}$ where $\mathcal{A} = \{q^* : G^*(q^*) = p_f\}$ By differentiating (11) we get

$$\dot{q^*} = \nabla D \dot{q} \tag{12}$$

and by using (9,7) in (12) we obtain

$$\dot{q^*} = \nabla D \cdot F(q) \cdot \Lambda(q) \cdot u^* = W(q^*)u^* \tag{13}$$

System Σ is *exactly steerable* if there is a control signal u^* that steers q^* to set $\mathcal{A}$, and this can be checked using classical control techniques ([5],[10]) *Example 1:* Consider two unicycles, that have to keep constant relative distance. We have $q_i = [x_i \ y_i \ \theta_i], i = \{1, 2\}$,

$$f_i(q) = \begin{bmatrix} cos(\theta) & sin(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}^T$$

$u_i = [v_i \ \omega_i]$, $H = (x_1 - x_2)^2 + (y_1 - y_2)^2 - d_{12}^2$. $H(q) = 0$ corresponds to the formation constraint of keeping the two uncicycles at constant (d_{ij} relative distance.

$$P(q) = [\partial H/\partial q] = \begin{matrix} (x_1 - x_2)\cos\theta_1 + (y_1 - y_2)\sin\theta_1 \\ 0 \\ -(x_1 - x_2)\cos\theta_2 - (y_1 - y_2)\sin\theta_2 \\ 0 \end{matrix}^T \tag{14}$$

The constraint $H(q)$ gives rise to $q^* = (x_1, y_1, \theta_d, \theta_1, \theta_2)$. Functions C and D are respectively

$$C(q^*) = [x_1, y_1, \theta_1, x_1 + d_{12}\cos(\theta_d), y_1 + d_{12}\sin(\theta_d), \theta_2]^T \tag{15}$$

$$D(q) = [x_1, y_1, atan2(y_2 - y_1, x_2 - x_1), \theta_1, \theta 2]^T, \forall q \in M_H] \tag{16}$$

The $\Lambda(q)$ is easily computed as

$$\Lambda(q) = [\upsilon_1 \ \upsilon_2 \ \upsilon_3]$$

with $\upsilon_1 = [0 \ 1 \ 0 \ 0]^T$, $\upsilon_2 = [0 \ 0 \ 0 \ 1]^T$ and, by denoting as $\zeta_1 = (x_1 - x_2)\cos(\theta_1) + (y_1 - y_2)\sin(\theta_1)$, and as $\zeta_2 = -(x_1 - x_2)\cos(\theta_2) - (y_1 - y_2)\sin(\theta_2)$ we have that

$$\upsilon_3(q) = [-\zeta_2 \quad 0 \quad \zeta_1 \quad 0]^T$$

Set $\mathcal{A}$ is defined as $\mathcal{A} = \{x, y, \theta_d, \theta_1, \theta_2 | x = x_{1_f}, y = y_{1_f}\}$ in q^* coordinates. Steerability of the system is determined by checking function $W(q^*)$, and its controllability properties.

■

Remark 1: It is interesting here to note the complexity of the resulting constrained free system, even for a simple system of two smoothly moving unicycles, that have to hold a constant distance. Systems with a large number of agents, or systems that include agents with switched kinematics,exhibit a rather large increase in the necessary computations.

B. Switched Systems

We are particulary interested in the feasibility of motion, when the kinematics of each agents are not smooth. The motivation comes from the field of micro-robotic systems.

Micro-Locomotion systems sometimes need independent activation of rotation and translation, either because of structural(i.e. Locomotion structure is not "rich" enough to allow simultaneous rotation and translation), power (activation of both constraints results in high power consumption) or navigational (simultaneous activation of 2 D.O.F. results in increased navigational error) constraints.

We assume the following agent kinematics

$$\dot{q}_i = \sum^{j} \sigma_i^j f^j(q_i) u^j \tag{17}$$

with $\sigma_i^j \in \{0, 1\}, \sum_j \sigma_i^j \leq 1 \forall i$ We assume that each subsystem can switch in arbitrary time, i.e. that we control the switching timing.

A problem posed with such systems is if the overall system has any motion capabilities, satisfying the constraints.

1) Feasibility Conditions for switched systems:

To resolve this problem, we consider the system without the switching constraints, i.e. we set all $\sigma_i^j = 1$, and we study the kinematics of the new system, which corresponds to a smooth system.Using the procedure presented above for smooth systems, we derive the following equation describing the solution space of the system without the switching constraints: $u = \Lambda(q)u^*$ We define as

$$C_i(q) = \cup_j c_j^i, c_j^i = \{u^* : \sigma_i^k = 0 \forall k, k \neq j\} \tag{18}$$

c_j^i is the subset of u^* that nullify all except the j^{th} input of i agent, making C_i is the subset of u^* that is valid for agent i, i.e. the subset of the actuation space the respects the switching nature of the kinematics of agent i. Thus, the intersection of all C_i is the subset of the actuation space that respects the real (switching) kinematics of all agents:

$$C(q) = \cap_i C_i(q) \tag{19}$$

If $C(q) \neq \emptyset$ then system Σ is *motion feasible* in M_H.

Equation (13) under the constraint that $u^* \in C^*(q^*)$ resolves the second question posed for Σ.

V. STEERABILITY OF STLC SYSTEMS

Given a system Σ, and using the procedure outlined,the mobility and controllability questions of this system can be answered with a systematic manner. But this direct approach has a number of limitations

- The complex structure of the sets on which the motion of the system evolves
- The non-smooth nature of the resulting expressions, especially when dealing with switched agents
- Scalability issues

We will present another approach, showing that for a large class of agents, we can decompose the planning of the formation with the motion of individual agents, and impose the kinematics of the agents on a latter step. For small time locally controllable agents, we can follow the planned trajectory, with arbitrary precision in finite time. The kinematics of the agents are of not particular interest, in the higher level planning phase, allowing thus use of heterogeneous system. Fig. 1 depicts a diagram of this control paradigm. In the higher level, the formation trajectory is planned assuming fully actuated agents. The kinematics of the agents are incorporated in the lower level, ensuring that the executed trajectory keeps bounded deviation from the calculated. This approach, is much easier to implement, scales extremely well with the number of agents, and provides very simple algorithms that can be used in real time environments. Finally , this approach can be used for systems that are not *exactly steerable*

A. Tracking a trajectory

We will try to decompose the problem of satisfying the differential constraints (1), (5) with the problem of satisfying the formation constraints (2) or the task. We will show that if each f_i corresponds to a small time local controllable

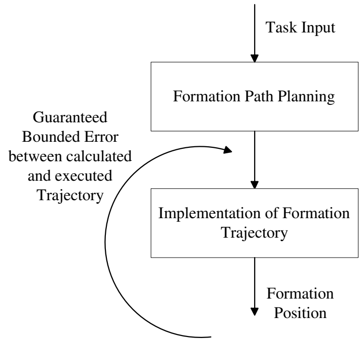

Fig. 1. Basic Block Diagram of control system

system, and if Σ is a *consistent* system, then Σ $\forall \epsilon_r$, Σ is *approximately* ϵ_r *steerable* in finite time for every r.If in addition the reachable area in time t is $\mathcal{O}(t)$, then the system is *exactly steerable* in bounded time.

System (1) will be called small time, locally controllable, if

$$\forall T > 0, \forall q_i \in M_i, \exists \epsilon > 0 : R(q_i, \mathcal{U}_i, T) \cap B(q_i, \epsilon) \neq \emptyset$$

We will need the following Lemma.

Lemma 1: Let (1) be a small time, local controllable system. We will show that $\forall q_0$ the radius of a ball around q belonging in $R(q_0, \mathcal{U}, t_0)$ is a strictly increasing function of time t_0.

Proof: Let

$$\dot{q} = f(q, u)$$

be a STLC system. We denote as ϵ_m^t the 'radius' of the maximum ball around q_I that can fit inside $R(q_I, \mathcal{U}, t)$. If $t_2 = t_1 + \delta t$, then we have that

$$R(q_I, \mathcal{U}, t_2) \supset (B(q_I, \epsilon_m^{t_1}) \cup_{q_i \in B(q_I, \epsilon_m^{t_1})} R(q_i, \mathcal{U}, \delta t)) \\ \supset B(q_I, \epsilon_m^{t_1}) \cup_{q_i \in B(q_I, \epsilon_m^{t_1})} B(q_i, \epsilon_m^{\delta t})$$

Let $\epsilon_{min}^{\delta t} = \min_{x_i} \epsilon_m^{\delta t}$ which is not zero, since system is STLC.
But then it is obvious that

$$R(q_I, \mathcal{U}, t_2) \supset B(q_I, \epsilon_m^t + \epsilon_{min}^{\delta t})$$

Therefore, sine for $t = 0, \epsilon_m^0 = 0$, there is a class $\mathcal{K}$ function $\beta(.)$ s.t. $\epsilon_m^t = \beta(t)$. ■

Assuming that $R(q_i, \mathcal{U}_i, t)$ is bounded, then, with the same argument, there exists a class $\mathcal{K}$ function $B(t)$, s.t. $R(q_i, \mathcal{U}_i, t) \subset B(q_i, B(t))$.

Definition 5: A *path or trajectory* p on M is a continuous function $p : A \rightarrow M$,where A is a closed interval beginning from 0 i.e.$A = [0, T]$.

Definition 6: Let $p_1(t) : [0, T_1] \rightarrow M$, $p_2(t) : [0, T_2] \rightarrow M$ be two trajectories on M. We say that these two are *in essence the same* iff there is a $a : [0, T_1] \rightarrow [0, T_2]$ class K function s.t. $p_1(t) = p_2(a(t)), \forall t \in [0, T_1]$.

Definition 7: We define the distance of two trajectories p_1,p_2 on M as

$$\mathcal{D}(p_1, p_2) = \min_{a(.)} \max_t ||p_1(t) - p_2(a(t))||$$

Obviously, if $\mathcal{D}(p_1, p_2) = 0$, trajectories p_1 and p_2 are *in essence the same*.

Theorem 1: We assume that we are given a nominal path on M, p_n lasting T_n time units. This path satisfies $H(p_n(t)) = 0, \forall t \leq T_n$, and moreover $G(p_n(T_n)) = p_f$.

Let $\epsilon > 0$. We will show that there is a p_ϵ, s.t. $\mathcal{D}(p_n, p_\epsilon) \leq \epsilon$, that can be followed by the n agents.

Proof:

Let $t_i > 0$ s.t. $B_i(t_i) < \epsilon/2$. Such t_i exists, since $B(.)$ is a continuous class $\mathcal{K}$ function. Let $t_m = \min_i t_i$ and $\epsilon_i = \beta_i(t_m)$. Let $\epsilon_s = \min_i \epsilon_i$. The path p_n corresponds to a path p_{n_i} for each agent according to $p_{n_i}(t) = \pi_i(q_n(t), \forall t \leq T_n$.

we construct a partition of T_n $\xi^j, j \in \{0, ..., N_s\}$ s.t. $||p_{n_i}(\xi^j) - p_{n_i}(\xi^{j+1})(T_n)||_\infty \leq \epsilon_s$ and $||p_{n_i}(\xi^j) - p_{n_i}(t)||_\infty \leq \epsilon/2, \forall t \in (\xi^j, \xi^{j+1}]$. Such a partition exists, since p_n is continuous, and therefore its projection on M_i is also continuous. Therefore, we can construct such a partition beginning from $p_n(0)$.For each i t_i is selected as

$$\max_{t_i} ||p_{n_i}(0) - p_{n_i}(t_i)||_\infty \leq \epsilon_s \wedge \quad (20)$$

$$||p_{n_i}(0) - p_{n_i}(t)||_\infty \leq \epsilon/2 \forall t \in (0, t_i] \quad (21)$$

$\xi^0 = \min_i t_i$.We continue satisfying the same inequality, from $p_{n_i}(\xi^0)$. The following algorithm steers the system of n agents, satisfying $\mathcal{D}(p_n, p_\epsilon) \leq \epsilon$:

Algorithm V.1: MOVETOPOINT$(n, N_s, \{\xi^j\})$

for $j \leftarrow 0$ **to** $N_s - 1$
do { **for** $i \leftarrow 1$ **to** n
  do MOVEAGENT$(i, q_i, p_{n_i}(\xi^j), p_{n_i}(\xi^{j+1}))$

where of course, the inner loop, is simultaneous for all agents. We denote as p_{ϵ_s} the trajectory generated by this algorithm. Obviously, at the end of each repetition of the outer loop, the position of the system coincides with $p_n(\xi^{j+1})$. Moreover, by construction we have that

$$\forall j, \forall t \in (\xi^j, \xi^{j+1}]||p_n - p_{\epsilon_s}||_\infty \leq \epsilon/2 + \epsilon/2 = \epsilon$$

and therefore $\mathcal{D}(p_n, p_\epsilon) \leq \epsilon$ ■

The time needed T_ϵ for the completion of p_ϵ can be estimated as

$$T_\epsilon = t_m \cdot N_s \quad (22)$$

which is bounded for any ϵ. But, while t_m is related almost linearly with ϵ, the relation of N_s with ϵ depends on function $\beta(t)$. When βt is $\mathcal{O}(t)$, then the $\lim_{t \to 0^+} \beta(t)/t > 0$ and the T_ϵ remains bounded as $\epsilon \rightarrow 0$. Such systems can follow a trajectory with arbitrary precision. Unfortunately, for every underactuated system, controlled locally,using Lie brackets of the input vector fields,$\lim_{t \to 0^+} \beta(t)/t = 0$. For these systems, decomposition of the formation trajectory and the kinematics, cannot give trajectories that can be exactly executed in finite time.

To accommodate this kind of behavior, we can allow a relaxation of the formation constraints, and our goal will be to control the multiagent system in such a way that $||H(q(t))||_\infty < \epsilon_H, \forall t$ Keeping the formation constraints in a bounded ball around zero, is adequate for most practical purposes, Moreover, if we take into account that even if we apply exact control,the system will almost always deviate from the exact position (due to noise,non-modeled phenomena e.t.c.) we come to the conclusion that bounded error on the formation constraints is the rule and not the exception. If p a trajectory on M let $p_s = \{q \in M | \exists t s.t. q = p(t)\}$. We restrict our attention to $O_p = cl(\bigcup_{q \in p_s} \bar{B}(q, \epsilon_r))$, which is, by construction, closed and bounded. The set O_p is a closed set, containing the sought system trajectory. Since H is smooth enough on M, it is smooth enough on O_p and hence $\exists N_p s.t. ||\nabla H(q)|| < N_p \forall q \in O_p$ By using the 'Mean Value Theorem' it easy to see that $\forall q \in Q_p, ||H(q)||_\infty \leq N_p \cdot \epsilon_r$. Since the goal is $||H(q)|| \leq \epsilon_H$, it suffices to take $N_q \cdot \epsilon_r \leq \epsilon_H$, or $\epsilon_r \leq \epsilon_H / N_q$.

Let us now assume that system (1) is not STLC, but instead, $\forall q \in M$ there is an admissible control signal s.t. the system can reach a ball $B(q, \epsilon)$ in time $\beta(\epsilon) + \gamma$, where $\beta(\epsilon)$ is a strictly increasing function with $\beta(0) = 0$ and $\gamma > 0$. This kind of timing could be be related either with the structure of the system, or with timing delays caused by the communication system. For example, consider a system of the form $\dot{q_u} = f_u(q_u, q_d, u_u)$, $\dot{q_d} = f_d(q_d, u_d)$ and assume that q_u represents the position and orientation of a vehicle, while q_d represents the internal state of the vehicle (for example the state of a legged locomotion system). The formation constraints would be typically imposed on q_u, so we would only be interested on checking the controllability of f_u. Suppose further on, that we can control q_u locally, by giving proper values to q_d, i.e. that for a proper value of $q_d = h(q_{u_1}, q_{u_2})$, q_u is STCL. Assuming q_d to be controllable in time T_d, it is obvious that q_u is (at least) controllable in the sense described above, and that we can only look at the lower dimensional switching system $\dot{q_u} = f_{u_i}(q_u, u)$ knowing that it is locally controllable in time $T_d + \beta(\epsilon)$. Systems of this kind are abundant and therefore it is significant to check if it is possible to utilize them in formations. For such systems it holds that $\forall q_i, \forall t > \gamma, \exists \epsilon R(q, \mathcal{U}_i, t) \supset B(q_i, \epsilon)$ and $\forall q_i, \forall t < \gamma R(q_i, \mathcal{U}_i, t) = \emptyset$ Obviously, the same algorithm described for STLC systems can be used here, since we assume that the reachability properties are similar. The difference lies in the time necessary for executing a given path. This difference will become more lucid, when we discuss how these systems can track a reference.

B. Tracking a reference

We can pose a similar but more demanding problem. Consider the system Σ, but defining the task as tracking, with bounded error, a time signal, i.e. find $u(t) \in \mathcal{U}$ such that $||G(q) - p(t)|| \leq \epsilon_1 \wedge ||H(q)||_\infty \leq \epsilon_2$ assuming the $\forall p, \exists q^* s.t. p = G^*(q^*)$, i.e. that the evolution of p lies on the consistent with the constraints subspace. The increased difficulty, comes form the fact the the tracking must be 'real time', i.e. at every time instant the distance between system and reference must be bounded. We want to be able to characterize how quickly the reference can change values,since we are talking about a moving reference. Assuming that M_H is a metric space, we can define as mean velocity of the reference $V_r = \frac{||q_r^*(t+\Delta t) - q_r^* t||}{\Delta t}$

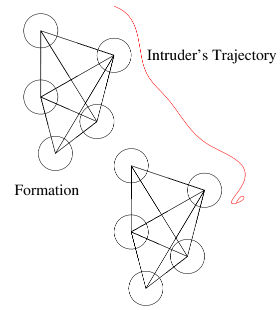

Fig. 2. Formation Tracking Enemy

Example 2: Consider a formation of micro-agents that have to remain in constant distances from each other, since by this way

- Communication is preserved: small d_{ij}
- Maximum Sensor Coverage: large d_{ij}

The goal of the system is to follow an foreign, independently moving object. Fig. 2 depicts this scenario. The red line corresponds to the reference trajectory, which is not know a priori, and is consider an input to the system. The multiagent system has to keep a coherent formation (depicted in the picture by lines joining the centers of the agents). The kinematics of these agents are complex and non-linear ([3],[6]) One possible optimized configuration would be for the micro-agents to keep a constant distance (d_{ig}) from one-another. If we set as formation directive 'Keep interagent distances steady', then we can describe all possible valid configurations using the position of one agent, and an the orientation of one inter-agent vector and the orientations of all agents, i.e. $q^* = (x_1, y_1, \theta_d, \theta_i)$. The task specification can be cast as $p = G(q) = (x_1, y_1)$, and $p(t) = r_{intr}(t) + s$, where r_{intr} is the position of the intruder, and s is a safety vector. The problem posed is under what constraints can we design a set of control laws for all the vehicles, so that the formation can follow the intruder?

It is obvious that we can easily solve this problem, if we disregard the kinematics of the agents . Just choose $q^*(t) = (r^x_{intr} + s^x, r^y_{intr} + s^y, 0, 0, ..., 0)$

■

We can use the same control paradigm used for steering the system to a point, for the more demanding task of tracking a referecne. Tracking the reference might result q leaving M_H, and since evaluation of the task function G outside M_H is of no use, our controller should always steer q from $q^{current} \in M_H$ to $q^{new} \in M_H$ in such a way that If $u : [T_{current}, T_{new}] \rightarrow \mathcal{U}$ the control signal for steering q then :

- $|||H(\Phi(q, u, t))|| \leq \epsilon_2, \forall t \in [T_{current}, T_{new}]$

- $||G(\Phi(q, u, T_{new})) - p(T_{new})|| \leq \epsilon_1$

We assume that for $t = 0$ we have $q(0) \in M_H$ and $G(q(0)) = p(0)$, i.e. the system begins tracking while being on the reference.We also assume that $||\nabla G^*|| \leq N_G, \forall q \in M_H$. This assumption may seem restricting, but if M_H is compact it comes naturally. Moreover, in many practical occasions G^* is simply an identity function on M_H (in the aforementioned example, G^* is a projection of q^* onto its two first coordinates). Since $||\nabla G^*|| \leq N_G$, $||q_1^* - q_2^*|| \leq a \rightarrow ||G^*(q_1^*) - G^*(q_2^*)|| \leq N_G \cdot a$. When $q_r^*(t)$ assumes a value such that $||q_r^*(t) - q_{current}^*|| = \epsilon_3$, set as $q_{goal} = C(q_r^*(t))$, $t = t_1$ and move the system from $q_{current}$ to q_{goal} using the algorithm described for a stationary system, setting as $\epsilon = \epsilon_2$. Since no path is given, we use the shortest path, which is a straight line, when M is a linear space. The time needed for this transition to q_{goal} is $T_{tr} = t_m \cdot N_s$. At this time instant, the reference shall be at a point $q_r^*(t_1 + T_{tr})$, while $q_{current}^* = q_r^*(t_1)$.If the velocity of the reference is at most V_r, then $||q_r^*(t + T_{tr}) - q_{current}^*|| \leq T_{tr} \cdot V_r$. If $\epsilon_3 \geq T_{tr} \cdot V_r$, then the error between reference, and system shall remain constant. The value of ϵ_3 is bounded by ϵ_1 / N_G, since the objective is to keep $||G(q) - p|| \leq \epsilon_1$

To obtain bounds on the velocity of the reference that we can track with this method, we need explicit bounds on the terms of eq. (22). We will assume that $B(t) = V_{max} \cdot t$ i.e. that the radius of the reachable ball grows linearly with time, while we will assume that $\beta(t) = V_{min} \cdot t^2$, i.e. the radius of the largest reachable ball grows with t^2. (Obviously this crude estimation is valid only for small times t). Essentially we assume a STLC system controllable locally using first order Lie brackets. Therefore we have that $t_m = \epsilon_2 / V_{max}$. N_s is evaluated as $N_s \leq \epsilon_3 / \beta(t_m) = \frac{\epsilon_3}{V_{min}(\epsilon_2/V_{max})^2}$. Therefore, $T_{tr} \leq \epsilon_3 \frac{V_{max}}{V_{min}} \frac{1}{\epsilon_2}$. So

$$V_r \cdot T_{tr} \leq \epsilon_3 \rightarrow V_r \leq \frac{V_{min}}{V_{max}} \epsilon_2 \tag{23}$$

This equation essentially relates the speed of the reference with ϵ_2 and the ratio of the two quantities V_{max}, and V_{min} corresponding respectively to how fast the error grows , and how fast can we move towards the goal.Also note, that if $\beta(t)$ is of $\mathcal{O}(t)$, then we obtain $V_r \leq V_{min}$, which is of course the expected result.

Eq. (23) essentially reveals that the system can track any reference, moving with velocity $\leq V_r$, always keeping bounded deviation from the formation zero level. The parameter ϵ_3 corresponds to the accuracy w.r.t the goal, and does not affect the velocity of the references the system can track with this control paradigm.

We now assume that $\{f_i\}$ do not correspond to STLC systems, but to systems locally controllable, in bounded time, i.e. that $t(\epsilon) = \beta(\epsilon) + \gamma$, where $t(\epsilon)$ is the time necessary for the radius of the maximum reachable ball to become ϵ.In this case the time for a ϵ_3 step is $\gamma \cdot \frac{\epsilon_3}{\epsilon_2}$, and hence in this case the reference velocity bound becomes $V_r \leq \epsilon_2 / \gamma$

VI. CONCLUSION

In this work we have presented a control paradigm for use in multi-agent formation problems. Our control strategy utilizes the local controllability properties of the agents to decompose the formation planning problem from the movement of each agent, providing provable error bounds between the actually and the planned formation trajectory. Our approach can easily accommodate strange agent kinematics and heterogeneous formations as it relies on general agent properties , while it scales very well with the size of the formation, making easy to adopt it for use in environments with hundreds of micro-robots. Moreover, the implementation simplicity of this control strategy allows use in environments where computational power is scarce (i.e. micro-robots).

REFERENCES

[1] C. Belta and V. Kumar. Abstraction and control of groups of robots. *IEEE Transactions on Robotics*, 20(5):865–875, 2004.

[2] Dimos V. Dimarogonas and Kostas J. Kyriakopoulos. Decentralized stabilization and collision avoidance of multiple air vehicles with limited sensing capabilities. *Proceedings of the 2005 American Control Conference*, 2005.

[3] W. Driesen, A. Bergander, T. Varidel, and J.M. Breguet. Energy consumption of piezoelectric actuators for inertial dirves. *Proceedings of the IEEE Intl. Symp. on Miceomechatronics and Human Science*, pages 51–58, 2003.

[4] J. A. Fax and R. M. Murray. Information flow and cooperative control of vehicle formations. *Proceedings of the 15th IFAC World Congress*, pages 2360–2365, 2002.

[5] A. Isidori. *Nonlinear Control Systems*. Springer, 1995.

[6] Project ISWARM. http://microrobotics.ira.uka.de/.

[7] S. G. Loizou and K. J. Kyriakopoulos. Closed loop navigation for multiple holonomic vehicles. *Proc. of IEEE/RSJ Int. Conf. on Intelligent Robots and Systems*, pages 2861–2866, 2002.

[8] S.G. Loizou, D.V. Dimarogonas, and K.J. Kyriakopoulos. Decentralized feedback stabilization of multiple nonholonomic agents. Tech. report, NTUA, 2003.

[9] S.G. Loizou and K.J. Kyriakopoulos. Closed loop navigation for multiple non-holonomic vehicles. *IEEE Int. Conf. on Robotics and Automation*, pages 420–425, 2003.

[10] H. Nijmeijer and A.J.van der Schaft. *Nonlinear Dynamical Control Systems*. Springer-Verlag, 1990.

[11] R. Olfati-Saber and R.M. Murray. Flocking with obstacle avoidance: Cooperation with limited communication in mobile networks. *42st IEEE Conf. Decision and Control*, 2003.

[12] P.Ogren, M.Egerstedt, and X. Hu. A control lyapunov function approach to multiagent coordination. *IEEE Transactions on Robotics and Automation*, 18(5):847–851, 2002.

[13] P.Tabuada, G.J.Pappas, and P.Lima. Feasible formations of multi-agent systems. *Proceedings of the American Control Conference*, 2001.

[14] C. L. Robinson, D. Block, S. Brennan, F. Bullo, and J. Cortes. Nonsmooth analysis and sonar-based implementation of distributed coordination algorithms. *Proceedings of the 2004 IEEE International Conference on Robotics and Automation*, pages 3000–3005, 2004.

[15] H.G. Tanner, A. Jadbabaie, and G.J. Pappas. Stable flocking of mobile agents,part i : Fixed topology. *42st IEEE Conf. Decision and Control*, 2003.

[16] Z.Lin, B.Francis, and M.Maggiore. Feasibility for formation stabilization of multiple unicycles. *Proceedings of the 43rd IEEE Conference on Decision and Control*, pages 2447–2452, 2004.