Proceedings of the
44th IEEE Conference on Decision and Control, and
the European Control Conference 2005
Seville, Spain, December 12-15, 2005

Robust Vertical/Lateral/Longitudinal Control of an Helicopter with Constant Yaw-Attitude

M. Bejar* , A. Isidori°†, L. Marconi°, R. Naldi°

* Departamento de Ingeniera de Sistemas y Automatica, Universidad de Sevilla, SPAIN

° C.A.SY. – Dipartimento di Elettronica, Informatica e Sistemistica, University of Bologna, 40136 Bologna, ITALY.

† Dipartimento di Informatica e Sistemistica, Università di Roma "La Sapienza", ITALY

Abstract— **We consider the problem of controlling the vertical, lateral and longitudinal motion of a nonlinear model of a helicopter maintaining a constant yaw-attitude. Given arbitrary references with suitable restrictions on the time derivatives, we design a nonlinear controller which succeeds in enforcing the desired trajectories robustly with respect to uncertainties characterizing the physical and aerodynamical parameters of the helicopter. Engine dynamic of the main rotor is also taken into account in deriving the control law. Simulation results are finally given showing the effectiveness of the method and its ability to cope with uncertainties on the plant and actuator model.**

I. Introduction

In this paper we address the design of an autopilot for the helicopter capable to let its vertical/lateral and longitudinal dynamics tracking arbitrary references (with only some bound requirements on the higher-order time derivatives) with a constant yaw-attitude. The design is carried out in presence of possible severe uncertainties characterizing the physical and aerodynamical parameters of the plant and in presence of engine dynamics actuating the main and tail rotor. This work enriches a number of recent works focused on autopilot design for helicopters (see, besides others, [10], [11], [12], [1]) motivated, on one hand, from the interest that this kind of air-vehicles have in practical applications ([16]) and, on the other hand, from some features of the helicopter model (such as the nonlinearity of the dynamics and the strong coupling between the forces and torques produced by the vehicle actuators) which render the system in question an ideal test-bed for testing and comparing nonlinear design techniques.

A crucial feature of the helicopter with respect to others air-vehicles is to be functionally controllable in the lateral/longitudinal and vertical directions with constant (or very small) yaw-attitude. This feature guarantees high maneuverability to the helicopter and it is somehow the distinguishing feature of this kind of systems with respect to fixed-wing air vehicles. In this paper we wish to employ this feature and investigate the design of an autopilot controlling the helicopter in the lateral/longitudinal and vertical direction by maintaining a constant yaw attitude. To this end we take advantage of the design techniques proposed in [2] (see also [1]) in which the problem was to track a vertical reference given by the sum of a certain number of sinusoidal functions (with unknown phases, amplitudes and frequencies) while stabilizing the lateral/longitudinal and attitude dynamics. Differently from [2], here we allow for arbitrary lateral/longitudinal and vertical references and we show how a modification of the control structure in [2] succeeds in asymptotically tracking the references provided that the latters fulfill specific bounds on the higher order time derivatives as better specified in the paper. The emphasis of the results here proposed is on the asymptotic robustness of the proposed control law to a number of physical parameters which are typically affected by strong uncertainties.

The paper is organized as follow. In section 2 we present the model of the helicopter and we fix the control problem. The control structure and the main results are then presented in Section 3. Simulation results are presented and shown in Section 4 while Section 5 concludes with final remarks.

This work was partially supported by ONR under grant N00014-03-1-0314 and by MIUR. Corresponding author: Ing. Roberto Naldi, CASY-DEIS University of Bologna, Via Risorgimento 2, 40136 Bologna, Italy. Tel: 0039 051 2093788, Fax: 0039 051 2093073, email: rnaldi@deis.unibo.it

II. Helicopter Model and Control Problem

A mathematical model of the helicopter can be derived using Newton-Euler equations of motion of a rigid body in the configuration space $SE(3) = \Re^3 \times SO(3)$. In particular, by fixing an inertial coordinate frame F_i and a coordinate frame F_b attached to the body, the model of the helicopter with respect to the inertia framework can be described as

$$\begin{aligned} M\ddot{p} &= Rf^b \\ J\dot{w} &= -Skew(w)Jw + \tau^b \end{aligned} \tag{1}$$

where f^b and τ^b represents respectively the vector of forces and torques applied to the helicopter expressed in the body frame, M and J the mass and the inertia matrix, the vector $p = \mathrm{col}(x, y, z)$ the position of the center of mass and R the rotation matrix relating the two reference frame. In order to describe rotation matrices we will take advantage of unit quaternion $q = (q_0, q) \in \Re^4$, where q_0 and $q = (q_1, q_2, q_3)^T$ denotes respectively the scalar and the vector part, satisfying the constraint $q_0^2 + \|q\|^2 = 1$. Accordingly R is given as

$$R = \begin{pmatrix} 1-2q_2^2-2q_3^2 & 2q_1q_2-2q_0q_3 & 2q_1q_3+2q_0q_2 \\ 2q_1q_2+2q_0q_3 & 1-2q_1^2-2q_3^2 & 2q_2q_3-2q_0q_1 \\ 2q_1q_3-2q_0q_2 & 2q_2q_3+2q_0q_1 & 1-2q_1^2-2q_2^2 \end{pmatrix}$$

while derivative of rotation matrix is replaced by quaternion propagation equation

$$\dot{q}_0 = -\frac{1}{2}q^{\mathrm{T}}\omega \qquad \dot{q} = \frac{1}{2}[q_0 I + \mathrm{Skew}(q)]\,\omega \tag{2}$$

The external wrench vector applied to the helicopter is a nonlinear function of the five *control inputs*

$$u = \mathrm{col}\begin{pmatrix} P_M & P_T & a & b & T_h \end{pmatrix} \tag{3}$$

where P_M and P_T denotes respectively the collective pitch of the main and of the tail rotor, a and b are respectively the longitudinal and lateral inclination of the tip path plane of the main rotor imposed controlling flapping dynamic by means of cyclic pitches and, finally, T_h is the throttle controlling the main engine power. In particular, following [2] and [3], it turns out that total force/torque can be modeled as

$$f^b = \begin{pmatrix} X_M \\ Y_M + Y_T \\ Z_M \end{pmatrix} + R^T \begin{pmatrix} 0 \\ 0 \\ Mg \end{pmatrix} \tag{4}$$

$$\tau^b = \begin{pmatrix} R_M \\ M_M \\ N_M \end{pmatrix} + \begin{pmatrix} Y_M h_m + Z_M y_m + Y_T h_t \\ -X_M h_m + Z_M l_m \\ -Y_M l_m - Y_T l_t \end{pmatrix}$$

in which g is the force of gravity, (l_m, y_m, h_m) and (l_t, y_t, h_t) denote respectively the coordinates of the main and tail rotor shafts relative to center of mass expressed in F_b, and

$$\begin{array}{rclrcl} X_M & = & -T_M \sin(a) & Y_M & = & -T_M \sin(b) \\ Z_M & = & -T_M \cos(a)\cos(b) & Y_T & = & -T_T \end{array} \tag{5}$$

and

$$\begin{array}{rclrcl} R_M & = & c_b^M b - Q_M \sin(a) & M_M & = & c_a^M a + Q_M \sin(b) \\ N_M & = & -Q_M \cos(a)\cos(b)\,. & & & \end{array} \tag{6}$$

In the previous expressions c_a^M,c_b^M are physical parameters modeling the flapping dynamic of the main rotor, Q_M is the total main rotor torque and T_M and T_T are the thrusts generated respectively by the main and the tail rotor given by

$$T_M = K_{T_M} P_M w_r^2 \qquad T_T = K_{T_T} P_T w_r^2 \tag{7}$$

where w_r denotes the angular velocity of the main rotor in the body frame and the coefficients K_{T_M} and K_{T_T} denote aerodynamic constants of the rotor's blades. It is supposed that the main rotor velocity may differ from the tail one only by a multiplicative constant coefficient $n_{wr} > 0$ included in K_{T_T}.

The angular velocity w_r of the main rotor is governed by the engine dynamics model which, according to [14], is modelled as

$$\dot{w}_r = \frac{Q_e - Q_M^R}{I_{rot}} \tag{8}$$

Q_e is the total engine torques and Q_M^R is a reaction torque due to aerodynamic resistance of rotor's blades given by

$$Q_M^R = c w_r^2 + d P_M^2 w_r^2 \tag{9}$$

with c and d physical parameters depending on wing geometry and other rotor's characteristics. As an approximation, torque acting on main rotor is assumed to be equal to engine torque, namely $Q_M = Q_e$. The expression of engine (and main) torque is then $Q_e = P_e/w_r$ where P_e denotes engine power which is assumed to be proportional to throttle

$$P_e = \bar{P}_e T_h \tag{10}$$

with $0 < T_h < 1$.

This completes the model of the helicopter which is a seventh order system with five control inputs. In the expressions above, however, we will make a number of assumptions in order to simplify the model for control purposes. First of all, as far as the external force f^b is concerned, we neglect the contribution of main rotor thrust along the x^b direction and assume that the contribution of tail rotor thrust along y^b is matched by the corresponding main rotor thrust component. Resultant equations are:

$$f^b = \begin{pmatrix} 0 \\ 0 \\ -T_M \end{pmatrix} + R^T \begin{pmatrix} 0 \\ 0 \\ Mg \end{pmatrix}. \tag{11}$$

Moreover, since the tilt angles a and b are small, we shall assume

$$\sin(a) \approx a\,,\ \sin(b) \approx b\,,\ \cos(a) \approx 1\,,\ \cos(b) \approx 1 \tag{12}$$

which, along with (7) and (10), yields a simplified total torque (4) given by

$$\tau^b = A\left(P_M, T_h, w_r\right) v + B\left(P_M, T_h, w_r\right) \tag{13}$$

in which $v = \mathrm{col}(a,\, b,\, P_T)$, whereas $A(\cdot)$ and $B(\cdot)$ are, respectively, a matrix and a vector of affine functions of the inputs P_M and T_h and of the angular rotor velocity w_r.

With an eye at (8), (11) and (13) the overall dynamic is described by the interconnection of four subsystems, sketched in figure 1, represented by the vertical, lateral-longitudinal, attitude and engine dynamics. In this interconnection the collective pitch of the main rotor P_M (which, as shown in the next section, is designed to control the vertical dynamics) influences all the others subsystems. The engine angular velocity w_r, which will be controlled through the throttle input T_h, influence both the vertical, lateral and attitude dynamic, with the latter influenced also by the engine torque Q_e. Finally the attitude dynamics, controlled through the inputs v, act as a "virtual" control input for lateral-longitudinal dynamics and influence the vertical dynamics.

One of the main goal of the controller to be designed is to deal with possibly large parameters uncertainties, including mass M and the inertia matrix J of the vehicle, the aerodynamic coefficients in (7) and the coefficients in (11), (13) and in engine model (8). In the following we shall denote

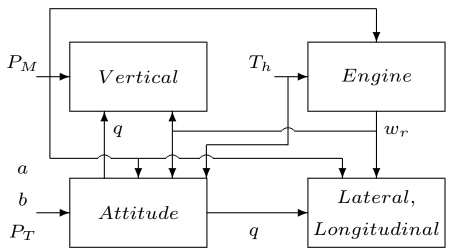

Fig. 1. Dynamical interconnections.

with the subscript "0" and "Δ" respectively the nominal and the uncertain values of these parameters, namely

$$\begin{aligned} M &= M_0 + M_\Delta\,, \qquad J = J_0 + J_\Delta \\ K_{T_i} &= K_{T_i0} + K_{T_i\Delta}\,, \qquad i \in \{M, T\} \\ c &= c_0 + c_\Delta\,, \qquad d = d_0 + d_\Delta \\ \overline{P}_e &= \overline{P}_{e0} + \overline{P}_{e\Delta} \end{aligned}$$

These uncertainties reflect into uncertainties of the matrix $A(\cdot)$ and vector $B(\cdot)$ introduced in (13) which will be accordingly written as

$$\begin{aligned} A\left(P_M, T_h, w_r\right) &= A_0\left(P_M, T_h, w_r\right) + A_\Delta\left(P_M, T_h, w_r\right) \\ B\left(P_M, T_h, w_r\right) &= B_0\left(P_M, T_h, w_r\right) + B_\Delta\left(P_M, T_h, w_r\right)\,. \end{aligned}$$

The ranges of the uncertainties of the physical parameters will be not constrained to be "small" but will be allowed in general to be "arbitrarily large" (fulfilling only physical constraints). The only mild requirement needed to support the results presented in this paper, is a restriction on the relative variation of $A(\cdot)$ with respect to its nominal value $A_0(\cdot)$. In particular it is required the existence of a positive number $m^\star$ such that $A_\Delta\left(P_M, T_h, w_r\right) A_0\left(P_M, T_h, w_r\right)^{-1} \leq m^\star I$ for all possible values of P_M, T_h, w_r within physical ranges.

The main control purpose addressed in this paper is to design the five control inputs (3) in order to asymptotically track vertical, lateral longitudinal references $z_{\mathrm{ref}}(t)$, $y_{\mathrm{ref}}(t)$, and $x_{\mathrm{ref}}(t)$ with q_3 fixed to zero. It will be shown throughout the paper that this indeed will be possible for generic references $z_{\mathrm{ref}}(t)$, $y_{\mathrm{ref}}(t)$, and $x_{\mathrm{ref}}(t)$ with only some restrictions (better expressed in the paper) on the bound of higher order-time derivatives.

We will assume that all state is accessible for control purpose, in particular w_r for engine dynamic, vectors $\mathbf{q}$ and w for the attitude dynamic, vectors p and $\dot{p}$ for the vertical, lateral and longitudinal dynamics. Furthermore the initial state is supposed to belong to any (arbitrarily large) compact set with the only restriction that $q_0(0) > 0$ (which implies that the helicopter is not overturned in the initial condition).

III. Control Structure and Main Results

We begin with the vertical dynamics which, elaborating (1), (7) and (11), are described by

$$M\ddot{z} = -(1 - 2q_1^2 - 2q_2^2) P_M K_{T_M} w_r^2 + Mg \tag{14}$$

and we choose the following preliminary feedback the control input P_M

$$P_M = \frac{-P_M' + M_0 g - M_0 z_{ref}^{(2)}}{K_{T_M0} \max_{\underline{w}}(w_r^2)\left(1 - \mathrm{sat}_c(2q_1^2 + 2q_2^2)\right)} \tag{15}$$

in which $\max_{\underline{w}}(s) := \max\{s, \underline{w}\}$, $\bar{w} > 0$, $\mathrm{sat}_c(s) := \mathrm{sgn}(s)\min\{|s|, c\}$, $0 < c < 1$, and P_M' is an auxiliary control input, whose goal is to decouple the vertical dynamics from the attitude and engine dynamics. The functions $\max(\cdot)$ and $\mathrm{sat}(\cdot)$ are clearly introduced to avoid singularities. Defining $e_z = z - z_{\mathrm{ref}}$ the vertical error dynamics is thus described by

$$\begin{aligned} M\ddot{e}_z &= \Psi_1(q)\Psi_2(w_r)\mu_1\left(P_M' - M_0 z_{\mathrm{ref}}^{(2)} - M_0 g\right) \\ &\quad + M z_{\mathrm{ref}}^{(2)} + Mg \end{aligned} \tag{16}$$

in which $\Psi_1(q) = (1 - 2q_1^2 - 2q_2^2)/(1 - \mathrm{sat}_c(2q_1^2 + 2q_2^2))$, $\Psi_2(w_r) = \omega_r^2/\max_{\underline{w}}(w_r^2)$ and $\mu_1 = K_{T_M}/K_{T_M0}$ is a positive uncertain parameter. Clearly, if $|q_1^2 + q_2^2| \leq c/2$, $w_r^2 \geq \underline{w}$ and $\Psi_1(q) = \Psi_2(w_r) = 1$ system (16) simplifies as

$$M\ddot{e}_z = \mu_1 P_M' + (M - \mu_1 M_0)(z_{\mathrm{ref}}^{(2)} + g) \tag{17}$$

Since we will be able to show, through a suitable design of the others control inputs, that $\Psi_2(w_r(t)) \equiv 1$ and that $\Psi_2 q(t) = 1$ in finite time, we design the residual input P_M' focusing on the simplified system (17). In particular we design P_M' as a *saturated PID* controller of the form

$$\begin{aligned} P_M' &= \mathrm{sat}_l(\xi - k_2\dot{e_z} - k_2 k_1 e_z) \\ \dot{\xi} &= -k_2\dot{e}_z - k_2 k_1 e_z + M_0\dot{e}_z \end{aligned} \tag{18}$$

where k_1, k_2 and l are design parameters. For the closed-loop system (17)-(18) we are able to prove the following.

Proposition 1: Consider system (17)-(18) and define

$$\chi = \xi - (M_0 - M/\mu_1)(z_{\mathrm{ref}}^{(2)} + g) \quad \varrho(s) = (M - M_0\mu_1) z_{\mathrm{ref}}^{(3)}$$

Let k_1 and R_z be arbitrary positive numbers. Then there exist $k_2^\star > 0$, $l^\star > 0$, $M > 1$, $\lambda > 0$ and $n > 0$ such that for all $k_2 \geq k_2^\star$ and $l \geq l^\star$ the trajectories of the system satisfy the ISS inequality

$$\begin{aligned} \|(\chi(t), e_z(t), \dot{e}_z(t))\| &\leq M e^{-\lambda t}\|(\chi(t), e_z(t), \dot{e}_z(t))\| \\ &\quad + n \max_{s\in[0,t]} |\varrho(s)| \end{aligned}$$

provided that $\|\varrho(\cdot)\|_\infty \leq R_z$.

We turn now our attention on the engine dynamics and we look for a control input T_h able to keep the value of w_r close to a desired value $w_r^\star$ and to guarantee that $w_r(t) \geq \underline{w}$ as desired by the previous analysis. To this end we choose a preliminary feedback, aiming to compensate for nominal value of Q_M^R, as

$$T_h = \frac{w_r^3}{\bar{P}_{e0}}(T_h' + c_0 + d_0 P_M^2) \tag{19}$$

in which T'_h is an additional control input designed as the *PI* control law

$$\tilde{w}_r = w - w_r^\star \qquad \dot{\xi} = k_3\, w_r^2\, \tilde{w}_r \qquad T'_h = -k_3\tilde{w}_r - k_4\xi \tag{20}$$

where k_3, and k_4 are design parameters. As a consequence of this choice the engine dynamics are readily seen to be an autonomous system in case of perfect knowledge of the parameters d and $\bar{P}_e$ or a system forced by the input P_M otherwise. The latter, as a consequence of the presence of the saturation function in (18), can be regarded as a *bounded* exogenous signal. This is a crucial property to be able to prove the following proposition underlying the tuning of the engine controller (19), (20).

Proposition 2: Let $k_4 > 0$ be an arbitrary positive number and let P_M be chosen as in (15), (18). Then for any positive ϵ and $T^\star$ there exists $k_3^\star > 0$ such that for all $k_3 \geq k_3^\star$ the following holds

a) $w_r(t) \geq \underline{w}$ for all $t \geq 0$;
b) $|\tilde{w}_r| \leq \epsilon$ for all $t \geq T^\star$

We address now the design of the inputs v in order to control the lateral-longitudinal and attitude dynamics. To this end note that, after few simple but annoying computations which make use of the previous choice of P_M, the lateral-longitudinal $(y - x)$ dynamics can be seen to be described by

$$M\begin{pmatrix} \ddot{y} \\ \ddot{x} \end{pmatrix} = MD(t,q)\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} + n(q)y_z(e_z, \dot{e}_z, \chi) \tag{21}$$

in which

$$D(t,q) = \frac{2(g - z_{\rm ref}^{(2)})}{1 - \mathrm{sat}_c(2q_1^2 + 2q_2^2)}\begin{pmatrix} q_0 & -q_3 \\ -q_3 & -q_0 \end{pmatrix}$$

$n(q) = -\begin{pmatrix} 2q_2q_3 - 2q_0q_1 & 2q_1q_3 + 2q_0q_2 \end{pmatrix}^{\rm T}/(1 - \mathrm{sat}_c(2q_1^2+2q_2^2))$ and $y_z(\cdot) = k_2\dot{e}_z + k_2k_1e_z + \chi$ is a coupling term with the vertical dynamics (where χ has been defined in Proposition 1). The idea is to look at the attitude variables (q_1, q_2) as *virtual control inputs* for the lateral-longitudinal dynamics and to start by designing a *virtual control law* for this system. To this purpose, inspired by [1], we augment system (21) with the bank of integrator

$$\dot{\eta_x} = x - x_{\rm ref} \qquad \dot{\eta_y} = y - y_{\rm ref} \qquad \dot{\eta_q} = q_3 \tag{22}$$

and we fix, for the virtual control input $\mathrm{col}(q_1, q_2, q_3)$, the virtual control law$q^\star = q_1^\star + q_2^\star$ in which $q_2^\star$, representing the control action motivated by the tracking of the lateral/longitudinal references, is defined as [1]

$$q_2^\star = \begin{pmatrix} D^{-1}(t,q) & 0 \\ 0 & 1 \end{pmatrix} r \quad r = \begin{pmatrix} y_{\rm ref}^{(2)} \\ x_{\rm ref}^{(2)} \\ 0 \end{pmatrix}$$

[1]Note that the matrix $D(\cdot)$ is always nonsingular provided that $z_{\rm ref}^{(2)} < g$.

and $q_1^\star$, representing a stabilization control action, is defined by the following *nested saturation* structure

$$\begin{aligned} q_1^\star &= -P_2\lambda_3\sigma(\frac{K_3}{\lambda_3}\zeta_3) \\ \zeta_3 &= \begin{pmatrix} y_2 & x_2 & \eta_q \end{pmatrix}^{\rm T} + P_1\lambda_2\sigma(\frac{K_2}{\lambda_2}\zeta_2) \\ \zeta_2 &= \begin{pmatrix} y - y_{\rm ref} \\ x - x_{\rm ref} \end{pmatrix} + \lambda_1\sigma(\frac{K_1}{\lambda_1}\zeta_1) \qquad \zeta_1 = \begin{pmatrix} \eta_y \\ \eta_x \end{pmatrix} \end{aligned} \tag{23}$$

in which K_i and λ_i, $i = 1, 2, 3$, are design parameters,

$$P_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \quad P_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

and $\sigma(\cdot)$ are vector valued saturation functions defined as $\sigma : \mathrm{col}(s_1, \ldots s_n) \mapsto \mathrm{col}(\mathrm{sat}_1(s_1), \ldots, \mathrm{sat}_1(s_n))$, in which $n = 3$ ($n = 2$) for the σ entering in the expression of $q_1^\star$ (ζ_3, ζ_2). The virtual control law $q^\star$ is then "step-back" to the real control input v. To this end we introduce a preliminary compensation of the nominal part of $B(\cdot)$ in the expression of τ^b in (13)

$$v = A_0^{-1}(P_M, T_h, w_r)\left[\tilde{v} - B_0(P_M, T_h, w_r)\right] \tag{24}$$

and we fix, for the residual input $\tilde{v}$, the following law

$$\tilde{v} = -K_PK_D(\omega - \omega^\star) + K_P(q - q^\star) + J_0\dot{\omega}^\star \tag{25}$$

in which K_P, K_D are design parameters and

$$\omega^\star = -2[q_0I + \mathrm{Skew}(q)]^{-1}\dot{q}_2^\star$$

As a consequence of this choice, the ω^b dynamics are described by

$$\begin{aligned} J\dot{\omega} &= -\mathrm{Skew}(\omega)J\omega + L(P_M, T_h, w_r)(-K_PK_D(\omega - \omega^\star) \\ &\quad + K_P(q - q^\star) + J_0\dot{\omega}^\star) + \Delta(P_M, T_h, w_r) \end{aligned} \tag{26}$$

in which $L(\cdot) = I + A_\Delta(\cdot)A_0^{-1}(\cdot)$, $\Delta(\cdot) = B_\Delta(\cdot) - A_\Delta(\cdot)A_0^{-1}(\cdot)B_0(\cdot)$, and the overall lateral-longitudinal-attitude dynamics are described by (2), (21), (22), (26). This system can be easily recognized to be a 13-th order uncertain system forced by the exogenous inputs $y_z(\cdot)$, $\Delta(\cdot)$, $\omega^\star$, $q_2^\star$ and $\dot{\omega}^\star$. For this system it is possible to prove that there exists a tuning of the overall design parameters (λ_i, K_i), $i = 1, 2, 3$, K_P and K_D which succeeds in enforcing the reference signals *provided* that the lateral-longitudinal references are sufficiently slow as more precisely stated in the next proposition. In the latter the restrictions on the references signals are given with respect to the following vectors

$$\mathbf{x}_{\rm ref} = \begin{pmatrix} x_{\rm ref}^{(2)} & x_{\rm ref}^{(3)} & x_{\rm ref}^{(4)} \end{pmatrix} \qquad \mathbf{y}_{\rm ref} = \begin{pmatrix} y_{\rm ref}^{(2)} & y_{\rm ref}^{(3)} & y_{\rm ref}^{(4)} \end{pmatrix}$$

Furthermore the tuning of the design parameters (λ_i, K_i), $i = 1, 2, 3$, entering in the expression of $q^\star$, is reduced to only two parameters, denoted by ϵ and ℓ, having preliminary made the choice

$$\begin{gathered} (\lambda_i, K_i) = (\epsilon^{i-1}\lambda_i^\star, \epsilon K_i^\star) \\ \lambda_i^\star = K^ic_i \quad K_i^\star = K\ell^i \quad c_{i+1} = \frac{\delta\ell^{i+1}}{2}c_i \end{gathered} \tag{27}$$

where δ and K are arbitrary positive numbers.

Proposition 3: Let R_Δ, $T^\star$, $\nu < 1$ be arbitrary positive numbers. There exist positive numbers ε, $K_D^\star$, $\epsilon^\star(K_D)$, $K_P^\star(K_D)$, $\ell^\star(K_D)$, R_z, γ_1 and γ_2 such that for all $K_D \geq K_D^\star$, $K_P \geq K_P^\star(K_D)$, $\ell \geq \ell^\star(K_D)$ and $0 < \epsilon \leq \epsilon^\star(K_D)$ the following holds:

a) $q_0(t) \leq \nu$ for all $t \geq 0$ and $q_0(t) > 1 - \nu$ for all $t \geq T^\star$;

b) the system (2), (21), (22), (26), (27) is Input-to-State Stable with restrictions on the exogenous inputs. In particular if $\|z^{(2)}_{\rm ref}\|_\infty \leq g/2$ and

$$\|(\mathbf{x}_{\rm ref}, \mathbf{y}_{\rm ref})\|_\infty \leq \varepsilon\,, \quad \|\Delta_e\|_\infty \leq R_\Delta\,, \quad \|y_z\|_\infty \leq R_z \epsilon^2 \tag{28}$$

then the following asymptotic estimate holds [2]

$$\|(\zeta_1, \zeta_2, \zeta_3, \eta_q)\|_a \leq \max\{\gamma_1 \|y_z\|_a \; \frac{\gamma_2}{K_P} \|\Delta_e\|_a\} \tag{29}$$

where $\Delta_e(P_M, T_h, w_r) = \Delta(\cdot) + (L(\cdot)J_0 - J)\dot{\omega}^\star \triangleleft$

Note that item (a) of the proposition guarantees that the helicopter never overturns (condition $q_0(t) \leq \nu$ for all $t \geq 0$) and that the vector q reaches arbitrarily small values (condition $q_0(t) > 1 - \nu$ for all $t \geq T^\star$) in an arbitrarily small time. This, in particular, guarantees that $\Psi_1(q(t)) = 1$ for all $t \geq T^\star$ and thus that system (16) reduces to (17) to which Proposition 1 applies. As a comment to item (b) of the proposition, note that the restrictions on $(\mathbf{x}_{\rm ref}, \mathbf{y}_{\rm ref})$ dictated by (28) can be always fulfilled by designing sufficiently smooth lateral and longitudinal references (with small acceleration, jerk and quirk). As far as the exogenous input $y_z(\cdot)$ is concerned not that, by bearing in mind the result of Proposition 1, it is possible to conclude that the restriction can be always fulfilled in finite time provided that the vertical reference has a small jerk or, eventually, if the physical parameters M and T_{T_M} are known with good accuracy (so that $(M - M_0\mu_1)$ is small). On the other hand note that the restriction on Δ_e does not impose limitations as the number R_Δ can be taken arbitrarily large. Moreover note that, by (29) and by definition of ζ_i, $i = 1, 2, 3$, the "goodness" of the asymptotic tracking performances of the lateral/longitudinal dynamics are governed by the amplitude of the signal $y_z(\cdot)$ and Δ_e, with the influence of the latter which can be rendered negligible by increasing the design parameter K_P. In view of this, a high precision asymptotic tracking can be achieved by enforcing zero-jerk vertical reference trajectories (yielding, by Proposition 1, $\|y_z\|_a = 0$). If, additionally, the reference signals are taken sufficiently smooth so that $\dot{\omega}^\star = 0$ and the matrices $A(\cdot)$ and $B(\cdot)$ are perfectly known so that $\Delta(\cdot) = 0$, the proposed control law guarantees perfect asymptotic tracking.

We conclude this section with few remarks about the control structure derived above to control the lateral/longitudinal/attitude dynamics (see fig. 2). It is constituted by two nested control loops. The inner loop, controlling the attitude dynamics, is designed as the high-gain feedback

[2]We define $\|s\|_a = lim_{t\to\infty} sup \|s(t)\|$.

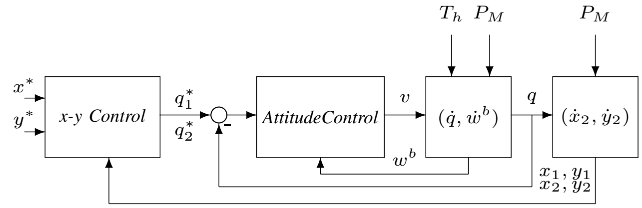

Fig. 2. lateral/longitudinal/attitude control structure.

(24)-(25) and acts as a "servo-loop" for the outer loop, designed using the nested saturation structure in (23) and whose goal is to control the lateral/longitudinal dynamics. The two loops are characterized by two-time scale dynamics (with the inner loop characterized by faster dynamics) enforced through the design parameters K_P and ϵ which are respectively very large and very small. For further comments and properties about this structure the interested reader is referred to [1].

IV. Simulation Results

We present in this section simulation results concerning a specific model of a small unmanned autonomous helicopter described in [3]. The nominal value of the helicopter parameters are given in Table 1, whereas control parameters are reported in table 2. We assume parametric uncertainties up to 30% of the nominal values.

TABLE I
NOMINAL PARAMETERS OF THE PLANT

$J_x = 0.142413$	$J_y = 0.271256$	$J_z = 0.271256$
$l_m = -0.015$	$y_m = 0$	$h_m = 0.2943$
$l_t = 0.8715$	$h_t = 0.1154$	$m = 4.9$
$c_M^{Q,T} = 25.23$	$K_{T_M} = 0.01$	$K_{T_T} = 0.00165$
$P_e = 1200$	$c = 0.00042$	$d = 0.005$

TABLE II
CONTROLLER PARAMETERS

Vertical	$k1 = 0.6$	$k2 = 35$	
Engine	$k_3 = 1.5/(w_r^*)^2$	$k_4 = 10/(w_r^*)^2$	$w_r^* = 167$
Attitude	$K_P = 30$	$K_D = 0.2$	
Lat. and Lon.	$K_1 = 0.4$	$K_2 = 0.2$	$K_3 = 1$
Saturation	$\lambda_1 = 2000$	$\lambda_2 = 10$	$\lambda_3 = 0.3$

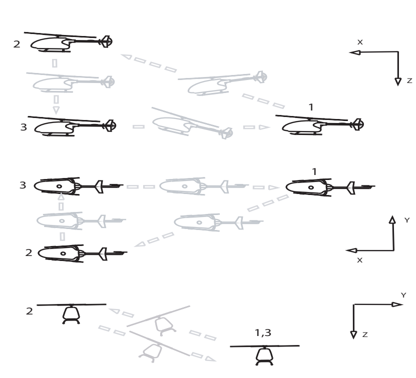

Fig. 3. Trajectory scheme. Top: lateral view. Center: top view. Bottom: front view.

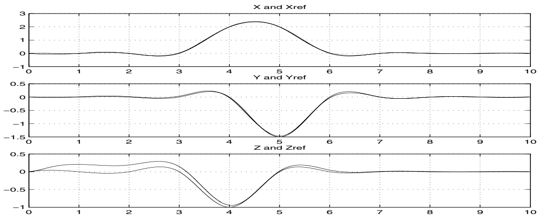

Fig. 4. Trajectory followed over desired path in X, Y and Z.

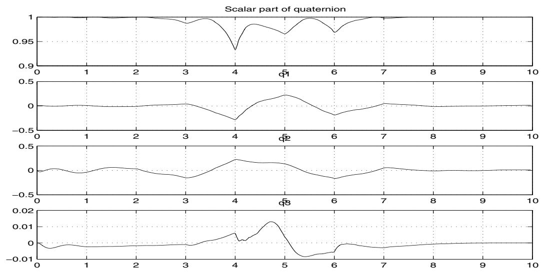

Fig. 5. Quaternion scalar part q_0 and vector part q_1, q_2 and q_3.

As it is shown in figure 3, the reference given to the control algorithm is designed in order to complete a closed path in three separated movements. The path is generated using a 3^{rd} order spline interpolating the desired end points. Figures 4 shows the trajectory followed over the desired path for each coordinate x, y and z. Note that even considering large uncertainties on the plant parameters the designed controller is able to track with very small error the desired path. Figure 5 shows the time history of attitude parameters. As expected, any changes in x and y position is reflected in q_1 and q_2 values whereas q_3 is closed to zero. Figure 6 shows the angular rotor velocity w_r which is kept close to the desired value w_r^* by control input T_h. Figure 7 show the other control inputs. With an eye at P_M time history observe that vertical dynamics influence engine dynamics, whereas both vertical and engine dynamics influence the attitude dynamics. The first influence is visible in both w_r and T_h time history, whereas the latter is clearly visible looking at control input P_T.

V. Conclusions

In this paper we presented the design of an autopilot for helicopters to track arbitrary lateral/longitudinal and vertical references in presence of uncertainties in the controlled system. The references are arbitrary signals with some limitations in the higher order time derivatives. The control structure is composed by a mix of high gain and nested saturation feedback control laws and feedforward control actions. Future works on this subject are focalized on experimental validation of the proposed design techniques on a small scale helicopter.

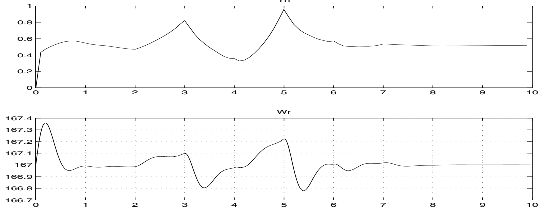

Fig. 6. Engine dynamic: T_h control input and angular rotor velocity w_r.

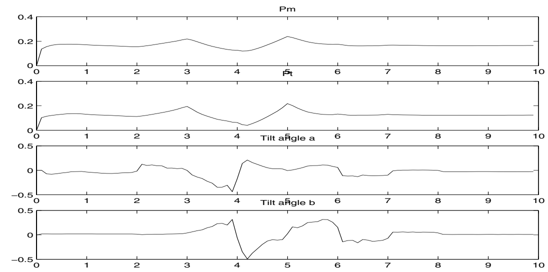

Fig. 7. P_M, P_T, a and b control inputs.

References

[1] A. Isidori, L. Marconi and A. Serrani, "Robust Autonomous Guidance", *Springer* ,2003.

[2] A. Isidori, L. Marconi and A. Serrani, "Robust Nonlinear Motion Control of a Helicopter", *IEEE Trans. Automat. Control* ,vol 48, no.3, pp 413-426, march 2003.

[3] T. Koo and Sastry, "Output tracking control design of a helicopter model based on approximate linearization", *in 37th IEEE Conf. Decision Control*, Tampa, FL, 1998.

[4] A. Isidori, L. Marconi and A. Serrani, "Computation of thero-error manifold for a problem of smooth vertical landing of a helicopter", *in 6th Eur. Control Conf.*,Porto,Portugal,2001.

[5] L. Marconi and A. Isidori, "Robust global stabilization of a class of uncertain feedfordward nonlinear systems", *Sys. Control Lett.*, vol.41, pp. 281-290, 2000.

[6] A. Isidori and C.I. Byrnes, "Output regulation of nonlinear systems", *IEEE Trans. Automat. Control*,vol 35, pp 131-140, feb. 1990.

[7] A. Isidori, L. Marconi and A. Serrani, "Autonomous landing of unmanned helicopters", *Dipartimento di Elettronica, Informatica e Sistemistica, Univ. degli Studi di Bologna, Bologna*, Italy, 2001.

[8] S. Lane and R. Stengel, "Flight control design using nonlinear inverse dynamics", *Automatica*, vol.24, no. 4, pp. 471-484, 1998.

[9] G. Meyer and L. Cicolani, "Application of nonlinear systems inverses to automatic flight control design: System concepts and flight evaluations", *AGARDograph AG-251 Theory Appl. Optimal Control Aerosp. Syst.*, pp. 10-1-10-29, 1980.

[10] O. Shakernia, Y. Ma, T. Koo and S. Sastry "Landing an unmanned air vehicle: vision based motion estimation and nonlinear control", *Asian J. Guid., Control, Dyna.*, vol 1, no.3, pp 128-144, 1999.

[11] H. Sira-Ramirez, R. Castro-Linares and E. Liceaga-Castro "A liouvillian system approach for the trajectory palnning-based control of helicopter models", *Int J. Robust Nonlinear Control*, vol 10, no.4, pp 301-320, 2000.

[12] S. Snell D. Enns and W. Garrard "Control of a twin lift helicopter system using nonlinear state feedback", *J. Amer. Helicopter Soc.*, vol 36, no.4, pp 57-65.

[13] J. Prasad M. M. and D. Schrage "Nonlinear inversion flight control for supermaneuverable aircraft", *AIA J. Guid., Control, Dyna.*, vol 15, no.1, pp 976-984, 1992.

[14] V. Gavrilets, "Aerobatic Maneuvering of Miniature Helicopters", *Ph.D. Thesis*, Massachussetts Institute of Technology, 2003.

[15] B. Mettler, M.B. Tishler and T. Kanade "System Identification of Small-Size Unmanned Helicopter Dynamics", *American Helicopter Society 55th Forum*, may 1999.

[16] K. Sprague, V. Gavrilets, D. Dugail, B. Mettler and E. Feron, I. Martinos " Design and applications of an avionics system for a miniature acrobatic helicopter", *Digital Avionics Systems, 20th Conference*, vol. 1 ,pp. 14-18, oct. 2001.