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Discontinuous Sliding Surface For a General Electromechanical System with Time-Invariant Uncertainties

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Abstract— **A new design approach of a sliding mode controller for a generalized electromechanical system with mismatched time-varying uncertainties is presented in this paper. A discontinuous sliding surface can be designed to satisfy a new matching condition, and two layers of sliding surfaces are considered. By maintaining the sliding mode on the first surface, the second layer can be reached in finite time. It is shown that the system is invariant on the sliding surface, and the stability of the closed-loop system is guaranteed. A magnetic levitation example is used to prove the effectiveness of the proposed control scheme.**

I. Introduction

Control algorithms using the variable structure control (VSC) system have been recently developed by many researchers [1], [2], [3]. In the VSC system, the control structure around the plant is intentionally changed by using a viable high speed switching gain to obtain a desired system behavior or response. Using the switching feedback control, the VSC system drives the trajectory of the system onto a specified and user-chosen surface, which is termed the sliding surface or the switching surface, and maintain the trajectory on this manifold for all subsequent time. The salient future of the VSC system is the so-called sliding mode on the sliding surface within which the system remains insensitive to internal parameter variations and extraneous disturbances. Thus, not only the existence of a sliding mode on the switching surface but the design of the proper switching surface is key problems in VSC system design. Further, most of the uncertain systems with the traditional variable structure control requires that the uncertainties satisfies the matching condition. This would limit the application of the variable structure control systems [4], [5], [8].

In this paper, the problem of designing a variable structure control law of uncertain electromechanical system where the uncertainties in the state model do not satisfy the matching conditions is considered. As usual the time-varying uncertainties are assumed to be bounded. Since the conventional matching conditions are no longer matched, a discontinuous sliding surface can be designed to satisfy a new matching condition.

The control problem we address in this paper is the asymptotic regulation of the mechanical angular position to a desired one where the uncertainty in the state model does not satisfy the matching condition for the generalized electromechanical system. It is shown that the electromechanical system is invariant on the sliding surface, and the stability of the closed-loop system is guaranteed. To illustrate the effectiveness of the proposed sliding mode controller, the problem of stabilizing a magnetic levitation system is considered. Agreement between analytical studies and simulation results confirms the validity of the proposed control system.

This paper is organized as follow. The generalized electromechanical system model considered in this paper is presented in Section 2. Section 3 presents the design of a discontinuous sliding mode surface. Sliding mode control with stability analysis for overall electromechanical system with the mismatched time-varying uncertainties are presented in Section 4. Section 5 is devoted to simulation results of the illustrative example. Finally, conclusions are provided in Section 6.

II. The system model description

We will consider the generalized electromechanical system described in [6], see also Chapters 8 and 9 of [7]. It consists of in all n_e windings, with possible permanent magnets or a salient rotor. Only linear magnetic materials are considered, and it is further assumed that all parameters are constant.

Under these assumptions, application of Gauss's law and Ampere's law leads to the following affine relationship between the *flux linkage* vector $\lambda \in R^{n_e}$ and the *current* vector $i \in R^{n_e}$

$$\lambda = L(\theta)i + \mu(\theta) \tag{1}$$

with $\theta \in \mathbb{R}$ the *mechanical angular position*, and $L(\theta) = L^\top(\theta) > 0$ the $n_e \times n_e$ multiport inductance matrix of the windings. The vector $\mu(\theta)$ represents the flux linkages due to the possible existence of *permanent magnets*. Notice that capacitive effects are not taken into account.

With the considerations above, the voltage balance equation yields

$$\dot{\lambda} + Ri \quad = \quad Bu \tag{2}$$

where $u \in R^{n_s}$ is the vector of voltages applied to some of the windings, $R = R^\top > 0$ is the matrix of electrical resistance of the windings, and B is a constant matrix that defines the actuated coordinates. If $n_s = n_e$ then, $B = I$, and we say that the electromechanical system is fully actuated. On the other hand, if $n_s < n_e$, then $B = \begin{bmatrix} I & 0 \end{bmatrix}^\top \in R^{n_e \times n_s}$, and we will call the electromechanical system underactuated.

The coupling between the electrical and the mechanical subsystems is established through the torque of electrical origin [1]

$$\tau(i,\theta) = \frac{1}{2} i^\top \nabla_\theta L(\theta)\, i + i^\top \nabla_\theta \mu(\theta) \tag{3}$$

The model is completed replacing the latter in the mechanical dynamics

$$J\ddot{\theta} = -r_m\dot{\theta} + \tau(i,\theta) - \nabla_\theta V(\theta) \tag{4}$$

where $J > 0$ is the rotational inertia of the rotor, $r_m \geq 0$ is the viscous friction coefficient, and the scalar function $V(\theta)$ is the potential energy of the mechanical subsystem.

As shown in [14] the model (1)–(4) contains, as particular cases, permanent magnet synchronous, as well as induction, magnetic levitated systems and stepping motors.

For simplicity sake, we will assume $B = I$, and $n_s = n_e = 1$. The general electromechanical system is then written as

$$(\Sigma) : \begin{cases} \dot{\lambda} & = u - R\, i \\ J\, \ddot{\theta} & = \tau(i,\theta) - r_m\, \dot{\theta} - \nabla_\theta\, V(\theta) + \delta(t) \end{cases} \tag{5}$$

where $\delta(t)$ represents a mismatched time-varying uncertainties.

III. Design of sliding surface

A. The control problem

The control problem we address in this paper is the asymptotic *regulation* of θ to a constant position θ_d where the uncertainty in the state model does not satisfy the matching conditions.

We note $p \triangleq J\dot{\theta}$ the mechanical momentum, and the state vector, x, is chosen as

$$x \triangleq [\lambda,\ \theta,\ p]^\top$$

[1]In the following, we will use the notation $\nabla_w H(z,w) \triangleq \frac{\partial H}{\partial w}(z,w)$.

Equilibrium of the system (5) is then given by

$$x_e = [\lambda_d,\ \theta_d,\ 0]^\top \tag{6}$$

where $\lambda_d = L(\theta_d)i_d + \mu(\theta_d)$ and i_d are the solutions of equations (3) and (4) for $\theta = \theta_d$

$$\frac{1}{2}\, i_d^\top\, \nabla_{\theta_d}\, L(\theta_d)\, i_d + i_d^\top\, \nabla_{\theta_d}\, \mu(\theta_d) - \nabla_\theta\, V(\theta_d) - \delta(t) = 0 \tag{7}$$

B. Sliding surface

Let us define the sliding surface as follow

$$\Omega \triangleq \{x : \sigma(x) = \tau(i,\theta) - \tau_d(i,\theta) = 0\}, \tag{8}$$

where $\tau(i,\theta)$ represents the electromechanical torque, and $\tau_d(i,\theta)$ is the *desired torque*. Let $\varepsilon_\theta = \theta - \theta_d$ represents the desired position error, $\dot{\varepsilon}_\theta = \dot{\theta} - \dot{\theta}_d$ denotes the desired velocity error, and $\ddot{\varepsilon}_\theta$ the desired acceleration error. Consider the second-order dynamic equation

$$\ddot{\varepsilon}_\theta + k_1\dot{\varepsilon}_\theta + k_2\varepsilon_\theta \tag{9}$$

By appropriate selection of k_1 and k_2, the desired rate of convergence of $\varepsilon \longrightarrow 0$ may be obtained. Then, mechanical part of system (Σ) gives,

$$\ddot{\theta} = \ddot{\theta}_d - k_1\dot{\varepsilon}_\theta - k_2\varepsilon_\theta$$

If the electromechanical torque $\tau(i,\theta)$ is given by the desired value

$$\begin{aligned} \tau(i,\theta) &= \tau_d(i,\theta) \\ &= J(\underbrace{\ddot{\theta}_d - k_1\dot{\varepsilon}_\theta - k_2\varepsilon_\theta}_{\theta}) + r_m\dot{\theta} + \nabla_\theta V(\theta) - \delta(t) \end{aligned}$$

Then, sliding surface $\sigma(x)$ is given by

$$\begin{aligned} \sigma(x) &= \tau(i,\theta) - \tau_d(i,\theta) \\ &= \tau(i,\theta) - J\,(\ddot{\theta}_d - k_1\dot{\varepsilon}_\theta - k_2\varepsilon_\theta) \\ &- \ r_m\,\dot{\theta} - \nabla_\theta\ V(\theta) + \delta(t) \end{aligned} \tag{10}$$

It is known that the discontinuity of sliding surface will cancel the effect of uncertainty when the system is in sliding mode. This is valid when the uncertainty satisfies the so called "matching condition" [4], [9].

Unfortunately, sliding surface function $\sigma(x)$ contains unknown mismatched time-varying uncertainties $\delta(t)$. The attribute of disturbances rejection for the sliding mode is lost. Let modify $\tau(i,\theta)$ by adding a pseudo-control term $\hat{\delta}(t)$

$$\tau_d(i,\theta) = J\,(\ddot{\theta}_d - k_1\dot{\varepsilon}_\theta - k_2\varepsilon_\theta) + r_m\,\dot{\theta} + \nabla_\theta V(\theta) + \hat{\delta}(t) \tag{11}$$

Mechanical part of the system (Σ) is given by

$$\begin{aligned} J\ddot{\theta} &= \quad \tau_d(i,\theta) - r_m\,\dot{\theta} - \nabla_\theta\, V(\theta) + \delta(t) \\ &= \ J\,(\ddot{\theta}_d - k_1\dot{\varepsilon}_\theta - k_2\varepsilon_\theta) \\ &+ \ r_m\,\dot{\theta} + \nabla_\theta V(\theta) + \hat{\delta}(t) - r_m\,\dot{\theta} - \nabla_\theta V(\theta) + \delta(t) \end{aligned}$$

Finally,

$$J\,(\ddot{\varepsilon}_\theta + k_1\dot{\varepsilon}_\theta + k_2\varepsilon_\theta) = \delta(t) + \hat{\delta}(t) \tag{12}$$

We observe that the equation (12) is an ordinary second-order system, while the mismatched time-varying uncertainties, $\delta(t)$, belongs to the range space of the new control $\hat{\delta}(t)$. Following proposition gives expression of a variable structure law, $\hat{\delta}(t)$, that stabilize θ.

Proposition 2.1: Consider the second layer of sliding surface, $s = 0$, defined as

$$s = \dot{\varepsilon}_\theta + k_s \varepsilon_\theta$$

and a variable structure law

$$\hat{\delta}(t) = J\,(k_1 \dot{\varepsilon}_\theta + k_2 \varepsilon_\theta - k_s \dot{\varepsilon}_\theta) - k_{ss}\, sgn(s) \qquad (13)$$

where

$$k_{ss} > |\delta(t)|_{max} + \eta, \qquad \eta > 0$$

Then, the second layer, $s = 0$, can be reached in finite time.

Proof: We have

$$\begin{aligned} J\,(\ddot{\varepsilon}_\theta + k_1 \dot{\varepsilon}_\theta + k_2 \varepsilon_\theta) &= \delta(t) + \hat{\delta}(t) \\ &= \delta(t) + J\,(k_1 \dot{\varepsilon}_\theta + k_2 \varepsilon_\theta - k_s \dot{\varepsilon}_\theta) - k_{ss}\, sgn(s) \end{aligned}$$

Then,

$$J\,(\ddot{\varepsilon}_\theta + k_s \dot{\varepsilon}_\theta) = \delta(t) - k_{ss}\, sgn(s)$$
$$J\,(\dot{s}) = \delta(t) - k_{ss}\, sgn(s)$$

Since

$$k_{ss} > |\delta(t)|_{max} + \eta, \qquad \eta > 0$$

then,

$$J\dot{s} < -\eta\, sgn(s)$$

The sliding condition becomes

$$s\dot{s} < -\eta_1\, |s|, \qquad \eta_1 = \frac{\eta}{J} \qquad (14)$$

The sliding mode for the variable structure control for the system (Σ) is guaranteed. ■

Now, we will give, for the overall electromechanical system (Σ), an explicit formula of discontinuous sliding surface , $\sigma(x) = 0$, and a variable structure law, u, that stabilize the system.

It follows from proposition (2.1), that the sliding mode function described on equation (10) can be written as

$$\sigma(x) = \tau(i,\theta) - J\,\ddot{\theta}_d + J\,k_s \dot{\varepsilon}_\theta - \nabla_\theta\, V(\theta) + k_{ss}\, sgn(s) = 0. \qquad (15)$$

Given equation (11). We replace, $\hat{\delta}(t)$ by its expression given by the proposition (2.1). Then,

$$\begin{aligned} \tau_d(i,\theta) &= J(\ddot{\theta}_d - k_1 \dot{\varepsilon}_\theta - k_2 \varepsilon_\theta) + r_m\, \dot{\theta} + \nabla_\theta V(\theta) + \hat{\delta}(t) \\ &= J(\ddot{\theta}_d - k_1 \dot{\varepsilon}_\theta - k_2 \varepsilon_\theta) + r_m\, \dot{\theta} \\ &+ \nabla_\theta V(\theta) + J(k_1 \dot{\varepsilon}_\theta + k_2 \varepsilon_\theta - k_s \dot{\varepsilon}_\theta) - k_{ss}\, sgn(s) \\ &= J(\ddot{\theta}_d - k_s \dot{\varepsilon}_\theta) + r_m\, \dot{\theta} + \nabla_\theta\, V(\theta) - k_{ss}\, sgn(s) \end{aligned}$$

Consequently, discontinuous sliding surface, $\sigma(x)$, takes the following form

$$\begin{aligned} \sigma(x) &= \tau(i,\theta) - \tau_d(i,\theta) \\ &= \tau(i,\theta) - J\,(\ddot{\theta}_d - k_s \dot{\varepsilon}_\theta) - r_m\, \dot{\theta} - \nabla_\theta\, V(\theta) + k_{ss}\, sgn(s) \end{aligned} \qquad (16)$$

IV. Sliding mode control law

Theorem 0.2: Consider the generalized electromechanical system with mismatched time-varying uncertainties (Σ), and a sliding surface

$$\sigma(x) = \tau(i,\theta) - \tau_d(i,\theta) = 0$$

with the control law

$$\begin{aligned} u &= \lambda^{-1}[L^{-2}(\theta)\, \nabla_\theta\, L^{-1}(\theta)]^{-1}[2\dot{\tau}_d(\lambda,\theta) \\ &- \lambda^2 \frac{d}{dt}(L^{-2}(\theta)\, \nabla_\theta\, L(\theta)) \\ &+ Ri - K_\sigma\, sgn(\sigma(x)) - K_p\, \sigma(x)] \end{aligned} \qquad (17)$$

where

$$\sigma(x) = \tau(i,\theta) - J\,(\ddot{\theta}_d - k_s \dot{\varepsilon}_\theta) - r_m\, \dot{\theta} - \nabla_\theta\, V(\theta) + k_{ss}\, sgn(s)$$

$$\tau_d(\lambda,\theta) = \frac{1}{2} i_d^2\, \nabla_\theta\, L(\theta)$$

$K_\sigma > \mu$, $\mu > 0$ and $K_p > 0$ are a positive constants.

Then the sliding surface, $\sigma(x) = 0$, is a globally attractive and invariant set.

Proof: We will show that the control law u drives the system trajectory onto the sliding surface and maintain the trajectory on the sliding surface for all subsequent time, i.e. the reachability condition is satisfied.

Consider a scalar Lyapunov function candidate $V(\sigma) = \frac{1}{2}\sigma^2(x)$. The Time derivatives of $\sigma(x)$ is given as

$$\dot{\sigma}(x) = \dot{\tau}(\lambda,\theta) - \dot{\tau}_d(\lambda,\theta) \qquad (18)$$

where $\tau(i,\theta)$ is given by

$$\tau(i,\theta) = \frac{1}{2} i^\top \nabla_\theta\, L(\theta)\, i + i^\top\, \nabla_\theta \mu(\theta)$$

Wa adopt the standard assumption of unsaturated flux, that is $\lambda \approx L(\theta)\, i$. The electromechanical torque becomes

$$\tau(\lambda,\theta) = \frac{1}{2}\, i^2 \nabla_\theta L(\theta) = \frac{1}{2}\, \lambda^2 L^{-2}(\theta)\, \nabla_\theta L(\theta)$$

Its time derivative is given by

$$\begin{aligned} \dot{\tau}(\lambda,\theta) &= \frac{1}{2}[\lambda\, \dot{\lambda}\, L^{-2}(\theta)\, \nabla_\theta L(\theta) && (19) \\ &+ \lambda^2\, \frac{d}{dt}[L^{-2}(\theta)\, \nabla_\theta L(\theta)]] \\ &= \frac{1}{2}[\lambda\, (u - R\, i)\, L^{-2}(\theta)\, \nabla_\theta L(\theta) && (20) \\ &+ \lambda^2\, \frac{d}{dt}[L^{-2}(\theta)\, \nabla_\theta L(\theta)]] \end{aligned}$$

Then, $\dot{\sigma}(x)$ takes the following form

$$\begin{aligned} \dot{\sigma}(x) &= \dot{\tau}(\lambda,\theta) - \dot{\tau}_d(\lambda,\theta) \\ &= \frac{1}{2}[\lambda\, (u - R\, i)\, L^{-2}(\theta)\, \nabla_\theta L(\theta) \\ &+ \lambda^2\, \frac{d}{dt}[L^{-2}(\theta)\, \nabla_\theta L(\theta)]] - \dot{\tau}_d(\lambda,\theta) \end{aligned} \qquad (21)$$

The time derivative of Lyapunov function can be written as

$$\begin{aligned}\dot{V}(\sigma) &= \sigma(x)\dot{\sigma}(x) \quad (22)\\ &= \frac{1}{2}\,\sigma(x)\{[\lambda\ (u-R\,i)\,L^{-2}(\theta)\,\nabla_\theta L(\theta)\\ &+\ \lambda^2\,\frac{d}{dt}[L^{-2}(\theta)\,\nabla_\theta L(\theta)]-\dot{\tau}_d(\lambda,\theta)]\}\end{aligned}$$

Using the explicit formula of control law u given by the theorem (0.2), the time derivative of $V(\sigma)$ is given as

$$\dot{V}(\sigma) = \frac{1}{2}\sigma(x)(-K_\sigma\,sgn(\sigma(x))-K_p\,\sigma(x))$$

and thus $\dot{V}(\sigma(x))$ is negative definite if $K_\sigma > 0,\ K_p > 0$. The system trajectories are thereby guaranteed to approach the sliding surface $\sigma(x)=0$ from any initial state plane. ■

A. Stability Analysis

Sliding mode controller given by theorem (0.2) ensures that the system trajectories will approach the equilibrium manifold, $\sigma(x)=0$, from any initial state in the state plane. Proposition (2.1) demonstrates that sliding mode control law maintains the system trajectories on the equilibrium manifold. In this part, after deriving the sliding surface dynamics, the stability of the overall system can be guaranteed by the following proposition.

Substituting the control law u into equation (16), one can obtain the dynamics

$$\dot{\sigma}(x)+K_p\sigma(x) = -K_\sigma sgn(\sigma(x)) \quad (23)$$

Proposition 1.1: Given the dynamics described in equation (23). Then

1) The reaching law (23) is bounded, and
2) given an initial state $\sigma(t_0)$, $\sigma(t)\rightarrow 0$ in a finite reaching time, T_r, given by

$$T_r = \frac{1}{K_p}Ln(\frac{K_\sigma+K_p|\sigma(t_0)|}{K_\sigma})$$

Proof: Consider a scalar Lyapunov function $V(\sigma)=\frac{1}{2}\sigma^2(x)$. From equation (23), the time derivatives of $V(\sigma)$ is given by

$$\begin{aligned}\dot{V}(\sigma) &= \frac{1}{2}\sigma(x)(-K_\sigma sgn(\sigma(x))-K_p\sigma(x))\\ &< \frac{1}{2}(-\mu|\sigma(x)|-K_p\sigma^2(x))\end{aligned}$$

Hence, $\sigma(t)\longrightarrow 0$ in a finite time T_r. Then $\dot{\sigma}(t)=0, \forall t\geq T_r$ Integrate the dynamics (23) from $\sigma(t_0)$ to $\sigma(t)=0$, the reaching time, T_r is given by

$$T_r = \frac{1}{K_p}Ln(\frac{K_\sigma+K_p|\sigma(t_0)|}{K_\sigma})$$

■

Remark 1.2: Constant plus proportional reaching law described in equation (23) gives an effective way for reducing chattering, improving transient, and steady state performance as compared as constant rate reaching law ($\dot{\sigma}=-K_\sigma\,sgn(\sigma)$ that is traditionally used on VSC literature [3], [9].

Chattering can be reduced by tuning parameters K_σ and K_p in the reaching law equation. Given equation (23), near the equilibrium manifold, $\sigma\approx 0$, so $|\dot{\sigma}|\approx K_\sigma$. By choosing the gain μ small, the momentum of the motion will be reduced as the system trajectory approaches the equilibrium manifold. A large value for K_p increases the reaching rate when the state is not near the equilibrium manifold.

We can also see that in light of equation (24). Hence, it is seen that $\dot{V}(\sigma) < -\mu|\sigma|-K_p\sigma^2(x)$. When the system trajectories are far from the equilibrium manifold, $\dot{V}(\sigma)$ is dominated by $K_p\sigma^2$. The larger value of K_p will shorten the reaching time, T_r. When the system trajectories are near the equilibrium manifold $\dot{V}(\sigma)$ is dominated by the term $-\mu|\sigma|$. The chattering will be attenuate if we choose a small μ.

V. Application: magnetic levitation system

A. Non linear model

In this section, a magnetic levitation system represented in Figure 1 will be used to prove the effectiveness of the proposed control scheme.

For this purpose, we use a non linear mathematical model [14]; the dynamics of a magnetic suspension consisting of an iron rotor in a vertical magnetic field created by a single electromagnet with $n_e=1, V(\theta)=mg\theta, \mu(\theta)=0$, and $L(\theta)$ some function, which is bounded from below in the domain of operation. (A classical choice for this function is $L(\theta)=\frac{k}{c-\theta}$, where k is a magnetic constant and c is the gap between the electromagnet and the rotor.)

In this case θ represents the rotor position with respect to the nominal position and J is replaced by the mass of the rotor m. The electromechanical torque is given by $\tau=\frac{k}{(c-\theta)^2}i^2$. Non linear model of this system is then given by

$$\begin{aligned}\dot{\lambda}+R\,L(\theta)^{-1} &= u \quad (24)\\ m\ddot{\theta} &= F-mg+\delta(t)\end{aligned}$$

where F denotes the magnetic active force, given as follow

$$F(\lambda,\theta) = -\frac{1}{2}\frac{\partial L(\theta)^{-1}}{\theta}\lambda^2 = \frac{1}{2k}\lambda^2 \quad (25)$$

and a mismatched time-varying uncertainty, $\delta(t)$ will be chosen as

$$\delta(t) = 0.25+0.75^{-03}sin(2\times\pi\times 100\times t)$$

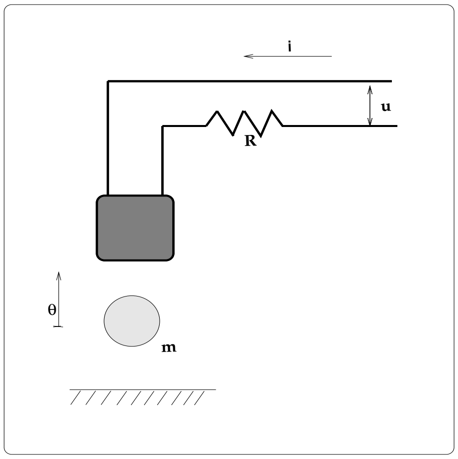

Fig. 1. Magnetic levitation system

B. Simulation results

Following gain parameters were used for the test: $k_1 = 10$, $k_2 = 4$, $k_{ss} = 50$, et $k_\sigma = 5$, and $k_p = 10$.
The follow values have been taken from a real magnetic levitation system.

System parameters	*Value*	*Units*
m	$0.0844 \pm 16\%$	[Kg]
g	9.81	$[m/s^2]$
k_a	$2.69.10^{-4} \pm 9\%$	$[Nm/A]$
c	0.005	$[m]$
R	10.05	$[\Omega]$

Table 1: System parameters

The proposed sliding mode control law (17) was applied to the dynamic system (24) for a small step change in the displacement θ from $1 \times 10^{-03} m$ to $1.5 \times 10^{-03} m$. Simulations will be given for both nominal system (solid line) and when the uncertainty acts on the system (dash line).
A case results is given in Figure 2, 3 where the actual position and velocity were shown.
With reference to Figure (5), the proposed discontinuous controller, however, showed chattering in the control signal. Therefore, it is ideal that when a discontinuous controller is implemented, the discontinuous terms are replaced with continuous functions (Southward *et al*. [13], Slotine and Li [11], Corless [10]). It is then to be investigated if the replacement of discontinuous terms can still maintain stability of the entire system. Particularly it is important to study trade-off between tracking precision and robustness. Some of these issues have already been discussed by Slotine [12]; they need to be investigated for the system under investigation in this paper and remains our future work. The sliding surface variable, $\sigma(x)$, and a sketch of the phase portrait is shown is Figures 6, and 7 .

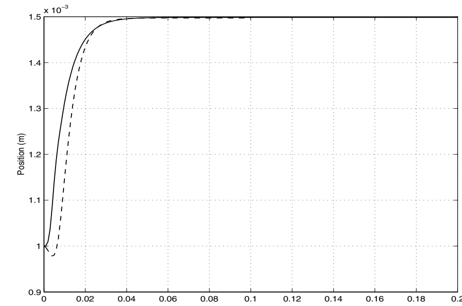

Fig. 2. The position θ

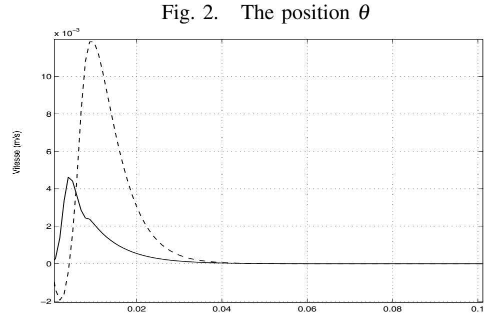

Fig. 3. The velocity

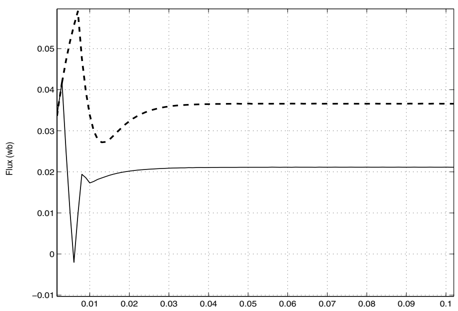

Fig. 4. The flux λ

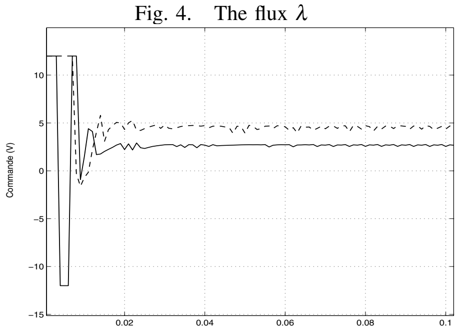

Fig. 5. The control action u

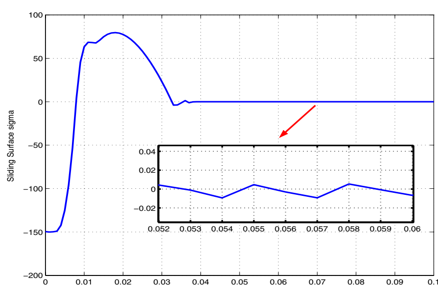

Fig. 6. Sliding surface σ

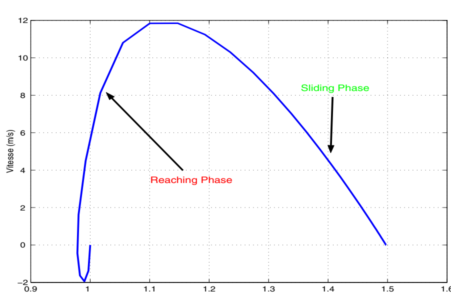

Fig. 7. Phase portrait of the system

VI. Conclusion

In this paper, a new design approach of a sliding mode controller for a generalized electromechanical system with mismatched time-varying uncertainties was presented. A discontinuous sliding surface was designed to satisfy a new matching condition, and two layers of sliding surfaces were considered. It was shown that the system is invariant on the sliding surface, and the stability of the closed-loop system is guaranteed. A magnetic levitation example was used to prove the effectiveness of the proposed control scheme.

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