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Optimal Reconfiguration of Formation Flying Satellites

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Abstract— **We employ a recently developed new technique for the numerical treatment of optimal control problems for mechanical systems in order to compute optimal open loop control laws for the reconfiguration of a group of formation flying satellites. The method is based on a direct discretization of a variational formulation of the dynamical constraints. We extend the method by linearizing around a given nominal trajectory and incorporate certain collision avoidance strategies. We numerically illustrate the approach for a certain reconfiguration maneuver in the context of future formation flying missions and compare our method to a standard finite difference approach.**

I. Introduction

In the upcoming space missions *Darwin*[1] and *Terrestrial Planet Finder (TPF)*[2], a group of formation flying spacecraft will build up an infrared interferometer in order to detect and analyze planets outside our solar system. One of the many challenges of these missions is to develop techniques for controlling the spacecraft such that their relative motion meets certain very demanding accuracy requirements: while their mutual distance is on the order of several 100 m, this distance has to be kept constant up to an error of 10^{-6} m during a measurement period. In [4], numerical evidence has been given that it is indeed possible to keep the spacecraft in formation using a low thrust control strategy only.

Additionally, in regular time intervals, the group of spacecraft will have to be reconfigured such that another planetary system can be analyzed. In light of the tight mass budget of these missions it is of great interest to minimize the propellant consumption in performing these reconfigurations. In this paper we employ a recently developed new technique [3] for the numerical treatment of optimal control problems for mechanical systems in order to compute optimal control laws for the reconfiguration of the spacecraft. The method bases on a variational formulation of the mathematical model of the mechanical system and uses a direct discretization of this formulation instead of a discretization of the associated ordinary differential equations (the Euler-Lagrange equations). This leads to a scheme that involves only half as many unknowns as a standard reversible finite difference (or collocation) approach, while showing the same nice energy conservation properties.

In our computations, we focus on the concrete setting of the Darwin and TPF missions: the group of six spacecraft will be placed in the vicinity of a certain periodic orbit near the Lagrange point L_2. The group is required to acquire a planar formation with each of the spacecraft being loacted at the vertex of a regular hexagon in that plane, while their common attitude must be aligned with the normal to the plane. For the purpose of this paper we restrict ourselves to the unperturbed case. We note, however, that in light of the tight error tolerances mentioned above, in order to account for the effects of perturbations, in an implementation the computed optimal trajectories will have to be stabilized by, e.g., suitable feedback laws.

Similar work has been done by Milam, Petit and Murray [8] where the reorientation of a cluster of fully actuated low-thrust micro-satellites is considered. The authors use an optimization-based control methodology and the problem is solved using the Nonlinear Trajectory Generation (NTG) software package which is based on a traditional discretization of the equations of motion. Garcia and Masdemont [2] use a Galerkin finite element method to discretize the equations of motion, yielding twice as many unknowns as our approach. In the work of Beard and Hadaegh [1] the goal is to rotate a formation of satellites from one orientation to another, using an optimal amount of fuel. In contrast to our setting the authors do not consider the translation along a halo orbit. They do, however, additionally balance the fuel consumption of the spacecraft.

An outline of the paper is as follows: based on the model for the dynamics of the spacecraft that is introduced in Section II we formalize the optimal control problem in Section III. We recall the numerical method that is used for its solution in Section V and finally present our numerical results in Section VI.

II. The Model

We are dealing with a group of n identical spacecraft. Each spacecraft is modeled as a rigid body with six degrees of freedom (position and orientation), i.e. its configuration manifold is $SE(3)$. We assume that each spacecraft can be

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[1]http://www.esa.int/science/darwin
[2]http://planetquest.jpl.nasa.gov/TPF

controlled in this configuration space by a force-torque pair (F, τ), acting on its center of mass.

In the current mission concepts for Darwin and TPF, it is planned to position the group of spacecraft in the vicinity of a Libration orbit around the L_2 Lagrange point. Correspondingly, for each spacecraft the dynamical model for the motion of its center of mass is given by the *circular restricted three body problem*:

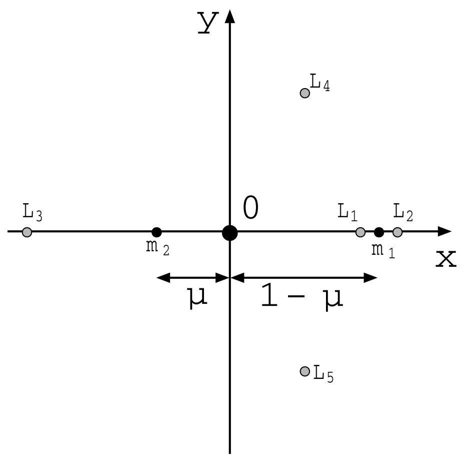

Fig. 1. Rotating coordiante system: location of the primaries and the Lagrange points.

Two large bodies (the *primaries*, i.e. Sun and Earth in our case) with masses m_1 and m_2 rotate on circles with common angular velocity ω around their center of mass. A third body, the spacecraft, moves within their gravitational potential without affecting the motion of the primaries. We neglect gravitational forces between the spacecraft. Figure 1 shows the position of the primaries and the equilibria (the *Lagrange points*) $L_1, \ldots, L_5$ in the xy-plane of a rotating coordinate system. In Figure 2 we plot a family of periodic orbits (*Halo orbits*) in the vicinity of the L_2 Lagrange point. This family has been computed by a predictor corrector method on an initial orbit found by a shooting technique (see [4]).

In a normalized, rotating coordinate system (cf. Figure 1), the potential energy of the spacecraft at position $x = (x_1, x_2, x_3) \in \mathbb{R}^3$ is given by

$$V(x) = -\frac{1-\mu}{|x-(1-\mu,0,0)|} - \frac{\mu}{|x-(-\mu,0,0)|}, \tag{1}$$

where $\mu = m_1/(m_1+m_2)$ is the normalized mass. Its kinetic energy is the sum of

$$K_{\text{trans}}(x, \dot{x}) = \frac{1}{2}((\dot{x_1} - \omega x_2)^2 + (\dot{x_2} + \omega x_1)^2 + \dot{x}_3^2),$$

(assuming that is mass is equal to 1 for simplicity) and

$$K_{\text{rot}}(\Omega) = \frac{1}{2}\Omega^T J \Omega,$$

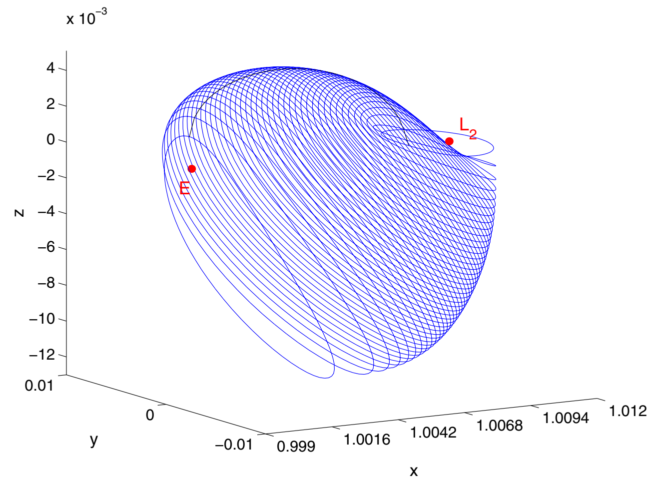

Fig. 2. Family of periodic orbits in the circular restricted three body problem in the vicinity of the L_2-Lagrange point.

where $\Omega \in \mathbb{R}^3$ is the angular velocity and J the inertia tensor of the spacecraft (again, for simplicity, we normalize $J = I$).

III. The Control Problem

Our goal is to compute control laws $(F^{(i)}(t), \tau^{(i)}(t))$, $i = 1, \ldots, n$, for each spacecraft, such that the group moves from a given initial state $(x^{(i)}, p^{(i)}, \dot{x}^{(i)}, \dot{p}^{(i)})_{i=1}^n$ into a prescribed target manifold within a prescribed time interval, where the unit quaternion $p^{(i)} \in \mathbb{R}^4$ represents the orientation of the i-th spacecraft. In our application context, the target manifold will be defined by prescribing the relative positioning of the spacecraft, their common velocity as well as a common orientation. We additionally require the resulting controlled trajectory to minimize a given cost functional which typically is related to the associated fuel consumption of the spacecraft.

More precisely, for their target state, we require the spacecraft to be located in a planar regular polygonal configuration with center on a Halo orbit. Let $\nu \in \mathbb{R}^3$ be a given unit vector (the "line of sight" of the spacecraft). The target manifold $M \subset TSE(3)^n$ is the set of all states $(x^{(i)}, p^{(i)}, \dot{x}^{(i)}, \dot{p}^{(i)})_{i=1}^n$ such that

1) all spacecraft lie in a plane with normal ν, i.e.

$$\langle x^{(i)} - x^{(j)}, \nu \rangle = 0, \quad i,j = 1, \ldots, n; \tag{2}$$

2) within that plane, the spacecraft are located at the vertices of a regular polygon with a prescribed center on a Halo orbit. Let $r_0 \in \mathbb{R}$ be a given radius and $\bar{x} \in \mathbb{R}^3$ a certain point on a Halo orbit and let $\nu_1^\perp \perp \nu_2^\perp \in \mathbb{R}^3$ be two perpendicular unit vectors that are perpendicular to ν. For $i = 1, \ldots, n$ we consider the vector

$$z^{(i)} = [\nu_1^\perp \ \nu_2^\perp]^T (x^{(i)} - \bar{x}) \in \mathbb{R}^2$$

and require that

$$\left(\widehat{z^{(i)}}\right)^n = r_0^n e^{i\varphi},$$

$i = 1, \ldots, n$, where $\widehat{z} \in \mathbb{C}$ denotes the complex number associated to a vector $z \in \mathbb{R}^2$ and φ describes the attitude of the polygon in the plane determined by $\nu_1^\perp$ and $\nu_2^\perp$.
The idea of this formulation is not to prescribe a fixed vertex on the polygon for each spacecraft but rather to let the optimization process find the best possible arrangement (if possible). In order to avoid that more than one spacecraft attains the same vertex, we employ a collision avoidance strategy as described in Section IV);

3) all spacecraft have their "line of sight" aligned with ν. For simplicity we here impose a more restrictive condition, namely that each spacecraft is rotated according to a prescribed unit quaternion $p_0^{(i)}$, i.e. we require that

$$p^{(i)} = p_0, \quad i = 1, \ldots, n;$$

4) all spacecraft have the same prescribed linear velocity,

$$\dot{x}^{(i)} = \dot{x}_0, \quad i = 1, \ldots, n,$$

where $\dot{x}_0$ will typically be determined on basis of the Halo orbit under consideration, and they have zero angular velocity, i.e.

$$\Omega^{(i)} = 2\dot{p}^{(i)}\bar{p}^{(i)} = 0, \quad i = 1, \ldots, n,$$

where $\bar{p}^{(i)}$ is the conjugate quaternion to $p^{(i)}$.

As mentioned, in addition to controlling to the target manifold, we would like to minimize the fuel consumption of the spacecraft. Here we simply consider the cost function

$$J(F, \tau) = \sum_{i=1}^n \int_{t_0}^{t_f} |F^{(i)}(t)|^2 + |\tau^{(i)}(t)|^2 \, dt, \tag{3}$$

where $F(t) = (F^{(1)}(t), \ldots, F^{(n)}(t))$ and $\tau(t) = (\tau^{(1)}(t), \ldots, \tau^{(n)}(t))$ denote the force and torque functions for the system.

IV. Collision avoidance

In order to avoid collisions between the spacecraft, we follow two different approaches:

1) we add an artificial potential to the gravitational potential V (i.e. we change the dynamics of the spacecraft);
2) we add a penalty term to the cost function of the problem.

A. Artificial potential

We consider an artificial potential V_a (cf. [6]) defined by

$$V_a(x^{(i)}, x^{(j)}) = \begin{cases} C_a \left(\ln r_{ij} + d_0/r_{ij}\right), & 0 < r_{ij} < d_0, \\ C_a \left(\ln d_0 + 1\right), & r_{ij} \geq d_0, \end{cases} \tag{4}$$

where $r_{ij} = |x^{(i)} - x^{(j)}|$ is the distance between the i-th and the j-th spacecraft, $C_a > 0$ and $d_0 > 0$ is a safety distance. For $r_{ij} < d_0$, the resulting force $f_a = \nabla V_a$ acts such that the i-th and the j-th spacecraft are repelled from each other, while for $r_{ij} \geq d_0$ no artifical force is in effect.

In this case, the Lagrangian of the full system reads

$$\begin{aligned} L \;=\; & \sum_{i=1}^n K_{\text{trans}}(x^{(i)}, \dot{x}^{(i)}) + K_{\text{rot}}(\alpha^{(i)}) \\ & - \sum_{i=1}^n V(x^{(i)}) - \sum_{\substack{i,j=1 \\ i \neq j}}^n V_a(x^{(i)}, x^{(j)}). \end{aligned}$$

B. Penalty term

We use the term

$$P(x^{(i)}, x^{(j)}) = C_a(\exp\left(-50(r_{ij} - d_0)^2\right) + 5\exp\left(-2r_{ij}^2\right)),$$

$r_{ij} = |x^{(i)} - x^{(j)}|$, in order to penalize a too close approach of the spacecraft. The new cost function accordingly reads

$$\begin{aligned} J(F, \tau) \;=\; & \sum_{i=1}^n \int_{t_0}^{t_f} |F^{(i)}(t)|^2 + |\tau^{(i)}(t)|^2 \, dt \\ & + \sum_{\substack{i,j=1 \\ i \neq j}}^n \int_{t_0}^{t_f} P(x^{(i)}(t), x^{(j)}(t)) \, dt. \end{aligned}$$

V. The Numerical Method

In order to solve the above formulated optimal control problem, we adapt a recently developed technique [3] that relies on a direct discretization of a variational formulation of the problem:

A mechanical system with configuration space Q is to be moved on a curve $q(t) \in Q$, $t \in [0, 1]$, from a state $(q^0, \dot{q}^0)$ to a state $(q^1, \dot{q}^1)$ under the influence of a force f. The curves q and f shall minimize a given cost functional

$$J(q, f) = \int_0^1 C(q(t), \dot{q}(t), f(t)) \, dt. \tag{5}$$

If $L : TQ \to \mathbb{R}$ denotes the Lagrangian of the mechanical system, the motion $q(t)$ of the system satisfies the *Lagrange-d'Alembert principle*, which requires that

$$\delta \int_0^1 L(q(t), \dot{q}(t)) \, dt + \int_0^1 f(t) \cdot \delta q(t) \, dt = 0 \tag{6}$$

for all variations δq with $\delta q(0) = \delta q(1) = 0$.

Using a global discretization of the states and the controls we directly obtain, via the *discrete Lagrange-d'Alembert principle*, equality constraints for the resulting finite dimensional nonlinear optimization problem, which can be solved by standard methods.

Discretization

We replace the state space TQ of the system by $Q \times Q$ and a path $q : [0,1] \to Q$ by a *discrete path* $q_d : \{0, h, 2h, \dots, Nh = 1\} \to Q$, $N \in \mathbb{N}$, where we view $q_k = q_d(kh)$ as an approximation to $q(kh)$ [7]. Analogously, we approximate the continuous force $f : [0,1] \to T^*Q$ by a discrete force $f_d : \{0, h, 2h, \dots, Nh = 1\} \to T^*Q$ (writing $f_k = f_d(kh)$).

The Discrete Lagrange-d'Alembert Principle

Based on this discretization, we approximate the action integral in (6) on a time slice $[kh, (k+1)h]$ by a *discrete Lagrangian* $L_d : Q \times Q \to \mathbb{R}$,

$$L_d(q_k, q_{k+1}) := hL\left(\tfrac{q_{k+1}+q_k}{2}, \tfrac{q_{k+1}-q_k}{h}\right) \approx \int_{kh}^{(k+1)h} L(q(t), \dot{q}(t))\, dt,$$

and the virtual work by

$$\int_{kh}^{(k+1)h} f(t) \cdot \delta q(t)\, dt \approx h\frac{f_{k+1} + f_k}{2} \cdot \frac{\delta q_{k+1} + \delta q_k}{2} = \frac{h}{4}(f_{k+1} + f_k) \cdot \delta q_k + \frac{h}{4}(f_{k+1} + f_k) \cdot \delta q_{k+1},$$

i.e. we have used $f_k^- = f_k^+ = \frac{h}{4}(f_{k+1} + f_k)$ as the *left and right discrete forces*.

The discrete version of the Lagrange-d'Alembert principle (6) then requires one to find discrete paths $\{q_k\}_{k=0}^N$ such that for all variations $\{\delta q_k\}_{k=0}^N$ with $\delta q_0 = \delta q_N = 0$, one has

$$\delta \sum_{k=0}^{N-1} L_d(q_k, q_{k+1}) + \sum_{k=0}^{N-1} f_k^- \cdot \delta q_k + f_k^+ \cdot \delta q_{k+1} = 0. \quad (7)$$

This is equivalent to the system

$$D_2 L_d(q_{k-1}, q_k) + D_1 L_d(q_k, q_{k+1}) + f_{k-1}^+ + f_k^- = 0, \quad (8)$$

$k = 1, \dots, N-1$. These are the *forced discrete Euler-Lagrange equations*.

Discrete Cost Function

We approximate the cost functional (5) on the time slice $[kh, (k+1)h]$ by

$$C_d(q_k, q_{k+1}, f_k, f_{k+1}) := hC\left(\tfrac{q_{k+1}+q_k}{2}, \tfrac{q_{k+1}-q_k}{h}, \tfrac{f_{k+1}+f_k}{2}\right), \approx \int_{kh}^{(k+1)h} C(q(t), \dot{q}(t), f(t))\, dt,$$

yielding the *discrete cost functional*

$$J_d(q_d, f_d) = \sum_{k=0}^{N-1} C_d(q_k, q_{k+1}, f_k, f_{k+1}). \quad (9)$$

Boundary Conditions

Finally, we need to incorporate the boundary conditions $q(0) = q^0$, $\dot{q}(0) = \dot{q}^0$ and $q(1) = q^1$, $\dot{q}(1) = \dot{q}^1$ into our discrete description. To this end, we link the description in $Q \times Q$ to one in TQ using the *discrete Legendre transforms* $\mathbb{F}^{f+}L_d : Q \times Q \to T^*Q$ and $\mathbb{F}^{f-}L_d : Q \times Q \to T^*Q$ for forced systems:

$$\begin{aligned} \mathbb{F}^{f+}L_d &: (q_{k-1}, q_k) \mapsto (q_k, p_k), \\ p_k &= D_2 L_d(q_{k-1}, q_k) + f_{k-1}^+ \quad \text{and} \\ \mathbb{F}^{f-}L_d &: (q_{k-1}, q_k) \mapsto (q_{k-1}, p_{k-1}), \\ p_{k-1} &= -D_1 L_d(q_{k-1}, q_k) - f_{k-1}^-. \end{aligned}$$

Using the *standard Legendre transform* $\mathbb{F}L : TQ \to T^*Q$

$$\mathbb{F}L : (q, \dot{q}) \mapsto (q, p) = (q, D_2 L(q, \dot{q})),$$

this leads to the two *discrete boundary conditions*

$$\begin{aligned} D_2 L(q_0, \dot{q}_0) + D_1 L_d(q_0, q_1) + f_0^- &= 0, \\ -D_2 L(q_N, \dot{q}_N) + D_2 L_d(q_{N-1}, q_N) + f_{N-1}^+ &= 0. \end{aligned}$$

The Discrete Constrained Optimization Problem

To summarize, after performing the above discretization steps, we are faced with the following equality constrained nonlinear optimization problem: Minimize

$$J_d(q_d, f_d) = \sum_{k=0}^{N-1} C_d(q_k, q_{k+1}, f_k, f_{k+1}) \quad (10)$$

with respect to f_d, subject to the constraints $q_0 = q^0$, $q_N = q^1$ and

$$\begin{aligned} D_2 L(q_0, \dot{q}_0) + D_1 L_d(q_0, q_1) + f_0^- &= 0, \\ D_2 L_d(q_{k-1}, q_k) + D_1 L_d(q_k, q_{k+1}) + f_{k-1}^+ + f_k^- &= 0, \\ -D_2 L(q_N, \dot{q}_N) + D_2 L_d(q_{N-1}, q_N) + f_{N-1}^+ &= 0, \end{aligned}$$

$k = 1, \dots, N-1$.

Linearization

One major numerical problem in a direct application of the numerical scheme described above (of any scheme, in fact) lies in the fact that the scales of interest differ by a factor of around 10^{17}: the distance between the Sun and the Earth is of the order of 10^{11} m, while the distances between the spacecraft are of the order of several 100 m and have to be kept constant up to an error of 10^{-6} m. When using standard double-precision floating point arithmetic, rounding errors will notably influence any corresponding computation. On the other hand, we are interested in the relative positions of the spacecraft with respect to each other only.

We will therefore perform our computations in a local coordinate system by linearizing the system around a Halo-orbit. Let $(q_k^H, \dot{q}_k^H)$, $k = 1, \dots, N$, be points on given Halo-orbit (a Halo-orbit of the family shown in Figure 2). Writing

$\tilde{q}_k = q_k - q_k^H$, the linearized constraints for the discrete optimization problem read

$$D_1 D_2 L(q_0^H, \dot{q}_0^H)\tilde{q}_0 + D_2 D_2 L(q_0^H, \dot{q}_0^H)(\dot{q}_0 - \dot{q}_0^H)$$
$$+ D_1 D_1 L_d(q_0^H, q_1^H)\tilde{q}_0 + D_2 D_1 L_d(q_0^H, q_1^H)\tilde{q}_1 + f_0^- = 0,$$

$$D_1 D_2 L_d(q_{k-1}^H, q_k^H)\tilde{q}_{k-1} + D_2 D_2 L_d(q_{k-1}^H, q_k^H)\tilde{q}_k$$
$$+ D_1 D_1 L_d(q_k^H, q_{k+1}^H)\tilde{q}_k + D_2 D_1 L_d(q_k^H, q_{k+1}^H)\tilde{q}_{k+1}$$
$$+ f_{k-1}^+ + f_k^- = 0,$$

for $k = 1, \ldots, N-1$, and finally

$$-D_1 D_2 L(q_N^H, \dot{q}_N^H)\tilde{q}_N$$
$$-D_2 D_2 L(q_N^H, \dot{q}_N^H)(\dot{q}_N - \dot{q}_N^H) + D_1 D_2 L_d(q_{N-1}^H, q_N^H)\tilde{q}_{N-1}$$
$$+ D_2 D_2 L_d(q_{N-1}^H, q_N^H)\tilde{q}_N + f_{N-1}^+ = 0.$$

VI. Example computations

As mentioned in the introduction, we focus on an application scenario that is directly motivated by the Darwin and TPF missions: We consider a group of six spacecraft in the vicinity of a Halo-orbit and require the spacecraft to adopt a planar hexagonal formation with center on the orbit. Figure 3 shows the Halo (in normalized coordinates) that we have chosen for this computation and the part of the orbit that we used for the linearization of the problem. In our computations

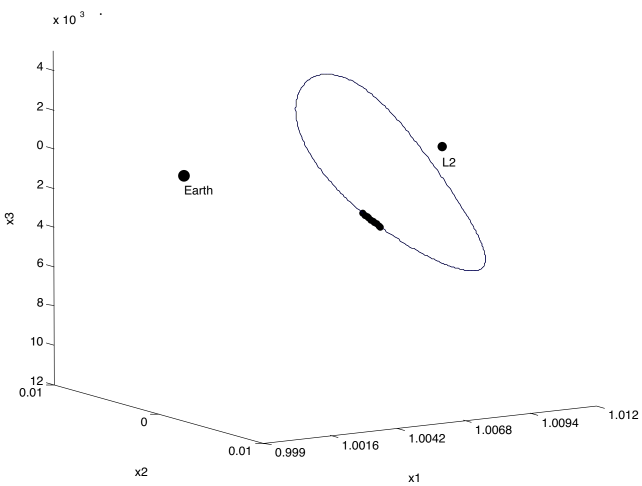

Fig. 3. L_2 Halo-orbit as chosen for the example computations (thin line) and the part used for the linearization (thick line).

we used $N = 10$ time intervals and solved the resulting finite dimensional constrained optimization problems by the SQP-method as implemented in the routine `E04UEF` of the NAG-library.

Discrete Mechanics vs. Midpoint Rule Discretization

We first compare our discretization scheme to a finite difference approach, where the dynamical constraints are discretized by applying the Midpoint Rule to the associated ordinary differential equations of the system (i.e. the forced Euler-Lagrange equations). In this case the constraints read

$$x_{k+1} - x_k - h\,F\left(\frac{x_{k+1}+x_k}{2}, \frac{f_{k+1}+f_k}{2}\right) = 0,$$

$k = 0, \ldots, N-1$, where $x_k = (q_k, \dot{q}_k)$ and F denotes the vector field of the forced Euler-Lagrange equations. Since these equations are dependent on $x_k \in TQ$ instead of $q_k \in Q$, we obtain an optimization problem with twice as many unknowns as in our approach. Therefore, the application of the midpoint rule results in a significantly higher numerical effort.

As the collision avoidance strategy we here used the approach based on the artifical potential.

Both methods result in almost identical solutions. Figure 4 shows (in normalized coordinates) the inital positions ($\times$), the optimal trajectories as well as the final positions ($\circ$)—here we only plot the data for the discrete mechanics solution since the one for the other method is visually identical. The group initially is located along a line with initial orientation $p_0^{(i)} = (cos(\frac{\pi}{2}), sin(\frac{\pi}{2}) \cdot (1,0,0))$ for each spacecraft i (i.e. a rotation of $\theta = \pi$ around the x_1-axis) and ends in a hexagonal formation in the plane with normal $n = (1,0,1)$ and final orienation $p_f^{(i)} = (cos(\pi), sin(\pi) \cdot (0,1,0))$ for each spacecraft i (i.e. a rotation of $\theta = 2\pi$ around the x_2-axis). In Figure 5 we compare the cost within each time interval for the individual spacecraft. With both methods, the overall cost is equal to $J = 1.792 \cdot 10^{-12}$.

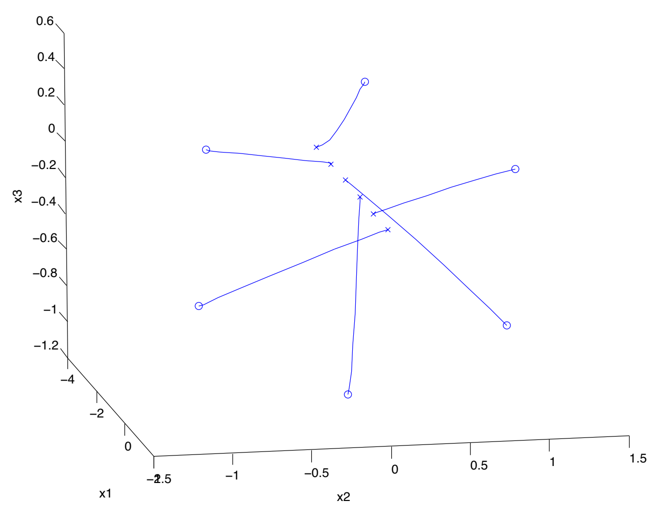

Fig. 4. Initial positions ($\times$), optimal trajectories and final positions ($\circ$) for a reconfiguration of six spacecraft.

As second example we computed optimal trajectories for a randomly chosen initial configuration and a final hexagonal formation in the $x_1 - x_2$-plane. Figure 6 shows the trajectories of the group for the discrete mechanics method. The overall cost is $J = 2.269 \cdot 10^{-12}$ for both methods.

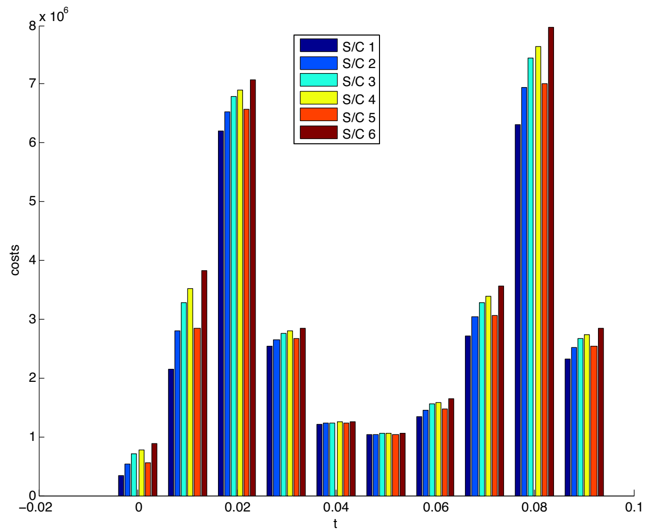

Fig. 5. Cost accumulated over each time interval of the individual spacecraft for the reconfiguration maneuver in Figure 4.

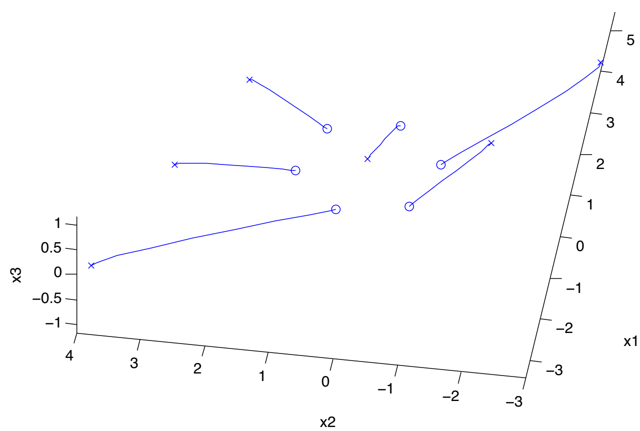

Fig. 6. Time evolution of the positions of the spacecraft (discrete mechanics discretization).

Comparing Collision Avoidance Strategies

As a second numerical test we compare the two collision avoidance strategies described in Section IV. As shown in Figure 7, all strategies result in the same final configuration. Also, the overall cost based on the penalty term in the cost function (excluding the contribution from the penalty term) is equal to the one obtained using the artificial potential function.

VII. Conclusion

We employed a recently developed approach for the numerical treatment of optimal control problems for mechanical systems in order to compute optimal open loop control laws for the reconfiguration of a group of formation flying satellites. Our numerical results indicate that the new method performs equally well as a standard finite difference approach, while the numerical effort is significantly lower since the number of state variables is only half as large.

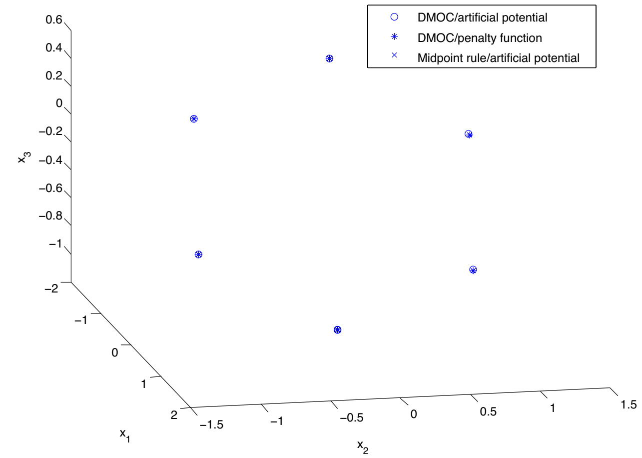

Fig. 7. Comparing the collision avoidance strategies: Final positions of the spacecraft for the reconfiguration maneuver in Figure 4.

VIII. Acknowledgments

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