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# Higher Order Sliding Mode Control of wheeled mobile robots in the presence of sliding effects

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*Abstract*— **This paper addresses the trajectory tracking problem for a wheeled mobile robot (WMR), considering the presence of sliding effects that violate the nonholonomic constraints. Using the singular perturbation approach, the unicycle robot is modelled as a dynamic system which is referenced to the trajectory to be tracked. Indeed, due to the transgression of the nonholonomic constraints, the dynamic model becomes time varying and highly non linear which impose the use of a robust stabilizing control. For this purpose, we propose a solution based on a second order sliding mode control, ensuring the asymptotic convergence of the unicycle about the reference trajectory. Simulation results are reported showing the efficiency of the proposed control with respect to the non ideal constraints and to the nonlinearities.**

## I. Introduction

Most of research works addressing the trajectory tracking problem are based on WMR that satisfy the ideal nonholonomic constraints during the motion, namely the pure rolling and the non slipping condition and this, even in presence of disturbances or parameter variations [1], [12]. However, in practical situations, owing to the sliding of the wheels, the ideal constraints are never strictly satisfied. So various control approaches that perform well in the presence of sliding effects have been developed. In [3], a singular perturbation formulation is derived, representing as fast dynamics the slipping effects, and leads to robustness results for linearizing feedback laws ensuring trajectory tracking in presence of sufficiently small sliding effects. Using the slow manifold method [10], a new control law achieving convergent output tracking for robots not satisfying kinematic constraints is derived. A time varying stabilizing control law based on the linear quadratic theory is proposed in [8]. This feedback control law ensures the asymptotic convergence of error dynamics of the unicycle but only under some conditions on the reference trajectory (accelerations should be sufficiently small). As an approach for robust control, a discrete-time sliding mode control has been proposed for trajectory tracking of a wheeled mobile robot in the presence of skidding effects [2]. A robust control law against decoupled skidding and slipping effects has been proposed for solving a velocity tracking problem [4].

In this paper, the robust trajectory tracking problem for a dynamic wheeled mobile robot in the presence of sliding effects is solved by means of a higher order sliding mode control law. Using the singular perturbation approach [3], the dynamic tracking error model of the unicycle type robot is derived. Then, a second order sliding mode control law is used. The proposed control law is based on two nonlinear sliding manifolds ensuring the asymptotic tracking of the output variables in spite of the transgression of the nonholonomic constraints during the motion.

The paper is organized as follow. The problem statement and the dynamic tracking model of the unicycle are reported in Section 2. The second order sliding mode control law is presented in Section 3, while Section 4 shows simulation results.

## II. Problem statement and model of the unicycle

Let us first consider a wheeled mobile robot whose configuration is described by the vector of generalized coordinates $q \in \mathbb{R}^n$. In the ideal case, when the nonholonomic constraints of pure rolling and non slipping are satisfied along the motion, the wheeled mobile robot satisfies a set of $m$ independent velocity constraints of the form:

$$A^T(q)\dot{q} = 0 \tag{1}$$

where $A^T(q) \in \mathbb{R}^{m\times n}$ is a full-rank matrix.

Let $S(q) \in \mathbb{R}^{n\times(n-m)}$ be a full-rank matrix such that

$$A^T(q)S(q) = 0, \ \forall q \in \mathbb{R}^n. \tag{2}$$

The constraints are equivalent to the fact that at each instant the vector $\dot{q}$ belongs to the space generated by the columns of $S(q)$. Thus, the kinematic model of the WMR can be given by:

$$\dot{q} = S(q)\eta. \tag{3}$$

where $\eta \in \mathbb{R}^{n-m}$ is a velocity vector.

However, in the realistic case the constraints are not satisfied, due to the various effects such as sliding, deformability or flexibility of the wheels. So, the interaction forces and slipping effects have to be modelled. Since the constraints (1) are not satisfied, $\dot{q}$ does not belong to the space generated by the columns of $S(q)$. Thus, the kinematic model can be expressed as:

$$\dot{q} = S(q)\eta + A(q)\epsilon\mu \tag{4}$$

where $\mu \in \mathbb{R}^{n\times(n-m)}$ is the vector reflecting the violation of the constraints and $\epsilon$ is a small scaling factor, which is the inverse of the largest stiffness coefficient.

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In this paper, the considered unicycle robot (see Figure 1) is made of two independent fixed driving wheels and one off-centered wheel. Let $q = (x, y, l\theta, r\varphi_1, r\varphi_2)^T$ be the generalized position vector.

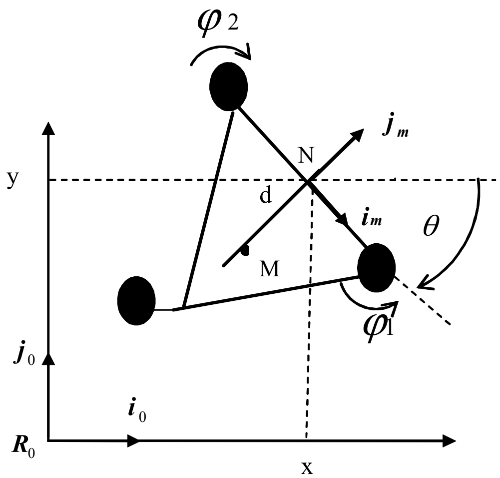



Fig. 1. The unicycle WMR.

$x$ and $y$ are the coordinates of the center gravity of the robot (point $M$), $\theta$ is the orientation of the car with respect to the $x$-axis, and $\varphi_1$, $\varphi_2$ are the orientation angles of the two fixed wheels. $l$ is the length between the two fixed wheels, and $r$ is the radius of the wheels. $d$ is the distance between the center of gravity and the middle point of the common axle of the fixed wheels (point $N$). The matrix $A$ and $S$ are given by:

$$A = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & \sin(\theta) \\ \sin(\theta) & \cos(\theta) & -\cos(\theta) \\ 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

and

$$S = \begin{pmatrix} -\sin(\theta) & 0 \\ \cos(\theta) & 0 \\ 0 & 1 \\ -1 & -1 \\ 1 & -1 \end{pmatrix}.$$

Note that robots of type $(\delta_m, \delta_s)$ with a degree of mobility $\delta_m = 2$ and a degree of steerability $\delta_s = 0$ present only three independent constraints, the non skidding constraint being the same for both driving wheels.

Let $v_x$ and $v_y$ be the lateral and the longitudinal velocities of point $N$ (see Figure 1), respectively, and $v_\theta$ the angular velocity of the platform. Let us also denote the following reference trajectory:

$$(x_r(t), y_r(t), \theta_r(t), v_{xr}(t), v_{yr}(t), v_{\theta r}(t)),$$

and the tracking error vector:

$$\begin{bmatrix} \widetilde{x}(t) \\ \widetilde{y}(t) \\ \widetilde{\theta}(t) \\ \widetilde{v}_x(t) \\ \widetilde{v}_y(t) \\ \widetilde{v}_\theta(t) \end{bmatrix} = \begin{bmatrix} x(t) - x_r(t) \\ y(t) - y_r(t) \\ \theta(t) - \theta_r(t) \\ v_x(t) - v_{xr}(t) \\ v_y(t) - v_{yr}(t) \\ v_\theta(t) - v_{\theta r}(t) \end{bmatrix}$$

The problem addressed in this paper is to find a state-feedback controller that solves the tracking problem of the unicycle robot when the nonholonomic constraints are violated, i.e.:

$$\lim_{t\to\infty} \widetilde{x}(t) = 0, \lim_{t\to\infty} \widetilde{y}(t) = 0, \lim_{t\to\infty} \widetilde{\theta}(t) = 0$$

In [8], the authors derived a partially linearized dynamical model based on a singular perturbation formalism for the unicycle robot when slipping effects are considered:

$$\begin{cases} \frac{d\widetilde{x}}{dt} = p_1(t)\widetilde{y} + p_2(t,\widetilde{\theta})\widetilde{\theta} + \widetilde{v}_x + \frac{\widetilde{y}-d}{l}\widetilde{v}_\theta \\ \frac{d\widetilde{y}}{dt} = -p_1(t)\widetilde{x} + p_3(t,\widetilde{\theta})\widetilde{\theta} + \widetilde{v}_y - \frac{\widetilde{x}}{l}\widetilde{v}_\theta \\ l\frac{d\widetilde{\theta}}{dt} = \widetilde{v}_\theta \\ \frac{d\widetilde{v}_x}{dt} = -\frac{2\overline{G}_x}{\epsilon m v} v_x + \frac{v_y v_\theta}{l} - \frac{d\widetilde{v}_{xr}}{dt} \\ \frac{d^2\widetilde{v}_y}{dt^2} = \widetilde{w}_1 \\ \frac{d^2\widetilde{v}_\theta}{dt^2} = \widetilde{w}_2 \end{cases} \tag{5}$$

where

$$v = \sqrt{\dot{x}^2 + y^2 + l^2\dot{\theta}^2}$$

$$p_1(t,\widetilde{\theta}) = \frac{v_{\theta r}}{l}$$

$$p_2(t,\widetilde{\theta}) = \frac{(1-\cos(\widetilde{\theta}))}{\widetilde{\theta}}\left(v_{xr} - \frac{d}{l}v_{\theta r}\right) - \frac{\sin(\widetilde{\theta})}{\widetilde{\theta}}v_{yr}$$

$$p_3(t,\widetilde{\theta}) = \frac{\sin(\widetilde{\theta})}{\widetilde{\theta}}\left(v_{xr} - \frac{d}{l}v_{\theta r}\right) + \frac{(1-\cos(\widetilde{\theta}))}{\widetilde{\theta}}v_{yr}$$

and where $\widetilde{w} = [\widetilde{w}_1, \widetilde{w}_2]^T$ is an auxiliary control vector obtained after application of a non stationary linear state feedback.

*Remark 1:* It has be shown in [8] that, for solving the tracking trajectory problem, the reference trajectory must satisfy a constraint on the lateral velocity $v_{xr}$, given by the skidding dynamic equation:

$$\dot{v}_{x,r} = -\frac{2\overline{G}_x}{\epsilon m v_r} v_{x,r} + \frac{v_{y,r}v_{\theta,r}}{l}, \ \forall t \geq 0 \tag{6}$$

where $v_r = \sqrt{v_{x,r}^2 + v_{y,r}^2 + v_{\theta,r}^2}$, and $\overline{G}_x$ is the normalized cornering stiffness coefficient.

The trajectory tracking problem becomes a stabilization problem which consists in stabilizing system (5) about the origin. For this, a robust nonlinear control law based on second order sliding mode is derived.

## III. SECOND ORDER SLIDING MODE CONTROLLER

Sliding mode control laws for nonlinear systems have been widely studied since they were introduced in [11]. The objective of this method is, by means of a discontinuous control, to constrain the system to evolve and stay, after a finite time, on a sliding manifold where the resulting behavior has some prescribed dynamics. Sliding mode control exhibits relative simplicity of design and some robustness properties with respect to matching perturbations.

Emel'yanov et al. [5] generalized the basic sliding mode idea to higher order sliding modes (HOSM). They are characterized by a discontinuous control acting on the higher time derivatives of the sliding constraint (instead of the first time derivative in classical sliding mode). Preserving the main advantages of the former approach (robustness properties, relative simplicity of design), they can reduce the chattering phenomenon (for the same sliding variable) and guarantee better convergence accuracy (see [6], [9] for a survey).

Consider a system whose dynamics is given by:

$$\dot{x} = f(t,x) + g(t,x)u$$

where $x \in \mathbb{R}^n$ is the state system, $u \in \mathbb{R}$ is the control and $f,\ g$ are sufficiently smooth functions. The sliding manifold $\mathcal{S}^r$ is defined by the vanishing of a corresponding *sliding variable* $S : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}$, and its successive time derivatives up to a certain order

$$\mathcal{S}^r = \left\{(t,x) \in \mathbb{R}^+ \times \mathbb{R}^n : S = \dot{S} = ..... = S^{(r-1)} = 0\right\}.$$

The resulting behavior is called *r-th order ideal sliding mode* with respect to $S$.

Here, we are interested by the case when the system has relative degree two with respect to $S$. Then, the second time derivative $\ddot{S}$ can be written as:

$$\ddot{S} = \phi(t,x) + \varphi(t,x)u$$

Assume that there exist positive constants $S_0,\ k_m,\ K_M,$ $C_0$ such that inside the domain $|S(t,x)| < S_0$, the system satisfies the following relation:

$$0 < k_m \leq |\varphi(t,x)| \leq K_M$$

$$|\phi(t,x)| < C_0$$

Then, it is then possible to generate different kinds of algorithms (ideal twisting, sampled twisting, super twisting, sub-optimal...) such that the system evolve featuring a second order sliding mode, after a finite time, i.e. the trajectories lie in the second order sliding set defined by:

$$\mathcal{S}^2 = \left\{(t,x) \in \mathbb{R}^+ \times \mathbb{R}^n : S = \dot{S} = 0\right\}.$$

The design approach of sliding mode control comprises two steps: the design of a surface in the state space so that the sliding motion satisfies the designer specifications, and then the selection of a discontinuous control law so that the sliding manifold is made (at least locally) attractive to the system trajectories.

In what follows, a nonlinear sliding manifold is made attractive by the means of a second order sliding mode algorithm called the twisting algorithm. Then, the resulting motion on this manifold is analyzed and it is shown that both the position and the angular tracking errors of the robot are asymptotically stabilized in an arbitrarily small neighborhood of the origin.

### A. Design and attractivity of the sliding manifold

Let us define the sliding constraint $s = [s_1, s_2]^T$ as:

$$s_1 = \widetilde{v}_y + \lambda_2\widetilde{y} + p_3(t,\widetilde{\theta})\widetilde{\theta}, \tag{7}$$

$$s_2 = \widetilde{v}_\theta + \lambda_1\widetilde{\theta} - \frac{l\lambda_3}{d}\widetilde{x} \tag{8}$$

where $\lambda_1,\ \lambda_2,\ \lambda_3$ are positive parameters. Note that the system has a relative degree two with respect to both $s_1$ and $s_2$. The task is to generate a second order sliding mode on the second order sliding manifold given by the equalities: $s = \dot{s} = 0$. The second time derivatives of $s_1$ and $s_2$ can be written as:

$$\begin{aligned}\ddot{s}_1 &= \varphi_1(z,t) + \widetilde{w}_1\\ \ddot{s}_2 &= \varphi_2(z,t) + \widetilde{w}_2\end{aligned}$$

with

$$\begin{aligned}\varphi_1(z,t) = {}& \lambda_2\left(\frac{dp_1(t)}{dt}\widetilde{x} - p_1(t)^2\widetilde{y} - p_1(t)\widetilde{v}_x\right)\\ &- \lambda_2\left(-p_1(t)p_2(t,\widetilde{\theta}) + p_3(t) + \lambda_2\frac{d^2p_3(t)}{dt^2}\right)\widetilde{\theta}\\ &+ \left(\lambda_2 p_3(t) + 2\frac{dp_3(t)}{dt} + d\lambda_2 p_1(t)\right)\frac{\widetilde{v}_\theta}{l}\\ &+ \left(-2p_1(t)p_2(t,\widetilde{\theta})\widetilde{y} - \frac{p_2(t,\widetilde{\theta})\lambda_2}{l}\widetilde{\theta} - \frac{\lambda_2}{l}\widetilde{v}_x\right)\frac{\widetilde{v}_\theta}{l}\\ &+ (d-y)\frac{\lambda_2\widetilde{v}_\theta^2}{l^2} + \left(\frac{-\lambda_2\tilde{x} + p_3(t)}{l}\right)\frac{d\widetilde{v}_\theta}{dt} + \lambda_2\frac{d\widetilde{v}_y}{dt}\end{aligned}$$

$$\begin{aligned}\varphi_2(z,t) = {}& \frac{l\lambda_3}{d}\left(p_1(t)^2\widetilde{x} - \frac{dp_1(t)}{dt}\widetilde{y} - \left(\widetilde{v}_y + \frac{p_2(t)}{l}\widetilde{v}_\theta\right)\right)\\ &+ \frac{l\lambda_3}{d}\left(\frac{dp_2(t)}{dt} + p_1(t)p_3(t)\right)\widetilde{\theta}\\ &+ \frac{\lambda_3}{d}\widetilde{v}_\theta\left(2p_1(t)x - p_3(t)\widetilde{\theta} - \widetilde{v}_y\right) + \frac{\lambda_3}{dl}\widetilde{x}\widetilde{v}_\theta^2\\ &- \frac{l\lambda_3}{d}\frac{d\widetilde{v}_x}{dt} + \left(\frac{\lambda_1 + l\lambda_3}{l} - \frac{\lambda_3}{d}\widetilde{y}\right)\frac{d\widetilde{v}_\theta}{dt}.\end{aligned}$$

Assume that the reference velocities $(v_{xr},\ v_{yr},\ v_{\theta r})$ and their first and second time derivatives are bounded and that the functions $\varphi_i(z,t)$ are bounded such that

$$|\varphi_i(z,t)| \leq K_i,\ i = 1,2$$

where the $K_i$ are positive constants.

Then, let us apply the following control laws, called the ideal twisting algorithm:

$$\widetilde{w}_1 = \begin{cases} -\lambda_{m_1}\operatorname{sgn}(s_1) \text{ if } s_1\dot{s}_1 \leq 0\\ -\lambda_{M_1}\operatorname{sgn}(s_1) \text{ if } s_1\dot{s}_1 > 0\end{cases} \tag{9}$$

$$\widetilde{w}_2 = \begin{cases} -\lambda_{m_2}\,\mathrm{sgn}(s_2) \text{ if } s_2\dot{s}_2 \le 0 \\ -\lambda_{M_2}\,\mathrm{sgn}(s_2) \text{ if } s_2\dot{s}_2 > 0 \end{cases} \quad (10)$$

where $\lambda_{m_i}$, $\lambda_{M_i}$ are positive constants that satisfy, for $i = 1,\ 2$, the following conditions:

$$0 < \lambda_{m_i} < \lambda_{M_i},$$

$$\lambda_{m_i} > K_i,$$

$$\lambda_{M_i} > \lambda_{mi} + 2K_i.$$

The control laws (9), (10) ensure a finite time convergence of the trajectories onto the sliding manifold $\{s = \dot{s} = 0\}$. In particular, this implies that:

$$\widetilde{v}_y = -p_3(t,\widetilde{\theta})\widetilde{\theta} - \lambda_2\widetilde{y} \quad (11)$$

$$\widetilde{v}_\theta = -\lambda_1\widetilde{\theta} + \frac{l\lambda_3}{d}\widetilde{x} \quad (12)$$

*B. Asymptotic stability of the sliding motion*

In order to show that, once in sliding mode, the posture errors of the robot are vanishing asymptotically, let us introduce the following candidate Lyapunov function:

$$V = \frac{1}{2}\left(\widetilde{x}^2 + \widetilde{y}^2 + l\widetilde{\theta}^2\right).$$

The time derivative of $V$ along the trajectories of the system is given by:

$$\frac{dV}{dt} = p_2(t,\widetilde{\theta})\widetilde{x}\widetilde{\theta} + \widetilde{x}\widetilde{v}_x - \frac{d}{l}\widetilde{x}\widetilde{v}_\theta + p_3(t,\widetilde{\theta})\widetilde{y}\widetilde{\theta} + \widetilde{y}\widetilde{v}_y + \widetilde{\theta}\widetilde{v}_\theta. \quad (13)$$

Replacing the expressions (11) and (12) of $\widetilde{v}_y$ and $\widetilde{v}_\theta$ in (13), one gets:

$$\begin{aligned} \frac{dV}{dt} &= -\lambda_3\widetilde{x}^2 - \lambda_2\widetilde{y}^2 - \lambda_1\widetilde{\theta}^2 \\ &\quad + \widetilde{x}\left(\widetilde{v}_x + \left(p_2(t,\widetilde{\theta}) + \frac{d\lambda_1}{l} + \frac{l\lambda_3}{d}\right)\widetilde{\theta}\right). \end{aligned}$$

As $p_2(t,\widetilde{\theta})$ is bounded for all $\widetilde{\theta}$, one can write:

$$\left|\left(p_2(t,\widetilde{\theta}) + \frac{d\lambda_1}{l} + \frac{l\lambda_3}{d}\right)\right| \le \Pi_3$$

where $\Pi_3$ is a positive constant.

Let us define $\lambda^* = \min_{i=1,3}(\lambda_i)$ and suppose that

$$\|\widetilde{v}_x\| \le \Pi_1 + \Pi_2\|X\|,$$

where $\Pi_1$, $\Pi_2$ are positive constants and $X = \begin{bmatrix} \widetilde{x} & \widetilde{y} & \widetilde{\theta} \end{bmatrix}^T$.

Then one can write:

$$\frac{dV}{dt} \le -\lambda^*\|X\|^2 + \|X\|^2\left(\frac{\Pi_1}{\|X\|} + \Pi_2 + \Pi_3\right).$$

Taking $\|X\| \ge \frac{\varepsilon}{2}$, this implies that

$$\frac{dV}{dt} \le -\lambda^*\|X\|^2 + \|X\|^2\left(\frac{2\Pi_1}{\epsilon} + \Pi_2 + \Pi_3\right).$$

Define the ball $\mathcal{B}_{\epsilon/2} = \left\{X : \|X\| \le \frac{\epsilon}{2}\right\}$. It results that outside the ball $\mathcal{B}_{\epsilon/2}$, one has

$$\frac{dV}{dt} \le -\widetilde{\lambda}\|X\|^2 = -2\widetilde{\lambda}V$$

with

$$\widetilde{\lambda} = \lambda^* - \left(\frac{2\Pi_1}{\epsilon} + \Pi_2 + \Pi_3\right).$$

Thus $\frac{dV}{dt}$ will be negative definite if the the parameter $\lambda^*$ is chosen as:

$$\lambda^* > \left(\frac{2\Pi_1}{\epsilon} + \Pi_2 + \Pi_3\right).$$

The solution of the system is given by

$$\|X(t)\| \le \|X(0)\|\exp\left(-\widetilde{\lambda}t\right)$$

and there exists a finite time $t_1$ such that $\forall\ t > t_1 : X(t) \in \mathcal{B}_\varepsilon$. Thus, $\widetilde{x}$, $\widetilde{y}$, $\widetilde{\theta}$ are stabilized in an arbitrarily small neighborhood of the origin.

## IV. Simulation results

The unicycle WMR is described by the dynamical model (5), with the following values for the physical parameters:

$$\begin{aligned} l &= 1m \\ d &= 0.2m \\ r &= 0.35m \\ \epsilon &= 10^{-4} \end{aligned}$$

In order to illustrate the effectiveness of the proposed control law, the unicycle was required to track two kinds of reference trajectories:

- a simple path which consists of a straight-line:
$$v_{yr}(t) = 1m.s^{-1},\ v_{\theta r}(t) = 0rd.s^{-1},\ \forall t \ge 0$$
- a circular path with :
$$v_{yr}(t) = 1m.s^{-1},\ v_{\theta r}(t) = 0.1rd.s^{-1},\ \forall t \ge 0.$$

The speed $v_{xr}$ is given by equation (6) with $v_{xr}(0) = 0\ m.s^{-1}$. In both cases, the initial position and velocity conditions are:

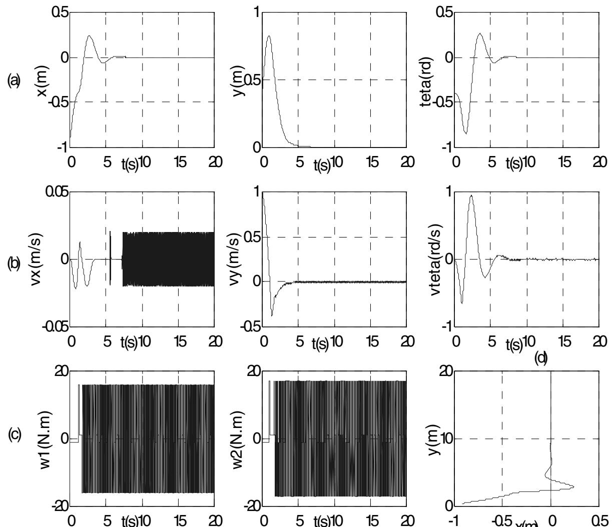
$$\begin{aligned} x(0) &= -0.92m,\ y(0) = 0.4m,\ \theta(0) = -\pi/8rd, \\ v_y(0) &= 2m.s^{-1},\ v_x(0) = 0m.s^{-1}. \end{aligned}$$

The sampling time $t_e$ is set to be $0.01s$ and the control gains are

$$\begin{aligned} \lambda_1 &= 1,\ \lambda_2 = 0.35,\ \lambda_3 = 0.6, \\ \lambda_{m_1} &= \lambda_{m_2} = 1,\ \lambda_{M_1} = 16,\ \lambda_{M_2} = 17. \end{aligned}$$

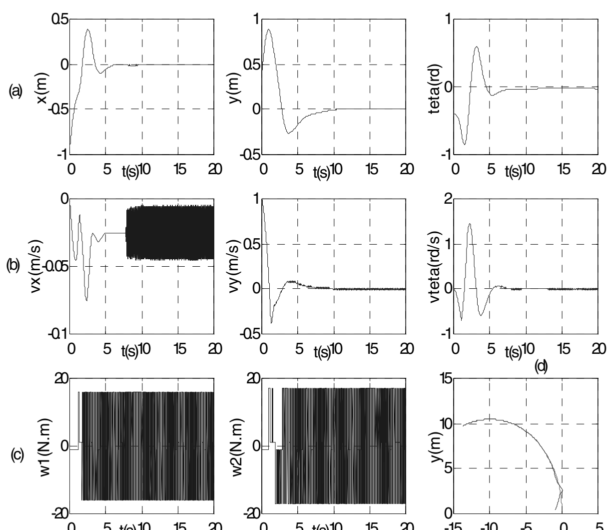
Simulated tracking responses of the unicycle for sliding mode control are given in the base frame by Figure 2 and 3. The first line of both of them gives the position and orientation error time plot (a). The second line provides the lateral and the longitudinal velocity error time, plot (b), i.e. the violation of the non skidding and the non pure rolling constraints. The input time response is given plot (c) and plot (d) shows the behavior of the WMR in the phase plane $(x, y)$.

It can be seen that, although there were forward and backward fluctuations, all the position and velocity posture converged to the desired trajectories. Figure 2 and 3 show that the controller performance is satisfactory for tracking errors but exhibits a chattering in control inputs. To reduce the chattering, it is needed to use a third order sliding mode controller since the relative degree of the system is equal to 2 with respect to the chosen manifold.




(a) Position and orientation errors, (b) Constraints violation : velocity errors.
(c) Control inputs, (d) Position phase.

Fig. 2. Tracking of a straight trajectory.




(a) Position and orientation errors, (b) Constraints violation : velocity errors.
(c) Control inputs, (d) Position phase.

Fig. 3. Tracking of a circular trajectory.

## V. CONCLUSION

The trajectory tracking problem for the dynamical model of a unicycle-type robot in presence of sliding effects has been solved using a second order sliding mode control. The dynamic tracking error model of the unicycle is derived from the singular perturbation approach and the proposed control policy is based on two nonlinear sliding surfaces. The asymptotic vanishing of both lateral and longitudinal error dynamics has been theoretically proved. However due to the nonlinearities, those errors are only stabilized in an arbitrarily small neighborhood of the origin. Simulation results are reported, showing the robustness of the proposed control law against nonlinearities and sliding effects. It will be interesting to extend this approach to car-like mobile robots. Further research aims at developing $r^{th}$ order sliding mode control schemes with $r > 2$, for instance, the third order sliding mode control dealing with robust practical tracking of wheeled mobile robots that are not satisfying the nonholonomic constraints.

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