Proceedings of the
44th IEEE Conference on Decision and Control, and
the European Control Conference 2005
Seville, Spain, December 12-15, 2005

Using Meta-Heuristics In The Control Of A Non-Linear Input Delay Laboratory Helicopter System

Asif Iqbal, Otto Roesch, Hubert Roth, and Asad Rasool

Abstract— **Stabilizing controllers for non-linear input delay system of a lab helicopter are designed by first using a 3D search in the controller parameter space, the bounds of which are estimated by over-conservative controller parameters given by Lyapunov Razumikhin theory. Simulated Annealing and Genetic Algorithms are then used to find near-optimal controller gains that ensure empirical stability while minimizing an objective function consisting of design goals like settling time, overshoot, etc. Both linearized and actual non-linear models for helicopter elevation control are used with third and second order stabilizing controllers. Performance of the resulting control schemes is evaluated and compared.**

I. INTRODUCTION

Time delays occur in many physical systems and generally contribute to unstability and poor control performance of such systems. In stability analysis and controller design of these systems, time delay issues need to be carefully considered. Such delays could occur because of data communication or by material flow in machines. They can be classified as constant or time-varying, bounded or unbounded, and deterministic or stochastic. The delays may appear either in the state, or in the input (or output) of the corresponding systems. Stability of such systems becomes hard to establish keeping in mind the fact that introduction of time-delay in the states or input causes the system to have infinite number of poles or zeros for closed loop systems, see [1]. The reported stability could either be delay-independent or delay-dependent, the former being more restrictive than the later and more difficult to ensure, [2]. A good mathematical overview of stability issues in time-delay systems is given in [3].

However time delays have a positive effect as well for a class of oscillatory systems which are not stabilizable otherwise. In [4], authors derive stabilizing controllers for such system with delayed output feedback using matrix pencils. Switches between stable and unstable regions for varying time delays with a given controller are also discussed. Yue [7] used LMIs to derive less conservative controllers for uncertain systems when input delay is unknown but within pre-specified interval.

Zhang et al. [8] give an excellent overview of stability issues in networked control systems. They divide the stability issue into two categories, i) developing new network protocols for control applications, and ii) to take the network as it is, and focus on control algorithms to achieve stability. For a general class of systems, the authors suggest to mark stability regions using simulations while continuing to increase the time delay.

If the control-system and the plant are spatially separated, the delays occur two times in the control loop, once from the plant to the controller (forward direction), and secondly from the controller back to the plant (backward direction), see Fig. 1. Since time delays in the control loop have a major influence on stability of control systems, they must be carefully considered in the mathematical description of the system dynamics. Fridman and Shaked [9] discuss the sources of conservatism in delay-dependant stabilization of retarded systems. It is shown that transformations to achieve stabilization introduce additional dynamics into the system thus resulting in over-conservative controllers.

A. Overview of the work

In this paper, signal delays due to computations, or data-transmission, etc. are considered. If the output of the controller towards the plant is delayed, an input delay comes into play introducing the concept of stability margin based on time-delay, which will be the focus of this work. Lyapunov-Razumikhin theory based approaches are the only that deal with continuously uniform delays, see [9], which are of the type that we have in our case. In this work, first a conservative controller using the Razumikhin approach given in [10] is found and then, applying meta-heuristics, near-optimal controller parameters with the help of a simulation setup for the helicopter model are identified. In order to use LMIs in Razumikhin approach, we first linearize the plant and use it with a 3rd order controller with added integral control of the error. Afterwards, the actual non-linear plant is used with a 2nd order state feedback controller which provides excellent stability results.

B. Organization

Section II defines the problem while modelling of the system is carried out in Section III. Proposed solution approaches are given in Section IV followed by discussion of results in Section V. We conclude in Section VI.

II. PROBLEM DEFINITION

In contrast to systems free of delays, a time-delay system considers not only the current system states, but also the past

A. Iqbal is with IPP Multi Sensorics, Center for Sensor Systems (ZESS), University of Siegen, D-57068 Siegen, Germany iqbal@ipp.zess.uni-siegen.de

O. Roesch, H. Roth, and A. Rasool are with Institute of Automatic Control Engineering, University of Siegen, D-57068 Siegen, Germany {otto.roesch, hubert.roth}@uni-siegen.de asad.rasool@student.uni-siegen.de

0-7803-9568-9/05/$20.00 ©2005 IEEE

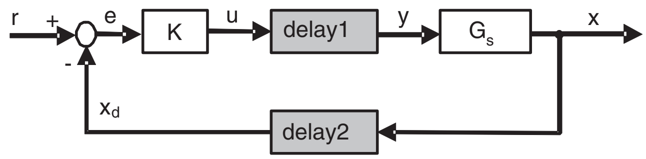

Fig. 1. System with Input and Feedback Delays

states which lead to a system described by delay differential equation (DDE):

$$\dot{x}(t) = f(t, x(t), x(t - \tau(t))) \tag{1}$$

with $x \in \mathbb{R}^n$, $\tau > 0$ and appropriate initial conditions derived on the interval $[t_0 - \tau, t_0]$. Due to tremendously increasing usage of tele-control over the internet, such time delay control systems get more and more important in practical applications. A general representation is shown in Fig. 1, with K the controller, G_s the plant, r the desired input and x the plant output. The time delay occurs from the controller to the plant and again from the plant backward to the control unit.

In general, control design algorithms do not consider time delay aspects in the control loop, like root-locus, Nyquist plots or pole placement approaches. Stability analysis of time delay systems are easier to handle in the time domain than in the frequency domain, especially in the case of time-varying uncertainties and nonlinearities (see [5]). Based on the second Method of Lyapunov, the theorems such as the Krasovskii and the Razumikhin provide stability conditions for time-delay systems in the time-domain. These methods deal exclusively with state space descriptions and the stability tests amount to sufficient conditions that can be posed as solutions of LMI problems. These robust control approaches are very conservative and have large stability margins, as will be shown in simulations.

For a system with only input delay, the problem can be formulated as:

Given a strictly proper transfer function $G(s) \in C^{p \times m}(p, m \geq 1)$ with a state-space representation ($u \in \mathbb{R}^m$, $y \in \mathbb{R}^p$, and $x \in \mathbb{R}^n$):

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \tag{2}$$

find all pairs $(K, \tau) \in \mathbb{R}^{m \times n} \times \mathbb{R}^+$ such that the delayed input $u(t) = Kx(t - \tau(t))$ stabilizes the system.

This control law results in a closed loop description of the system given as:

$$\dot{x}(t) = Ax(t) + BKx(t - \tau(t)) \tag{3}$$

As a start, a LMI (Linear Matrix Inequality) based approach is used to find the parameters of a third-order state-feedback controller. A Lyapunov-Razumikhin function maximizes the time-delay (in sub-optimal sense) of the closed loop system for a stable system. Once this sub-optimal controller achieves stability, it becomes of practical interest to see how much the parameters of this stabilizing controller can be varied while remaining within the stability margin of the system. Furthermore, can the upper bound on the input delay be improved with such a perturbation? The answers to these questions will be attempted using Simulated Annealing and Genetic Algorithms and performance will be evaluated of 2nd and 3rd order controllers for the given plant. In addition, specific consideration on the performance of the plant, such as settling-time, over-shoot, etc., can also be taken care of in the design of the controller as will be shown later.

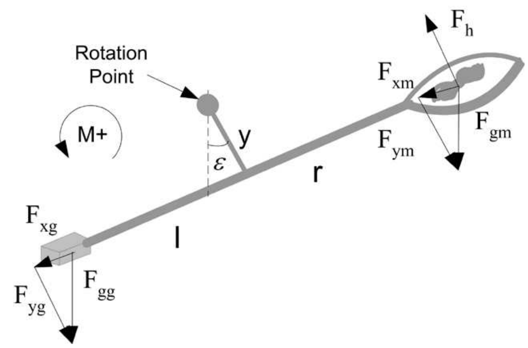

Fig. 2. Helicopter Model with Fixed Travel and Pitch Axis

In this work, delay τ is assumed to be constant during one set of experiments, then its value is increased to check if the system is as well stable at the new higher value or not. This process is repeated till the system can no more be stabilized in the given upper bound of settling time.

III. SYSTEM MODELLING

The system under consideration is represented by a helicopter model from Quanser [6], see Fig. 2. It consists of a fixed base, on which a rotary arm is mounted. The arm carries the helicopter body on one end, and a counter-weight on the other. The arm can make an elevation angle (ϵ) movement around the rotating point. Two motors with propellers mounted on the helicopter arm can generate a force proportional to the voltage applied to the motors. The corresponding nonlinear mathematical model is given by:

$$\ddot{\epsilon}(t) = \psi sin(\epsilon(t)) + \phi v(t) \tag{4}$$

where $\psi = \frac{-gy(M+m)}{J_g}$ and $\phi = 2K_t r^*$, and

K_t : Motor Constant (0.5 N/V)
v : Motor Volts
r : Distance between rotor and the rotation point
M : Counterweight
m : Mass of helicopter excluding counterweight
J_g : Moment of inertia
y : Distance between rotating point and helicopter frame

Stabilization of the delayed system was attempted using 3rd and 2nd order controllers respectively.

$^*K_t$ is multiplied by two because of two rotors.

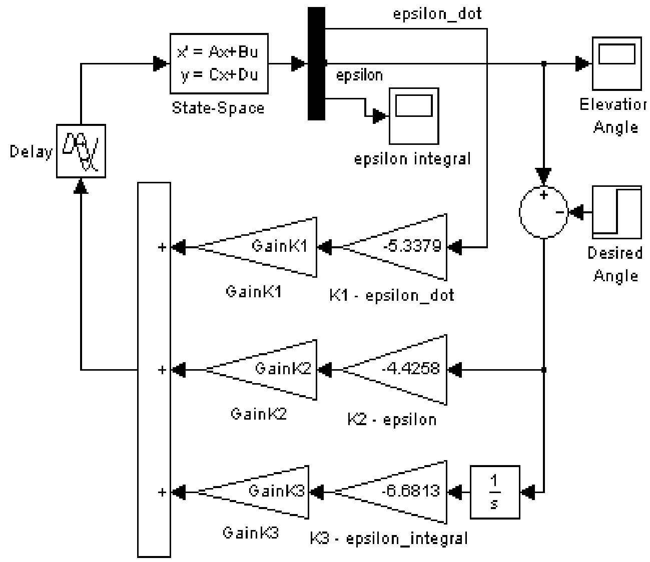

Fig. 3. Linearized Plant with Third Order Controller

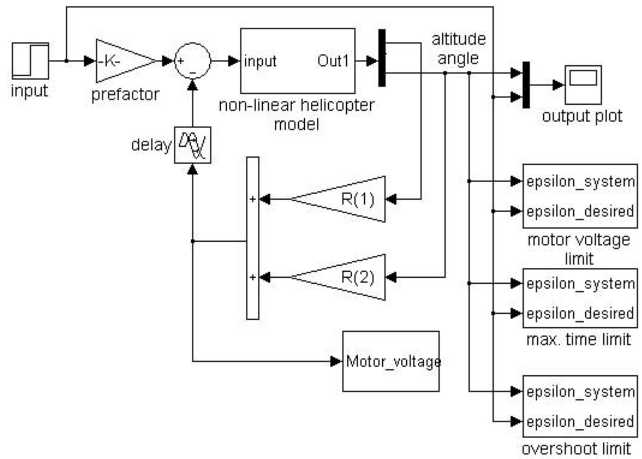

Fig. 4. Non-linear Plant with Second Order Controller

A. 3rd Order Controller with Additional Error Integrator

The system is first solved for a 3rd order controller which includes an integral of the error in elevation angle to achieve minimal steady state error as given in Fig. 3. To apply the Razumikhin approach based control algorithm presented [10], the model must be linearized around the quiescent point $\epsilon = 0$ and transferred into the state space form. System matrices for linearized plant according to description in (2) are given as:

$$A = \begin{bmatrix} -0.08 & -2.97 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 0.55 \\ 0 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

A Lypunov-Razumikhin function is used to find near-optimal solution to the problem given in Section II by solving a set of LMIs for $W \in \mathbb{R}^{m \times n}$ and $Q \in \mathbb{R}^{n \times n}$, where $Q > 0$, see [10] for further details. The controller output is then given as:

$$\begin{aligned} u(t) &= WQ^{-1}x(t-\tau(t)) \quad (5) \\ &\text{or} \\ u(t) &= Kx(t-\tau(t)) \end{aligned}$$

For the given plant, it results in a maximum stabilizable delay of $\tau = 0.116$ seconds with the following controller:

$$K = \begin{bmatrix} -5.3379 & -4.4258 & -6.6813 \end{bmatrix} \quad (6)$$

B. 2nd Order Controller using Non-Linear Plant

In this case, inspired by the results of meta-heuristics on 3rd order controller design, we select the original non-linear model of the plant as in (4) and choose a second order state-feedback controller as shown in Fig. 4. This improves the system performance resulting in faster dynamics as discussed in Section V.

IV. SOLUTION APPROACHES

By applying the control vector in (6) to the real plant, we observed that the controller was very conservative and showed a stable behavior even for time delays up to 0.4 second. The analysis of the stability margins in the controller design thus becomes necessary for a further improvement of the system, even for small delays. For practical stability analysis, a simulation of the nonlinear mathematical model of the helicopter is carried out in Simulink. The simulation is carried out to achieve the following optimal objective function:

$$J^* = \arg\min_{K \in [\underline{K}, \overline{K}]} (\alpha t_s + \beta \Delta\theta + \gamma v) \quad (7)$$

subject to

$$\begin{aligned} t_s &\leq t_{max} \\ \Delta\theta &\leq \Delta\theta_{max} \\ v &\leq v_{max} \end{aligned}$$

where α,β, and γ are weighting factors, on settling-time(t_s), overshoot($\Delta\theta$), and motor volts (v). The values of these parameters are chosen empirically. The range of gain vector K is chosen in the sufficiently large vicinity of solution produced by Lyapunov-Razumikhin function for third order controller design. In the second order case, Genetic Algorithm provides near optimal results by searching for all possible values of gains. This result is then used to define controller gain boundaries for Simulated Annealing. It is to be noted that effect of varying gain vector K is implicit in the values of the variables $t_s, \Delta\theta$. Maximum settling time (t_{max}) is a plant dependant property and is set to a value of 20s.

A. 3-D Combinatorics Search

The controller given in Section III-A calculated from the Razumikhin approach is used as a starting point. By varying these controller parameters successively with additional gain

TABLE I

PERFORMANCE OF THIRD ORDER CONTROLLER WITH LINEARIZED PLANT

Delay (sec)	Settling Time (sec)		
	3D	SA	GA
0.1	4.251	3.536	5.251
0.2	2.752	5.670	7.419
0.3	3.622	6.687	8.713
0.4	6.133	9.194	10.203
0.5	14.998	14.940	17.073
0.6	unstable	unstable	unstable

TABLE II

COMPUTATION TIME

Delay (secs)	Computation Time (mins)			
	3rd Order		2nd Order	
	SA	GA	SA	GA
0.1	7.111	0.634	0.950	0.529
0.2	6.753	0.621	0.946	0.901
0.3	9.016	0.387	0.996	0.508
0.4	9.186	0.409	1.001	1.612
0.5	8.335	1.121	1.038	0.490
0.6	unstable	unstable	1.013	1.203

factors and considering the settling time and the overshoot, a further improvement of the control system is achieved. These additional gain factors were increased from 0.1 to 3 in a sequential order to consider all possible combinations.

B. Meta-Heuristics

Genetic Algorithms and Adaptive Simulated Annealing are used to search for near optimal gains using objective function given in (7). Optimizer evaluates a particular set of gains by simulating the plant models, as in Figs. 3, and 4, and then measuring t_s, $\Delta\theta$, and v. Before the objective function is evaluated, the values of these variables are checked against the given constraints in (7). If any of the constraints is violated, the computed objective function is made extraordinarily large to exclude it from the list of optimal candidates.

V. RESULTS AND DISCUSSION

First the third order plant is used and the results of three different approaches in terms of settling-time are given in Table I. We can see that 3D combinatorics search is almost always better but computationally is very exhaustive as it amounts to almost complete characterization of gain vector space. While you can see the computation times for meta-heuristic approaches in Table II, a single run for 3D search on third-order controller design took about 6 hours on a Pentium-IV 2GHz machine with 512 MB RAM. Thus complete parametrization of stabilizing gains for delays up to 0.6 sec required about two days of computations. If the resolution of the step-size is chosen to be coarse, for faster results, the performance of 3D search is degraded and no satisfactory results are obtained.

Looking again at Table II, it is clear that increasing delays have little or no effect on computation times in the case of

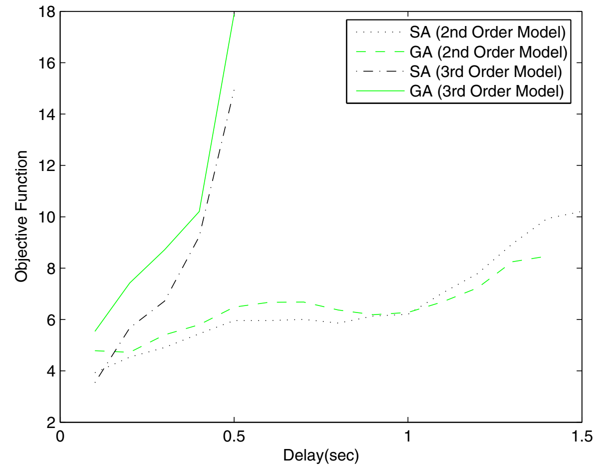

Fig. 5. Objective Functions vs. Time Delay

TABLE III

PERFORMANCE OF SECOND ORDER CONTROLLER WITH NON-LINEAR PLANT

Delay (sec)	Settling Time (sec)		Overshoot (rad)	
	SA	GA	SA	GA
0.1	3.926	4.782	0.00012	0.00012
0.2	4.037	3.848	0.00048	0.00000
0.3	5.643	5.532	0.00000	0.00001
0.4	5.798	5.724	0.00000	0.00000
0.5	5.138	7.115	0.00000	0.00053
0.6	6.609	6.752	0.00000	0.00000
0.7	6.614	7.280	0.00000	0.00000
0.8	5.625	6.485	0.00000	0.00000
0.9	6.030	5.759	0.00092	0.00000
1	4.414	5.545	0.00032	0.00091
1.1	7.979	5.864	0.00025	0.00017
1.2	6.952	7.644	0.00088	0.00000
1.3	9.609	8.231	0.01788	0.03295
1.4	9.471	7.952	0.03123	0.05150
1.5	9.355	unstable	0.08607	unstable
1.6	unstable	unstable	unstable	unstable

Genetic Algorithms but shows a steady increase in the case of Simulated Annealing. While on the other hand, a look at plot in Fig. 5 shows that Simulated Annealing is always doing a better job in terms of overall objective function, another manifestation of NFLT (No Free Lunch Theorem, see [11]), which implies that saying "strategy x is better (or even good)" is problematical if the particular class of problems is not identified for which strategy x is intended as a solution.

From Fig. 5 and Table I, we can see that the linearized plant with third order controller is not stabilizable beyond a delay of 0.5 seconds while staying in the settling time constraint of 20 seconds. This however is a considerable improvement over Laypunov-Razumikhin solution which gave a max. allowable delay of 0.166 seconds. The real performance gain is obtained when we directly use the non-linear model with a second order state-feedback control using meta-heuristics for controller design. Fig. 5 shows that for

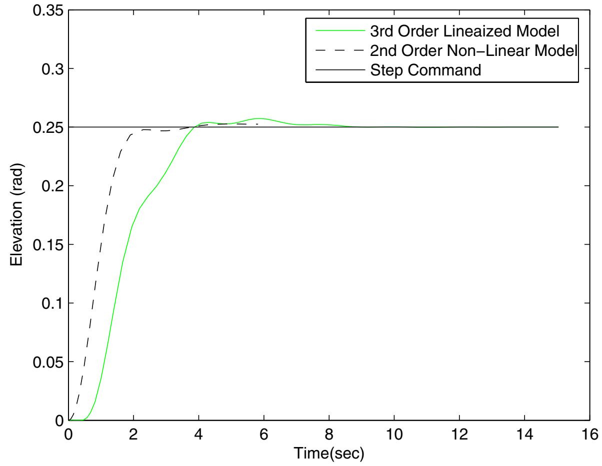

Fig. 6. Comparison of Second and Third Order Models, Delay = 0.4 sec

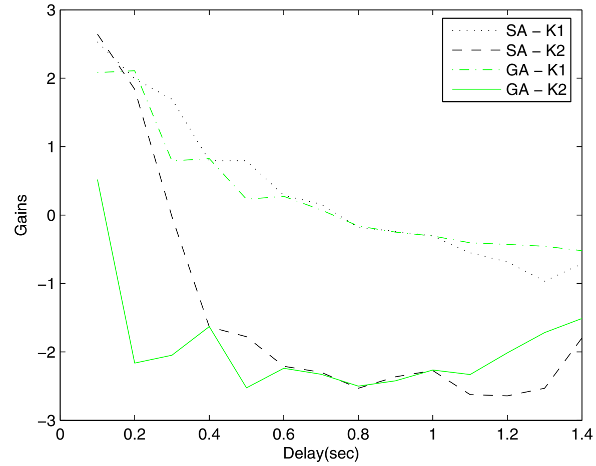

Fig. 7. Gains vs. Time Delay (Second Order Model)

a delay of 0.4 seconds, we gain a 160% improvement in objective function value as it drops from 15 for 3rd order case to mere 5.79 using the non-linear model and reduced order controller. Also we can see that the improved scheme allows us to stabilize the plant for delays up to 1.5 seconds which suffices for most time-delayed real-time applications provided the delay can be calculated in a sufficiently precise manner. System responses of both 3rd order linearized and 2nd order non-linear plants to a step input with a delay of 0.4 second are shown in Fig. 6. Superior performance of second order non-linear scheme is obvious.

Gain vectors for the second order controller are shown in Fig. 7 for Simulated Annealing and Genetic Algorithms. We can see a continuous decrease in the gains because controller is adding more and more damping to overcome the increasing delays. Near to the end of the graphs, we see an apparent increase in gain vectors which is due to the fact that in order to remain within the maximum settling time constraints, the controller strives to put more energy into the system which increases the overshoot and thus renders system as unstable in given constraints. A rather closer view of the results sheds more light into the dynamics of the system and optimization routines because we see that both Simulated Annealing and Genetic Algorithms are relaxing Gain K1 more than Gain K2 which seems to be natural choice because Gain K1 can be attributed to be a proportional gain whereas Gain K2 modifies the derivative of elevation ($\dot{\epsilon}$). See R(1) and R(2) in Fig. 4 which are obtained by merely scaling Gains K1 and K2 with constant factors.

Simulation tests were carried out with the optimized controllers to check for any stability/unstability switches as discussed in [4]. But no such switch was observed, that is, once the system was *rendered* unstable with a given controller within the constraints specified in (7), it never switched back to stability for any higher delays.

The proposed framework using either of the meta-heuristic approaches can be used for the control of an input-delay plant with variable time delays as well. In this case we can use gain scheduling to switch between the stabilizing controllers when time delay changes from one value to next. Because it will be too exhaustive to characterize all the stabilizing controllers with a given precision of calculated time delay, so a function $f : \mathbb{R}^+ \rightarrow \mathbb{R}^2$ can be computed from the experimental data that will map time delays to appropriate gain vectors. Using Simulated Annealing, following 5th order map is computed for Gains K1 and K2:

$$\begin{aligned} K_1(\tau) &= 11.5173\tau^5 - 42.3281\tau^4 + 55.1374\tau^3 \quad (8)\\ &\quad -28.0984\tau^2 + 0.7305\tau + 2.6697 \\ K_2(\tau) &= 39.1881\tau^5 - 140.0434\tau^4 + 173.1226\tau^3 \quad (9)\\ &\quad -78.1564\tau^2 + 0.1614\tau + 3.3882 \end{aligned}$$

Second order controller gains using Simulated Annealing and the ones estimated using maps (8) and (9) are shown in Fig. 8. Using these functions, gains for intermediate delay values can be easily generated and applied to the plant. To compare the performance of estimated gains, the system is simulated using optimized and estimated gains for an input delay of 1.1 seconds. System output for both approaches is shown in Fig. 9. It can be seen that the output of estimated gains closely matches that of the optimized ones so (8) and (9) can be easily used in gain scheduling control design.

VI. CONCLUSIONS

Meta-heuristics are effectively used to obtain stabilizing controllers for the given input-delay non-linear helicopter model. The use of an objective function comprising of design parameters like settling time, overshoot, etc. helps in reaching efficient design trade-offs. Both linear and non-linear model structures are compared against second and third order controllers. Performance of Genetic Algorithms and Simulated Annealing is also evaluated on the given problem.

In addition, a framework is provided so that the given approach may be used for time-varying delay plants by

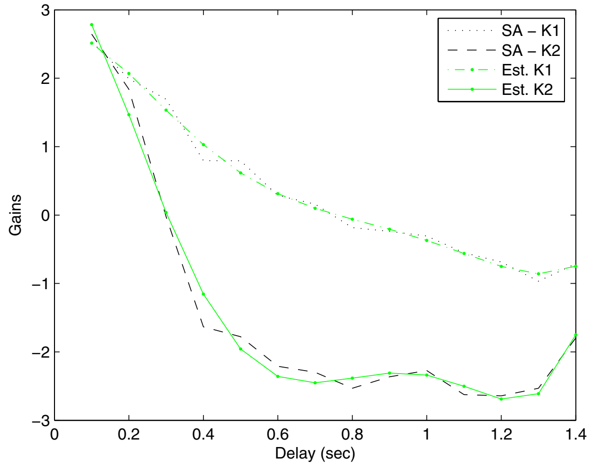

Fig. 8. Optimized and Estimated Gains vs. Time Delay (Second Order Model)

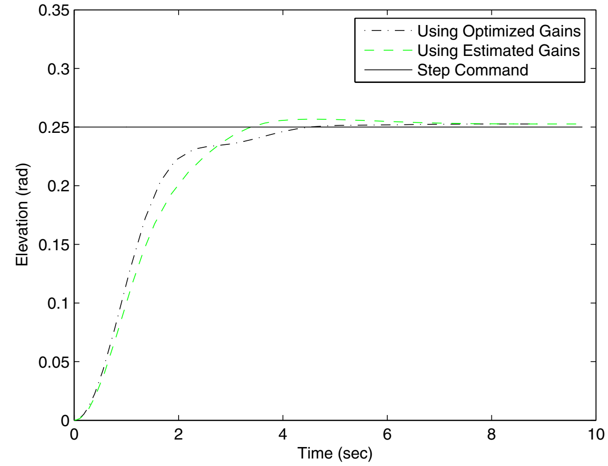

Fig. 9. Output of Non-Linear Model using Optimized and Estimated Controllers

making use of gain scheduling. This approach however requires computation of time-delay, by external means, in real-time operation. As a future work, a statistical model of delay can be incorporated in the system to relax this requirement.

REFERENCES

[1] J.E. Marshall, H. Gorecki, A. Korytowski, and K. Walton, "Time Delay Systems, Stability and Performance Criteria with Applications", *Ellis Horwood Press*, 1992.

[2] E.K. Boukas and Z.K. Liu, "Deterministic and Stochastic Time Delay Systems", *Control Engineering, Birkhaeuser*, 2002.

[3] J.-P. Richard, "Time-delay systems: an overview of some recent advances and open problems", *Automatica*, vol. 39, pp. 1667-1694, 2003.

[4] S.I. Niculescu and C. Abdallah, "Delay effects on static output feedback stabilization", *in 39th IEEE Conference on Decision and Control, Sidney, Australia*, 12-15 December, 2000.

[5] K. Gu, V.L. Kharitonov, and J. Chen, "Stability of Time-Delay Systems", *Birkenhuser*, 2003.

[6] Quanser, "3D Helicopter Experiment with DAQ Board and WinCon Software", *http://www.quanser.com*, 2004.

[7] D. Yue, "Robust stabilization of uncertain systems with unknown input delay", *Automatica*, vol. 40, pp. 331-336, 2004.

[8] W. Zhang, M.S. Branicky, and S.M. Phillips, "Stability of Networked Control Systems", *IEEE Control Systems Magazine*, February, 2001.

[9] E. Fridman and U. Shaked, "Delay-dependent stability and H_∞ control: constant and time-varying delays", *International Journal of Control*, vol. 76(1), pp. 48-60, 2003.

[10] S.I. Niculescu and W. Michiels, "Delay Dependant Closed Loop Stability of Linear Systems with Input Delay: An LMI Approach", *in 36th Conference on Decision and Control*, San Diego, California, USA, 1997.

[11] Y.C. Ho and D.L. Pepyne, "Simple Explanation of the No-Free-Lunch Theorem and Its Implications", *Journal of Optimization Theory and Applications*, Vol. 115, No. 3, pp. 549570, December 2002.