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Joint Integrated PDA Avoiding Track Coalescence

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Abstract— **Joint PDA approach is very effective in tracking multiple targets in data association environment, with clutter measurements and missed detections. Joint IPDA has built upon this by including the probability of target existence as a track quality measure to enable automatic tracking and track maintenance in data association environment. Both JPDA and JIPDA suffer from the problem of track coalescence, where tracks following targets whose trajectories don't separate for extended time start following ″center″ of these targets. JPDA* is an extension of JPDA which combats coalescence by pruning possible measurement to track allocation hypotheses. Following JPDA* derivation, this paper derives JIPDA*, an extension of JIPDA which also combats track coalescence. JIPDA* updates the probability of target existence as the track quality measure. A simulation study verifies the effectiveness of this approach and compares JIPDA* with JIPDA and IPDA when tracking crossing targets in an environment of heavy clutter.**

I. Introduction

In many radar, sonar and other target tracking applications, measurements (detections) may originate from targets, whose existence and trajectory are generally not known *a priori* as well as from other random sources, usually termed clutter. Target measurements are only present in each scan with some probability of detection < 1. In a multi-target situation, the measurements may also have originated from one of various targets. The number of targets in the surveillance area is unknown. Automatic tracking in this environment initiates and maintains tracks using both target and clutter measurements. If a track follows a target, we call it a true track otherwise we call it a false track. To discriminate between true and false tracks, an appropriate track quality measure has to estimated simultaneously with track maintenance. Moreover the possibility that a measurement may have originated from another target not being followed by the current track has to be taken into account.

Single target tracking (STT) algorithms which use an appropriate track quality measure are for example IPDA and related algorithms [1], [2], [4], [5], GPB1-PDA [6], [7] and IMM-PDA [3]. These however ignore the challenges posed by other targets. Multi-target tracking (MTT) algorithms allow for the possibility that measurements may have arisen from the targets being followed by other tracks. Optimum all-neighbours MTT forms all possible joint measurement-to-track assignment hypotheses and recursively calculates their *a posteriori* probabilities. Joint PDA (JPDA) [8], [9] is one of the best known algorithms following this approach. JPDA incorporates all-neighbours single Gaussian probability density function PDA approximation with the Bayesian data association paradigm. However, JPDA essentially assumes that all tracks are true tracks. Using the probability of target existence paradigm [1], JIPDA [11] integrates the probability of target existence estimation with JPDA track updating.

Both JPDA and JIPDA show remarkable resistance to clutter and missed detections. However when tracking near targets, they both tend to coalesce tracks. A recent algorithm for multi target tracking in clutter, JPDA* [10] improves JPDA considerably in this respect, while still retaining resistance to clutter and missed detections. The aim of this paper is to combine the JIPDA development of [11] with the JPDA* development of [10]. In order to accomplish this, the JIPDA problem formulation is first integrated with the descriptor system approach of [10]. Subsequently the paper develops JIPDA* algorithm for tracking multiple targets in clutter, with integrated track quality measure and with track coalescence resistance of JPDA*.

The paper is organized as follows. Section II defines the problem considered. Section III embeds the tracking problem into one of filtering for a jump linear descriptor system with stochastic i.i.d. coefficients. In section IV exact Bayesian and JPDA* filter equations are developed. Section V illustrates the advantages of JIPDA* using a simulation study. Section VI draws conlusions.

II. Stochastic Modelling

This section describes the target existence model, the potential target model and the measurement model.

A. Target existence model

Tracks may be established using clutter measurements, or they can start following clutter measurements. Thus, the existence of a target being followed by each track is a random event. We note two models for target existence [1] propagation. Markov Chain One model assumes that existing target is always detectable. Markov Chain Two model also allows for the possibility that the target exists and is temporarily not detectable. In this text we will derive formulae for the case of Markov Chain One model for track existence propagation. For each track i, random event $\xi_{i,t}$ will describe target existence at time t:

$$\begin{aligned} \xi_t^i &= 1 \quad \text{if the target exists} \\ \xi_t^i &= 0 \quad \text{if the target does not exist.} \end{aligned}$$

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Markov Chain One transition probability of target existence for track i satisfies

$$\begin{aligned}
&P\{\xi_t^i = 1|\xi_{t-1}^i = 1\} = p_{11} \in (0,1)\\
&P\{\xi_t^i = 0|\xi_{t-1}^i = 1\} = 1 - p_{11}\\
&P\{\xi_t^i = 1|\xi_{t-1}^i = 0\} = 0\\
&P\{\xi_t^i = 0|\xi_{t-1}^i = 0\} = 1
\end{aligned}$$

The last two equations mean that from the moment t on that an existing target i becomes non-existing, it will remain forever non-existing i.e. if $\xi_t^i = 0$ then $\xi_s^i = 0$ for all $s \geq t$.

B. Potential target model

Consider $M_t = M$ potential targets (tracks) at moment t. Denote with $\xi_t \triangleq \text{Col}\{\xi_{1,t}, \cdots, \xi_{M,t}\}$ target existence indicator vector at moment t. Assume that the state of the i-th potential target is modelled as a linear system:

$$x_t^i = a^i x_{t-1}^i + b^i w_t^i, \quad i = 1, \cdots, M, \tag{1}$$

where x_t^i is the n-vectorial state of the i-th potential target, a^i and b^i are $(n \times n)$- and $(n \times n')$-matrices, and w_t^i is a sequence of i.i.d. standard Gaussian variables of dimension n' with w_t^i , w_t^j independent for all $i \neq j$ and w_t^i ,x_0^i, x_0^j independent for all $i \neq j$. Let $x_t \triangleq \text{Col}\{x_t^1, \cdots, x_t^M\}$, $A \triangleq \text{Diag}\{a^1, \cdots, a^M\}$, $B \triangleq \text{Diag}\{b^1, \cdots, b^M\}$, and $w_t \triangleq \text{Col}\{w_t^1, \cdots, w_t^M\}$. If we assume that from moment t to moment $t+1$ none of the tracks is deleted, and there are no track births, then $M_{t+1} = M_t = M$ and the state of our M potential targets as follows:

$$x_t = Ax_{t-1} + Bw_t \tag{2}$$

with A of size $Mn \times Mn$ and B of size $Mn \times Mn'$.

C. Measurement Model

A set of measurements consists of measurements originating from potential targets and measurements originating from clutter.

1) Potential measurements originating from potential targets: We assume that a potential measurement associated with state x_t^i (which we will denote by z_t^i) is modelled as a linear system:

$$z_t^i = h^i x_t^i + g^i v_t^i, \quad i = 1, \cdots, M \tag{3}$$

where z_t^i is an m-vector, h^i is an $(m \times n)$-matrix and g^i is an $(m \times m')$-matrix, and v_t^i is a sequence of i.i.d. standard Gaussian variables of dimension m' with v_t^i and v_t^j independent for all $i \neq j$. Moreover v_t^i is independent of x_0^j and w_t^j for all i,j. Next with $z_t \triangleq \text{Col}\{z_t^1, \cdots, z_t^M\}$, $H \triangleq \text{Diag}\{h^1, \cdots, h^M\}$, $G \triangleq \text{Diag}\{g^1, \cdots, g^M\}$, and $v_t \triangleq \text{Col}\{v_t^1, \cdots, v_t^M\}$, we obtain:

$$z_t = Hx_t + Gv_t \tag{4}$$

with H and G of size $Mm \times Mn$ and $Mm \times Mm'$ respectively. Because of notational simplicity we assume h^i, g^i, H and G to be time-invariant.

2) Detections originating from existing targets: We next introduce a model that takes into account that not all targets have to be detected at moment t, which implies that not all potential measurements z_t^i have to be available as true measurements at moment t. To this end, let $\phi_{i,t} \in$\{0,1\} be the existence and detection indicator for potential target i, which satisfies:

$$\begin{aligned}
\phi_{i,t} &= 1 \quad \text{if } \xi_{1,t} = 1 \text{ \underline{and} } z_t^i \text{ is detected}\\
\phi_{i,t} &= 0 \quad \text{if } \xi_{1,t} = 1 \text{ \underline{and} } z_t^i \text{ is undetected}\\
\phi_{i,t} &= 0 \quad \text{if } \xi_{1,t} = 0
\end{aligned}$$

We capture this through the following equation

$$\phi_{i,t} = \xi_{i,t}\delta_{i,t} \tag{5}$$

with the $(0,1)$ valued $\{\delta_{i,t}\}$ a sequence of i.i.d. random variables satisfying

$$\begin{aligned}
\text{Prob}\{\delta_{i,t} = 1\} &= P_{\text{d}}^i \in (0,1)\\
\text{Prob}\{\delta_{i,t} = 0\} &= 1 - P_{\text{d}}^i
\end{aligned}$$

where P_{d}^i is the conditional detection probability of potential target i given target i exists. Hence the conditional probability distribution of ϕ_t^i given $\xi_t^i = \xi^i \in \{0,1\}$ satisfies

$$P\{\phi_t^i = \phi^i|\xi_t^i = \xi^i\} = (1 - \xi^i P_{\text{d}}^i)^{1-\phi^i}(\xi^i P_{\text{d}}^i)^{\phi^i} \tag{6}$$

This approach yields the following existence and detection indicator vector ϕ_t of size M:

$$\phi_t \triangleq \text{Col}\{\phi_{1,t}, ..., \phi_{M,t}\}.$$

The number of existing and detected targets is $D_t \triangleq \sum_{i=1}^{M} \phi_{i,t} = \xi_t^T \delta_t$.
In order to link the existence and detection indicator vector with the measurement model, we introduce the following operator Φ: for an arbitrary (0,1)-valued M'-vector ϕ' we define $D(\phi') \triangleq \sum_{i=1}^{M'} \phi_i'$ and the operator Φ producing $\Phi(\phi')$ as a $(0,1)$-valued matrix of size $D(\phi') \times M'$ of which the ith row equals the ith non-zero row of $\text{Diag}\{\phi'\}$. Next we define, for $D_t > 0$, a vector that contains all measurements originating from targets at moment t in a fixed order.

$$\tilde{z}_t \triangleq \underline{\Phi}(\phi_t)z_t, \text{ where } \underline{\Phi}(\phi_t) \triangleq \Phi(\phi_t) \otimes I_m,$$

with I_m a unit-matrix of size m, and $\otimes$ the Kronecker product.

In reality we do not know the order of the targets. Hence, we introduce the stochastic $D_t \times D_t$ permutation matrix χ_t, which is conditionally independent of $\{\phi_t\}$. We also assume that $\{\chi_t\}$ is a sequence of independent matrices. Hence, for $D_t > 0$,

$$\tilde{\tilde{z}}_t \triangleq \underline{\chi}_t \tilde{z}_t, \text{ where } \underline{\chi}_t \triangleq \chi_t \otimes I_m,$$

is a vector that contains all measurements originating from targets at moment t in a random order.

3) Measurements originating from clutter: Let the random variable F_t be the number of false measurements at moment t. We assume that F_t has Poisson distribution:

$$\begin{aligned} P_{F_t}\{F\} &= \tfrac{(\lambda V)^F}{F!}\exp\big(-\lambda V\big), \qquad F \geq 0 \\ &= 0, \qquad \text{else} \end{aligned}$$

where λ is the spatial density of false measurements and V is the volume covered by the sensor. Hence, λV is the expected number of false measurements. A column-vector v_t^* of F_t i.i.d. false measurements has the following probability density function:

$$p_{v_t^*|F_t}(v^*|F) = V^{-F}.$$

Furthermore we assume that the process $\{v_t^*\}$ is a sequence of independent vectors, which are independent of $\{x_t\}, \{w_t\}, \{v_t\}$ and $\{\phi_t\}$.

4) Random insertion of clutter measurements: Let the random variable L_t be the total number of measurements at moment t. Thus,

$$L_t = D_t + F_t$$

With $\tilde{\mathrm{y}}_t \triangleq \mathrm{Col}\{\tilde{\tilde{z}}_t, v_t^*\}$, it follows with the above defined variables that

$$\tilde{\mathrm{y}}_t = \begin{bmatrix} \underline{\chi}_t \underline{\Phi}(\phi_t) z_t \\ \cdots\cdots\cdots \\ v_t^* \end{bmatrix}, \text{ if } L_t > D_t > 0 \tag{7}$$

whereas the upper and lower subvector parts disappear for $D_t = 0$ and $L_t = D_t$ respectively. With this equation, the measurements originating from clutter still have to be randomly inserted between the measurements originating from the detected targets. To do so, we first define target indicator and clutter indicator processes, denoted by $\{\psi_t\}$ and $\{\psi_t^*\}$, respectively:

$$\begin{aligned} \psi_t^i &= 1 \quad \text{if measurement } i \text{ is detection} \\ \psi_t^i &= 0 \quad \text{if measurement } i \text{ is clutter} \\ \psi_t^{*i} &= 1 \quad \text{if measurement } i \text{ is clutter} \\ \psi_i^{*i} &= 0 \quad \text{if measurement } i \text{ is detection} \end{aligned}$$

Thus $\psi_{i,t}^* = 1 - \psi_{i,t}$. This approach yields the following indicator vectors

$$\begin{aligned} \psi_t &\triangleq \mathrm{Col}\{\psi_{1,t}, \cdots, \psi_{L_t,t}\} \\ \psi_t^* &\triangleq \mathrm{Col}\{\psi_{1,t}^*, \cdots, \psi_{L_t,t}^*\}. \end{aligned}$$

The measurement vector with clutter inserted is:

$$\mathrm{y}_t = \left[\underline{\Phi}(\psi_t)^T \vdots \underline{\Phi}(\psi_t^*)^T\right] \tilde{\mathrm{y}}_t \quad \text{if } L_t > D_t > 0 \tag{8}$$

Substituting (7) into (8) yields the following model for the observation vector y_t at moment t:

$$\mathrm{y}_t = \left[\underline{\Phi}(\psi_t)^T \vdots \underline{\Phi}(\psi_t^*)^T\right] \begin{bmatrix} \underline{\chi}_t \underline{\Phi}(\phi_t) z_t \\ \cdots\cdots\cdots \\ v_t^* \end{bmatrix} \text{ if } L_t > D_t > 0 \tag{9}$$

This, together with equations (2) and (4), forms a complete characterization of the multitarget scenario in terms of a system of stochastic difference equations.

III. EMBEDDING INTO A DESCRIPTOR SYSTEM WITH STOCHASTIC COEFFICIENTS

Because $\left[\underline{\Phi}(\psi_t)^T \vdots \underline{\Phi}(\psi_t^*)^T\right]$ is a permutation matrix for $L_t > D_t > 0$, its inverse equals its transpose and satisfies

$$\left[\underline{\Phi}(\psi_t)^T \vdots \underline{\Phi}(\psi_t^*)^T\right]^{-1} = \begin{bmatrix} \underline{\Phi}(\psi_t) \\ \cdots \\ \underline{\Phi}(\psi_t^*) \end{bmatrix} \tag{10}$$

Premultiplying (9) by such inverse yields

$$\begin{bmatrix} \underline{\Phi}(\psi_t) \\ \cdots \\ \underline{\Phi}(\psi_t^*) \end{bmatrix} \mathrm{y}_t = \begin{bmatrix} \underline{\chi}_t \underline{\Phi}(\phi_t) z_t \\ \cdots\cdots\cdots \\ v_t^* \end{bmatrix} \quad \text{if } L_t > D_t > 0 \tag{11}$$

From (11), it follows that

$$\underline{\Phi}(\psi_t)\mathrm{y}_t = \underline{\chi}_t \underline{\Phi}(\phi_t) z_t \quad \text{if } D_t > 0 \tag{12}$$

Substitution of (4) into (12) yields:

$$\underline{\Phi}(\psi_t)\mathrm{y}_t = \underline{\chi}_t \underline{\Phi}(\phi_t) H x_t + \underline{\chi}_t \underline{\Phi}(\phi_t) G v_t \quad \text{if } D_t > 0 \tag{13}$$

Notice that (13) is a linear Gaussian descriptor system [12] with stochastic i.i.d. coefficients $\underline{\Phi}(\psi_t)$ and $\underline{\chi}_t \underline{\Phi}(\phi_t)$. Because χ_t has an inverse, (13) can be transformed into

$$\underline{\chi}_t^T \underline{\Phi}(\psi_t)\mathrm{y}_t = \underline{\Phi}(\phi_t) H x_t + \underline{\Phi}(\phi_t) G v_t \quad \text{if } D_t > 0 \tag{14}$$

Next we introduce an auxiliary indicator matrix process $\tilde{\chi}_t$ of size $D_t \times L_t$, as follows:

$$\tilde{\chi}_t \triangleq \chi_t^T \Phi(\psi_t) \quad \text{if } D_t > 0.$$

With this we get a simplified version of (14):

$$\underline{\tilde{\chi}}_t \mathrm{y}_t = \underline{\Phi}(\phi_t) H x_t + \underline{\Phi}(\phi_t) G v_t \quad \text{if } D_t > 0 \tag{15}$$

where $\underline{\tilde{\chi}} \triangleq \tilde{\chi} \otimes I_m$. Size of $\underline{\tilde{\chi}}_t$ is $D_t m \times L_t m$ and size of $\underline{\Phi}(\phi_t)$ is $D_t m \times Mm$.

IV. TRACKING FILTER EQUATIONS

In this section we present a Bayesian characterization of the track state in (2), conditional on the σ-algebra generated by measurements y_t up to and including moment t, denoted here by Y_t.

From (15), it follows that for $D_t > 0$ all relevant associations and permutations can be covered by $(\phi_t, \tilde{\chi}_t)$-hypotheses. We extend this to $D_t = 0$ by adding the combination $\phi_t = \{0\}^M$ and $\tilde{\chi}_t = \{\}^{L_t}$. Hence, through defining the weights

$$\beta_t(\xi, \phi, \tilde{\chi}) \triangleq P\{\xi_t = \xi, \phi_t = \phi, \tilde{\chi}_t = \tilde{\chi} | Y_t\},$$

the law of total probability yields:

$$P\{\xi_t^i = 1 | Y_t\} = \sum_{\substack{\xi,\phi,\tilde{\chi} \\ \xi^i = 1}} \beta_t(\xi, \phi, \tilde{\chi}) \tag{16}$$

$$\begin{aligned} p_{x_t^i|\xi_t^i=1,Y_t}(x^i) \;&=\; \sum_{\tilde{\chi},\phi} P\{\phi_t = \phi, \tilde{\chi}_t = \tilde{\chi} | \xi_t^i = 1, Y_t\} \cdot \\ &\cdot p_{x_t^i|\xi_t^i,\phi_t,\tilde{\chi}_t,Y_t}(x^i|1, \phi, \tilde{\chi}) \end{aligned} \tag{17}$$

Our goal is to characterize the terms in the last summation.

$$P\{\phi_t=\phi,\tilde{\chi}_t=\tilde{\chi}|\xi_t^i=1,Y_t\} = \frac{P\{\phi_t=\phi,\tilde{\chi}_t=\tilde{\chi},\xi_t^i=1|Y_t\}}{P\{\xi_t^i=1|Y_t\}}$$

$$P\{\phi_t=\phi,\tilde{\chi}_t=\tilde{\chi},\xi_t^i=1|Y_t\}=\sum_{\xi:\xi^i=1}\beta_t(\xi,\phi,\tilde{\chi})$$

Proposition 1: For any $\xi,\phi\in\{0,1\}^M$, such that $D(\phi)\leq L_t$, and any $\tilde{\chi}_t$ matrix realization $\tilde{\chi}$ of size $D(\phi)\times L_t$, the following holds true:

$$\begin{aligned} p_{x_t|\xi_t,\phi_t,\tilde{\chi}_t,Y_t}(x\mid\xi,\phi,\tilde{\chi}) &= \\ &= \frac{p_{\tilde{z}_t|x_t,\xi_t,\phi_t}(\underline{\tilde{\chi}}\mathrm{y}_t\mid x,\xi,\phi)\cdot p_{x_t|\xi_t,Y_{t-1}}(x\mid\xi)}{F_t(\xi,\phi,\tilde{\chi})} \end{aligned} \quad (18)$$

$$\begin{aligned} &\beta_t(\xi,\phi,\tilde{\chi})=F_t(\xi,\phi,\tilde{\chi})\lambda^{(L_t-D(\phi))}\cdot \\ &\cdot[\prod_{i=1}^{M}(1-\xi^iP_{\mathrm{d}}^i)^{(1-\phi^i)}(\xi^iP_{\mathrm{d}}^i)^{\phi^i}]\cdot p_{\xi_t|Y_{t-1}}(\xi)/c_t \end{aligned} \quad (19)$$

where $\underline{\tilde{\chi}}\triangleq\tilde{\chi}\otimes I_m$, and $F_t(\xi,\phi,\tilde{\chi})$ and c_t are such that they normalize $p_{x_t|\xi_t,\phi_t,\tilde{\chi}_t,Y_t}(x|\xi,\phi,\tilde{\chi})$ and $\beta_t(\xi,\phi,\tilde{\chi})$ respectively.
Proof: Omitted

Next we assume per track independent target existence and state density given existence:

$$p_{\xi_t|Y_{t-1}}\{\xi\}=\prod_{i=1}^{M}p_{\xi_t^i|Y_{t-1}}\{\xi^i\}$$

$$p_{x_t|\xi_t,Y_{t-1}}(x|\xi)=\prod_{i=1}^{M}p_{x_t^i|\xi_t^i,Y_{t-1}}\{x^i|\xi^i\}$$

This leads to the following Theorem and Corollary.

Theorem 1: Let $p_{\xi_t|Y_{t-1}}(\xi)=\prod_{i=1}^{M}p_{\xi_t^i|Y_{t-1}}(\xi^i)$ and let $p_{x_t|\xi_t,Y_{t-1}}(x|\xi)=\prod_{i=1}^{M}p_{x_t^i|\xi_t^i|Y_{t-1}}(x^i|\xi^i)$, then $\beta_t(\xi,\phi,\tilde{\chi})$ of proposition 1 satisfies:

$$\begin{aligned} \beta_t(\xi,\phi,\tilde{\chi}) &= \lambda^{L_t-D(\phi)}\cdot \\ \cdot\prod_{i=1}^{M}&\left[f_t^i(\phi,\tilde{\chi})(1-\xi^iP_{\mathrm{d}}^i)^{(1-\phi^i)}(\xi^iP_{\mathrm{d}}^i)^{(\phi^i)}\cdot p_{\xi_t^i|Y_{t-1}}(\xi^i)\right]/c_t \end{aligned}$$

with for $\phi^i=0$: $f_t^i(\phi,\tilde{\chi})=1$, and for $\phi^i=1$

$$f_t^i(\phi,\tilde{\chi})=p_{z_t^i|x_t^i,\phi_t}\left(\left[\Phi(\phi)^T\tilde{\chi}\right]_{ik}\mathrm{y}_t^k|x^i,\phi\right)$$

Moreover

$$p_{x_t|\xi_t,\phi_t,\tilde{\chi}_t,Y_t}(x|\xi,\phi,\tilde{\chi})=\prod_{i=1}^{M}p_{x_t^i|\xi_t^i,\phi_t,\tilde{\chi}_t,Y_t}(x^i|\xi^i,\phi,\tilde{\chi})$$

with:

$$\begin{aligned} &p_{x_t^i|\xi_t^i,\phi_t,\tilde{\chi}_t,Y_t}(x^i|\xi^i,\phi,\tilde{\chi})= \\ &=\frac{p_{z_t^i|x_t^i,\phi_t}\left(\left[\Phi(\phi)^T\tilde{\chi}\right]_{ik}\mathrm{y}_t^k|x^i,\phi\right)\cdot p_{x_t^i|\xi_t^i,Y_{t-1}}(x^i|1)}{f_t^i(\phi,\tilde{\chi})} \\ &\qquad\qquad\qquad\qquad\text{if } \phi^i=1 \text{ and } \xi^i=1 \\ &=p_{x_t^i|\xi_t^i,Y_{t-1}}(x^i|\xi^i)\qquad\quad\text{if } \phi^i=0 \text{ and/or } \xi^i=0 \end{aligned}$$

Proof: Omitted

Corollary 1: For each potential target i, the probability density function of trajectory state estimate, $p_{x_t^i|\xi_t^i=1,Y_t}$ is a mixture of probability density functions of trajectory state estimates, each calculated with the assumption that one of the measurements is the detection of the potential target i:

$$\begin{aligned} p_{x_t^i|\xi_t^i,Y_t}(x^i|1) &= \sum_{k=1}^{L_t}p_{x_t^i|\xi_t^i,z_t^i,Y_{t-1}}(x^i|1,\mathrm{y}_t^k)\cdot\beta_t^{ik}+ \\ &+p_{x_t^i|\xi_t^i,Y_{t-1}}(x^i|1)\cdot\beta_t^{i0} \end{aligned} \quad (20)$$

where $p_{x_t^i|\xi_t^i,z_t^i,Y_{t-1}}(x^i|1,\mathrm{y}_t^k)$ is the estimation pdf of potential target i given that it exists and that its' detection at time t was y_t^k. β_t^{ik} is the *a posteriori* probability that measurement k is a detection of an existing target i; and β_t^{i0} is the a posteriori probability that there is no detection of potential target i, given target i existence:

$$\begin{aligned} \beta_t^{ik} &\triangleq P\{[\Phi(\phi_t)^T\tilde{\chi}_t]_{ik}=1|\xi_t^i=1,Y_t\}= && (21)\\ &= \sum_{\substack{\phi,\tilde{\chi}\\ \xi:\xi^i=1}}[\Phi(\phi)^T\tilde{\chi}]_{ik}\beta_t(\xi,\phi,\tilde{\chi})/P\{\xi_i=1|Y_t\} \\ \beta_t^{i0} &\triangleq P\{\phi_t^i=0|\xi_t^i=1,Y_t\}= && (22)\\ &= \sum_{\substack{\tilde{\chi}\\ \phi:\phi^i=0\\ \xi:\xi^i=1}}\beta_t^1(\xi,\phi,\tilde{\chi})/P\{\xi_i=1|Y_t\} \end{aligned}$$

where $[\Phi(\phi_t)^T\tilde{\chi}_t]_{ik}=1$ under the hypotheses that potential target i exists, is detected, and measurement k applies.
Proof: Omitted

A. JIPDA

For each potential target, JIPDA approximates the track trajectory estimate pdf with a Gaussian function:

$$\begin{aligned} p_{x_t^i|\xi_t^i,Y_{t-1}}(x^i|1) &\approx N(x^i;\bar{x}_t^i,\bar{P}_t^i) \\ p_{x_t^i|\xi_t^i,Y_t}(x^i|1) &\approx N(x^i;\hat{x}_t^i,\hat{P}_t^i). \end{aligned}$$

Thus, $p_{x_t|\xi_t^i,Y_{t-1}}(x|1)$ is Gaussian with mean $\bar{x}_t=\mathrm{Col}\{\bar{x}_t^1,\cdots,\bar{x}_t^M\}$ and covariance $\bar{P}_t=\mathrm{Diag}\{\bar{P}_t^1,\cdots,\bar{P}_t^M\}$. Then $p_{x_t^i|\xi_{i,t}^i,Y_t}(x^i|1)$ is a Gaussian mixture, approximated with a single Gaussian which will preserve the overall mean $\hat{x}_t^i$ and its overall covariance $\hat{P}_t^i$:

$$\hat{x}_t^i = \bar{x}_t^i+W_t^i\cdot\left(\sum_{k=1}^{L_t}\beta_t^{ik}\mu_t^{ik}\right) \quad (23)$$

$$\begin{aligned} \hat{P}_t^i &= \bar{P}_t^i-W_t^ih^i\bar{P}_t^i\left(\sum_{k=1}^{L_t}\beta_t^{ik}\right)+ && (24)\\ &+W_t^i\left(\sum_{k=1}^{L_t}\beta_t^{ik}\mu_t^{ik}(\mu_t^{ik})^T\right)\cdot(W_t^i)^T- \\ &-W_t^i\left(\sum_{k=1}^{L_t}\beta_t^{ik}\mu_t^{ik}\right)\cdot\left(\sum_{k'=1}^{L_t}\beta_t^{ik'}\mu_t^{ik'}\right)^T(W_t^i)^T \end{aligned}$$

with:

$$\begin{aligned} W_t^i &\triangleq \bar{P}_t^i (h^i)^T [h^i \bar{P}_t^i (h^i)^T + g^i (g^i)^T]^{-1} \\ \mu_t^{ik} &= y_t^k - h^i \bar{x}_t^i \end{aligned}$$

B. JIPDA*

A shortcoming of JPDA and JIPDA is its sensitivity to track coalescence. Following the approach of JPDA* [10], JIPDA* filter equations are obtained from JIPDA algorithm by keeping only the strongest hypotheses with the common (ϕ, ψ), prior to updating track state, probability of target existence and estimate of trajectory state conditioned on target existence. In other words, keep the strongest hypotheses from each set of hypotheses having common set of detected tracks and allocated measurements.

For every ϕ and ψ find

$$\hat{\chi}_t(\xi, \phi, \psi) \triangleq \underset{\chi}{\text{Argmax}} \ \beta_t(\xi, \phi, \chi^T \Phi(\psi))$$

where the maximization is over all permutation matrices χ of size $D(\phi) \times D(\phi)$. Then the following values for data association probabilities are used

$$\hat{\beta}_t(\xi, \phi, \chi^T \Phi(\psi)) = \begin{cases} \hat{c}_t^{-1} \beta_t^1(\xi, \phi, \chi^T \Phi(\psi)), & \chi_t = \hat{\chi}_t(\phi, \psi) \\ 0, & \text{otherwise} \end{cases}$$

with $\hat{c}_t$ normalization constant such that

$$\sum_{\xi, \phi, \psi} \hat{\beta}_t(\xi, \phi, \chi^T \Phi(\psi)) = 1$$

V. Simulation Study

The purpose of these simulations are to compare the JIPDA* algorithm with JIPDA [11] and IPDA [1]. The focus of this study is on the false track discrimination and target crossing outcomes, in a heavy and non-homogeneous clutter environment. Non parametric versions [11] of the algorithms are compared.

A two-dimensional surveillance situation was considered. The area under surveillance was $1000m$ long and $400m$ wide. The false measurements satisfied a Poisson distribution with density $1.0 \cdot 10^{-4}$ /scan /m^2.

The experiments consisted of 1000 runs, with each run consisting of 50 scans. There are two targets in the surveillance region whose trajectories cross at scan 35 with the crossover angle of 10^o. Targets appear in scan one and move with the uniform motion, with target one having an initial state of $x_0 = [130m \ 15m/s \ 200m \ 0m/s]$. The other target also moves with an uniform motion with speed of $15m/s$. The true track situation is observed on scan 20 and then again on scan 50 to evaluate the crossover results.

False tracks are carried over from one simulation run to the other, while the true tracks are terminated at the end of each simulation run. The motion of each target is modeled in Cartesian coordinates as

$$x_t^i = a x_{t-1}^i + w_t^i \quad (25)$$

where x_t^i is the target state vector at time t and consists of the position and the velocity in each of the 2 coordinates

$$x' = \left[\begin{array}{cccc} x & \dot{x} & y & \dot{y} \end{array} \right] \quad (26)$$

with the transition matrix a

$$a = \left[\begin{array}{cc} a_T & 0 \\ 0 & a_T \end{array} \right]; \qquad a_T = \left[\begin{array}{cc} 1 & T \\ 0 & 1 \end{array} \right] \quad (27)$$

where T is the sampling period of 1s. The plant noise w_t is the zero mean white Gaussian noise with known covariance

$$E\left[w_t^i w_s^{iT}\right] = Q\, \delta\,(t - s) \quad (28)$$

where δ is the Dirac function and

$$Q = q \left[\begin{array}{cc} Q_T & 0 \\ 0 & Q_T \end{array} \right]; \ Q_T = \left[\begin{array}{cc} T^4/4 & T^3/2 \\ T^3/2 & T^2 \end{array} \right] \quad (29)$$

with $q = 0.75$. The detection probability was 0.9 throughout the experiment and the sensor introduced independent errors in the x and y coordinates with a root mean square of $5m$ in each coordinate. The tracking estimation filter was a simple Kalman filter based on the described trajectory and sensor models. The selection probability was set to $P_W = 0.99$. Tracks are initiated automatically in every scan, using two point differencing and initial track probability assignment as described in [5]. Selection gates of the algorithms were overlapping for more than 15 scans in each run.

In the experiments labelled "JIPDA", and "JIPDA*", IPDA was applied to unconfirmed tracks and JIPDA and JIPDA* were applied to the set of confirmed tracks respectively. Tracks are confirmed when the probability of target existence exceeds the confirmation threshold and are terminated if the probability falls below the termination threshold. The sum of confirmed false track scans was approximately equal for each simulation experiment and in the vicinity of 1 per 800 scans in each of the simulation experiments.

The crossover performance is presented in Table I. Only cases where two confirmed tracks were following each of the two targets at scan 20 were considered. The "Ambiguous" row in Table I lists the number of simulation runs in which one or both tracks were following both targets at scan 50. In this experiment, JIPDA* was unambiguously better than JIPDA. The percentage of successful crossovers was substantially higher. The frequency of merging was order of magnitude smaller, indicating coalescence resistance. Finally, IPDA did not have a single successful cross over case, showing the weakness of using single target tracking filter in complex multi target situations.

The true track confirmations are presented in Figure 1. Each curve shows the number of cases in which a confirmed track was following a target. For two targets and a thousand runs, 2000 indicates 100% success rate. The horizontal axis depicts the time in scans from the start of the simulation run. Again, JIPDA* shows the best performance, having lost

TABLE I

TARGET CROSSING OUTCOMES

	JIPDA*	JIPDA	IPDA
Total	816	815	811
both OK	579	372	0
one OK	105	244	453
both switch	76	2	0
one switch	51	196	358
both lost	5	1	0
Ambiguous	2	1	0
Merged	35	406	801

only 5% of tracks after crossover, compared to 19% and 32% in the case of JIPDA of IPDA respectively. Figure 2 shows estimation errors over time. Again, JIPDA* shows the smallest estimation errors after the crossover.

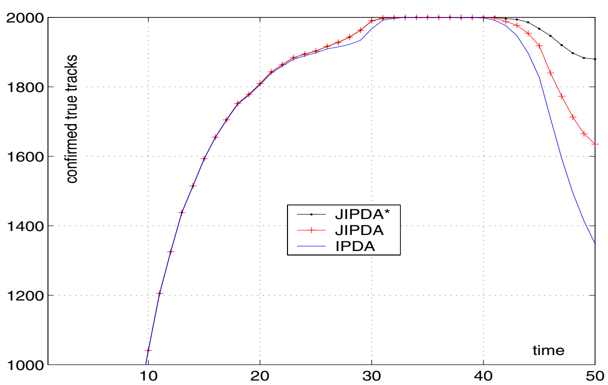

Fig. 1. Confirmed target tracks

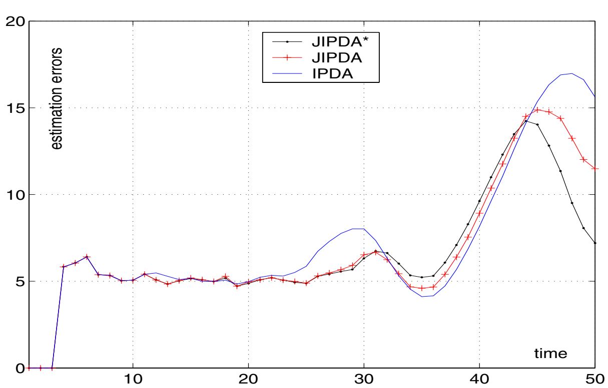

Fig. 2. RMS errors

VI. Conclusions

This paper has formulated the multi target tracking including track existence estimation within the descriptor system modelling approach of [10]. Subsequently this formulation has been used to characterize Bayesian filter recursions of the joint conditional density for multi target state and existence (Proposition 1 and Theorem 1). It has also been shown how the JIPDA filter equations of [11] can be obtained from this. Subsequently we applied the JPDA* hypotheses reduction approach of [10] to the descriptor formulated version of JIPDA. This comes down to pruning JIPDA permutation hypotheses in the sense of keeping the best permutation hypothesis only per combination of existing and detected targets and allocated measurements. In [10] it was shown that this kind of pruning results into an avoidance of track coalescence. Hence, the resulting filters are referred to as JPDA* and JIPDA*, where the * stand for "avoiding track coalescence".
Through Monte Carlo simulations with IPDA, JIPDA and JIPDA* on an illustrative example, the coalescence avoidance property of JIPDA* has been confirmed, and the potential benefits of using JIPDA* in difficult target crossing scenarios have been demonstrated.

There are many directions in which the results of this paper can be extended. One of our priorities is to incorporate IMM for maneuvering target tracking with JIPDA*, in a similar way as the descriptor system approach has been used to incorporate IMM with JPDA* [13], [14], [15].

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