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A nonlinear controller design for embedded electrical networks

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***Abstract*— This paper is devoted to the design of a nonlinear controller well suited for the voltage regulation of embedded AC electrical networks, such as plane or ship power systems. The approach may also be applied without fundamental restriction to high voltage or distribution power networks. The here-proposed approach relies on the nonlinear differential-algebraic dynamics of an aggregated electrical network based on the one-axis model. The design is based on a linearization technique of this nonlinear differential-algebraic system coupled to a state estimator which reconstructs the full state vector of the network from easily measurable variables (voltages at network nodes). Simulations on the aggregated network demonstrate the effectiveness of the approach. Application of this approach to large-scale electrical networks is also discussed.**

Keywords. electrical networks, nonlinear control design, nonlinear differential-algebraic systems, differential nonlinear inversion.

I. INTRODUCTION

Today there is a big challenge in both design and control of secure electrical networks for various embedded systems, such as planes, ships, or cars. For example, the new generations of planes (such as the Airbus A380) are more and more electrically actuated and the reduction of the embedded masses of the electrical devices (generator, actuators, converters, etc) is of the greatest importance. It is commonly admitted that this mass optimization can be partially achieved by a better design of the network control system, since in this case, over-dimensioning of the electrical devices can be reduced if performing control systems are implemented to face disturbances acting on the network, such as important electrical load variations [1]. In this paper, we will focus our attention on an aggregated model of an embedded AC network. The system consists in an aggregated generator connected to an aggregated nonlinear load through a bus. A so-called SVC (a thyristor-controlled reactance) is also connected to the load node. This simplified system is very similar to the single machine power network studied in [5] (see chap. 7) for high voltage applications. This means that the here-proposed approach may be easily extended to the case of high voltage or distribution electrical networks. Figure 1 describes both the aggregated network and the controller design objective of the paper. The problem is to design

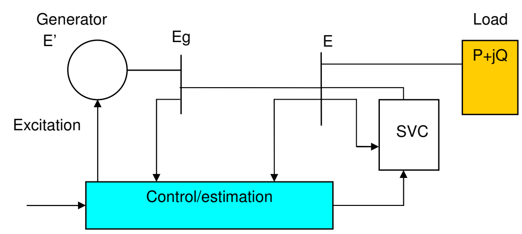

Fig. 1. Network Topology

a nonlinear control scheme for the goal of voltage regulation in the presence of load variations. One important issue is to reduce the number of measurements to a minimum and to use some easily measurable state variables. More precisely, we aim at designing a controller, using only the measurement of voltages E_g and E, and acting simultaneously on both the excitation of the generator (first control input) and the SVC control input (second control input). The SVC, which is located near the load, is introduced as an additional degree of freedom for the stabilization of voltages E_g and E. The use of advanced control techniques for power system control has been one of the most promising application areas of automatic control, see [3], [4], [8]. Compared with the use of traditional linear control theory where the operating domain of the controlled system is restricted to a small operation domain ("small signal" approach), or the structural properties of the system are lost or partially used, the design of excitation controls with nonlinear techniques allows the controlled system to face large disturbances and to recover a steady-state post-fault situation (see [2]). For that reason, we consider the control design of some nonlinear differential-algebraic equations (DAE) of networks. As a consequence, we cannot in general explicitly eliminate the algebraic equations resulting from the network structure (Kirchhoff laws), by computing the algebraic variables as explicit functions of the differential variables. Numerical elimination would require the use of Newton-Raphson techniques, that is not suitable for control purpose. Notice that in the linear case (with use of the DAE linearized around an equilibrium point), it is always possible to make this algebraic elimination (under some regularity assumption) and, therefore, to work with a conventional differential linear system. The paper is now organized as follows: section 2 is devoted to the dynamical modelling of the aggregated network. We will consider the design of the nonlinear control scheme in section 3. Some simulation results are provided in section 4. Section 5 is

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devoted to a discussion on the possible extension of the approach to more complex embedded networks based on the interconnections of several generators and nonlinear loads. Finally, we sum up some conclusions.

II. AN AGGREGATED MODEL OF AN EMBEDDED ELECTRICAL NETWORK

The mechanical dynamics of the generator is defined by

$$\dot{\delta}' = 2\pi f_0 \omega \quad (1)$$

$$M\dot{\omega} = -D\omega + P_m - P_e, \quad (2)$$

where f_0 is the network frequency, δ' is the generator angle, ω is the generator speed deviation, M is the generator inertia coefficient, D is the generator damping coefficient, P_m is the mechanical power and $P_e = -E'E_g \sin(\delta_g - \delta')/x'_d$ is the electrical power (where δ_g denotes the angle at the generation node). We make the assumption that the mechanical power of the generator is controlled by a PI regulator of the form

$$P_m = -k_1\omega - k_2 \int_0^t \omega(\tau) d\tau, \quad (3)$$

which ensures stability of ω around zero for any constant P_e, by an appropriate choice of k_1 and k_2. As a consequence, generator angle δ' remains bounded for any bounded electrical power demand $P_e = P$. This decouples voltage dynamics from mechanical dynamics. In this case, the dynamics of the generator reduces to the following electrical equations:

$$T'_{d0}\dot{E}' = -\frac{x_d}{x'_d}E' + \frac{(x_d - x'_d)}{x'_d}E_g \cos(\delta_g - \delta') + E_f. \quad (4)$$

The dynamics of the SVC is given by:

$$T\dot{B}_s = -B_s + E_r. \quad (5)$$

The Kirchhoff laws expressed at each node in terms of active and reactive powers lead to the following algebraic equations:

$$E'E_g \sin(\delta_g - \delta')/x'_d + E_g E \sin(\delta_g - \delta)/x = 0, \quad (6)$$

$$(E_g^2 - E_g E' \cos(\delta_g - \delta'))/x'_d + (E_g^2 - E_g E \cos(\delta_g - \delta))/x = 0, \quad (7)$$

$$-EE_g \sin(\delta_g - \delta)/x + P = 0, \quad (8)$$

$$(E^2 - EE_g \cos(\delta_g - \delta))/x + Q_0 + HE + (B + B_s)E^2 = 0, \quad (9)$$

where δ_g and δ represent the power angle related to the generation and load nodes, respectively. Using the implicit function theorem, we can eliminate these 4 algebraic equations by expressing $(\delta_g - \delta')$, $(\delta_g - \delta)$, E and E_g as functions of the dynamical variables E' and B_s. Although in this case we could obtained explicit expressions (and eliminate $\delta_g - \delta'$ and $\delta_g - \delta$ for example), we cannot follow this way for more complex systems. The 4 algebraic equations will be now denoted as $G(E_g, E, \delta_g - \delta', \delta_g - \delta, E', B_s) = 0$. The overall system can be expressed by the following DAE:

$$\dot{X} = F(X, Z, U) = F_1(X, Z) + F_2 U, \quad (10)$$

$$G(X, Z) = 0 \quad (11)$$

where $X = (E', B_s)^T$, $Z = (\delta_g - \delta', \delta_g - \delta, E_g, E)^T$, $U = (E_f, E_r)^T$,

$$F_1 = \begin{pmatrix} -\frac{x_d}{x'_d}E' + \frac{(x_d - x'_d)}{x'_d}E_g \cos(\delta_g - \delta') \\ -\frac{B_s}{T} \end{pmatrix},$$

$$F_2 = \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{T} \end{pmatrix},$$

and

$$G = \begin{pmatrix} E'E_g \sin(\delta_g - \delta')/x'_d + E_g E \sin(\delta_g - \delta)/x \\ (E_g^2 - E_g E' \cos(\delta_g - \delta'))/x'_d \\ +(E_g^2 - E_g E \cos(\delta_g - \delta))/x \\ -EE_g \sin(\delta_g - \delta)/x + P \\ (E^2 - EE_g \cos(\delta_g - \delta))/x \\ +Q_0 + HE + (B + B_s)E^2 \end{pmatrix}$$

III. A NONLINEAR CONTROLLER DESIGN

The idea is to control both voltages E_g and E through the generator excitation E_f and the SVC control input E_r. The control design is based on input/ouput linearization by considering the two outputs: $Y = (E_g, E)^T$.

Proposition 1: In order to get a linear closed-loop dynamics of the form $\dot{Y} = -K(Y - Y_d)$, $K > 0$ which ensures closed-loop exponential stability around the set-point Y_d, for all trajectories such that $\frac{\partial G}{\partial Z}$ has full rank, U should be chosen as:

$$U = (\left[\frac{\partial G}{\partial Z}^{-1}\right]_y \frac{\partial G}{\partial X}F_2)^{-1}[-\left[\frac{\partial G}{\partial Z}^{-1}\right]_y \frac{\partial G}{\partial X}F_1 + K(Y - Y_d)]. \quad (12)$$

Proof. The time-derivative of the output vector $\dot{Y}$ is obtained by considering that if $G(E_g, E, \delta_g - \delta', \delta_g - \delta, E', B_s) = 0$, necessarily $\dot{G}(E_g, E, (\delta_g - \delta'), (\delta_g - \delta), E', B_s) = 0$, where:

$$\dot{G}(X, Z) = \frac{\partial G}{\partial X}\dot{X} + \frac{\partial G}{\partial Z}\dot{Z} = 0 \quad (13)$$

along the trajectory of Z. This means that the trajectory of Y can be obtained (outside of the singularities of $\frac{\partial G}{\partial Z}$) by integrating:

$$\dot{Y} = -\left[\frac{\partial G}{\partial Z}^{-1}\right]_y \frac{\partial G}{\partial X}(F_1 + F_2 U), \quad (14)$$

where $[.]_y$ stands for the matrix containing the lines of $[.]$ related to Y and $\dot{X} = F_1 + F_2 U$, with $U = (E_f, E_r)^T$, which corresponds to (4) and (5). We can notice that there is no zero dynamics since the relative degrees of each output is one, while order of system (4)-(5) is two (since the algebraic variables can be eliminated according to the implicit function theorem).

Proposition 2: Under the assumption that the second mechanical equation (2) is stabilized around $\omega = 0$, thanks to a speed controller P_m (3), dynamics (2) remains bounded under (12).

Proof. The closed-loop system (4),(5), (6)-(9), with (12) is such that the states $E_g, E, \delta_g - \delta', \delta_g - \delta, E', B_s$ are bounded for any bounded P and $P_e = P$. Since the closed-loop system (1)-(2) with controller (3) is bounded for any bounded input P_e, the result holds. Furthermore in practice, it is convenient to introduce an integral term in the control design which will guarantee that the closed-loop system will reach the equilibrium $Y_e = Y_d$ for any disturbance that will not render the closed-loop system unstable. We seek for an exponentially stable closed-loop dynamics of the form

$$\dot{I} = Y - Y_d, \quad (15)$$

$$\dot{Y} = -K_1(Y - Y_d) - K_2 I, \quad (16)$$

$K_1, K_2 > 0$. The associated integral controller is given by

$$\dot{I} = Y - Y_d, \quad (17)$$

$$U = (\left[\frac{\partial G}{\partial Z}^{-1}\right]_y \frac{\partial G}{\partial X} F_2)^{-1}[-\left[\frac{\partial G}{\partial Z}^{-1}\right]_y \frac{\partial G}{\partial X} F_1 + K_1(Y - Y_d) + K_2 I]. \quad (18)$$

An important issue remains the accessability to the state variables. The algebraic equations (6)-(9) can be rewritten under the form

$$G(X_r, Y) = 0, \quad (19)$$

where $X_r = (\delta_g - \delta', \delta_g - \delta, E', B_s)^T$ are not known. In practice, $Y = (E_g, E)^T$ are easily measurable. To overcome the problem of computing X_r from Y by using a Newton-Raphson procedure, that is not suitable for real-time purpose, we introduce the idea of differential inversion, which is based on the following differential system:

$$\dot{G}(X_r, Y) + \Lambda G(X_r, Y) = 0, \quad (20)$$

where Λ is a symmetric positive definite matrix. The solution of (20) is given by $G(\hat{X}_r(t), Y(t)) = e^{-\Lambda t} G(\hat{X}_r(0), Y(0))$, that ensures manifold G is attractive and thus, if $\hat{X}_r(t = 0)$ is such that $G(\hat{X}_r(0), Y(0)) = 0$, trajectory $\hat{X}_r(t)$ remains on the manifold, $\forall t > 0$. When $G(\hat{X}_r(0), Y(0)) \neq 0$, $G(\hat{X}_r(t), Y(t))$ tends to zero, when $t \to +\infty$ Therefore, $\hat{X}_r$, solution of (20), is an estimate of $X_r(t)$, satisfying $G(X_r(t), Y(t)) = 0$. (20) is equivalent to

$$\frac{\partial G}{\partial X_r}(\hat{X}_r, Y)\dot{\hat{X}}_r + \frac{\partial G}{\partial Y}(\hat{X}_r, Y)\dot{Y} + \Lambda G(\hat{X}_r, Y) = 0. \quad (21)$$

Or, equivalently, if again $\frac{\partial G}{\partial X_r}$ has full rank, $\hat{X}_r(t)$ is obtained through integration of

$$\dot{\hat{X}}_r = -(\frac{\partial G}{\partial X_r})^{-1}(\hat{X}_r, Y)[\frac{\partial G}{\partial Y}(\hat{X}_r, Y)\dot{Y} + \Lambda G(\hat{X}_r, Y)]. \quad (22)$$

It is important to point out that output time-derivative $\dot{Y}$ are required. In practice, these time-derivatives are not available. A way to estimate these output time-derivatives is the use of derivation filters of the form

$$H(s) = \frac{s}{\epsilon s + 1}. \quad (23)$$

The overall state estimator is then given by

$$\dot{\hat{X}}_r = -(\frac{\partial \hat{G}}{\partial X_r})^{-1}[\frac{\partial \hat{G}}{\partial Y}\hat{\dot{Y}} + \Lambda G(\hat{X}_r, Y)], \quad (24)$$

$$\dot{x}_d = -\frac{1}{\epsilon}x_d + \frac{1}{\epsilon}Y, \quad (25)$$

$$\hat{\dot{Y}} = -\frac{1}{\epsilon}x_d + \frac{1}{\epsilon}Y, \quad (26)$$

where $\hat{G}$ stands for G evaluated for $\hat{X}$. Finally, the overall output integral feedback controller is given by

$$\dot{\hat{X}}_r = -(\frac{\partial \hat{G}}{\partial X_r})^{-1}[\frac{\partial \hat{G}}{\partial Y}\hat{\dot{Y}} + \Lambda G(\hat{X}_r, Y)], \quad (27)$$

$$\dot{x}_d = -\frac{1}{\epsilon}x_d + \frac{1}{\epsilon}Y, \quad (28)$$

$$\hat{\dot{Y}} = -\frac{1}{\epsilon}x_d + \frac{1}{\epsilon}Y, \quad (29)$$

$$\dot{I} = Y - Y_d, \quad (30)$$

$$U = (\left[\frac{\partial \hat{G}}{\partial Z}^{-1}\right]_y \frac{\partial \hat{G}}{\partial X}\hat{F}_2)^{-1}[-\left[\frac{\partial \hat{G}}{\partial Z}^{-1}\right]_y \frac{\partial \hat{G}}{\partial X}\hat{F}_1 + K_1(Y - Y_d) + K_2 I], \quad (31)$$

where $U = (U_1, U_2)^T = (E_f, E_r)^T$.

Proposition 3: Under large enough Λ and small enough ϵ, the closed loop system (4)-(5) and (27)-(31) is exponentially stable.

Proof. It is a direct application of the singular perturbation stability theorem (see [6], chap. 11, pp. 456-457 for example), where the slow dynamics is given by

$$\dot{X} = F(X, Z, U), \quad (32)$$

$$\dot{I} = Y - Y_d, \quad (33)$$

$$U = (\left[\frac{\partial \hat{G}}{\partial Z}^{-1}\right]_y \frac{\partial \hat{G}}{\partial X}\hat{F}_2)^{-1}[-\left[\frac{\partial \hat{G}}{\partial Z}^{-1}\right]_y \frac{\partial \hat{G}}{\partial X}\hat{F}_1 + K_1(Y - Y_d) + K_2 I], \quad (34)$$

while the fast dynamics corresponds to the differential inversion associated to the time-derivation filters:

$$\dot{\hat{X}}_r = -(\frac{\partial \hat{G}}{\partial X_r})^{-1}[\frac{\partial \hat{G}}{\partial Y}\hat{\dot{Y}} + \Lambda G(\hat{X}_r, Y)], \quad (35)$$

$$\dot{x}_d = -\frac{1}{\epsilon}x_d + \frac{1}{\epsilon}Y, \quad (36)$$

$$\hat{\dot{Y}} = -\frac{1}{\epsilon}x_d + \frac{1}{\epsilon}Y, \quad (37)$$

when some large enough Λ and small enough ϵ are chosen.

IV. SIMULATION RESULTS

We consider simulations of the controller design for the aggregated electrical network of figure 1. Two case studies have been performed: figure 2 presents results obtained with controller (27)-(31) without integral action ($K_2 = 0$), while figures 3-5 present results obtained with controller (27)-(31) with integral action. All the variables are expressed in p.u. Figures 2 and 3 present the responses of voltages E_g and E in response to (P, Q_0) variations. Figures 4 presents the responses of $\delta_g - \delta$ (denoted DeltaaGa) and $\delta_g - \delta'$ (denoted DeltaaGap), and their respective estimates (denoted DeltaaGa-est and DeltaaGap-est, respectively). Figure (5) presents the responses of E' and B_s and their estimates (denoted Ep-est and Bs-est, respectively). In both cases, state estimation appears to be biased, since P and Q_0 are unknown. Furthermore, some transients appear due to filters (23) (see the transients just after $t = 0$ on figures 4 and

5). The electrical parameters of the system used in the simulations are defined as follows:

$$T'_{d0} = 10, T = 1.75, x_d = 1, x'_d = 0.2; x = 0.1, \quad H = 0.84, B = 0.84$$

The system starts from the equilibrium state defined by $X_e = (E'_e, Bs_e)^T = (0.7823, 0)^T$ and $Z_e = ((\delta_g - \delta')_e, (\delta_g - \delta)_e, E_{g_e}, E_e)^T = (-0.3004, 0.2135, 0.6049, 0.7823)^T$.

Simulations have been performed, for the controller without integral action, with gain matrix $K_1 = diag(5,5)$ ($K_2 = 0$), and for the controller with integral action, with $K_1 = diag(10.5, 10.5)$ and $K_2 = diag(56.25, 56.25)$, both gains corresponding to closed-loop damping coefficient $\zeta = 0.7$ and $\omega_n = 7.5\ rad/s$. In both cases, the state estimator gains are $\lambda = diag(50, 50, 50, 50)$ and $\epsilon = diag(0.01, 0.01)$.

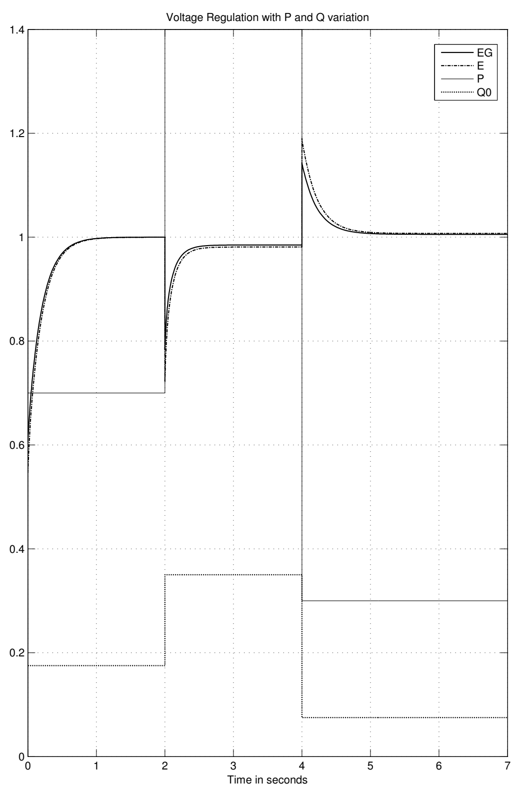

Fig. 2. Voltage regulation without integral action

The simulations show a small sensitivity of the closed-loop system to (P, Q_0) variations when the controller has no integral term (less than 2 % when the variations of P and Q_0 are greater than 100 %). Clearly, no measurements of both P and Q_0, nor integral action appear to be necessary to ensure a robust closed-loop behavior with respect to (P, Q_0) variations.

V. DISCUSSION ON THE EXTENSION TO MORE COMPLEX NETWORKS

The n synchronous generators of an interconnected electrical network can be described by the following set of $3n$

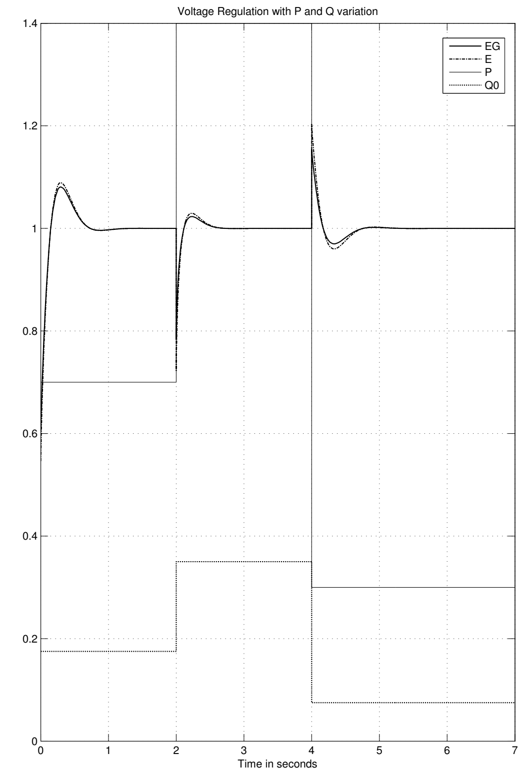

Fig. 3. Voltage regulation with integral action

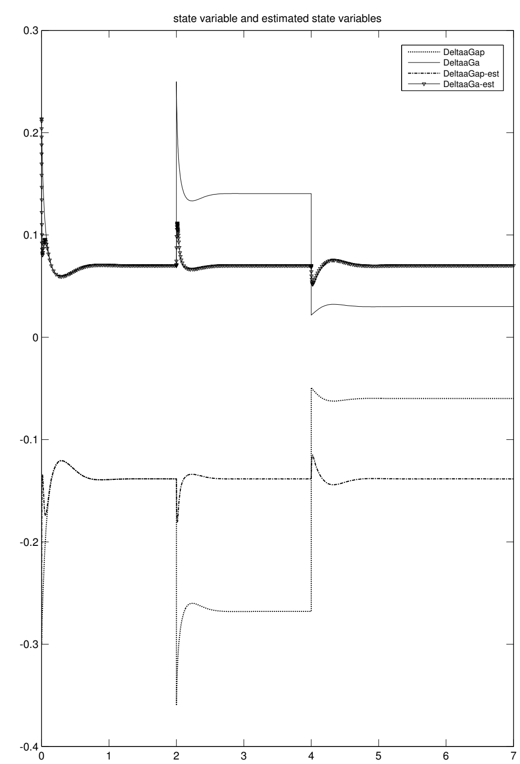

Fig. 4. State estimation with integral action

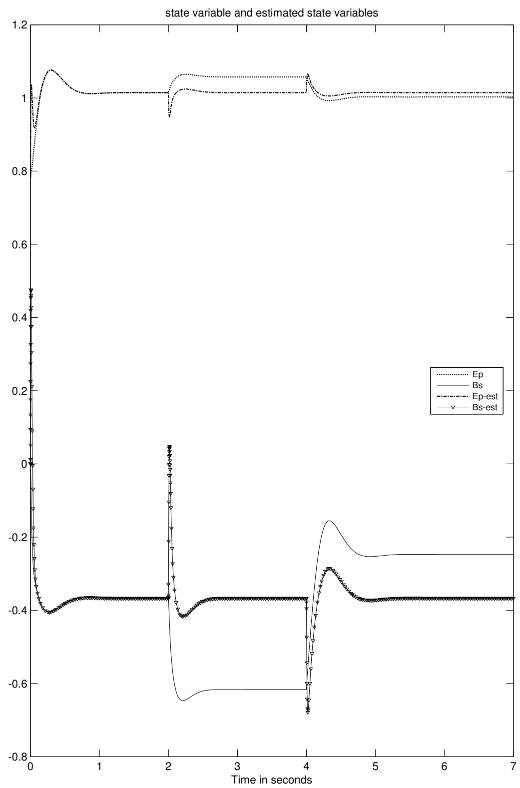

Fig. 5. State estimation with integral action

nonlinear dynamical equations [7], [9] (one-axis model):

$$\dot{\delta}' = 2\pi f_0 \omega \tag{38}$$

$$M\dot{\omega} = -D\omega + P_m - P_g \tag{39}$$

$$T'_q \dot{E}' = E_f - E' - \Delta_d {E'}^{-1} Q_g, \tag{40}$$

where δ', ω, E', E_f and P_m are $n \times 1$ vectors defining the rotor angles, the speed deviations, the internal voltages due to the field flux linkages, the excitation voltages and the mechanical power of the m generators, respectively, M and D are some $n \times n$ diagonal matrices defining the inertia coefficients and the damping coefficients and the field open circuit time constants respectively. f_0 is the synchronous speed. The functions P_g and Q_g are two $n \times 1$ vectors representing the active and reactive powers of the n generators respectively. E' denotes the diagonal matrix of terms E'_i, $i = 1, ..., n$. Δ_d is the diagonal matrix of terms $x_{di} - x'_{di}$, with x_{di} and x'_{di} are the direct axis reactance and direct axis transient reactance of each generator i respectively. T'_q is the diagonal matrices of the quadrature axis transient short-circuit time constant of each generator respectively. Again, each generator is controlled in speed deviation by a PI regulator of the form

$$P_m = -k_1 \omega - k_2 \int_0^t \omega(\tau) d\tau, \tag{41}$$

which ensures stability of ω around zero for any constant P_e, by an appropriate choice of k_1 and k_2. We suppose now that the n generators are connected to a grid defined by N nodes and m buses interconnecting the nodes together. In addition to the n generators, we suppose that m_1 loads and m_2 SVC are also connected to the nodes of this grid. If we define A as the incidence matrix of the graph of the grid, whose rows correspond to the node indices of the grid, we get the following power flux conservation laws:

$$AP = 0, \ AQ = 0, \tag{42}$$

where P and Q represent the $n + m + m_1 + m_2 \times 1$-vectors of active and reactive powers respectively. These two vectors can be partitioned into four sub-vectors corresponding to the generator variables, the line variables, the load variables and the SVC variables, respectively: $P = (P_g^T, P_l^T, P_c^T, P_f^T)^T$ and $Q = (Q_g^T, Q_l^T, Q_c^T, Q_f^T)^T$. We can introduce the same partitioning for the incidence matrix A: $A = (A_g \ \ A_l \ \ A_c \ \ A_f)^T$. Finally equations (42) can be expressed as follows:

$$\begin{aligned} A_g P_g + A_l P_l + A_c P_c + A_f P_f = 0, \\ A_g Q_g + A_l Q_l + A_c Q_c + A_f Q_f = 0. \end{aligned} \tag{43}$$

Moreover, the flow variables on the grid (P_l, Q_l) have to be defined. If we consider two nodes i and j, the active and reactive powers P_{ij} and Q_{ij} between node i and node j are given by:

$$\begin{aligned} P_{ij} = V_i V_j B_{ij} \sin(\delta_i - \delta_j), \\ Q_{ij} = -V_i V_j B_{ij} \cos(\delta_i - \delta_j), \end{aligned} \tag{44}$$

where δ_i is the voltage angle at node i, V_i is the voltage magnitude at node i and B_{ij} is the admittance of line ij. The incidence matrix A defines the mapping between vertices and flows, while the transpose matrix A^T defines the mapping between the flows and the potentials at vertices (voltage magnitude and angle at the grid and generator nodes). For that reason and by using (44), we can define vectors P_l and Q_l as follows:

$$P_l = F_1^{A_l^T}(\delta_l, V_l), \tag{45}$$

$$Q_l = F_2^{A_l^T}(\delta_l, V_l), \tag{46}$$

where δ_l and V_l are the $m \times 1$-vectors of voltage angles and magnitudes at the nodes of the grid, respectively. According to the same principle, P_g and Q_g are given by:

$$P_g = G_1^{A_g^T}(\Delta', V_l, E'), \tag{47}$$

$$Q_g = G_2^{A_g^T}(\Delta', V_l, E'), \tag{48}$$

where $\Delta' = [A_g^T]^{n+N} \begin{pmatrix} \delta' \\ \delta_l \end{pmatrix}$, with $[A_g^T]^{n+N}$ representing the $n + N$ columns of A_g^T, whose indices correspond to $\begin{pmatrix} \delta' \\ \delta_l \end{pmatrix}$. On the other hand, the active and reactive power vectors of the SVC are given by:

$$P_f = 0, \tag{49}$$

$$Q_f = G_3^{A_f^T}(\delta_l, V_l, x_f), \tag{50}$$

where B_s are the state vector of the SVC, solution of the state-space equations:

$$T\dot{B}_s = -B_s + E_r, \quad (51)$$

where E_r is the SVC reference input vector. We are now ready to state the overall model as the following set of differential-algebraic equations:

$$\dot{\delta}_g = 2\pi f_0 \omega \quad (52)$$

$$M\dot{\omega} = -D\omega - k_1\omega - k_2\int_0^t \omega(\tau)d\tau - G_1^{A_g^T}(\Delta', V_l, E') \quad (53)$$

$$T_q\dot{E}' = E_f - E' - \Delta_d {E'}^{-1} G_2^{A_g^T}(\Delta', V_l, E') \quad (54)$$

$$A_g G_1^{A_g^T}(\Delta', V_l, E') + A_l F_1^{A_l^T}(\delta_l, V_l) + A_c P_c = 0 \quad (55)$$

$$A_g G_2^{A_g^T}(\Delta', V_l, E') + A_l F_2^{A_l^T}(\delta_l, V_l) + A_c Q_c + A_f G_3^{A_f^T}(\delta_l, V_l, x_f) = 0. \quad (56)$$

The control input vectors are P_m, E_f and E_r, while P_c and Q_c are the disturbance input vectors representing the active and reactive powers of the loads. Again, controllers (41) ensure decoupling of the generator mechanical dynamics from the electrical differential-algebraic equations of the network, which can be described by the following DAE:

$$\dot{X} = F(X, Z, U) = F_1(X, Z) + F_2 U \quad (57)$$

$$G(X, Z) = 0 \quad (58)$$

$$Y = V_l \quad (59)$$

where $X = ({E'}^T, B_s^T)^T$, $Z = ({\Delta'}^T, \delta_l^{cT}, V_l^T)^T$, $U = (E_f^T, E_r^T)^T$,

$$F_1 = \begin{pmatrix} \frac{1}{T_q}(E_f - E' - \Delta_d {E'}^{-1} G_2^{A_g^T}(\Delta', V_l, E')) \\ -\frac{B_s}{T} \end{pmatrix},$$

$$F_2 = \begin{pmatrix} I_n & 0 \\ 0 & \frac{I_{m_2}}{T} \end{pmatrix},$$

and

$$G = \begin{pmatrix} A_g G_1^{A_g^T}(\Delta', V_l, E') + A_l F_1^{A_l^T}(\delta_l, V_l) + A_c P_c \\ A_g G_2^{A_g^T}(\Delta', V_l, E') + A_l F_2^{A_l^T}(\delta_l, V_l) \\ + A_c Q_c + A_f G_3^{A_f^T}(\delta_l, V_l, x_f) \end{pmatrix}$$

Again we seek for estimating non easily-measurable state vector $X_r = ({\Delta'}^T, \delta_l^c, {E'}^T, B_s)^T$ from grid voltage vector Y only. In order to use nonlinear differential inversion (20),we need that $\frac{\partial G}{\partial X_r}$ is a regular square matrix. Since the dimension of $X_r = ({\Delta'}^T, \delta_l^c, {E'}^T, B_s)^T$ is $N+n+m_2$, $\frac{\partial G}{\partial X_r}$ is a square matrix if and only if $N + n + m_2 = 2N$. The number of SVC m_2 to be implemented in the network can be chosen as a degree of freedom to get $N+n+m_2 = 2N$. Furthermore, the location of the $m_2 = N-n$ SVC should be performed in order to obtain regularity of $\frac{\partial G}{\partial X_r}$. Under the assumption that $\frac{\partial G}{\partial X_r}$ has full rank (the main open issue remains to established if location of $N-n$ SVC is sufficient to ensure regularity), it is straightforward to show that the nonlinear output feedback controller design of proposition (3) can be applied in this case:

$$\dot{\hat{X}} = -(\frac{\partial \hat{G}}{\partial X_r})^{-1}[\frac{\partial \hat{G}}{\partial Y}\dot{\hat{Y}} + \Lambda G(\hat{X}_r, Y)], \quad (60)$$

$$\dot{x}_d = -\frac{1}{\epsilon}x_d + \frac{1}{\epsilon}Y, \quad (61)$$

$$\dot{\hat{Y}} = -\frac{1}{\epsilon}x_d + \frac{1}{\epsilon}Y, \quad (62)$$

$$\dot{I} = Y - Y_d, \quad (63)$$

$$U = (\left[\frac{\partial \hat{G}}{\partial Z}^{-1}\right]_y \frac{\partial \hat{G}}{\partial X}\hat{F}_2)^{-1}[-\left[\frac{\partial \hat{G}}{\partial Z}^{-1}\right]_y \frac{\partial \hat{G}}{\partial X}\hat{F}_1 + K_1(Y - Y_d) + K_2 I], \quad (64)$$

where $U = (U_1, U_2)^T = (E_f^T, E_r^T)^T$.

VI. CONCLUSIONS AND FUTURE WORKS

A nonlinear output feedback controller has been proposed for the goal of voltage stabilization for aggregated embedded AC electrical networks, but also for power systems. We have shown that stabilization can be obtained through the measurements of network voltages, without the need of power measurements. We have also discussed the extension of this control design to multi-machine interconnected electrical networks. Future work will be devoted to sensitivity analysis of the closed-loop system with respect to (P, Q) set-point changes (bifurcation analysis), with comparison to conventional linear controllers and extension to larger networks.

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