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$\mathcal{H}_\infty$ Model Approximation for Discrete-time Markovian Jump Systems with Mode-dependent Time Delays

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Abstract— **This paper considers the problem of computing an approximation system for a discrete-time Markovian jump system with mode-dependent time delays such that the H_∞ norm of the error system is less than a prescribed scalar. It can be shown that the approximation system is constructed by the solutions of linear matrix inequalities (LMIs) with inverse constraints. An efficient algorithm is used to obtain the approximation system and an example is employed to demonstrate the effectiveness of the model approximation algorithm.**

I. INTRODUCTION

Discrete-time Markovian jump linear system is a hybrid system with state comprised of two components: a discrete part, which is a discrete-time Markov chain representing the mode of the system, and a continuous part, which is the physical state of the system. This class of systems is suitable to model the plants whose structures are subject to random abrupt parameters changes due to, for instance, component and/or interconnections failures, sudden environment changes, change of the operating point of a linearized model of a nonlinear system, etc [6]. The application of Markovian jump systems can be found in manufacturing systems, aircraft control, target tracking, robotics, solar thermal receiver control, and power system [11]. For more information on discrete-time Markovian jump systems, the reader is referred to [1], [4], [10], [15], [17] and the references therein.

In many physical, industrial and engineering systems, delays occur due to the finite capabilities of information processing and data transmission among various parts of the systems. Delay could arise also from inherent physical phenomena like mass transport flow recycling and the presence of large displacements and forces [9], [12], [13], [16]. Time-delay can be constant or time-varying, known or unknown, deterministic or stochastic, and mode-dependent or mode-independent depending on the system under consideration. The robust H_∞ control for discrete-time Markovian jump system with time-varying norm-bounded parameter uncertainty and unknown time delay have been studied in [14]. The stochastic stability and stochastic stabilizability of discrete-time Markovian jumps systems with mode-dependent time delays have been addressed in [2]. The robust H_∞ control for uncertain discrete-time Markovian jump system with constant time delay has been studied in [3], while the extension to mode-dependent time-delay is considered in [1].

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The purpose of this paper is to construct a reduced order discrete-time Markovian jump linear systems with mode-dependent time delays to approximate a given full order one such that the H_∞ norm of error system between the full one and the reduced one is less than a given scalar. The result given in this paper is an extension of existing result on the H_∞ model reduction problem for discrete-time Markovian jump linear systems without time delay in [17] to mode-dependent time delays case. A simple algorithm to compute the reduced order system is given. Moreover, a numerical example is given to demonstrate the effectiveness of theoretical result.

II. PRELIMINARIES AND PROBLEM FORMULATION

Given a probability space $(\Omega, \mathcal{F}, \mathcal{P})$ where Ω is the sample space, $\mathcal{F}$ is the algebra of events, and $\mathcal{P}$ is the probability measure defined on $\mathcal{F}$. Let $\{r_k, k \geq 0\}$ be a discrete-time homogeneous Markov chain with finite state space $S = \{1, \ldots, N\}$ and stationary transition probability matrix $P = [p_{ij}]_{i,j \in S}$, where

$$\Pr\{r_{k+1} = j \,| r_k = i\} = p_{ij}, \quad \forall i, j \in S$$

with $p_{ij} \geq 0$, $\forall i$, $i \in S$ and $\sum_{j=1}^{N} p_{ij} = 1$, for $i \in S$. For the probability space $(\Omega, \mathcal{F}, \mathcal{P})$, consider a discrete-time Markovian jump linear system Σ_d with mode-dependent time delays:

$$\begin{aligned}
x_{t+1} &= A(t, r_t)x_t + A_d(t, r_t)x_{t-\tau(r_t)} + B(t, r_t)u_t \\
y_t &= C(t, r_t)x_t + C_d(t, r_t)x_{t-\tau(r_t)} + D(t, r_t)u_t \\
x_s &= \alpha_s, \quad s = -\tau, \ldots, -1, 0
\end{aligned}$$

where $x_t \in \mathbb{R}^n$ is the state, $u_t \in \mathbb{R}^m$ is the input, and $y_t \in \mathbb{R}^p$ is the output. $\tau(r_t)$ denotes the time delay when system Σ_d is in mode r_t. $A(t, r_t)$, $A_d(t, r_t)$, $B(t, r_t)$, $C(t, r_t)$, $C_d(t, r_t)$ and $D(t, r_t)$ are matrices with appropriate dimensions. The set S comprises the operation models of system Σ_d. For each possible value $r_t = i$, $i \in S$, we denote the matrices associated with the ith mode by

$$\begin{aligned}
A_i &= A(t, i), \quad A_{di} = A_{di}(t, i), \quad B_i = B(t, i) \\
C_i &= C(t, i), \quad C_{di} = C_d(t, i), \quad D_i = D(t, i)
\end{aligned}$$

An infinite sequence $a = \{a_t,\ t \geq 0\} \in l_{E2}$ if $\|a\|_{E2}^2 = \lim_{M\to\infty} \mathcal{E}\left\{\sum_{t=0}^{M} \|a_t\|^2\right\} < \infty$.

Definition 2.1: [2] Markovian jump system Σ_d is said to be stochastically stable if for all $\alpha_s \in \mathbb{R}^n$ defined on $\{-\tau, \ldots, -1, 0\}$ and initial mode r_0,

$$\lim_{M\to\infty} \mathcal{E}\left\{ \sum_{t=0}^{M} \|x_t\|^2 \,\middle|\, \alpha, r_0 \right\} < \infty$$

Definition 2.2: [1] Markovian jump system Σ_d is said to be stochastically stable and $\|\Sigma_d\|_\infty < \gamma$ if Markovian jump system Σ_d is stochastically stable and $\|y\|_{E2} \leq \gamma \|u\|_2$ for $u \in l_2$.

By removing the terms related to uncertainties in Theorem 6 of [1], we have the following sufficient condition for the H_∞ norm of system Σ_d to be less than γ.

Lemma 2.3: Markovian jump system Σ_d is stochastically stable and $\|\Sigma_d\|_\infty < \gamma$ if there exist matrices $X_i > 0$, $i \in S$ and $U > 0$ such that

$$\begin{bmatrix} -X_i & 0 & 0 & X_i C_i^T & X_i A_i^T W_i & X_i \\ * & -U & 0 & U C_{di}^T & U A_{di}^T W_i & 0 \\ * & * & -\gamma^2 I_m & D_i^T & B_i^T W_i & 0 \\ * & * & * & -I_p & 0 & 0 \\ * & * & * & * & -\mathcal{X} & 0 \\ * & * & * & * & * & -\frac{1}{\rho} U \end{bmatrix} < 0$$

where $*$ represents the symmetric part and

$$\begin{aligned} W_i &= \begin{bmatrix} \sqrt{p_{i1}} I_n & \cdots & \sqrt{p_{iN}} I_n \end{bmatrix} \\ \mathcal{X} &= \mathrm{diag}(X_1, \ldots, X_N) \\ \rho &= 1 + (1 - p_m)(\tau_M - \tau_m) \\ p_m &= \min_{i \in S} \{p_{ii}\} \\ \tau_M &= \max_{i \in S} \{\tau(i)\} \\ \tau_m &= \min_{i \in S} \{\tau(i)\} \end{aligned}$$

This paper is concerned with the following H_∞ *model approximation problem*: Given a scalar γ, find a stochastically stable system $\hat{\Sigma}_d$ given by

$$\begin{aligned} \hat{x}_{t+1} &= \hat{A}(t, r_t)\hat{x}_t + \hat{A}_d(t, r_t)\hat{x}_{t-\tau(r_t)} + \hat{B}(t, r_t) u_t \\ \hat{y}_t &= \hat{C}(t, r_t)\hat{x}_t + \hat{C}_d(t, r_t) x_{t-\tau(r_t)} + \hat{D}(t, r_t) u_t \\ \hat{x}_s &= \psi_s, \quad s = -\tau, \ldots, -1, 0 \end{aligned}$$

where $\hat{x}_t \in \mathbb{R}^{\hat{n}}$, $\hat{y}_t \in \mathbb{R}^p$ and $\hat{n} < n$, such that the H_∞ norm of the error system satisfies $\left\| \Sigma_d - \hat{\Sigma}_d \right\|_\infty < \gamma$ under zero initial condition.

III. Characterization of Approximation Models

Theorem 3.1: If there exist matrices $P_i > 0$, $X_i > 0$, $i \in S$, $Q > 0$, and $U > 0$ such that

$$\begin{bmatrix} -X_i & 0 & 0 & X_i \bar{A}_i^T \Upsilon_i^T & X_i \\ * & -U & 0 & U \bar{A}_{di}^T \Upsilon_i^T & 0 \\ * & * & -\gamma^2 I_m & \bar{B}_i^T \Upsilon_i^T & 0 \\ * & * & * & -\mathcal{Y}_i & 0 \\ * & * & * & * & -\frac{1}{\rho} U \end{bmatrix} < 0 \qquad (1)$$

$$\begin{bmatrix} \begin{matrix} -\mathcal{I}_1 P_i \mathcal{I}_1^T \\ +\rho \mathcal{I}_1 Q \mathcal{I}_1^T \end{matrix} & 0 & C_i^T & \mathcal{I}_1 \bar{A}_i^T W_i \\ * & -\mathcal{I}_1 Q \mathcal{I}_1^T & C_{di}^T & \mathcal{I}_1 \bar{A}_{di}^T W_i \\ * & * & -I_p & 0 \\ * & * & * & -\mathcal{X} \end{bmatrix} < 0 \qquad (2)$$

$$P_i X_i = I_{\tilde{n}}, \quad QU = I_{\tilde{n}} \qquad (3)$$

where

$$\Upsilon_i = \begin{bmatrix} \mathcal{I}_1^T & 0 & \cdots & 0 & \cdots & 0 \end{bmatrix}^T, \quad \tilde{n} = n + \hat{n}$$

$$\mathcal{Y}_i = \begin{bmatrix} \frac{1}{p_{i1}} \mathcal{I}_1 X_1 \mathcal{I}_1^T & \frac{1}{p_{i1}} \mathcal{I}_1 X_1 & 0 & \cdots & 0 & 0 \\ \frac{1}{p_{i1}} X_1 \mathcal{I}_1^T & \begin{matrix} \frac{1}{p_{iN}} X_N \\ +\frac{1}{p_{i1}} X_1 \end{matrix} & \frac{1}{p_{iN}} X_N & \cdots & \frac{1}{p_{iN}} X_N & \frac{1}{p_{iN}} X_N \\ 0 & \frac{1}{p_{iN}} X_N & \begin{matrix} \frac{1}{p_{i2}} X_2 \\ +\frac{1}{p_{iN}} X_N \end{matrix} & \cdots & \frac{1}{p_{iN}} X_N & \frac{1}{p_{iN}} X_N \\ \cdots & \cdots & \cdots & \ddots & \cdots & \cdots \\ 0 & \frac{1}{p_{iN}} X_N & \frac{1}{p_{iN}} X_N & \cdots & \begin{matrix} \frac{1}{p_{iN}} X_N \\ +\frac{1}{p_{i(N-2)}} X_{N-2} \end{matrix} & \frac{1}{p_{iN}} X_N \\ 0 & \frac{1}{p_{iN}} X_N & \frac{1}{p_{iN}} X_N & \cdots & \frac{1}{p_{iN}} X_N & \begin{matrix} \frac{1}{p_{iN}} X_N \\ +\frac{1}{p_{i(N-1)}} X_{N-1} \end{matrix} \end{bmatrix}$$

then there exists a stochastically stable system $\hat{\Sigma}_d$ that solves the H_∞ model approximation problem with $\left\| \Sigma_d - \hat{\Sigma}_d \right\|_\infty < \gamma$. In this case, a desired approximation system corresponding to a feasible solution ($P_i > 0$, $X_i > 0$, $i \in \mathcal{S}$, $Q > 0$, and $U > 0$) to (1)–(3) is parametrized as

$$\begin{aligned} G_i &= \begin{bmatrix} \hat{D}_i & \hat{C}_i & \hat{C}_{di} \\ \hat{B}_i & \hat{A}_i & \hat{A}_{di} \end{bmatrix} \\ &= -U_i^{-1} \Omega_i^T V_i \Lambda_i^T \left(\Lambda_i V_i \Lambda_i^T \right)^{-1} \\ &\quad + U_i^{-1} K_i^{\frac{1}{2}} L_i \left(\Lambda_i V_i \Lambda_i^T \right)^{-\frac{1}{2}} \end{aligned} \qquad (4)$$

where L_i are any matrices satisfying $\|L_i\| < 1$, and $U_i > 0$ such that

$$\begin{aligned} V_i &= \left(\Omega_i U_i^{-1} \Omega_i^T - \Phi_i \right)^{-1} > 0 \\ K_i &= U_i - \Omega_i^T \left(V_i - V_i \Lambda_i^T \left(\Lambda_i V_i \Lambda_i^T \right)^{-1} \Lambda_i V_i \right) \Omega_i \end{aligned}$$

$$\Phi_i = \begin{bmatrix} -X_i & 0 & 0 & X_i \bar{C}_i^T & X_i \bar{A}_i^T W_i & X_i \\ * & -U & 0 & U \bar{C}_{di}^T & U \bar{A}_{di}^T W_i & 0 \\ * & * & -\gamma^2 I_m & \bar{D}_i^T & \bar{B}_i^T W_i & 0 \\ * & * & * & -I_p & 0 & 0 \\ * & * & * & * & -\mathcal{X} & 0 \\ * & * & * & * & * & -\frac{1}{\rho} U \end{bmatrix} \qquad (5)$$

$$\Omega_i = \left[\begin{array}{c|c|c|c|c|c} 0 & 0 & 0 & \bar{J}^T & \bar{F}^T W_i & 0 \end{array} \right]^T \qquad (6)$$

$$\Lambda_i = \left[\begin{array}{c|c|c|c|c|c} \bar{H} X_i & \bar{M} U & \bar{N} & 0 & 0 & 0 \end{array} \right] \qquad (7)$$

$$\bar{A}_i = \begin{bmatrix} A_i & 0 \\ 0 & 0 \end{bmatrix}, \ \bar{A}_{di} = \begin{bmatrix} A_{di} & 0 \\ 0 & 0 \end{bmatrix}, \ \bar{B}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix} \qquad (8)$$

$$\bar{C}_{di} = \begin{bmatrix} C_{di} & 0 \end{bmatrix}, \ \bar{C}_i = \begin{bmatrix} C_i & 0 \end{bmatrix}, \ \bar{D}_i = D_i \qquad (9)$$

$$\bar{F} = \begin{bmatrix} 0 & 0 \\ 0 & I_{\hat{n}} \end{bmatrix}, \ \bar{H} = \begin{bmatrix} 0 & 0 \\ 0 & I_{\hat{n}} \\ 0 & 0 \end{bmatrix}, \ \mathcal{I}_1 = \begin{bmatrix} I_n & 0 \end{bmatrix} \qquad (10)$$

$$\bar{M} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & I_{\hat{n}} \end{bmatrix}, \ \bar{N} = \begin{bmatrix} I_m \\ 0 \\ 0 \end{bmatrix}, \ \bar{J} = \begin{bmatrix} -I_p & 0 \end{bmatrix} \quad (11)$$

Proof. From systems Σ_d and $\hat{\Sigma}_d$, the error system $\Sigma_d - \hat{\Sigma}_d$ can be described by

$$\begin{aligned} \tilde{x}_{t+1} &= \tilde{A}(t,r_t)\tilde{x}_t + \tilde{A}_d(t,r_t)\tilde{x}_{t-\tau(r_t)} + \tilde{B}(t,r_t)u_t \\ \tilde{y}_t &= \tilde{C}(t,r_t)\tilde{x}_t + \tilde{C}_d(t,r_t)\tilde{x}_{t-\tau(r_t)} + \tilde{D}(t,r_t)u_t \end{aligned}$$

where $\tilde{x}_t = \begin{bmatrix} x_t^T & \hat{x}_t^T \end{bmatrix}^T$, $\tilde{y}_t = y_t - \hat{y}_t$ and

$$\begin{aligned} \tilde{A}(t,r_t) &= \begin{bmatrix} A(t,r_t) & 0 \\ 0 & \hat{A}(t,r_t) \end{bmatrix} \\ \tilde{A}_d(t,r_t) &= \begin{bmatrix} A_d(t,r_t) & 0 \\ 0 & \hat{A}_d(t,r_t) \end{bmatrix} \\ \tilde{B}(t,r_t) &= \begin{bmatrix} B(t,r_t) \\ \hat{B}(t,r_t) \end{bmatrix} \\ \tilde{C}(t,r_t) &= \begin{bmatrix} C(t,r_t) & -\hat{C}(t,r_t) \end{bmatrix} \\ \tilde{C}_d(t,r_t) &= \begin{bmatrix} C_d(t,r_t) & -\hat{C}_d(t,r_t) \end{bmatrix} \\ \tilde{D}(t,r_t) &= D(t,r_t) - \hat{D}(t,r_t) \end{aligned}$$

From Lemma 2.3, it can be established that $\left\| \Sigma_d - \hat{\Sigma}_d \right\|_\infty < \gamma$ if there exist matrices $X_i > 0$, $i \in S$ and $U > 0$ such that

$$\begin{bmatrix} -X_i & 0 & 0 & X_i\tilde{C}_i^T & X_i\tilde{A}_i^T W_i & X_i \\ * & -U & 0 & U\tilde{C}_{di}^T & U\tilde{A}_{di}^T W_i & 0 \\ * & * & -\gamma^2 I_m & \tilde{D}_i^T & \tilde{B}_i^T W_i & 0 \\ * & * & * & -I_p & 0 & 0 \\ * & * & * & * & -\mathcal{X} & 0 \\ * & * & * & * & * & -\frac{1}{\rho}U \end{bmatrix} < 0 \quad (12)$$

with G_i defined in (4) and

$$\begin{aligned} \tilde{A}_i &= \bar{A}_i + \bar{F}G_i\bar{H}, \quad \tilde{A}_{di} = \bar{A}_{di} + \bar{F}G_i\bar{M} \quad (13) \\ \tilde{B}_i &= \bar{B}_i + \bar{F}G_i\bar{N}, \quad \tilde{C}_i = \bar{C}_i + \bar{J}G_i\bar{H} \quad (14) \\ \tilde{C}_{di} &= \bar{C}_{di} + \bar{J}G_i\bar{M}, \quad \tilde{D}_i = \bar{D}_i + \bar{J}G_i\bar{N} \quad (15) \end{aligned}$$

with $\bar{A}_i$, $\bar{A}_{di}$, $\bar{B}_i$, $\bar{C}_i$, $\bar{C}_{di}$, $\bar{D}_i, \bar{F}$, $\bar{H}$, $\bar{M}$, $\bar{N}$ and $\bar{J}$ defined in (8)–(11). From the expressions in (13)–(15), matrix inequality (12) can be rewritten as

$$\Phi_i + \Omega_i G_i \Lambda_i + (\Omega_i G_i \Lambda_i)^T < 0 \quad (16)$$

where Φ_i, Ω_i, and Λ_i are defined in (5)–(7). The necessary and sufficient conditions for LMIs (16) to have solutions are

$$\Omega_i^\perp \Phi_i \Omega_i^{\perp T} < 0, \quad \Lambda_i^{T\perp} \Phi_i \Lambda_i^{T\perp T} < 0 \quad (17)$$

$\Omega_i^\perp$ and $\Lambda_i^{T\perp}$ can be selected as follows:

$$\begin{aligned} \Omega_i^\perp &= \begin{bmatrix} I_{\tilde{n}} & 0 & 0 & 0 & 0 & 0 \\ 0 & I_{\tilde{n}} & 0 & 0 & 0 & 0 \\ 0 & 0 & I_m & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & T_i & 0 \\ 0 & 0 & 0 & 0 & 0 & I_{\tilde{n}} \end{bmatrix} \\ \Lambda_i^{T\perp} &= \begin{bmatrix} \mathcal{I}_1 P_i & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathcal{I}_1 Q & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_p & 0 & 0 \\ 0 & 0 & 0 & 0 & I_{N\tilde{n}} & 0 \\ 0 & 0 & 0 & 0 & 0 & I_{\tilde{n}} \end{bmatrix} \end{aligned}$$

where

$$T_i = \begin{bmatrix} \frac{1}{\sqrt{p_{i1}}}\mathcal{I}_1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ \frac{1}{\sqrt{p_{i1}}}I_{\tilde{n}} & 0 & 0 & \cdots & 0 & 0 & -\frac{1}{\sqrt{p_{iN}}}I_{\tilde{n}} \\ 0 & \frac{1}{\sqrt{p_{i2}}}I_{\tilde{n}} & 0 & \cdots & 0 & 0 & -\frac{1}{\sqrt{p_{iN}}}I_{\tilde{n}} \\ 0 & 0 & \frac{1}{\sqrt{p_{i3}}}I_{\tilde{n}} & \cdots & 0 & 0 & -\frac{1}{\sqrt{p_{iN}}}I_{\tilde{n}} \\ \cdots & \cdots & \cdots & \ddots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & \frac{1}{\sqrt{p_{i(N-2)}}}I_{\tilde{n}} & 0 & -\frac{1}{\sqrt{p_{iN}}}I_{\tilde{n}} \\ 0 & 0 & 0 & \cdots & 0 & \frac{1}{\sqrt{p_{i(N-1)}}}I_{\tilde{n}} & -\frac{1}{\sqrt{p_{iN}}}I_{\tilde{n}} \end{bmatrix}$$

and $\mathcal{I}_1$ is defined in (10). Then, by Schur complement and condition (3), we obtain that $\Omega_i^\perp \Phi_i \Omega_i^{\perp T} < 0$ is equivalent to (1). $\Lambda_i^{T\perp} \Phi_i \Lambda_i^{T\perp T} < 0$ is equivalent to (2). From LMIs (1) and (2), and condition (3), if there exist matrices $P_i > 0$, $X_i > 0$, $Q > 0$ and $U > 0$, $i \in S$ satisfying (17), there exists matrix G_i such that (16) holds. By Lemma 2.3, we have $\left\| \Sigma_d - \hat{\Sigma}_d \right\|_\infty < \gamma$. All the parameters of the approximation models satisfying (12) can be obtained by the parametrization method of Gahinet and Apkarian in [8]. This completes the proof. □

IV. H_∞ Model Approximation Algorithm

It can be indicated from Theorem 3.1 that the construction of the approximation system resides in solving LMIs (1) and (2) and the inverse constraints in (3). This can be solved by the cone complementarity linearization (CCL) algorithm [7]. To utilize the CCL algorithm to solve the H_∞ model approximation problem, a convex set of all the feasible solutions of LMIs (1) and (2) is defined as

$$\mathcal{C} := \{\mathcal{X} \mid \mathcal{X} \text{ satisfies LMIs (1) and (2)}\}$$

and a nonconvex set of all feasible solutions satisfying the inverse constraints in (3) is defined as

$$\mathcal{T} := \{\mathcal{X} \mid \mathcal{X} \text{ satisfies (3)}\}$$

where

$$\mathcal{X} = (P_i > 0, X_i > 0, Q > 0, U > 0, i \in \mathcal{S})$$

The solvability of the H_∞ model approximation problem can be translated into the following nonconvex feasibility problem:

$$\text{Find } \mathcal{X} \in \mathcal{C} \quad \text{subject to } \mathcal{X} \in \mathcal{T} \quad (18)$$

by the sufficient condition in Theorem 3.1. Then, by the CCL approach, the above nonconvex problem (18) has a solution if and only if the minimization problem

$$\min_{\mathcal{X} \in \mathcal{C} \cap \mathcal{C}_{in}} \left\{ \text{trace}\left(\sum_{i=1}^{N} P_i X_i + QU\right) \right\} \quad (19)$$

where

$$\mathcal{C}_{in} = \left\{ \begin{bmatrix} P_i & I \\ I & X_i \end{bmatrix} \geq 0, \ \begin{bmatrix} Q & I \\ I & U \end{bmatrix} \geq 0, \ i \in \mathcal{S} \right\}$$

achieves a minimum, $2(N+1)\tilde{n}$, that is, an optimal solution of problem (18) satisfying

$$\text{trace}(P_i X_i) = \text{trace}(QU) = \tilde{n}, \qquad i \in \mathcal{S}$$

Otherwise (18) is infeasible. Therefore, in order to solve the H_∞ model approximation problem for discrete-time Markovian jump systems with mode-dependent time delays, we transform it to a global solution of the minimization problem (18). The CCL algorithm solves problem like (18) effectively although it is still a nonconvex optimization problem [5]. Based on the above analysis, an algorithm is presented to solve the H_∞ model approximation problem.

Step 1 Given system Σ_d, the order of the approximation model $\hat{n}$, the prescribed model approximation error $\gamma > 0$ and a sufficiently small prescribed scalar $\delta > 0$ to control the convergence accuracy.

Step 2 Set $k = 0$ and choose any initial guess $\mathcal{X}_0 = (P_{i0}, X_{i0}, Q_0, U_0, i \in \mathcal{S}) \in \mathcal{C}$.

Step 3 Define

$$\begin{aligned} f_k(\mathcal{X}) &= \text{trace}(\sum_{i=1}^{N}(P_{ik}X_i + X_{ik}P_i) \\ &\quad + U_k Q + Q_k U) \end{aligned}$$

Solve the following convex minimization problem:

$$\min_{\mathcal{X} \in \mathcal{C} \cap \mathcal{C}_{in}} \{f_k(\mathcal{X})\}$$

and denote the minimizer $\mathcal{X}_k^*$ and compute the minimum value $f_k^* = f_k(\mathcal{X}_k^*)$.

Step 4 If $|f_k^* - 2(N+1)\tilde{n}| < \delta$, then construct an approximation model based on (4); otherwise set $k = k+1$, and assign $(P_{ik}, X_{ik}, Q_k, U_k, i \in \mathcal{S}) = \mathcal{X}_{k-1}^*$ and go to Step 3.

It can be indicated that Step 2 is a simple LMI feasibility problem, and Step 3 is a convex programming with LMI constraints. From the explanation in [7], $\{f_k^*\}$ is decreasing and bounded below by $2(N+1)\tilde{n}$. Once it converges, then (18) is feasible, which implies that the H_∞ model approximation problem is solvable for a given $\gamma > 0$.

V. Numerical Example

A discrete-time Markovian jump linear system Σ_d with mode-dependent time delays and two modes is given by

$$A_1 = \begin{bmatrix} 0.5 & 0.01 & 0.01 & 0 \\ 0 & 0.6 & 0 & 0.01 \\ 0 & 0 & 0.2 & 0 \\ 0 & 0 & 0 & 0.4 \end{bmatrix}$$

$$A_{d1} = \begin{bmatrix} 0.01 & 0 & 0.01 & 0 \\ 0 & 0.02 & 0 & 0.01 \\ 0 & 0 & 0.02 & 0 \\ 0 & 0 & 0 & 0.03 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0.1 \\ 0.7 \\ 1.3 \\ 0.5 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.3 \\ 0.7 \\ 1.2 \\ 0.1 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 1.2 & 0.4 & 0.6 & 0.9 \\ 0.4 & 0.5 & 0.6 & 0.1 \end{bmatrix}$$

$$C_{d1} = \begin{bmatrix} 0.12 & 0.04 & 0.06 & 0.09 \\ 0.04 & 0.05 & 0.06 & 0.01 \end{bmatrix}$$

$$D_1 = \begin{bmatrix} 0.01 \\ 0.02 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0.2 \\ 0.5 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -0.3 & 0.01 & 0 & 0 \\ 0 & -0.7 & 0 & 0.02 \\ 0.03 & 0 & 0.4 & 0 \\ 0 & 0 & 0 & -0.4 \end{bmatrix}$$

$$A_{d2} = \begin{bmatrix} 0.01 & 0.01 & 0 & 0 \\ 0 & -0.02 & 0 & 0 \\ 0 & 0 & -0.02 & 0 \\ 0 & 0 & 0 & -0.01 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} 1.1 & 0.5 & 0.7 & 1.9 \\ 0.1 & 0.3 & 0.4 & 0.4 \end{bmatrix}$$

$$C_{d2} = \begin{bmatrix} 0.012 & 0.004 & 0.006 & 0.009 \\ 0.004 & 0.005 & 0.006 & 0.001 \end{bmatrix}$$

$$\tau_1 = 1, \quad \tau_2 = 2, \quad \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} = \begin{bmatrix} 0.1 & 0.9 \\ 0.55 & 0.45 \end{bmatrix}$$

Here, $\gamma = 0.21$ is chosen and the 2nd-order model obtained using the result in Theorem 3.1 has system matrices given by

$$\hat{A}_1 = \begin{bmatrix} 0.4547 & -0.0718 \\ -0.1753 & 0.2282 \end{bmatrix}$$

$$\hat{A}_{d1} = \begin{bmatrix} 0.0148 & -0.0114 \\ -0.0116 & 0.0197 \end{bmatrix}$$

$$\hat{B}_1 = \begin{bmatrix} 0.6494 \\ -1.1711 \end{bmatrix}, \quad \hat{D}_1 = \begin{bmatrix} 0.0149 \\ -0.0100 \end{bmatrix}$$

$$\hat{C}_1 = \begin{bmatrix} 0.3790 & -1.3089 \\ 0.0836 & -1.2597 \end{bmatrix}$$

$$\hat{C}_{d1} = \begin{bmatrix} 0.0855 & -0.0956 \\ 0.0412 & -0.0867 \end{bmatrix}$$

$$\hat{A}_2 = \begin{bmatrix} -0.5322 & 0.0172 \\ 0.5959 & 0.3702 \end{bmatrix}$$

$$\hat{A}_{d2} = \begin{bmatrix} -0.0046 & 0.0030 \\ -0.0004 & -0.0106 \end{bmatrix}$$

$$\hat{B}_2 = \begin{bmatrix} 0.5065 \\ -0.9236 \end{bmatrix}, \quad \hat{D}_2 = \begin{bmatrix} 0.2296 \\ 0.4852 \end{bmatrix}$$

$$\hat{C}_2 = \begin{bmatrix} 0.7462 & -1.5517 \\ 0.1018 & -0.7670 \end{bmatrix}$$

$$\hat{C}_{d2} = \begin{bmatrix} 0.0071 & -0.0082 \\ 0.0071 & -0.0107 \end{bmatrix}$$

Figures 1 and 2 are the output trajectories of the error system between the original system and the 2nd order reduced system subjected to $u_t = e^{-t/8}\cos(2t)$. Figures 2 and 3 are the output trajectories of the original system (solid line), the 2nd order model (dash-dotted line) subjected to input u_t. It can been seen that the output due to the approximation system gives a reasonable approximation of the original system.

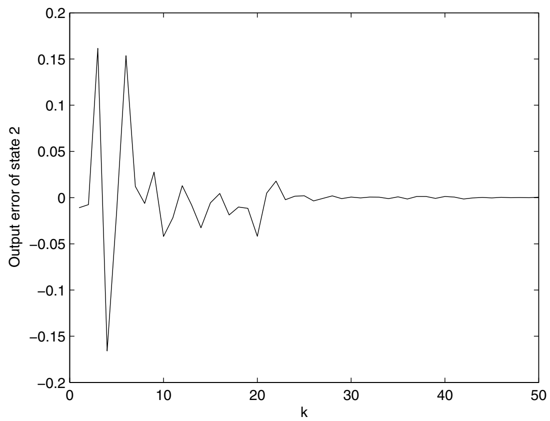

Fig. 1. Output trajectory of state 1 of the error system.

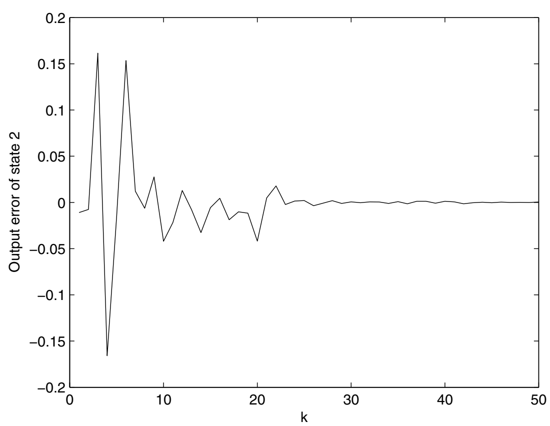

Fig. 2. Output trajectory of state 2 of the error system.

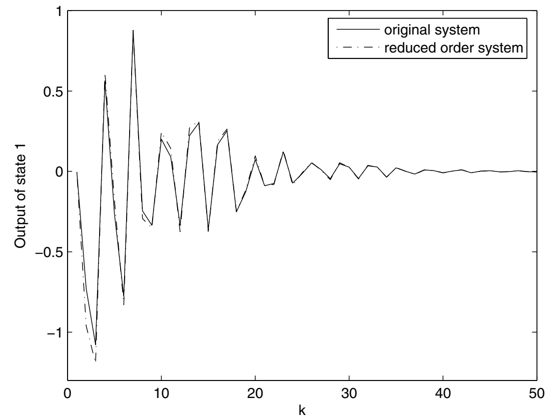

Fig. 3. Output trajectory of state 1 of original and reduced systems.

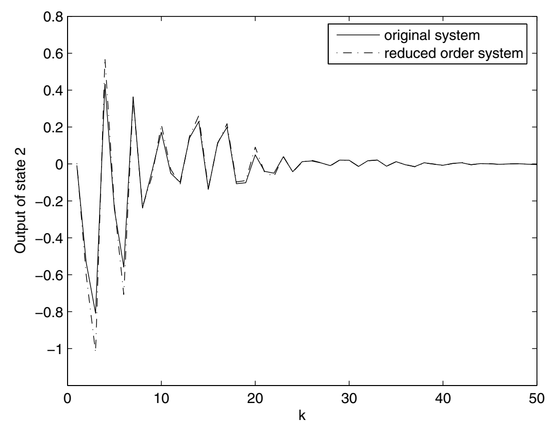

Fig. 4. Output trajectory of state 2 of original and reduced systems.

VI. Conclusion

In this paper, the H_∞ model approximation problem for discrete-time Markovian jump system with mode-dependent time delays has been addressed and solved via a sequential LMI minimization problem. An explicit formula for the construction of approximation systems has been given in terms of a set of LMIs with inverse constraints. An effective algorithm has been used to solve the LMI problems with inverse constraints. A numerical example has been given to demonstrate the validity of the theoretical result.

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