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Robust Fault Diagnosis using Parallelotope-based Set-membership Consistency Tests

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Abstract— The paper proposes robust fault detection methods that take advantage of a recently proposed set-membership identification procedure based on parallelotopes for systems linear in the parameters. It is shown that consistency checks indicating faults can be performed in a natural manner with a parallelotope description of the feasible parameter set. Fault detection algorithms are presented for systems with invariant parameters, with parameter variation bounded between samples and with unbounded variation. Finally, an example is given which demonstrates how the algorithms work on a multivariable process.

I. INTRODUCTION

Model-based fault detection is based on the use of mathematical models of the monitored system. Reliability and performance of fault diagnosis algorithms depend on the quality of the model used. Fault detection algorithms can often be improved by improving the models they are based on. However, high fidelity models are costly and modelling errors and disturbances in complex engineering systems are inevitable. Hence there is a need to develop robust fault detection algorithms where model uncertainty is explicitly taken into account. The robustness of a fault detection system indicates its ability to distinguish between faults and model uncertainty, see [1].

Investigation of robust fault diagnosis has roughly focused on two distinct approaches. In one of the approaches, characterized as active, the central idea is to decouple the effect of the uncertainty [1]. The other approach, known as passive, is based on enhancing the robustness of the fault detection system at the decision-making stage [2]. The aim with the passive approach is usually to determine, given a set of models, if there is any member in the set that can explain the measurements. A common approach to this problem is to propagate the model uncertainty to the alarm limits of the residuals. When the residuals are outside of the alarm limits it is argued that model uncertainty alone can not explain the residual and therefore a fault must have occurred, see [3] for

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a recent article using this approach. This approach has the drawback that faults that produce a residual deviation smaller than the residual uncertainty due to parameter uncertainty will not be detected.

Another approach to the passive robust fault detection problem is to explicitly calculate the set of parameters that are consistent with the measurements [4]. When a measurement is found to be inconsistent with this set, a fault is assumed to have occurred. As an exact representation of the set of parameters consistent with the measurements is hard to calculate, outer bounds are often used instead. This is the approach adopted in the present work. Other authors have used this approach, see for example [5]. There, the feasible parameter set was approximated with an ellipsoid. In [6] a similar fault detection scheme based on orthotopic sets was presented.

In the current article, the parameter set is bounded with parallelotopes. In [7] a set-membership identification algorithm was presented which results in a parallelotopic representation of the parameter uncertainty. In [8] an extension was presented to deal with time variant systems. It will be shown that this representation is particularly suitable for fault detection based on consistency tests.

Set-membership identification methods have been the subject of a number of publications. They can be classified according to how the approximation of the feasible set of parameters is represented or parameterized. In [9] the set was over bounded by an ellipsoid. Other authors have focused on orthotopic approximations, see [10].

When using set-membership identification there is a trade-off between set size (conservativeness) and complexity of the identification method. Simpler methods generally lead to more conservative set estimates. In [7] it was claimed that parallelotopic estimates may be consistently better than ellipsoidal estimates while complexity is similar. As better estimates lead directly to a better tradeoff between false alarms and missed detections, it is clear that parallelotopic estimates deserve attention from the fault detection community.

In [11] a consistency based fault detection scheme was presented which used the recursive optimal bounding parallelotope (ROBP) algorithm presented in [12]. They proposed a moving horizon strategy where an outer bound of the initial state was propagated using the ROBP and a fault was detected when no noise sequence within deterministic bounds could explain the observed data. Uncertainty in process parameters was not considered. Robust fault diagnosis using parallelotope estimates of parameter uncertainty has not been

proposed to the knowledge of the authors.

The problem of parameter tracking or "recovering" the parameter estimate is important to isolate the type and to estimate the size of fault that has occurred once a fault has been detected since allows to know where in the parameter space the parameter set has moved. In [13] this problems was discussed and will be addressed as well in the current article. In [7] it was claimed that the parallelotope method was more suitable to recover new parameter estimates after a large change in parameters had occurred.

The paper is organized in the following manner. In section II, the problem is introduced. In section III, polytopes are introduced and tools to manipulate them discussed. In section IV, the fault detection strategies are introduced. Finally in section V, an example is given.

II. MODEL-BASED CONSISTENCY DIAGNOSIS

The principle of *model-based consistency diagnosis* is to test whether the measured sequence of inputs U and outputs Y available for N points, available at every time instant k from the system lies within the behaviour described by a model of the faultless system. If the measurements are inconsistent with the model of the faultless system, the existence of a fault is proved concluding the fault detection task. For isolation and fault identification, information about the effects of the faults under consideration has to be available, allowing application of the same principle as in the case of fault detection.

In this paper it is assumed that the system output can be described by

$$y(k) = \varphi^T(k)\theta(k) + e(k) \qquad (1)$$
$$\theta(k+1) = \theta(k) + w(k) \qquad (2)$$
$$\theta(k) \in \Theta \qquad (3)$$

where $\theta(k) \in \mathbb{R}^n$ is the parameter vector whose values are assumed to be unknown but to belong to a compact bounded initial set Θ, $\varphi(k) \in \mathbb{R}^n$ is the regressor vector which can contain any function of inputs and outputs, and the two noise terms are limited as

$$|e(k)| \le \sigma \quad \text{and} \quad |w(k)| \le \lambda \qquad (4)$$

As the parameter vector is assumed to belong to $\mathbb{R}^n$ so does λ and the last inequality is an element wise inequality. Notice that this system description includes any system linear in the parameters. Parameter uncertainty comes from physical modelling or from the set-membership parameter estimation algorithms applied in a non-faulty situation.

Notice that Eq. (2) specifies the allowed variance of uncertain parameters θ. Depending on the value of λ, three different cases can be considered:

- Time invariant case, $\lambda = 0$
- Time-varying case 1, $\lambda = \bar{\lambda}$
- Time-varying case 2, $\lambda = \infty$

In the first case, the parameter is unknown within Θ but it is known that it will not vary. In the second case, the parameter variation is bounded specifically by a vector $\bar{\lambda}$ while in the

last case, the variation is implicitly bounded only by the initial parameter set Θ and can vary at will within that set.

The first case could represent situations when an initial variance comes from components specifications that are known only with a mean and variance in the beginning of the fault detection. The second case could represent a system that has been identified over a number of operation conditions, each with a different θ within Θ, but with the variance between samples bounded by $\bar{\lambda}$.

A. Fault detection

A definition of what constitute a fault for a series of data is now given. From the model description above the following sequences are defined.

$$\Phi_N = \{\varphi(k)\}_{k=1,\ldots,N} \quad Y_N = \{y(k)\}_{k=1,\ldots,N} \qquad (5)$$

To define what constitute a fault the feasible solutions set at time N is defined.

Definition 1: (**Feasible Solution Set**) For given data sequences Φ_N and Y_N, the parameter θ is said to belong to the *Feasible Solution Set* at time N, (denoted FSS_N), if there exist $\theta(1), \theta(2), \ldots, \theta(N)$ such that:

- $\theta = \theta(N)$ (6)
- $|y(k) - \varphi^T(k)\theta(k)| < \sigma \quad k = 1, \ldots, N$ (7)
- $|\theta(k) - \theta(k-1)| < \lambda \quad k = 2, \ldots, N$ (8)
- $\theta(k) \in \Theta \quad k = 1, \ldots, N$ (9)

Using the definition of the feasible solution set, a fault is now defined for the sequences Φ_N and Y_N and considering that each measurement gives a set of feasible parameters that are consistent with it:

$$F_k = \{\theta \in \mathbb{R}^n : -\sigma \le y(k) - \varphi(k)^T\theta \le \sigma\} \qquad (10)$$

F_k is the region between two hyperplanes. The normalized form of the strip is written as

$$F_k = \{\theta \in \mathbb{R}^n : |\frac{y(k)}{\sigma} - \frac{\varphi(k)}{\sigma}^T \theta| \le 1\}$$
$$= \{\theta \in \mathbb{R}^n : |c^T\theta - d| \le 1\} \qquad (11)$$

Definition 2: For given data sequences Φ_N and Y_N, a *fault* is said to have occurred if the set FSS_N is empty.

In practice, the computation of FSS_N is difficult. The fault detection algorithms presented in this paper are based on using an approximated feasible solution set, AFSS_N, that fulfills $\mathrm{FSS}_N \subseteq \mathrm{AFSS}_N$ for which consistency is checked. In the case when $\lambda > 0$, the set AFSS_N is expanded to take the allowed parameter variance into account in the next sample. The expanded set is denoted $\overline{\mathrm{AFSS}}_{N+1}$.

A general form of the suggested fault detection algorithms is to execute the following procedures at each sample:

1: Expand AFSS_{N-1} taking into account λ to obtain $\overline{\mathrm{AFSS}}_N$.
2: Obtain input-output data and build F_N.
3: If $\overline{AFSS}_N \cap F_N = \emptyset$ conclude that a fault has occurred
4: calculate AFSS_N that fulfills $F_N \cap \overline{\mathrm{AFSS}}_N \subset \mathrm{AFSS}_N$

In this paper, the approximate feasible solution set will be represented with a parallelotope.

B. Fault-isolation and estimation task

Once the fault has been detected, the next task consists in fault isolation and estimation. Fault isolation and estimation are very straightforward since it just consists in looking where in the parameter space the parameter set has moved. This can be done by resetting the feasible parameter set to a set that contains all possible values even in faulty situation and applying algorithm presented in section II-A. Then, by iteratively applying the refinement introduced in step 4 of such algorithm, the faulty parameter set can be identified (fault isolation) and the size of change can be estimated (fault estimation). This fault isolation scheme allows to isolate several faulty parameters at the same time without any problem.

III. POLYTOPES

To describe the fault detection tests different polytope set representations will be used. The treatment follows closely the presentation given by [12]. Given m strips as defined by Eq. (11), where $m \geq n$, the set of consistent parameters can be described as the polytope, $\mathcal{P}$, given by the intersection of the strips. This is written as

$$\mathcal{P} = \cap_{i=1}^{m} F_i \qquad (12)$$

In the case $n = m$ the simplest bounded polytope is called a parallelotope. In this case, $\mathcal{P} = \{\theta : |c_i^T \theta - d_i| \leq 1, i = 1, \dots n\}$. A parallelotope can also be expressed in an alternative form as

$$\mathcal{P} = \{\theta : \theta = p + H\tilde{\theta}, \|\tilde{\theta}\|_\infty \leq 1\} \qquad (13)$$

where $H = [h_1 \ h_2 \ \dots \ h_n] = P^{-1}$, $P = [c_1 \ c_2 \ \dots \ c_n]^T$ and $p = P^{-1}D$ where $D = [d_1 \ d_2 \ \dots \ d_m]^T$. Notice that by assuming the polytope to be bounded, the inverse of matrix P necessarily exists. Formally, a parallelotope can be thought of as a Minkowski sum of a finite set of line segments. A shorthand notation of parallelotopes is $p \oplus H\mathbf{B}^m$ where H is a $n \times n$ matrix with column vectors h_i and $\mathbf{B}^m$ is the unit cube in m dimensions.

If columns are added to H and the vector $\tilde{\theta}$ is in the same way expanded the resulting set falls in the group of zonotopes.

Notice that the bound on parameter variation can be expressed as $|\theta(k + 1) - \theta(k)| < \Lambda$ which in turn can be expressed as

$$\theta(k + 1) \in \theta(k) \oplus \Lambda\mathbf{B}^n \qquad (14)$$

where Λ is a square matrix with the diagonal equal to λ. One of the principal features of zonotopes is that the Minkowski sum of a box and a zonotope is another zonotope. Therefore, if at time k it is known that the parameter belongs to set $\mathcal{Z}_k = p \oplus H\mathbf{B}^m$ then using Eq. (14) the parameter set at time $k + 1$ can be expressed as

$$\mathcal{Z}_{k+1} = p \oplus H\mathbf{B}^m \oplus \Lambda\mathbf{B}^n = p \oplus [H \ \Lambda]\mathbf{B}^{m+n}$$

In what follows, certain properties of zonotopes and parallelotopes will be presented as well as tools to manipulate them. Due to space limitations, details of the algorithms will not be presented.

A. Supporting hyperplane of a parallelotope

A hyperplane $\mathcal{S} = \{x : c^T x = q\}$ is a supporting hyperplane of a parallelotope $\mathcal{P} = p \oplus H\mathbf{B}^m$ if either $c^T x \leq q, \forall x \in \mathcal{P}$ or else $c^T x \geq q, \forall x \in \mathcal{P}$ with equality occurring for some $x \in \mathcal{P}$. The two constants q_u and q_d characterizing the supporting hyperplanes are easily calculated as

$$q_u = c^T p + \|H^T c\|_1 \qquad (15)$$
$$q_d = c^T p - \|H^T c\|_1 \qquad (16)$$

where $\| \cdot \|_1$ is the 1-norm of a vector.

B. Checking consistency of a parallelotope with a strip

Here it is assumed that $FSS_k \subseteq \mathcal{P}$ where $\mathcal{P} = p \oplus H\mathbf{B}^m$ is a parallelotope. Given a new data point $\{y(k), \varphi(k)\}$, consistency can be assessed by checking if

$$\mathcal{P} \cap F_k = \emptyset \qquad (17)$$

Notice that this check is very easy to perform. Calculating the supporting hyperplane constant q_u and q_d the intersection is empty if and only if

$$q_u < y(k) - \sigma \quad \text{or} \quad q_d > y(k) + \sigma \qquad (18)$$

This condition of inconsistency was reported in [7].

C. Bounding the intersection between a parallelotope and a strip with a minimum-volume parallelotope

The method proposed in [7], [12] is used to bound the intersection of a parallelotope and a strip. First, a tight representation of the parallelotope and the strip is computed. Next, the minimum-volume parallelotope that bounds the intersection of $n + 1$ tight strips is obtained.

D. Calculating a parallelotope that bounds a zonotope

There are various ways to calculate a parallelotope that bounds a zonotope. A straight forward approach is to calculate the box M in which the zonotope is included by calculating the two hyperplanes for each dimension. An other approach which gives good compromise between complexity and conservativeness is to use singular value decomposition. Given a zonotope $\mathcal{Z} = p \oplus H\mathbf{B}^m$ a parallelotope $\mathcal{P} = p \oplus P\mathbf{B}^n$ is sought so that $\mathcal{Z} \subseteq \mathcal{P}$. Assume that a singular value decomposition has been calculated for H and is parameterized as $H = U\Sigma V^T$. Matrix P is assumed to have the form UD where $n \times n$ matrix D is diagonal with elements d_i. Then, the problem is reduced to finding the elements d_i so that the above set inclusion holds. Notice that as U is a unitary matrix, this problem is equivalent to guaranteeing that $\Sigma V^T \mathbf{B}^m \subseteq D\mathbf{B}^n$. This can be guaranteed by selecting $d_i = \|\sigma_i V_i\|_1$ where V_i is the column vector of matrix V and σ_i is the diagonal element of matrix Σ.

IV. IMPLEMENTATION OF THE FAULT DETECTION ALGORITHM USING POLYTOPES

Using the computational tools presented in section III the fault detection algorithm introduced in section II will implemented as follows:

1: $\overline{\mathcal{P}}_N = \text{Update}(\mathcal{P}_{N-1}, \lambda)$.
2: Obtain input-output data and build F_N.
3: If $\overline{\mathcal{P}}_N \cap F_N = \emptyset$ then a fault has occured
4: Else obtain an outer bound of $\overline{\mathcal{P}}_N \cap F_N$ and denote it $\mathcal{P}_N$

The initial value of $\mathcal{P}_N$ is the parameter set Θ. To check consistency in step 3, the procedure described in section III-B is used. What differentiates the algorithm for the three cases of allowed parameter variance is the update procedure in step 1. In all cases, $\overline{\mathcal{P}}_N$ is an outer bounding of the feasible solution set FSS_N.

Certain steps are performed the same way for all cases of parameter variance. The consistency check in step 3 is performed using the procedure presented in III-B. In step 4, the procedure presented in section III-C is used.

A. Time invariant case, $\lambda = 0$

In this case, the least conservative approach consists of solving a linear program with the $2N$ linear inequalities given by Eq. (10) in addition to the $2n$ inequalities given by the restriction of θ to belong to Θ. It is clear that the linear program would have a solution if and only if no fault was present in the series. As the size of the LP grows in each sample, this approach is not practical in actual applications and relaxations have to be introduced. One relaxation approach is to solve an LP for only the last M data points. As in this case not all data points are used, it is clear that a fault can go undetected.

Referring to the algorithm above, the procedure in step 1, returns the set unchanged as $\lambda = 0$. It is clear that if it would be possible to calculate an exact representation of $\overline{\mathcal{P}}_N \cap F_N$ to use as $\mathcal{P}_N$ then the procedure could be made equally stringent as the linear programming test described before. As this is generally not feasible, the approach proposed here is to calculate an outer bound of $\mathcal{P}_N \cap F_N$ in each sample, as in the other cases.

B. Time-varying case, $\lambda = \bar{\lambda}$

In this case, the update procedure in step 1 consists of two steps.

1: Expand parallelotope $\mathcal{P}_{N-1}$ to zonotope $\mathcal{Z}_N$ using Eq.(14).
2: Calculate $\overline{\mathcal{P}}_N$ as the parallelotope that bounds $\mathcal{Z}_N$ using the procedures presented in section III-D.

C. Time-varying case, $\lambda = \infty$

This case is the simplest of all cases. As the parameters can vary at will within the initial parameter set Θ, the update procedure consists of putting parallelotope $\overline{\mathcal{P}}_N$ equal to the initial parameter set Θ.

V. APPLICATION EXAMPLE

A simulation of a quadruple-tank process (see [14]) is used to illustrate the proposed fault diagnosis algorithm. A diagram of the process is shown in figure 1. The process inputs are v_1 and v_2 (input voltages to the pumps). The continuous model derived from the mass balances and the Bernoulli's law is:

$$\frac{d h_1}{dt} = -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1}v_1 \quad (19)$$

$$\frac{d h_2}{dt} = -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2}v_2 \quad (20)$$

$$\frac{d h_3}{dt} = -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2)k_2}{A_3}v_2 \quad (21)$$

$$\frac{d h_4}{dt} = -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1)k_1}{A_4}v_1 \quad (22)$$

where A_i is the cross section of the tank i and a_i is the cross section of the outlet hole of the tank i. The vector of measured outputs is composed by the levels of four tanks, denoted h_i. The parameters γ_1, $\gamma_2 \in (0,1)$ determine the flow to different tanks, that is, the flow to tank 1 is $\gamma_1 k_1 v_1$ and the flow to tank 4 is $(1-\gamma_1)k_1 v_1$ and similarly for tank 2 and tank 3.

The fixed values used in the simulations are $A_1 = A_3 = 28 \quad cm^2$, $A_2 = A_4 = 32 \quad cm^2$, $k_1 = 3.33 \quad cm^3/Vs$, $k_2 = 3.35 \quad cm^3/Vs$ and $g = 981 \quad cm/s^s$. To obtain a system of the form given by Eq. (1), an euler discretization with step size equal to 1 of two equations of the model is used:

$$h_1(k+1) = h_1(k) + 1*(-\frac{a_1}{A_1}\sqrt{2gh_1(k)}$$
$$+ \frac{a_3}{A_1}\sqrt{2gh_3(k)} + \frac{\gamma_1 k_1}{A_1}v_1(k)) + e_1(k) \quad (23)$$
$$h_2(k+1) = h_2(k) + 1*(-\frac{a_2}{A_2}\sqrt{2gh_2(k)}$$
$$+ \frac{a_4}{A_2}\sqrt{2gh_4(k)} + \frac{\gamma_2 k_2}{A_2}v_2(k)) + e_2(k) \quad (24)$$

where $|e_i(k)| \leq 1$ with $i = 1,2$ is a bounded random error. Then, the parameter vector θ is composed of: $\theta = [a_1 \; a_2 \; a_3 \; a_4 \; \gamma_1 \; \gamma_2]^T$, the regressor vector corresponding to the first output $y_1 = h_1$ is

$$\varphi_{y_1}(k) = \left[-\frac{\sqrt{2gh_1(k)}}{A_1} \; 0 \; \frac{\sqrt{2gh_3(k)}}{A_1} \; 0 \; \frac{k_1 v_1(k)}{A_1} \; 0\right]^T$$

while the regressor vector corresponding to the second output $y_2 = h_2$ is

$$\varphi_{y_2}(k) = \left[0 \; -\frac{\sqrt{2gh_2(k)}}{A_2} \; 0 \; \frac{\sqrt{2gh_4(k)}}{A_2} \; 0 \; \frac{k_2 v_2(k)}{A_2}\right]^T$$

Note that the results obtained in this section could be improved using the other two equations of the model.

Fig. 1. Quadruple-tank process

Fig. 2. Detection and fault diagnosis of parameter a_2

Fig. 3. Window of fault detection of parameter a_2
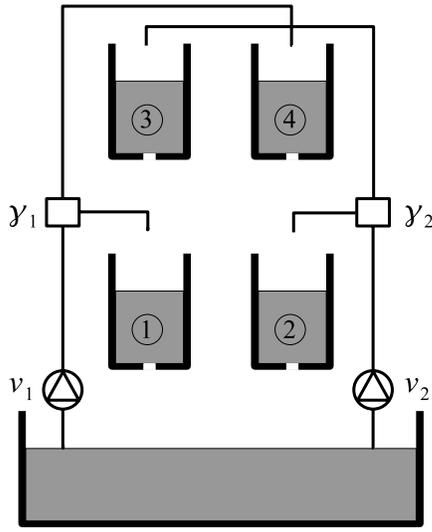
A. Time-invariant parameters

In this case, the initial parameter uncertainty set is assumed to be:

$$\Theta = \{\theta : \theta = p_0 + H_0\tilde{\theta}, \|\tilde{\theta}\|_\infty \leq 1\}$$

where: $p_0 = [0.4 \ 0.3 \ 0.4 \ 0.3 \ 0.5 \ 0.5]^T$, $H_0 = diag\left([0.4 \ 0.3 \ 0.4 \ 0.3 \ 0.5 \ 0.5]^T\right)$. Since uncertain parameters are considered time-invariant, then $\lambda = 0$ in Eq. (4), and fault detection algorithm is implemented as described in Section IV.A. The non-faulty system is simulated with parameters equal to $a_1 = a_3 = 0.71 \ cm^2$, $a_2 = a_4 = 0.57 \ cm^2$, $\gamma_1 = 0.7$ and $\gamma_2 = 6$. The first fault considered is a variation in the parameter a_2. Figure 2 shows the evolution of this parameter. The dotted line represents the exact value of parameter a_2. It can be seen that a_2 changes its value to 0.35 at sample time 516. The solid lines represent the outer bounds obtained by the detection algorithm. Before sample time 516, the detection algorithm bounds the parameter with $0.55 \leq a_2 \leq 0.58$. This bound is considered a non-faulty situation. The algorithm considers a window of five detected faults to overcome outliers. In the figures, the sample times where faults are detected are indicated with diamonds. As it can be seen in figure 3, at sample time 530 the window of faults is reached and the fault diagnosis process starts. The parallelotope is initialized and at sample time 645 the fault has been diagnosed because the diagnosis algorithm assures that the new value of parameter a_2 is less than 0.55. On the other hand, if we had looked at any of the other parameters, namely, a_1, a_3, a_4, γ_1 or γ_2 their associated fault isolation test is not activated. Here, these tests are not presented because of the lack of space. This allows to isolate a_2 as the faulty parameter. A second fault scenario is a variation in the parameter γ_2. Figure 4 shows the evolution of this parameter. The dotted line represents the exact value of parameter. The solid lines represent the outer bounds obtained by the detection algorithm. It can be seen that γ_2

B. Time-varying parameters

In this case, the initial parameter uncertainty set is assumed to be:

$$\Theta = \{\theta : \theta = p_0 + H_0\tilde{\theta}, \|\tilde{\theta}\|_\infty \leq 1\}$$

where: $p_0 = [0.696 \ 0.559 \ 0.696 \ 0.559 \ 0.686 \ 0.588]^T$ $H_0 = diag\left([0.036 \ 0.029 \ 0.036 \ 0.028 \ 0.035 \ 0.030]^T\right)$. Uncertain parameters are considered time-varying with $\lambda = \infty$, in Eq. (4). This means that variation is bounded only by the initial parameter set Θ varying at will within this set. Then, fault detection algorithm is implemented as described in Section IV.C. Figure 5 shows the evolution of parameter a_2. Dotted line represents the exact value of parameter a_2. The solid lines represent the outer bounds obtained by the detection algorithm. It can be seen that a_2 changes its value to 0.54 at sample time 68. The new value belongs to the bound of non-faulty situation and the algorithm does not detect fault. At sample time 260, parameter a_2 changes its

Fig. 4. Fault detection and isolation of parameter γ_2
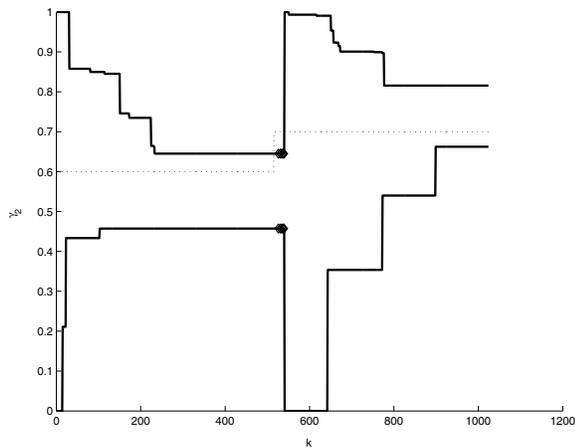
Fig. 6. Fault detection and isolation of parameter γ_2

value to 0.5. This value is out of considered non-faulty bound. The algorithm detect the fault at sample time 366 and diagnoses the fault at sample time 387. A bound of the new value of the parameter is obtained at sample 740. Figure 6 shows the evolution of parameter γ_2. Dotted line represents the exact value of parameter γ_2. The solid lines represent the outer bounds obtained by the detection algorithm. It can be seen that γ_2 changes its value to 0.7 at sample time 260. This value is out of considered non-faulty bound. The algorithm detect the fault at sample time 342 and diagnoses the fault at sample time 540. A bound of the new value of the parameter is obtained at sample 620.
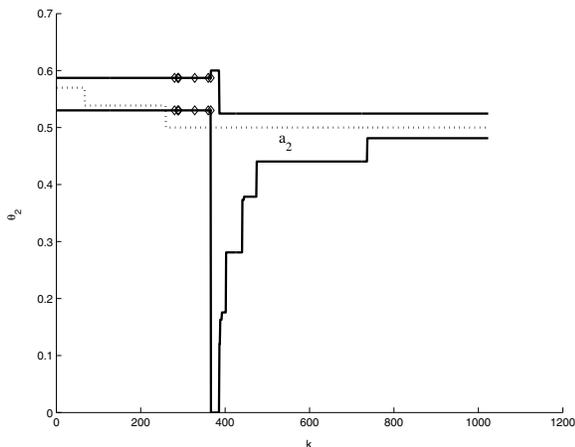
Fig. 5. Fault detection and isolation of parameter a_2

VI. Conclusions

A robust fault diagnosis method has been proposed that takes advantage of a recently proposed set-membership identification procedure based on parallelotopes for systems linear in the parameters. A general algorithm was presented based on proving that the feasible solution set of parameters for a series of data, is empty. Three distinct cases of allowed parameter variance have been considered. Computational procedures were given for each step in the algorithm. Finally, the method was applied to a simulation model of a multivariable process showing its effectiveness.
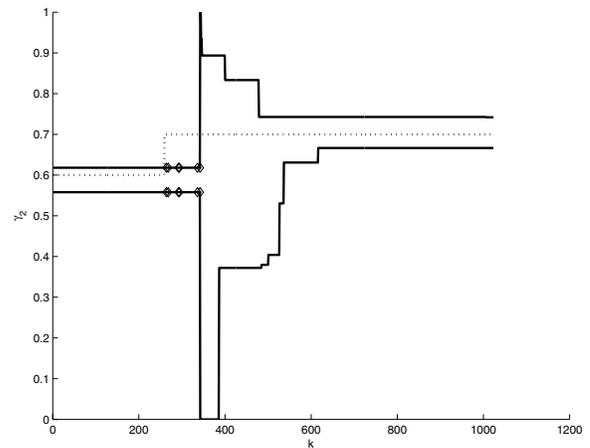
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