Proceedings of the
44th IEEE Conference on Decision and Control, and
the European Control Conference 2005
Seville, Spain, December 12-15, 2005

Rolling horizon for active fault detection

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Abstract— **This paper presents a feasible design of suboptimal active fault detection system in multiple-model framework. The optimal solution for finite horizon is approximated by means of well known rolling horizon scheme which belongs to the class of limited look-ahead policies. The suboptimal input signal which is chosen from given discrete set is obtained by l-step closed loop optimization. It is shown that such input signal can improve fault detection.**

I. INTRODUCTION

A change detection problem rises in countless applications, from time series analysis to fault detection and isolation (FDI). Primary aim of fault detection is early recognition of a change in behavior of observed system using available measurements. A survey of various methods to the detector design can be found within many of others in [1], [2], [3]. More complex methods were proposed with growing availability of faster and cheaper computers [4]. The detector design is usually based on a mathematical model of a real process. Such detector passively monitors available measurements and makes decisions.

A lot of the approaches consider that the detection consists of two steps. The first step is the generation of residuals which represent inconsistencies between the true, and expected behavior of the observed system and the decisions based on the residuals are made in the second step. A comprehensive review of methods used for residual generator design can be found in [5]. The work [6] is aimed to design of both parts of detector and this design is based on statistical approach mainly. In many papers great attention is aimed to robustness problem in the residual generator design [7], [8], [9]. On the other side, the problem of system excitation is often either omitted or only the request for appropriate excitation is stated even if change detection methods sensitive to the system excitation are used.

Design of a special input signal which can improve system recognition is well known from system identification [10]. This idea was consequently applied to change detection problem in multiple-model (MM) framework. In [11] the input signal maximizes the Baram's distance between considered models. More systematical approach was proposed in [12] where the optimal input signal is designed by the appropriate criterion minimization. A method for the input signal design is presented in [13] and the input signal enables to decide surely in which mode the system operates.

Most of the existing FDI methods do not consider the future information as a tool for minimizing possible losses caused by wrong decisions. They utilize all available information at the moment of decision but the fact that additional information will be obtained in the future is not considered. Some control policies (CEC, OLF, etc.) suffer from similar drawback as discussed and solved in [14]. If it is considered that decision can influence the observed process progression, then the future information can backwards affect this decision. This problem was elaborated in [15] where three different information processing strategies (IPS's) of change detection in MM framework are studied, namely open loop (OL), open loop feedback (OLF) and closed loop (CL) IPS's. The design procedure based on the CL IPS is superior to commonly used the Bayesian approach e.g. [16] which matches the OLF IPS. An extension of the idea was presented in [17] but the optimal generator and the optimal detector are infeasible because of computational and memory demands.

This paper concerns the active FDI in MM framework. The design based on the CL IPS [17] can provide an active fault detection system which generates better decisions in comparison with detection systems obtained using standard design approaches. Unfortunately, the optimal solution is infeasible and thus the aim of this paper is to propose a suboptimal detector and generator of input signal which should improve fault detection using the CL IPS.

The paper is organized as follows. In Section II the model of system together with description of a detection system are presented. The advantage of detector design by a criterion minimization is briefly discussed. The optimal solution of stated problem utilizing dynamic programming (DP) is presented in Section III. Further, a suboptimal solution based on the rolling horizon (RH) scheme is proposed. Moreover, the stated problem is also solved using the OLF IPS because most of known FDI methods utilize received information in this sense. The description of a suitable pruning and merging technique is presented at the end of section. In Section IV the RH scheme and OLF IPS are compared in a simple numerical example which provides good insight into algorithms.

II. PROBLEM STATEMENT

The active fault detection system consists of the observed system **S**, generator **G** and the detector **D** as shown in Fig. 1. The detector uses input-output data to make decisions d_k and generator should provide input signal u_k which improves fault detection. The MM approach is used for

This work was supported by the Ministry of Education, Youth and Sports of the Czech Republic, project No. 1M0572.

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0-7803-9568-9/05/$20.00 ©2005 IEEE

system description and it is supposed that one of models defines the safety behavior pattern (mode) and the others describe individual faulty modes.

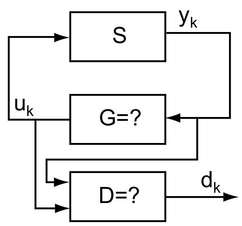

Fig. 1. Structure of an active fault detection system

Let the observed system **S** be described at each time $k \in \mathcal{T} = \{0, 1, \dots, F\}$ by jump Markov linear Gaussian model

$$\begin{aligned} \mathbf{x}_{k+1} &= A(\theta_k)\mathbf{x}_k + B(\theta_k)u_k + G(\theta_k)\mathbf{w}_k \\ \mathbf{y}_k &= C(\theta_k)\mathbf{x}_k + H(\theta_k)\mathbf{v}_k \end{aligned} \quad (1)$$

where $\mathbf{y}_k \in \mathcal{R}^{n_y}$ denotes the observation, $\mathbf{x}_k \in \mathcal{R}^{n_x}$ denotes the unknown state vector, $u_k \in \mathcal{U}_k \subset \mathcal{R}$ is a known input and $\theta_k \in \mathcal{M} = \{1, 2, \dots, N\}$ denotes unknown discrete state. It is assumed that a set of models is identical with set of all possible modes of system (i.e. model set cover all possible behavior patterns of the system **S**). Thus, the number of models N remains constant over time. The variable number of models governed by some model-set adaptation technique [18] is beyond the scope of this paper. The noises $\mathbf{w}_k \in \mathcal{R}^{n_x}$ and $\mathbf{v}_k \in \mathcal{R}^{n_y}$ are mutually independent zero-mean white Gaussian noises with identity covariance matrices, $\mathcal{N}\{0, I\}$. The initial state $\mathbf{x}_0$ is given as $\mathcal{N}\{\mathbf{x}_0^{'}, \Sigma_0^{'}\}$ and the probability of the parameter θ_0 is $P(\theta_0)$. The variables $\mathbf{x}_0$ and θ_0 are mutually independent and also independent of the noise processes. The switching between models is governed by the given transition probability $P_{i,j} = P(\theta_{k+1} = j|\theta_k = i)$ for $i, j \in \mathcal{M}$ and the each model θ_k is represented by known matrices $A(\theta_k)$, $B(\theta_k)$, $C(\theta_k)$, $G(\theta_k)$, $H(\theta_k)$ of appropriate dimensions. Finally, it is assumed that for each combination $\theta_k, \theta_{k+1} \in \mathcal{M}, k = 0, \dots, F-1$, it holds that $\text{rank}(C(\theta_{k+1})B(\theta_k)) > 0$, i.e. one-step input-output delay in the system (1). If the model uncertainty represented by unknown matrices in (1) is considered then the active FDI can be solved identically, however the state estimation will be more complicated.

Now, the description of an active fault detection system will be presented. At each time k the detection system should utilize all information retrieved up to this time. The deterministic description of the detector and the input generator is used and the design can be made independently [17] due to additivity of below introduced criterion.

The detector **D** is described by an unknown function of all available information as

$$d_k = \sigma_k\left(\mathbf{I}_0^k\right), \quad k \in \mathcal{T}, \quad (2)$$

where $d_k \in \mathcal{M}$ denotes decision of detector at time k, $\mathbf{I}_0^k = [\mathbf{y}_0^{k^T}, u_0^{k-1^T}]^T$ denotes all available information and the notation $\mathbf{y}_0^k = [\mathbf{y}_0{}^T, \mathbf{y}_1{}^T, \dots, \mathbf{y}_k{}^T]^T$ is used for description of whole history of considered variable or sequence of functions.

The auxiliary generator **G** should generate sequence of inputs u_0^F, which improve change detection in the sense of lowering of value of chosen criterion. The generator

$$u_k = \gamma_k\left(\mathbf{I}_0^k\right), \quad k \in \mathcal{T} \quad (3)$$

is described by unknown functions γ_0^F. In control problem statement the functions γ_0^F create control policy and they are designed to fulfil control objectives.

The goal of the design is to choose such detection system which achieves a minimum value of losses connected with decision. So, a loss function in the criterion should penalize wrong decision and the value of the criterion can be assumed as expectation of the sum of the loss function over the finite detection horizon F. The additive criterion is

$$J(\sigma_0^F, \gamma_0^F) = \mathrm{E}\left\{\sum_{k=0}^{F} L(\theta_k, d_k)\right\}, \quad (4)$$

where $\mathrm{E}\{\cdot\}$ denotes expectation operator over all included random variables and the loss function $L(\theta_k, d_k)$ is a real-valued non-negative function which is chosen with respect to real costs caused by the decisions. These costs can be given by economical, environmental and other factors. Such approach to detector design does not guarantee probability of missed and false alarm in contrary to the approaches based on statistical tests but it is very useful if it is possible to evaluate the real costs which are connected with right and wrong decisions.

III. Active detector design

The problem of active fault detection system design was formulated as a problem of the criterion minimization on the finite horizon. In general, the optimal solution utilizing the CL IPS can be found using standard DP algorithm [19]. This algorithm respects the fact that future observations will be used for fault detection and obtained value of the criterion is minimal. Unfortunately, it is not possible to find an analytical solution in closed form due to the intractable integrals and some numerical solution is necessary even for relatively simple problem considered in this paper. Further, the numerical computation required by the DP can be substantially reduced by applying a suitable approximation. The RH scheme with l-step CL optimization is proposed to obtained feasible algorithm.

Most of known FDI techniques are based on the OLF IPS because they do not count with receiving further observation in the future. Thus, the stated active FDI is also solved using the OLF IPS for comparison.

A. Closed loop information processing strategy

The basic optimal solution of stated problem together with some rearrangements will be presented.

The aim is to find functions σ_0^F and γ_0^F which minimize criterion (4) given constrains (1). The DP algorithm is general tool for solving such multi-step constrained optimization problems and the backward recursive equations are

$$\begin{aligned} V^*_{F+1} &= 0, \\ V^*_k(\mathbf{I}_0^k) &= \min_{d_k\in\mathcal{M}} \mathrm{E}\left\{L(\theta_k, d_k)|\mathbf{I}_0^k, d_k\right\} + \\ &\min_{u_k\in\mathcal{U}_k} \mathrm{E}\left\{V^*_{k+1}(\mathbf{I}_0^{k+1})|\mathbf{I}_0^k, u_k\right\}, \quad k = F,\dots,0 \\ J^* &= \mathrm{E}\left\{V^*_0(\mathbf{I}_0)\right\}, \end{aligned} \tag{5}$$

where $\mathrm{E}\{\cdot|\cdot\}$ denotes the conditional expectation, V^*_{F+1} is boundary condition, $V^*_k(\mathbf{I}_0^k)$ is so called cost-to-go function at time k and J^* is resulting value of the criterion. The minimum $V^*_k(\mathbf{I}_0^k)$ in (5) is achieved using the optimal decision d^*_k and the optimal input signal u^*_k

$$d^*_k = \arg\min_{d_k\in\mathcal{M}} \mathrm{E}\left\{L(\theta_k, d_k)|\mathbf{I}_0^k, d_k\right\}, \tag{6}$$

$$u^*_k = \arg\min_{u_k\in\mathcal{U}_k} \mathrm{E}\left\{V^*_{k+1}(\mathbf{I}_0^{k+1})|\mathbf{I}_0^k, u_k\right\}. \tag{7}$$

It is obvious that these expressions are in fact the searched functions σ_k and γ_k. Note that the introduced algorithm is not restricted to given system (1) but it is valid as well for more general system described in [17].

Now, the probability density functions (pdf's) needed to evaluate (5) will be expressed using properties of the system (1) and the DP algorithm will be rearranged to a form which is preferable for numerical computations.

It can be easily seen that for the expectations evaluation in (5) the probability $P(\theta_k|\mathbf{I}_0^k)$ and conditional pdf $p(\mathbf{y}_{k+1}|\mathbf{I}_0^k, u_k)$ are required. For clarity of the following explanations, the probability $P(\theta_k|\mathbf{I}_0^k)$ will be expressed as

$$P(\theta_k|\mathbf{I}_0^k) = \sum_{\theta_0\in\mathcal{M}} \cdots \sum_{\theta_{k-1}\in\mathcal{M}} P(\theta_0^k|\mathbf{I}_0^k) = \sum_{\theta_0^{k-1}} P(\theta_0^k|\mathbf{I}_0^k). \tag{8}$$

The joint probability $P(\theta_0^k|\mathbf{I}_0^k)$ is recursively computed as

$$\begin{aligned} P(\theta_0^k|\mathbf{I}_0^k) = &\frac{p(\mathbf{y}_k|\mathbf{I}_0^{k-1}, \theta_0^k, u_{k-1})P(\theta_k|\theta_{k-1})}{p(\mathbf{y}_k|\mathbf{I}_0^{k-1}, u_{k-1})} \times \\ &P(\theta_0^{k-1}|\mathbf{I}_0^{k-1}), \end{aligned} \tag{9}$$

where $p(\mathbf{y}_k|\mathbf{I}_0^{k-1}, u_{k-1})$ is only a normalization constant which will be cancelled out after some rearrangements and the likelihood term $p(\mathbf{y}_k|\mathbf{I}_0^{k-1}, \theta_0^k, u_{k-1})$ can be obtained as Gaussian predictive pdf of the observation from the Kalman filter corresponding to the sequence of models θ_0^k. The number of required Kalman filters and amount of memory are exponentially growing with time and this issue will be discussed later in Section III-C.

The following rearrangement of the DP algorithm allows to cancel out the pdf $p(\mathbf{y}_k|\mathbf{I}_0^{k-1}, u_{k-1})$ and thus simplify computations. Successive substituting in (9) leads to

$$P(\theta_0^k|\mathbf{I}_0^k) = \frac{Q_k(\mathbf{I}_0^k, \theta_0^k)}{R_k(\mathbf{I}_0^k)}, \tag{10}$$

where

$$\begin{aligned} Q_k(\mathbf{I}_0^k, \theta_0^k) &= \prod_{i=0}^{k} p(\mathbf{y}_i|\mathbf{I}_0^{i-1}, \theta_0^i, u_{i-1})P(\theta_i|\theta_{i-1}), \\ R_k(\mathbf{I}_0^k) &= \prod_{i=0}^{k} p(\mathbf{y}_i|\mathbf{I}_0^{i-1}, u_{i-1}). \end{aligned} \tag{11}$$

It will be shown by backward induction on k that the normalization constant $R_k(\mathbf{I}_0^k)$ in denominator of (10) will be subsequently cancel out. According to (5) the cost-to-go function at time $k = F$ is

$$V^*_F(\mathbf{I}_0^F) = \min_{d_F\in\mathcal{M}} \sum_{\theta_0^F} L(\theta_F, d_F)P(\theta_0^F|\mathbf{I}_0^F). \tag{12}$$

Substituting (10) in (12), it follows that

$$\begin{aligned} V^*_F(\mathbf{I}_0^F) &= \frac{1}{R_F(\mathbf{I}_0^F)} \min_{d_F\in\mathcal{M}} \sum_{\theta_0^F} L(\theta_F, d_F)Q_F(\mathbf{I}_0^F, \theta_0^F) \\ &= \frac{\overline{V}^*_F(\mathbf{I}_0^F)}{R_F(\mathbf{I}_0^F)} \end{aligned} \tag{13}$$

Thus, assuming that for all $V^*_{k+1}(\mathbf{I}_0^{k+1})$ holds

$$V^*_{k+1}(\mathbf{I}_0^{k+1}) = \frac{\overline{V}^*_{k+1}(\mathbf{I}_0^{k+1})}{R_{k+1}(\mathbf{I}_0^{k+1})}, \tag{14}$$

it is easy to shown that

$$\begin{aligned} V^*_k(\mathbf{I}_0^k) &= \frac{1}{R_k(\mathbf{I}_0^k)} \min_{d_k\in\mathcal{M}} \sum_{\theta_0^k} L(\theta_k, d_k)Q_k(\mathbf{I}_0^k, \theta_0^k) + \\ &\quad \min_{u_k\in\mathcal{U}_k} \int \frac{\overline{V}^*_{k+1}(\mathbf{I}_0^{k+1})}{R_{k+1}(\mathbf{I}_0^{k+1})} p(\mathbf{y}_{k+1}|\mathbf{I}_0^k, u_k) d\mathbf{y}_{k+1} \\ &= \frac{1}{R_k(\mathbf{I}_0^k)} \Bigg[\min_{d_k\in\mathcal{M}} \sum_{\theta_0^k} L(\theta_k, d_k)Q_k(\mathbf{I}_0^k, \theta_0^k) + \\ &\quad \min_{u_k\in\mathcal{U}_k} \int_{\mathbf{y}_{k+1}} \overline{V}^*_{k+1}(\mathbf{I}_0^{k+1}) d\mathbf{y}_{k+1} \Bigg] = \frac{\overline{V}^*_k(\mathbf{I}_0^k)}{R_k(\mathbf{I}_0^k)}. \end{aligned} \tag{15}$$

The transformed DP algorithm is described by backward recursive equation

$$\begin{aligned} \overline{V}^*_{F+1} &= 0, \\ \overline{V}^*_k(\mathbf{I}_0^k) &= \min_{d_k\in\mathcal{M}} \sum_{\theta_0^k} L(\theta_k, d_k)Q_k(\mathbf{I}_0^k, \theta_0^k) + \\ &\quad \min_{u_k\in\mathcal{U}_k} \int \overline{V}^*_{k+1}(\mathbf{I}_0^{k+1}) d\mathbf{y}_{k+1}, \quad k = F,\dots,0 \\ J^* &= \int \overline{V}^*_0(\mathbf{I}_0) d\mathbf{y}_0. \end{aligned} \tag{16}$$

Unfortunately, the functions $\overline{V}^*_k(\mathbf{I}_0^k)$ for $k \in \mathcal{T}$ can not be expressed analytically due to intractable integrals which result from the minimization over discrete set of decisions. Therefore some numerical integration method together with hypotheses pruning must be used. Given the sequence θ_0^F without any observations the function $Q_F(\mathbf{I}_0^F, \theta_0^F)$ is in fact weighted $n_y \times F$ dimensional Gaussian distribution. Solving the optimization problem at time k involves numerical integration over dimension $n_y \times (F-k)$ for one cost-to-go function $\overline{V}^*_k(\mathbf{I}_0^k)$ evaluation. It is obvious that such approach is applicable only for relatively short horizon.

Rolling horizon. The main aim of the following subsection is to present some reasonable suboptimal solution. A systematical approach to forward solution of the DP equations (5) based on the method of stochastic approximation is presented e.g. in [20]. In this paper the RH scheme using l-step closed loop optimization over discrete sets $\mathcal{M}$ and $\mathcal{U}_k$ will be considered. The terminal cost-to-go functions of the truncated horizon are replaced by zeros in the RH scheme. The length $l > 0$ of truncated horizon should be as short as possible to save computational demands but on other hand it has to preserve dependence of value of the minimized criterion on the input signal u_k. Note that the l-step CL optimization can reduce computational demands but it is not possible to guarantee that such strategy will be always better than the OLF IPS.

At each time $k \in \mathcal{T}$ the following criterion is minimized

$$\widetilde{J}(\sigma_k^{\bar{l}}, \gamma_k^{\bar{l}}) = E\left\{\sum_{i=k}^{\bar{l}} L(\theta_i, d_i)\right\}, \tag{17}$$

where $\bar{l} = \min\{k+l-1, F\}$ and only the decision d_k^{RH} and input signal u_k^{RH} will be used.

At time k the approximation of the cost-to-go function $\overline{V}^*_{\bar{l}+1}(\mathbf{I}_0^{\bar{l}+1}) \approx \widetilde{V}^*_{\bar{l}+1} = 0$ holds and following backward recursive equation has to be solved

$$\begin{aligned}\widetilde{V}_i(\mathbf{I}_0^i) = &\min_{d_i \in \mathcal{M}} \sum_{\theta_0^i} L(\theta_i, d_k) Q_i(\mathbf{I}_0^i, \theta_0^i) + \\ &\min_{u_i \in \mathcal{U}_i} \int \widetilde{V}_{i+1}(\mathbf{I}_0^{i+1}) d\mathbf{y}_{i+1},\ i = \bar{l}, \ldots, k+1.\end{aligned} \tag{18}$$

The decision d_k^{RH} and the input u_k^{RH} are then computed as

$$d_k^{RH} = \arg\min_{d_k \in \mathcal{M}} \sum_{\theta_0^k} L(\theta_k, d_k) Q_k(\mathbf{I}_0^k, \theta_0^k), \tag{19}$$

$$u_k^{RH} = \arg\min_{u_k \in \mathcal{U}_k} \int \widetilde{V}_{k+1}(\mathbf{I}_0^{k+1}) d\mathbf{y}_{k+1}. \tag{20}$$

The numerical solution of the integral in (18) which is carried out over vector of future measurement $\mathbf{y}_{i+1}$ can be computationally demanding for higher dimensions and must be performed for every considered input $u_k \in \mathcal{U}_k$. If the set $\mathcal{U}_k$ consists of discrete values then the growth of computations is directly proportional to cardinality of the set $\mathcal{U}_k$. The main computational difficulty which increases with time remains in the growing number of possible model sequences.

B. Open loop feedback information processing strategy

The OLF IPS is often used scheme where the information horizon is truncated up to time of decision. It requires to solve OL minimization problem for future stages $k+1, \ldots, F$ at each time k. Thus, the decision d_k and auxiliary input u_k are found as arguments which minimize

$$\bar{J} = \min_{u_k^F, d_k^F} \mathrm{E}\left\{\sum_{i=k}^{F} L(\theta_i, d_i) | \mathbf{I}_0^k, d_k^F, u_k^F\right\}. \tag{21}$$

Using property of expectation and by splitting the sum, the relation (21) can be written as

$$\begin{aligned}\bar{J} = &\min_{d_k \in \mathcal{M}} \sum_{\theta_k} L(\theta_k, d_k) P(\theta_k | \mathbf{I}_0^k, d_k) + \\ &\min_{d_{k+1}^F} \sum_{\theta_k^F} \sum_{i=k+1}^{F} L(\theta_i, d_i) P(\theta_k^F | \mathbf{I}_0^k, d_{k+1}^F),\end{aligned} \tag{22}$$

where $P(\theta_k^F | \mathbf{I}_0^k) = P(\theta_k | \mathbf{I}_0^k) \prod_{i=k}^{F-1} P(\theta_{i+1} | \theta_i)$. Only the first term can be minimized by the decision d_k and the second one represents future expected costs if no further measurements will be received. Thus, the the optimal decision is

$$d_k^{OLF} = \arg\min_{d_k \in \mathcal{M}} \sum_{\theta_k \in \mathcal{M}} L(\theta_k, d_k) P(\theta_k | \mathbf{I}_0^k, d_k). \tag{23}$$

Comparing (23) with (6) it is clear that the decision is chosen by the same procedure as in optimal case but there is not any suggestion how to choose input u_k when this approximation is used. This is obvious result because the input u_k can influence only the future measurements $\mathbf{y}_{k+1}^F$ which are not considered to be received. Thus, an additional criterion for input determination can be proposed or an arbitrary probing signal can be chosen. It is assumed that the Pseudo Random Binary Sequence (PRBS) will be used as input signal in this suboptimal scheme because it is well known and easily realizable probing signal.

C. Pruning and merging sequence of models

This subsection is devoted to the approximative estimation of the system state and observation prediction. The exact pdf $p(\mathbf{y}_{k+1} | \mathbf{I}_0^k, \theta_0^{k+1}, u_k)$ resulting from the pdf $p(\mathbf{x}_k | \mathbf{I}_0^k, \theta_0^k)$ and probability $P(\theta_k | \mathbf{I}_0^k)$ can be computed using N^{k+1} Kalman filters. The issue with growing computational and memory demands in switching MM have to be solved using some pruning and merging technique (GPB, IMM, RSA, etc.).

The pdf approximation is chosen with respect to proposed RH scheme. The number of model sequences is reset back to number of models N after each h time steps of estimation algorithm. So, it is supposed that standard MM estimation algorithm is used and merging and pruning are performed at each time step $k = h, 2h, \ldots$ as will be shown. It is assumed that the following filtering pdf and probability are known at a time of pruning and merging

$$p(\mathbf{x}_k | \mathbf{I}_0^k, \theta_{k-h}^k) = \mathcal{N}\{\hat{\mathbf{x}}_k(\theta_{k-h}^k), \Sigma_k(\theta_{k-h}^k)\}, \tag{24}$$

$$P(\theta_{k-h}^k | \mathbf{I}_0^k), \tag{25}$$

where pdf $p(\mathbf{x}_k | \mathbf{I}_0^k, \theta_{k-h}^k)$ was obtained from previous Gaussian filtering density $p(\mathbf{x}_{k-1} | \mathbf{I}_0^{k-1}, \theta_{k-h}^{k-1})$ by standard the Kalman filtering algorithm and the probability $P(\theta_{k-h}^k | \mathbf{I}_0^k)$ can be computed using a slightly modified relation (9) as

$$\begin{aligned}P(\theta_{k-h}^k | \mathbf{I}_0^k) = &\frac{p(\mathbf{y}_k | \mathbf{I}_0^{k-1}, \theta_{k-h}^k, u_{k-1}) P(\theta_k | \theta_{k-1})}{p(\mathbf{y}_k | \mathbf{I}_0^{k-1}, u_{k-1})} \times \\ &P(\theta_{k-h}^{k-1} | \mathbf{I}_0^{k-1}).\end{aligned} \tag{26}$$

The predictive pdf $p(\mathbf{y}_k|\mathbf{I}_0^{k-1},\theta_{k-h}^k,u_{k-1})$ of the observation can be generally computed from the pdf $p(x_{k-1}|\mathbf{I}_0^{k-1},\theta_{k-h}^{k-1})$ as

$$p(\mathbf{y}_k|\mathbf{I}_0^{k-1},\theta_{k-h}^k,u_{k-1}) = \int_{\mathbf{x}_{k-1}^k} p(\mathbf{y}_k|\mathbf{x}_k,\theta_k)\times \quad (27)$$
$$p(x_k|\mathbf{x}_{k-1},\theta_{k-1},u_{k-1})p(x_{k-1}|\mathbf{I}_0^{k-1},\theta_{k-h}^{k-1})d\mathbf{x}_{k-1}^k,$$

where the pdf's $p(x_k|\mathbf{x}_{k-1},\theta_{k-1},u_{k-1})$ and $p(\mathbf{y}_k|\mathbf{x}_k,\theta_k)$ are equivalent to the state and the observation equations of the system (1), respectively. Considering the Gaussian form of the filtering pdf, i.e. $p(x_{k-1}|\mathbf{I}_0^{k-1},\theta_{k-h}^{k-1}) = \mathcal{N}\{\hat{\mathbf{x}}_{k-1}(\theta_{k-h}^{k-1}),\Sigma_{k-1}(\theta_{k-h}^{k-1})\}$ and the system equations (1), the predictive pdf of the state $\mathbf{x}_k$ has the form

$$p(x_k|\mathbf{I}_0^{k-1},\theta_{k-h}^{k-1},u_{k-1})=\mathcal{N}\{\mathbf{x}_k^{'}(\theta_{k-h}^{k-1}),\Sigma_k^{'}(\theta_{k-h}^{k-1})\}, \quad (28)$$

where

$$\begin{aligned}\mathbf{x}_k^{'}(\theta_{k-h}^{k-1}) &= A(\theta_{k-1})\hat{\mathbf{x}}_{k-1}(\theta_{k-h}^{k-1}) + B(\theta_{k-1})u_{k-1},\\ \Sigma_k^{'}(\theta_{k-h}^{k-1}) &= A(\theta_{k-1})\Sigma_{k-1}(\theta_{k-h}^{k-1})A^T(\theta_{k-1})+\\ &\quad G(\theta_{k-1})G^T(\theta_{k-1}).\end{aligned} \quad (29)$$

The predictive pdf of the observation $\mathbf{y}_k$ is also Gaussian $p(\mathbf{y}_k|\mathbf{I}_0^{k-1},\theta_{k-h}^k,u_{k-1}) = \mathcal{N}\{\mathbf{y}_k^{'}(\theta_{k-h}^k),\Gamma_k^{'}(\theta_{k-h}^k)\}$ where

$$\mathbf{y}_k^{'}(\theta_{k-h}^k)=C(\theta_k)\mathbf{x}_k^{'}(\theta_{k-h}^{k-1}), \quad (30)$$

$$\Gamma_k^{'}(\theta_{k-h}^k)=C(\theta_k)\Sigma_k^{'}(\theta_{k-h}^{k-1})C^T(\theta_k)+H(\theta_k)H^T(\theta_k). \quad (31)$$

The probability of model $\theta_k \in \mathcal{M}$ can be easily computed as the following sum of the model sequence probability $P(\theta_{k-h}^k|\mathbf{I}_0^k)$, i.e.

$$P(\theta_k|\mathbf{I}_0^k) = \sum_{\theta_{k-h}^{k-1}} P(\theta_{k-h}^k|\mathbf{I}_0^k). \quad (32)$$

The filtering pdf of the state $\mathbf{x}_k$ corresponding to model θ_k is the Gaussian sum

$$p(\mathbf{x}_k|\mathbf{I}_0^k,\theta_k) = \sum_{\theta_{k-h}^{k-1}} \alpha(\theta_{k-h}^k)p(\mathbf{x}_k|\mathbf{I}_0^k,\theta_{k-h}^k), \quad (33)$$

where $\alpha(\theta_{k-h}^k) = P(\theta_{k-h}^k|\mathbf{I}_0^k)/P(\theta_k|\mathbf{I}_0^k)$ and the first two moments of the state $\mathbf{x}_k$ will be used for determination of a new single Gaussian distribution which approximates the Gaussian sum in (33). The first moment $\hat{\mathbf{x}}_k(\theta_k)$ is given by

$$\hat{\mathbf{x}}_k(\theta_k) = \sum_{\theta_{k-h}^{k-1}} \alpha(\theta_{k-h}^k)\hat{\mathbf{x}}_k(\theta_{k-h}^k) \quad (34)$$

and the second moment $\Sigma_k(\theta_k)$ is computed as

$$\begin{aligned}\Sigma_k(\theta_k) = \sum_{\theta_{k-h}^{k-1}} &\alpha(\theta_{k-h}^k)\big\{\Sigma_k(\theta_{k-h}^k)+\\ &[\hat{\mathbf{x}}_k(\theta_{k-h}^k)-\hat{\mathbf{x}}_k(\theta_k)][\hat{\mathbf{x}}_k(\theta_{k-h}^k)-\hat{\mathbf{x}}_k(\theta_k)]^T\big\}.\end{aligned} \quad (35)$$

Thus, the Gaussian sum (33) is replaced by the single Gaussian distribution by the moment matching technique

$$p(\mathbf{x}_k|\mathbf{I}_0^k,\theta_k) = \mathcal{N}\{\hat{\mathbf{x}}_k(\theta_k),\Sigma_k(\theta_k)\}. \quad (36)$$

Note that the same pruning and merging procedure is applied during the l-step CL optimization as well in order to keep consistency between the state estimation and prediction in the l-step CL optimization.

IV. Illustrative example

The presented active detector design based on the RH scheme with l-step optimization is compared with the design based on the OLF IPS with the PRBS probing signal in numerical example. The horizon for CL optimization is chosen $l = 2$ and the approximative state estimation is performed with $h = 1$. The example is simple enough to understand and intended to shown improvement of the fault detection when the RH scheme is used. Performance improvement can be significant if the models in the set $\mathcal{M}$ are close to each other, but the example shows that an improvement arises even in the case where the models are enough distinct. Further, it will be shown that obtained improvement considerably depends on magnitude of input signal.

Values of the criterion (4) can not be evaluated analytically thus they are estimated by Monte Carlo simulation and denoted $\hat{J}$. The estimate of mean $\mathrm{E}\{\hat{J}\}$ and variance $\mathrm{VAR}\{\hat{J}\}$ of random variable $\hat{J}$ are presented in Table I and Table II.

The set of models is $\mathcal{M} = \{1, 2\}$ and the system is described at each time k by one of the following models

$$\begin{aligned}\theta_k = 1: x_{k+1} &= 0.3x_k + u_k + \sqrt{0.25}w_k,\\ y_k &= -2x_k + \sqrt{0.25}v_k,\end{aligned} \quad (37)$$

$$\begin{aligned}\theta_k = 2: x_{k+1} &= 0.5x_k + 1.5u_k + \sqrt{0.25}w_k,\\ y_k &= 1.5x_k + \sqrt{0.25}v_k,\end{aligned} \quad (38)$$

where $\{w_k\}$, $\{v_k\}$ are mutually independent white sequences. Their distribution is zero mean Gaussian with variance $\sigma^2 = 1$. The initial condition x_0 has also Gaussian distribution with mean 0 and variance $\sigma_{x_0}^2 = 0.1$ and it is independent of the noises $\{w_k\}$ and $\{v_k\}$. A priori probabilities of models are $P(\theta_0 = 1) = P(\theta_0 = 2) = 0.5$. The transition probabilities are defined for all times k as

$$P_{i,j} = \begin{bmatrix} 0.2 & 0.8\\ 0.8 & 0.2\end{bmatrix}. \quad (39)$$

The loss function is chosen as zero–one loss function

$$\begin{aligned}\theta_k = d_k \quad &\Rightarrow \quad L_k(\theta_k,d_k) = 0,\\ \theta_k \neq d_k \quad &\Rightarrow \quad L_k(\theta_k,d_k) = 1.\end{aligned} \quad (40)$$

It leads to decision which is optimal in the sense of maximum a posteriori probability.

The system was simulated for $k = 0,\ldots,50$ and the set of possible inputs is chosen as

$$\mathcal{U}_k = \{-0.5, 0.5\}, \quad k \in \mathcal{T}. \quad (41)$$

The number of Monte Carlo simulation will be written as K_1/K_2 where K_1 denotes number of runs used for evaluation $\hat{J}$ and K_2 denotes number of trials used to compute statistics $\mathrm{E}\{\hat{J}\}$ and $\mathrm{VAR}\{\hat{J}\}$. The results obtained by 1000/20 Monte Carlo simulations are presented in Table I. The detector designed using the RH scheme achieves the considerably lower value of the criterion (4) in comparison with detector based on the OLF IPS with the PRBS input signal. Figure 2 shows one typical run of detection for steps $k = 0,\ldots,30$. Four faulty decisions were made with the

TABLE I

CRITERION VALUE COMPARISON

	OLF+PRBS	RH
$\mathrm{E}\{\hat{J}\}$	6.6142	2.7548
$\mathrm{VAR}\{\hat{J}\}$	0.0083	0.0028

TABLE II

INFLUENCE OF PROBING SIGNAL MAGNITUDE

$\bar{u}$	0.1	0.3	0.5	1	3
$\mathrm{E}\{\hat{J}^{OLF}\}$	18.5533	11.0350	6.6142	2.53	0.605
$\mathrm{VAR}\{\hat{J}^{OLF}\}$	0.0751	0.0977	0.0083	0.068	0.0018
$\mathrm{E}\{\hat{J}^{RH}\}$	17.0333	7.2633	2.7548	0.545	0.455
$\mathrm{VAR}\{\hat{J}^{RH}\}$	0.2687	0.1928	0.0028	0.0044	0.0024
D	1.52	3.7717	3.8594	1.985	0.15

PRBS signal and only one faulty decision was made using the RH scheme in the shown example.

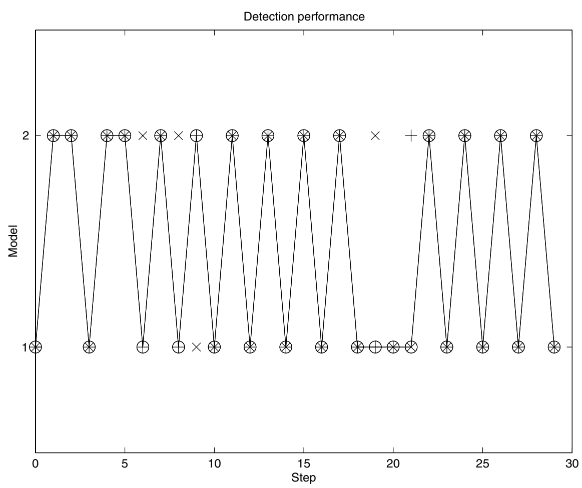

Fig. 2. Typical run of detection. True model (o-), estimate with detector based on the OLF (×), estimate with detector base on the RH scheme (+).

It was mentioned that degree of improvement depends on the magnitude of the used probing signal. Thus, the following simulation should demonstrate the influence of the various magnitudes of input. The estimates denoted by *OLF* and *RH* were obtained by using the OLF IPS with the PRBS input signal and using the RH scheme, respectively. The system again ran for $k = 0, \ldots, 50$ but the results were obtained only by 100/6 Monte Carlo runs/trials. The set of possible input is chosen as $\mathcal{U}_k = \{\pm\bar{u}\}, k \in \mathcal{T}$ and the simulation was performed for the following magnitudes of probing signal $\bar{u} \in \{0.1, 0.3, 0.5, 1, 3\}$. The results are presented in Table II. The diference $D = \mathrm{E}\{\hat{J}^{RH}\} - \mathrm{E}\{\hat{J}^{OLF}\}$ shows that there exists a region of magnitude for which the improvement is significant. So the higher computational demands connected with design of input using the RH scheme are acceptable only for certain magnitudes.

V. CONCLUSION

The new feasible active fault detection system was designed. The design of the detector and the generator of input signal was based on minimization of chosen criterion. Such formulation of the FDI is useful in situations where the costs can be easily assigned to the decisions. The optimal solution based on the CL IPS outperforms most of the known FDI methods. Unfortunately, such optimal solution is infeasible and thus the approximative design applying the rolling horizon scheme was proposed. The designed feasible detection system was compared with the detection system obtained using the OLF IPS and a better quality of the designed detection system was shown in the numerical example.

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