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A Multivariable Nonlinear $\mathcal{H}_\infty$ Controller for a Laboratory Helicopter

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Abstract— **This paper considers the problem of a nonlinear $\mathcal{H}_\infty$ design for a laboratory twin rotor system. This mechanical device features a highly nonlinear strongly coupled dynamics, which constitutes a challenge for many classical linear control techniques. The approach presented in this paper considers a nonlinear $\mathcal{H}_\infty$ disturbance rejection procedure on the reduced dynamics of the rotors, including integral terms on the tracking error to cope with persistent disturbances. The resulting controller exhibits the structure of a nonlinear PID, with time-varying constants according to the system dynamics. The methodology has been tested by experimental results using a laboratory helicopter.**

I. INTRODUCTION

This paper presents an application of the nonlinear $\mathcal{H}_\infty$ control framework to a laboratory helicopter consisting of a double rotor system in quadrature.

The system resembles a simplified behaviour of a real helicopter with fewer degrees of freedom. In real helicopters the control is generally achieved by tilting appropriately the blades of the rotors with the collective and cyclic actuators, while keeping constant rotor speed. These systems have been extensively investigated yielding a number of control applications that range from linear robust control techniques in [9],[10] to more recent nonlinear approaches as in [12] or [11].

In order to simplify the mechanical design of the system, the laboratory setup employed in this paper, is designed slightly differently. In this case, the blades of the rotors have a fixed angle of attack, and control is achieved by controlling the speeds of the rotors. As a first consequence of this, the laboratory helicopter presents higher coupling between dynamics of the rigid body and dynamics of the rotors than a conventional helicopter, and yields a highly nonlinear, coupled dynamics.

Additionally, it can be proved that characteristic dynamics of the system is nonminimum phase, exhibiting unstable zero dynamics, which makes the system not suitable for classical feedback linearization techniques [8]. This fact, jointly with important modelling uncertainties, specially on the high frequency range, makes the system hard to control by conventional strategies.

Some classical approaches to control this kind of systems, make use of simplificative assumptions to decouple the dynamics of the rigid body from the dynamics of the rotors such that partial feedback linearization can be applied. The system thus linearized, takes the form of a classical master-slave cascade linear system [16], where the simplified linear system thus obtained, can be then controlled with well known linear strategies as $\mathcal{H}_\infty$ [7], LQR optimal control [4] or switched control [8].

In the last few years, important advances in the theory of nonlinear control have been developed, yielding interesting tools for the control of a range of challenging problems. One of such control strategies is the nonlinear extension of classical robust $\mathcal{H}_\infty$ control to the nonlinear framework [13]. This control technique involves finding the solution of a HJBI partial differential equation (PDE), for which no explicit solutions can be computed for the general case. The present paper proposes a structure for this solution providing explicit expressions of the terms of the controller that solves the problem. Additionally, the control structure can be interpreted as a nonlinear PID controller with time varying gains, giving some insight on the controlled system.

The behaviour of the proposed control law is tested by experimental results on the above mentioned laboratory helicopter setup.

The remainder of the paper is organized as follows: a description of the system is exposed in Section II, followed by an introduction of the basic nonlinear $\mathcal{H}_\infty$ results in section III. In Section IV an application of the general framework is provided for the laboratory helicopter system, and experimental results are shown and discussed in Section V. Finally, the major conclusions to be drawn are given in Section VI.

II. SYSTEM DESCRIPTION

The twin rotor system used in this paper consists of two free joints (see Fig. 1) thrusted by two propellers placed in perpendicular planes [6]. As it has been mentioned, this equipment is a multivariable, nonlinear and strongly coupled system, with degrees of freedom on the pitch and yaw angles, for the reduced model.

The system is propelled by two crossed rotors with fixed angle of attack on the propeller's blades. Thus, the system is controlled by changing the angular velocities of the rotors which, according to the action-reaction principle, involves the generation of a resultant torque on the body of the double rotor system, that makes it rotate in the opposite direction of the rotor.

In this way, there are two kinds of forces applied to the system:

- velocity dependent forces
- acceleration dependent forces or inertial forces

Fig. 2 shows the notation used for the system modelling.

The equations of motion can be obtained by solving the Lagrange dynamic equation. The solution may be expressed

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Fig. 1. Laboratory double rotor system.

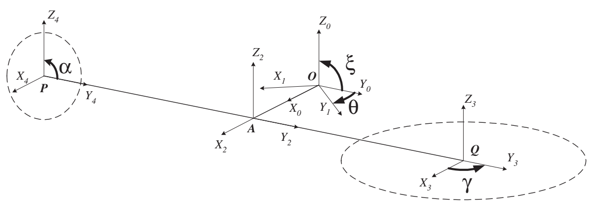

Fig. 2. Notation

in a matrix formulation as follows:

$$\begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \cdot \begin{pmatrix} \ddot{\theta} \\ \ddot{\xi} \end{pmatrix} =$$
$$\begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{pmatrix} \cdot \begin{pmatrix} \ddot{\alpha} \\ \ddot{\gamma} \end{pmatrix} + \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix} \cdot \begin{pmatrix} \dot{\theta}^2 \\ \dot{\xi}^2 \end{pmatrix} +$$
$$\begin{pmatrix} A_1 \\ A_2 \end{pmatrix} \dot{\xi}\dot{\theta} + \begin{pmatrix} \zeta_\theta \\ \zeta_\xi \end{pmatrix} + \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix} \cdot \begin{pmatrix} \dot{\theta} \\ \dot{\xi} \end{pmatrix} + \begin{pmatrix} Cte_1 \\ Cte_2 \end{pmatrix} +$$
$$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \end{pmatrix} \cdot \begin{pmatrix} \dot{\alpha}\dot{\theta} \\ \dot{\gamma}\dot{\theta} \\ \dot{\gamma}\dot{\xi} \end{pmatrix}$$

Where

- θ: Yaw angle. Rotation around the vertical axis, Z_0.
- ξ: Pitch angle. Rotation around the transverse axis X_0.
- $\dot{\alpha}$: Angular velocity of the tail rotor.
- $\dot{\gamma}$: Angular velocity of the main rotor.
- M(θ,ξ),K(θ,ξ) matrices: Inertial terms.
- C(θ,ξ,$\dot{\theta}$,$\dot{\xi}$) matrix: Centrifugal terms.
- A(θ,ξ,$\dot{\theta}$,$\dot{\xi}$),D(θ,ξ,$\dot{\theta}$,$\dot{\xi}$) matrices: Coriolis terms.
- V matrix: Friction terms.
- G matrix: potential derived forces.

A more detailed description of the system and parameters can be found in [4].

With this setup, the system can be shown to be underactuated and nonminumum phase, since there are two control actions, the main and tail rotor torques, ζ_θ and ζ_ξ, and four variables to be controlled, the pitch and yaw attitudes, θ and ξ, as well as the main and tail rotor angular velocities, $\dot{\alpha}$ and $\dot{\gamma}$.

In order to apply the proposed methodology, the system must conform the structure of a fully actuated system, so the following hypothesis is applied: The inertias of the main and tail thrusters are considered negligible with respect to the rigid body system inertias, which yields and much faster dynamics on the actuators. Additionally, the thrusters are assumed to accelerate smoothly, such that dominant forces on the dynamics are due to aerodynamics effects rather than inertial forces.

With these assumptions, the speeds of the rotors can be taken to be approximately constant, and the equations of the system can be be expressed as

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{12} & M_{21} \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{\xi} \end{bmatrix} + \begin{bmatrix} A_1\dot{\xi}\dot{\theta} + C_{12}\dot{\xi}^2 \\ C_{21}\dot{\theta}^2 \end{bmatrix} + \begin{bmatrix} 0 \\ G \end{bmatrix} +$$
$$+ \begin{bmatrix} V_1\dot{\theta} \\ V_2\dot{\xi} \end{bmatrix} + \begin{bmatrix} d_{11}\dot{\alpha}\dot{\theta} + d_{13}\dot{\gamma}\dot{\xi} \\ d_{22}\dot{\gamma}\dot{\theta} \end{bmatrix} = \begin{bmatrix} \zeta_\theta \\ \zeta_\xi \end{bmatrix} \quad (1)$$

Comparing these equations with those of a classical robot manipulator, it can be observed that all the terms can be easily identified, with exception of that depending on the speeds of the rotors. Nonetheless, under the hypothesis of speeds of the rotors approximately constant, the term

$$\begin{bmatrix} d_{11}\omega_t\dot{\theta} + d_{13}\omega_m\dot{\xi} \\ d_{22}\omega_m\dot{\theta} \end{bmatrix}$$

can be interpreted as an additional damping on the nominal viscous friction

$$\begin{bmatrix} V_1\dot{\theta} \\ V_2\dot{\xi} \end{bmatrix}$$

Thus, the dynamics of the reduced system resembles that of a two degrees of freedom robot manipulator, and appropriate methodologies can be applied to control it.

III. NONLINEAR H_∞ APPROACH

The dynamic equation of an nth order smooth nonlinear system which is affected by an unknown disturbance can be expressed as follows:

$$\dot{x} = f(x,t) + g(x,t)\,u + k(x,t)\,\omega \quad (2)$$

where $u \in \Re^p$ is the vector of control inputs, $\omega \in \Re^q$ is the vector of external disturbances and $x \in \Re^n$ is the vector of states. Performance can be defined using the cost variable $z \in \Re^{(m+p)}$ given by the expression

$$z = W \begin{bmatrix} h(x) \\ u \end{bmatrix} \quad (3)$$

where $h(x) \in \Re^m$ represents the error vector to be controlled and $W \in \Re^{(m+p)\times(m+p)}$ is a weighting matrix. If states x are assumed to be available for measurement (which is usual in mechatronic systems), then the optimal $\mathcal{H}_\infty$ problem can be posed as follows [1]:

Find the smallest value $\gamma^* \geq 0$ such that for any $\gamma \geq \gamma^*$ there exists a state feedback $u = u(x,t)$, such that the L_2 gain from ω to z is less than or equal to γ, that is,

$$\int_0^T \|z\|_2^2\,dt \leq \gamma^2 \int_0^T \|\omega\|_2^2\,dt \quad (4)$$

The integral expression on the left-hand side of inequality (4) can be written as

$$\|z\|_2^2 = z^T z = \left[\begin{array}{cc} h^T(x) & u^T \end{array} \right] W^T W \left[\begin{array}{c} h(x) \\ u \end{array} \right]$$

and the symmetric, positive definite matrix $W^T W$ can be partitioned as follows:

$$W^T W = \left[\begin{array}{cc} Q & S \\ S^T & R \end{array} \right]$$

Matrices Q and R are symmetric, positive definite and the fact that $W^T W > 0$ guarantees that $Q - SR^{-1}S^T > 0$.

The following structures, considered in this paper for matrices Q and S, constitute an extension of the original formulation [2]:

$$Q = \left[\begin{array}{ccc} Q_1 & Q_{12} & Q_{13} \\ Q_{12} & Q_2 & Q_{23} \\ Q_{13} & Q_{23} & Q_3 \end{array} \right] \qquad S = \left[\begin{array}{c} S_1 \\ S_2 \\ S_3 \end{array} \right]$$

Under these assumptions, an optimal control signal $u^*(x,t)$ may be computed for system (2) if there exists a smooth solution $V(x,t)$, with $V(x_0,t) \equiv 0$ for $t \geq 0$, to the following *HJBI* equation:

$$\frac{\partial V}{\partial t} + \frac{\partial^T V}{\partial x} f(x,t) + \frac{1}{2}\frac{\partial^T V}{\partial x}\left[\frac{1}{\gamma^2} k(x,t) k^T(x,t) \right.$$
$$\left. - g(x,t) R^{-1} g^T(x,t)\right] \frac{\partial V}{\partial x} - \frac{\partial^T V}{\partial x} g(x,t) R^{-1} S^T h(x)$$
$$+ \frac{1}{2} h^T(x) \left(Q - SR^{-1}S^T\right) h(x) = 0 \tag{5}$$

for each $\gamma > \sqrt{\sigma_{\max}(R)} \geq 0$, where $\sigma_{\max}$ stands for the maximum singular value. In such a case, the optimal state feedback control law is derived as follows [2]:

$$u^* = -R^{-1}\left(S^T h(x) + g^T(x,t) \frac{\partial V(x,t)}{\partial x}\right) \tag{6}$$

IV. APPLICATION TO A LABORATORY HELICOPTER

In this section, the general methodology described in the preceding development is applied to the laboratory helicopter in Section II.

The equations of motion 1 used to describe the behaviour of the helicopter can be posed as follows:

$$M(q)\ddot{q} + N(q,\dot{q}) = \tau + \tau_d \tag{7}$$

with

$$N(q,\dot{q}) = C(q,\dot{q})\dot{q} + F(\dot{q}) + G(q)$$

where $q = [\theta, \xi]^T \in \Re^2$, and $\dot{q}$ is its temporal derivative (joint speeds). It is assumed that these two vectors are available for measurements. Vector $\tau = [\zeta_\theta, \zeta_\xi]^T$ (generalized torques applied on the joint axes) is the input signal of the system and τ_d represents the total effect of system modelling errors and external disturbances.

The inertia matrix $M(q)$ is symmetric positive definite, $C(q,\dot{q})\dot{q} = \left[A_1\dot{\xi}\dot{\theta} + C_{12}\dot{\xi}^2, C_{21}\dot{\theta}^2\right]^T$ is the vector of centripetal and Coriolis terms, $F(\dot{q}) = \left[d_{11}\omega_t\dot{\theta} + d_{13}\omega_m\dot{\xi} + V_1\dot{\theta}, d_{22}\omega_m\dot{\theta} + V_2\dot{\xi}\right]^T$ represents the friction terms, and $G(q) = [0, G]^T$ denotes the gravity terms.

As is known ([14]), matrix $C(q,\dot{q})$ of the centripetal term is not unique. For the sake of convenience, in this paper this matrix will be computed through the following expression:

$$C(q,\dot{q}) = \frac{1}{2}\dot{M}(q,\dot{q}) + \mathcal{N}(q,\dot{q}) \tag{8}$$

where the terms $\dot{M}(q,\dot{q})$ and $\mathcal{N}(q,\dot{q})$ are given by:

$$\dot{M}_{ij} = \frac{d}{dt}M_{ij} = \frac{\partial M_{ij}}{\partial q}\dot{q} = \sum_{k=1}^{n} \frac{\partial M_{ij}}{\partial q_k}\dot{q}_k \tag{9}$$

$$\mathcal{N}_{ij} = \frac{1}{2}\sum_{k=1}^{n}\left(\frac{\partial M_{ik}}{\partial q_j} - \frac{\partial M_{jk}}{\partial q_i}\right)\dot{q}_k \tag{10}$$

Using q_r, $\dot{q}_r$ and $\ddot{q}_r$ to denote the desired joint position, speed and acceleration, respectively, the tracking error vector, x, and its derivative, $\dot{x}$, can be defined as follows:

$$x(t) = \left[\begin{array}{c} \dot{e}(t) \\ e(t) \\ \int e(t)\, dt \end{array} \right] \qquad \dot{x} = \left[\begin{array}{c} \ddot{e}(t) \\ \dot{e}(t) \\ e(t) \end{array} \right] \tag{11}$$

where

$$\ddot{e} = \ddot{q} - \ddot{q}_r, \qquad \dot{e} = \dot{q} - \dot{q}_r,$$
$$e = q - q_r, \qquad \int e dt = \int_o^t (q - q_r)\, dt$$

For system (7), a control law with the following structure is considered

$$\tau = M(q)\ddot{q} + N(q,\dot{q}) - \frac{1}{\rho}\left(M(q)T\dot{x} + C(q,\dot{q})Tx\right) + \frac{1}{\rho}u \tag{12}$$

This proposed control law can be split up into three different parts: the first one consists of the first three terms of that equation, which are designed in order to compensate for the robot dynamics (see Eq. 7). The second part consists of terms including error vector x and its derivative, $\dot{x}$. Assuming $\tau_d \equiv 0$, these two terms of the control law enable perfect tracking, which means that they represent the *essential* control effort needed to perform the task. Finally, the third part includes a vector u, which represents the *additional* control effort needed for disturbance rejection.

It can also be pointed out that, despite the preceding control law might seem a not well posed system, it will be shown afterwards that the computed torque does not rely on joint accelerations, but on their references.

Matrix T in Eq. (12) can be partitioned as follows:

$$T = \left[\begin{array}{ccc} T_1 & T_2 & T_3 \end{array} \right] \tag{13}$$

with $T_1 = \rho I$, where ρ is a positive scalar and I is the nth-order identity matrix.

Substituting the expression of the control law from (12) into Eq. 7 and defining $\omega = \rho\tau_d$, yields

$$M(q)T\dot{x}(t) + C(q,\dot{q})Tx(t) = u(t) + \omega(t) \tag{14}$$

This expression represents the *dynamic equation of the system error*. It is a *nonlinear* 6th order equation since its coefficients (matrices $M(q)$ and $C(q,\dot{q})$) vary with time. Taking into account this nonlinear equation, the nonlinear $\mathcal{H}_\infty$ control problem can be posed as follows:

"Find a control law $u(t)$ such that the ratio between the energy of the cost variable $z = W\left[h^T(x)\ u^T\right]^T$ and the energy of the disturbance signals ω is less than a given attenuation level γ".

To apply the theoretical results presented in Section III, it is necessary to rewrite the nonlinear dynamic equation of the error (Eq. 14) into the standard form of the nonlinear $\mathcal{H}_\infty$ problem (see Eq. 2). If the functional dependencies on $M(q)$ and $C(q,\dot{q})$ are dropped for compactness, this can be done by defining the following expressions [15]:

$$f(x,t) = T_o^{-1}\begin{bmatrix} -M^{-1}C & O & O \\ \frac{1}{\rho}I & I-\frac{1}{\rho}T_2 & -I-\frac{1}{\rho}(T_3-T_2) \\ 0 & I & -I \end{bmatrix} T_o x \tag{15}$$

$$g(x,t) = k(x,t) = T_o^{-1}\begin{bmatrix} M^{-1} \\ O \\ O \end{bmatrix} \tag{16}$$

where I is the identity matrix, O the zero matrix, both of n-th order and

$$T_o = \begin{bmatrix} T_1 & T_2 & T_3 \\ O & I & I \\ O & O & I \end{bmatrix} \tag{17}$$

As stated in Section III, the solution of the *HJBI* equation depends on the choice of the cost variable, z, and particularly on the selection of function $h(x)$ (see Eq. 3). In this paper, this function is taken to be equal to the error vector, that is, $h(x) = x$. Once this function has been selected, computing the control law, u, will require finding the solution, $V(x,t)$, to the *HJBI* equation posed in the previous section (see Eq. 5). A solution for that equation and for a sufficiently high value of γ is the following [15]:

$$V(x,t) = \frac{1}{2}x^T T_o^T \begin{bmatrix} M & O & O \\ O & Y & X-Y \\ O & X-Y & Z+Y \end{bmatrix} T_o x \tag{18}$$

where X, Y and $Z \in \Re^{n\times n}$ are constant, symmetric, and positive definite matrices such that $Z - XY^{-1}X + 2X > 0$, M is the inertia matrix, and T_o is as defined in (17). Let $T = \begin{bmatrix} T_1 & T_2 & T_3 \end{bmatrix}$ be the matrix appearing in dynamic equation (14). If these matrices verify the following equation:

$$\begin{bmatrix} O & Y & X \\ Y & 2X & Z+2X \\ X & Z+2X & O \end{bmatrix} + Q + \frac{1}{\gamma^2}T^T T - \left(S^T + T\right)^T R^{-1}\left(S^T + T\right) = 0$$

Then, substituting $V(x,t)$ in Eq. (6), control law u^* corresponding to the H_∞ optimal index γ is given by

$$u^* = -R^{-1}\left(S^T + T\right)x$$

A particular case can be obtained when the components of the weighting compound $W^T W$ verify:

$$Q_1 = w_1^2 I \quad Q_2 = w_2^2 I \quad Q_3 = w_3^2 I \quad R = w_u^2 I \tag{19}$$
$$Q_{12} = Q_{13} = Q_{23} = O \quad S_1 = S_2 = S_3 = O$$

In this case, if the expressions T, $\dot{x}$ and u^* are replaced for in (12), and after some manipulation, the optimal control law can be written as:

$$\tau^* = M(q)\ddot{q}_r + N(q,\dot{q}) - M(q)\left(K_d\ \dot{e} + K_p\ e + K_i \int e\ dt\right) \tag{20}$$

where:

$$K_d = \frac{\sqrt{w_2^2 + 2w_1w_3}}{w_1}I + M^{-1}\left(C + \frac{1}{w_u^2}I\right)$$
$$K_p = \frac{w_3}{w_1}I + \frac{\sqrt{w_2^2 + 2w_1w_3}}{w_1}M^{-1}\left(C + \frac{1}{w_u^2}I\right)$$
$$K_i = \frac{w_3}{w_1}M^{-1}\left(C + \frac{1}{w_u^2}\right)$$

It is easy to see that Eq. 20 represents a *computed-torque control law* with an *external PID controller*. This external *PID* is a nonlinear one since its gain matrices are time-varying, like matrices $M(q)$ and $C(q,\dot{q})$. Besides this, from Eq. 20 it can be concluded that the control law does not depend on the joint accelerations, preventing the system to be ill-posed as it could seem from Eq. 12.

The above gain matrices depend not only on the system model, but also on a few parameters: w_1, w_2 and w_3 which allows to weight the error, its integral and its derivative, and w_u that penalizes the increment of the control signal. However, since the values of the gains depend on the ratio among them, only three parameters has to be tuned to achieve the required performance. Thus, these gain matrices can be expressed as:

$$K_p = k_p + k_p^u \quad K_d = k_d + k_d^u \quad K_i = k_i + k_i^u$$

where the following diagonal matrices are defined:

$$w_{ij} \doteq \frac{w_i}{w_j} \quad w_{231} \doteq \sqrt{w_{21}^2 + 2w_{31}}$$
$$w_u^{-2} \doteq K_u$$

obtaining the time-varying gains

$$\begin{array}{ll} k_p = w_{31} + w_{231}M^{-1}C; & k_p^u = K_u w_{231} M^{-1} \\ k_d = w_{231} + M^{-1}C; & k_d^u = K_u M^{-1} \\ k_i = w_{31}M^{-1}C; & k_i^u = K_u w_{31} M^{-1} \end{array}$$

V. EXPERIMENTAL RESULTS

In this section, some experimental results on the performance of the control methodology proposed in this paper are presented.

In order to test a range of dynamical behaviours, the system was intended to follow a series of step references of different amplitudes and speeds. Thus, Fig. 3 shows the performance of a couple of positive simultaneous steps in both, the pith and yaw degrees of freedom at moderate speed, with the corresponding speeds of the rotors in Fig. 4. In this case, the nonminimum phase of the systems is hidden by the simultaneous positive references.

Inverse response associated to this property is better observed from Fig. 5 and 6, where positive and negative references are provided. Nonetheless, it can be observed that this fact significantly degrades the behaviour on pitch axis.

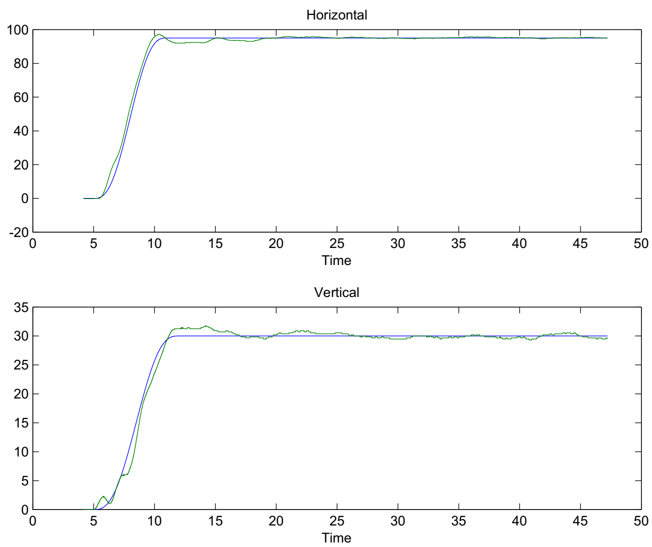

Fig. 3. Yaw and Pitch angles

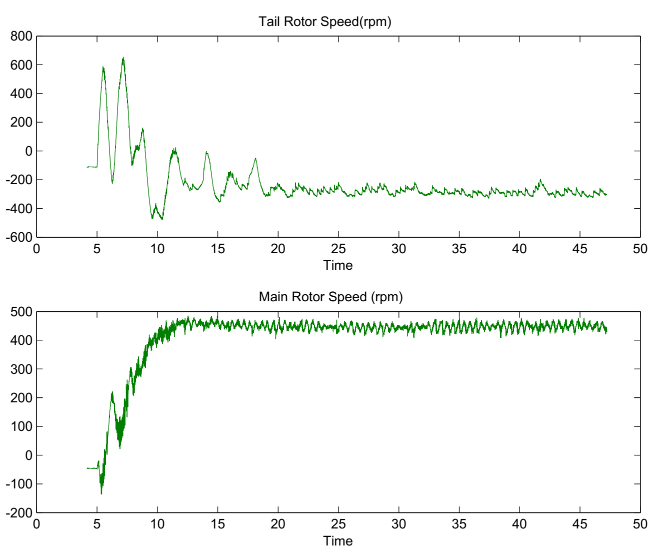

Fig. 4. Main and Tail Rotor Speeds

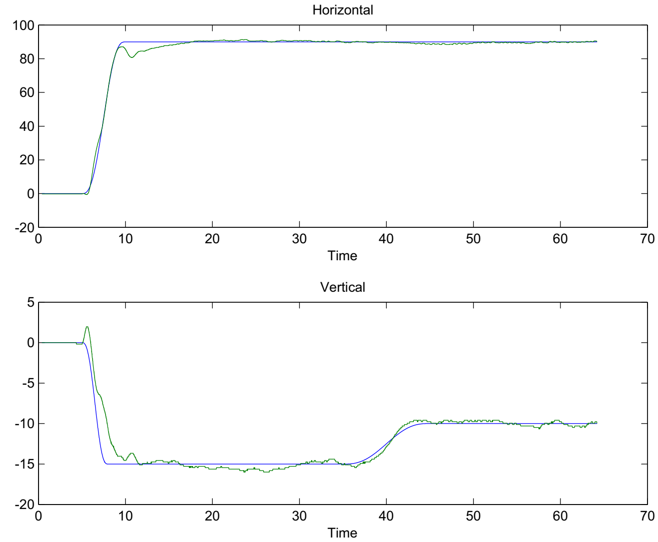

Fig. 5. Yaw and Pitch angles

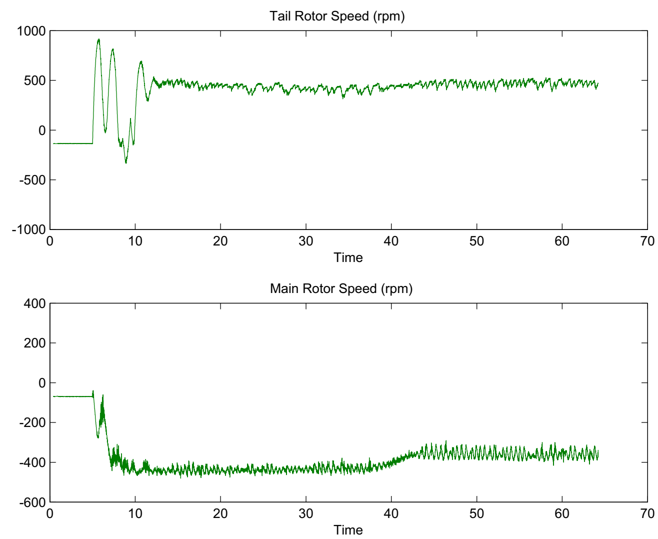

Fig. 6. Main and Tail Rotor Speeds

Fig. 7 and 8 show results for non simultaneous steps on both axis, where the remarkable decoupling abilities of the controller can be observed. Additionally, comparative results on the performance of the system for gradually increasing demanding references are shown. It can be noticed that, remarkably, the controller performs outstandingly for high speed references, partially due to the dominating rotor dynamics for high rotor speeds.

VI. CONCLUSIONS

This paper presents a nonlinear $\mathcal{H}_\infty$ controller for a highly nonlinear laboratory helicopter system. The control structure proposed gives a particular solution for the HJBI PDE associated to the nonlinear $\mathcal{H}_\infty$ formulation, yielding explicit expressions for a set of tuning parameter of the controller. Remarkably, the overall control structure can be interpreted as a nonlinear PID controller with time-varying

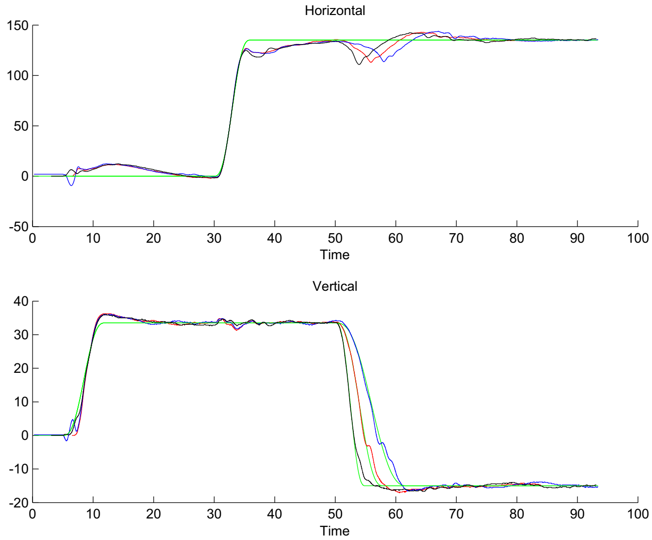

Fig. 7. Yaw and Pitch angles

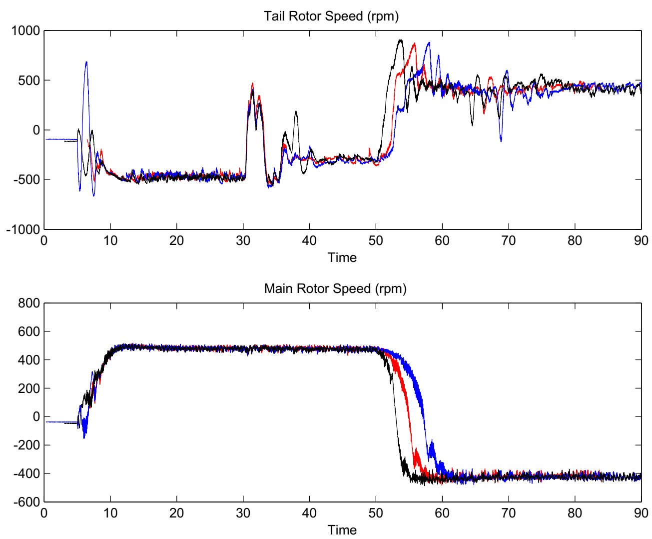

Fig. 8. Main and Tail Rotor Speeds

gains, plus a sort of computed-torque linearization strategy.

To test the controller structure proposed, a series of experimental results are given, showing good performance for a range of operating conditions.

VII. ACKNOWLEDGMENTS

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