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Robust PI controller design for an uncertain recycle system

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***Abstract*— In this paper, a new approach is proposed for design of a robust controller for interval process systems in the presence of parametric uncertainties. The proposed method uses a necessary condition and sufficient condition for stability of interval polynomial. These conditions are used to derive a set of inequalities in terms of the compensator parameters which can be solved to obtain a robust controller. The proposed method is simple, involves less computational complexity and provides an easy method to obtain a robust compensator. The proposed method is applied to design a robust proportional-integral controller for a recycle system in the presence of process gain uncertainties. The results show the efficacy of the proposed method.**

Keywords: Robust control, Robust stability and performance, Interval plants.

I. INTRODUCTION

The problem of designing robust controller for process plants having unknown but bounded parameter uncertainties, which often called the interval process systems, has received considerable attention [1],[2],[3],[4],[5],[6],[7],[8],[9],[10],[11]. Since control systems operate under large uncertainties it is important to study stability robustness in the presence of uncertainty. The uncertainty present in the control system causes degradation of system performance and destabilization. Various analysis and design are essentially meant for application to a "nominal" model. The resulting design is said to be robust if the system performs within acceptable limits in the face of significant parameter variations and model uncertainties. The need to incorporate robustness in design is necessitated by the fact that for most practical system, the model is known only approximately. For example, in modeling a chemical process, there unavoidably exist uncertainties due to poor process knowledge, nonlinearities, unknown internal or external noises, environment influence, time varying parameters, changing operating conditions, etc. Therefore, to build a very accurate model that describes the physical process exactly may be very costly and it may turn out to be impractical from the viewpoint of analysis and design. However, a simplified model may not adequately represent the actual process system and may result in an unacceptable design. Similarly in the aircraft industry, the aircraft model is constructed using the data obtained from the wind-tunnel experiments on the aircraft body. As a consequence, the parameters of the model would not have a specific value, rather they are known to lie within an interval. Since the actual flight data are not available the controller should be able to account for the unmodeled parameters that can be obtained only when the aircraft is airborne. The other example includes robotic manipulators, nuclear reactors, electrical machines and large power networks etc., which have parametric uncertainties for the entire range of operation. An important approach to this subject is via expressing the characteristic polynomial by an interval polynomial, i.e. a polynomial whose coefficient each vary independently in a prescribed interval. The stability analysis of polynomials subjected to parameter uncertainty have received considerable attention after the celebrated theorem of Kharitonov [12], which assures robust stability under the condition that four specially constructed "extreme polynomials", called Kharitonov polynomials are Hurwitz. The problem of robust stability of interval polynomial is also dealt in [13], [14], [8], [9], [10], [11]. Based on interval polynomial theory, Soh[15] proposed a technique for the design of a robust pole placement controller. Rotstein[3] proposed a computational method to design a robust characteristic polynomial placement controller. Several results have appeared in the literature which aims at reducing test of Hurwitz stability of entire family to a small subset of entire family. In this regard few extreme point results are available in the literature. These includes work due to Ghosh [16], where he has shown that a pure gain compensator $C(s) = k$ stabilizes entire interval plant family if and only if it stabilizes a distinguished set of eight of the extreme plants. Hollot and Fang [17] considered the same setup as Ghosh but allow the controller to be first order. They prove that to robustly stabilize the entire family, it is necessary and sufficient to stabilize the set of extreme plants which are obtained by taking all possible combinations of extreme values of the plant numerator coefficients with extreme values of the plant denominator coefficients. If the plant numerator has degree m and the plant denominator is monic with degree n, the number of extreme plants can be as high as $N_{ext} = 2^{m+n+1}$. In [18], Barmish proved that, it is necessary and sufficient to stabilize only *sixteen* of the extreme plants. A complete survey of these extreme point is given in [19]. The design of robust stabilizing controller for interval process

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systems is presented in [5] which is extended to ensure robust stability and performance in [6]. In [1] a necessary condition and sufficient condition for robust stability of interval polynomials using the results of Nie [20] for fixed polynomials is presented. In this paper, we propose a method which is based on our previous work [1] to guarantee robust stability in the presence of parametric uncertainties. The method is simple and involves less computational complexity compared to method in [5]. The paper is organized as: Section 2 gives a necessary condition and sufficient condition for robust Hurwitz stability of interval polynomial. In Section 3 a robust controller design methodology is proposed to ensure robust stability of interval process systems. In Section 4, the proposed method is applied to design a robust PI controller for an uncertain recycle system. Finally conclusion is given in Section 5.

II. Conditions for robust stability of interval polynomial

Consider the set of real polynomials of degree n of the form

$$\delta(s) = \delta_0 + \delta_1 s + \delta_2 s^2 + \delta_3 s^3 + \delta_4 s^4 + \cdots + \delta_n s^n \quad (1)$$

where the coefficients lie within given ranges,

$$\delta_0 \in [x_0, y_0]\,,\ \delta_1 \in [x_1, y_1]\,,\ \cdots, \delta_n \in [x_n, y_n]\,.$$

We assume that the degree remains invariant over the family, so that $0 \notin [x_n, y_n]$. Such a set of polynomials is called a real interval family and is referred as an interval polynomial. The set of polynomials given by (1) is stable if and only if each and every element of the set is a Hurwitz polynomial. In [1] a necessary condition and sufficient condition for the robust stability of interval polynomial (1) is proposed using the algebraic stability criterion for fixed polynomials due to Nie [20]. These conditions are stated in the following lemmas.

Lemma 2.1: The interval polynomial $\delta(s)$ defined in (1) is Hurwitz for all $\delta_i \in [x_i, y_i]$ where $i = 0, 1, 2, \cdots, n$ if the following necessary conditions are satisfied

$$y_i \geq x_i > 0 \quad i = 0, 1, 2, \cdots, n$$

$$x_i x_{i+1} > y_{i-1} y_{i+2} \quad i = 1, 2, \cdots n-2. \quad (2)$$

Proof: See [1]

Lemma 2.2: The interval polynomial $\delta(s)$ defined in (1) is Hurwitz for all $\delta_i \in [x_i, y_i]$ where $i = 0, 1, 2, \cdots, n$ if the following sufficient conditions are satisfied

$$y_i \geq x_i > 0 \quad i = 0, 1, 2, \cdots, n$$

$$0.4655 x_i x_{i+1} > y_{i-1} y_{i+2} \quad i = 1, 2, \cdots n-2. \quad (3)$$

Proof: See [1]

III. Design procedure for robust stabilization of interval process systems

Consider a strictly proper interval plant family consisting all plants of the form,

$$G(s,p,q) = \frac{N(s,p)}{D(s,q)} \quad (4)$$

where the numerator and denominator polynomials are of the form

$$N(s,p) = p_0 + p_1 s + p_2 s^2 + \cdots + p_{m-1} s^{m-1} + p_m s^m \quad (5)$$

$$D(s,q) = q_0 + q_1 s + q_2 s^2 + \cdots + q_{n-1} s^{n-1} + q_n s^n \quad (6)$$

and where vectors p and q lie in given rectangles P and Q, respectively, i.e.,

$$p \in P = \left\{ p : p_i^- \leq p_i \leq p_i^+ \ \text{ for } \ i = 0, 1, \cdots, m \right\} \quad (7)$$

and

$$q \in Q = \left\{ q : q_i^- \leq q_i \leq q_i^+ \ \text{ for } \ i = 0, 1, \cdots, n \right\} \quad (8)$$

where $q_n = [1, 1]$ and the bounds p_i^-, p_i^+, q_i^-, and q_i^+ are specified *a priori*.
To stabilize the interval plant family, we consider a proper first-order compensator of the form (any form for the controller can be considered),

$$C(s) = k_1 + \frac{k_2}{s} = \frac{N_c(s)}{D_c(s)} \quad (9)$$

We say that this compensator $C(s)$ robustly stabilizes the interval plant family if, for all $p \in P$ and all $q \in Q$, the resulting closed loop polynomial

$$\Delta(s,p,q) = N_c(s) N(s,p) + D_c(s) D(s,q) \quad (10)$$

has all its roots in the strict left half plane; that is $\Delta(s,p,q)$ is Hurwitz. This is being the case, $C(s)$ is said to be a robust stabilizer and the closed loop system is said to be robustly stable. Let the closed loop interval polynomial be in the form

$$\Delta(s,p,q) = [x_0, y_0] + [x_1, y_1] s + \cdots + [x_n, y_n] s^n + [1, 1] s^{n+1} \quad (11)$$

The stability conditions in (2) and (3) can be applied to closed loop characteristic polynomial in (11), which leads to inequalities in terms of compensator parameters. These inequalities can be solved to obtain compensator parameters. Even though the method in [5] is based on transforming the robust controller design problem into an equivalently non-linearly constrained optimization problem by making use of Kharitonov and Hermite-Biehler stability theorems the method in [1] involves less computational complexity and provides a simple way to obtain a robust controller. When the degree of the closed loop polynomial 'n' is greater than 6 then the constraints in [5] are $12([\frac{n}{2}] + [\frac{n-1}{2}] - 4)$ where the notation $[r]$ represents the greatest integer that does not exceed r. The constraints in [1] are $3n-4$ for any degree 'n' which are fewer compared to [5].

IV. Application to Uncertain Recycle System

Processes with a recycle operation are often encountered in practical process industries. However, there has been a limited number of papers exploring the challenge problem associated with the dynamics and control of recycle systems [21],[22] and references cited therein, especially for the case where uncertainties are involved [23]. The purpose of this section is to apply the proposed technique to design a robust controller for a recycle process system in the presence of uncertain but bounded parameters. The block diagram of the process under consideration is shown in Fig. 1. This process consists

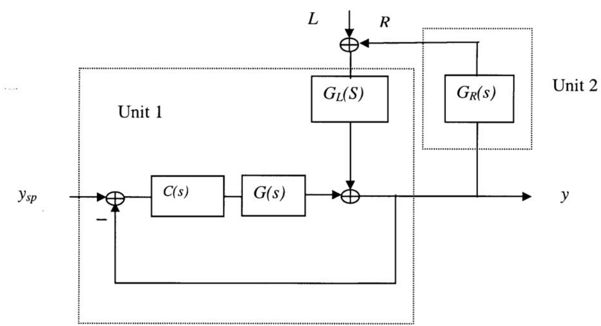

Fig. 1. Block diagram of a recycle system

of two units. The unit 1 has a feedback controller $C(s)$ that attempts to keep the output variable y near its set point value y_{set}. The manipulated variable $M(s)$ enters the process whose transfer function is $G(s)$. The unit 2 is a recycle process. The output of the plant, y, is fed back into the recycle unit. An output variable of the recycle process, R, is then summed with the load variable, L, and enters unit 1. The transfer functions $G_L(s)$ and $G(s)$ are assumed to be identical [22] and be given by

$$G(s) = G_L(s) = \frac{k}{(\tau s+1)^2}$$

In order to design a robust PI stabilizing feedback controller for interval plant, we first perform the nominal design. Consider a PI controller of the form

$$k_1 + \frac{k_2}{s} = \frac{N_c(s)}{D_c(s)}$$

to robustly stabilize given interval plant. The $C(s)$ will stabilize the given interval plant if the closed loop interval polynomial in (13) is stable. Before designing a robust controller, we will consider unit 1 and design a PI controller for it. The closed loop transfer function from the input $(L+R)$ of unit 1 to the output $y(s)$ is

$$G_{CL}(s) = \frac{y(s)}{L(s)+R(s)} = \frac{G_L(s)}{1+C(s)G(s)}$$

$$G_{CL}(s) = \frac{ks}{\tau^2 s^3 + 2\tau s^2 + (1+k_1 k)s + k_2 k}$$

which gives the characteristic equation as

$$\tau^2 s^3 + 2\tau s^2 + (1+k_1 k)s + k_2 k = 0 \qquad (12)$$

The optimal characteristic equation that minimizes the ITAE performance for a step input is given by [24]

$$s^3 + 1.75 w_n s^2 + 2.15 w_n^2 s + w_n^3 = 0 \qquad (13)$$

By comparing (12) and (13) we get the nominal PI controller parameter as $k_1^0 = \frac{1.8082}{k}$ and $k_2^0 = \frac{1.4927}{k\tau}$. We set the process parameters as $k=2$ and $\tau=1$ so we get controller parameters as $k_1^0 = 0.9041$ and $k_2^0 = 0.7464$. For this controller setting, the closed-loop poles are -0.8093 and -0.5954 ± 1.2207, and the damping ratio of the complex-conjugate roots is 0.438. Fig. 2 shows

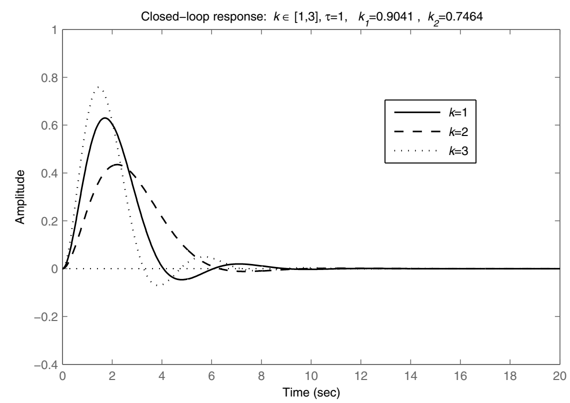

Fig. 2. Closed loop response of unit 1 to a step change in load subject to process gain uncertainties (without a recycle)

the actual response of unit 1 to a unit step change in load without a recycle. It is observed from this figure that the nominal controller can smooth out the step load disturbance without having a long-standing oscillation. It can therefore be inferred that the complex-conjugate poles are not the dominant poles though the damping ratio is small. Now, suppose that there is ± 50 parameter variation in the process gain k due to uncertainties, i.e., $k \in [1,3]$. The Fig. 2 also reveals the responses to step change in load by using ITAE PI controller for $k=1$ and $k=3$. As seen from the Fig. 2, for all variation in process gain uncertainties response settle out earlier and that there is no offset. This indicates that the obtained ITAE PI controller can stabilize unit 1 for all possible process gain parameter taken a value in the uncertainty bound $[1,3]$. If the recycle unit (unit 2) is incorporated into unit 1 and operate as a whole, the transfer function of the closed loop system from the load $L(s)$ to the output $Y(s)$ is

$$\frac{Y(s)}{L(s)} = \frac{G_L(s)}{1+C(s)G(s) - G_L(s)G_R(s)} \qquad (14)$$

where $G_R(s)$ is the trasfer function of the recycle system. The presence of the recycle unit contributes a negative term in the overall closed loop transfer function. Hence, the recycle is equivalent to a positive (negative gain) feedback controller which can destabilize a designed control system. Consider that recycle unit is described by the following first order transfer function

$$G_R(s) = \frac{k_R}{\tau_R s + 1} \quad (15)$$

Then the characteristic equation of the overall system using ITAE PI controller and with parameters as $\tau = 1, k_R = 1.6$ and $\tau_R = 1$ will be,

$$\begin{aligned}\Delta(s,p,q) \quad = \quad & [1,1]s^4 + [3,3]s^3 + \\ & +[k_1+3, 3k_1+3]s^2 \\ & +[k_1+k_2-0.6, 3k_1+3k_2-3.8]s + [k_2, 3k_2]\end{aligned} \quad (16)$$

In the light of the equation (16), for nominal values of $k = 2$ the nominal characteristics equation is

$$\Delta^0(s,p,q) = s^4+3s^3+(3+2k_1)s^2+(2k_1+2k_2-2.2)s+2k_2 \quad (17)$$

For the process under its nominal condition i.e. for $k = 2$ and with designed nominal ITAE PI controller, the response to unit step load change is shown in Fig. 3. It is observed that the entire dynamics of the recycle system is quite different from that of the system without having a recycle. The presence of recycle unit tends to increase he process sensitivity to disturbances, and therefore leads to a more oscillatory response and also process response time is increased. To examine the effect of uncertainties on the process with recycle stream, we again assume that there is ± 50 variation in the process gain, i.e. $k \in [1, 3]$. Fig. 4 shows that the process is stable when the process gain is $k = 1$, but unstable for $k = 3$ i.e. for its upper bound. This shows that the nominally designed ITAE PI controller is no longer valid for the recycle system with the process gain uncertainty $k \in [1, 3]$. It is therefore desired to redesign a robust controller for the uncertain recycle system starting from its nominal controller. Following the procedure of [1],

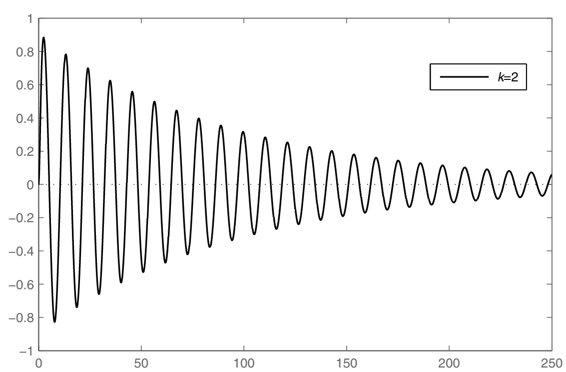

Fig. 3. Closed loop response of a recycle system to a unit step change in load

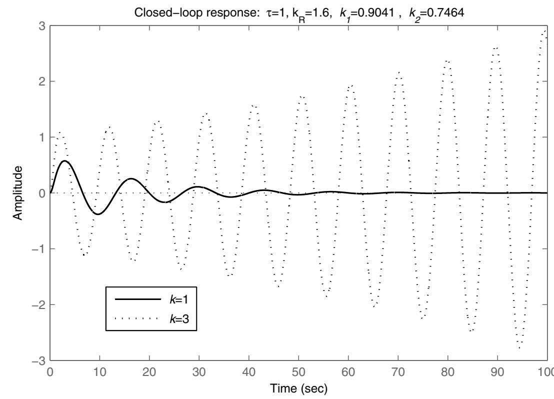

Fig. 4. Closed loop response of a recycle system to unit step change in load subject to uncertainities

and applying robust stability conditions (2),(3) to overall closed loop characteristic equation (16) we have the Non-Linear Programming (NLP) optimization problem stated as follows:
Find the controller parameters, k_1 and k_2 such that the objective function

$$J = (k_1 - k_1^0)^2 + (k_2 - k_2^0)^2 \quad (18)$$

is minimized subject to the following constraints.

$$\begin{gathered} -k_1^2 - 2.4k_1 + 6k_2 - k_1k_2 + 1.8 \le 0 \\ 3k_2 - 12.8 \le 0 \\ -k_1^2 - 2.4k_1 + 16.3340k_2 - k_1k_2 + 1.8 \le 0 \\ 1.6035k_1 + 3k_2 - 7.9895 \le 0 \\ -k_1 - k_2 + 0.6 \le 0 \\ -k_1 - 3 \le 0 \\ -3k_1 - 3k_2 + 3.8 \le 0 \end{gathered}$$

The above inequalities can be solved using software package MATLAB [25]. The table 1 shows the various values of the controller parameters for various values of ϵ used in NLP.

Controller set	ϵ	k_1	k_2
1	0.5	1.2992	0.1667
2	1	1.8133	0.3333
3	1.5	2.2337	0.5
4	1.8	2.4561	0.6
5	2.5	2.4242	0.6931

TABLE I
ROBUST PI CONTROLLERS FOR UNCERTAIN RECYCLE SYSTEM

It can be verified that each of the obtained PI controllers is able to stabilize the whole closed-loop recycle system subject to the process gain uncertainty in the bounded interval $k = [1, 3]$. We will use the controller set 3 for which we have $k_1 = 2.2337$ and $k_2 = 0.5$ to show

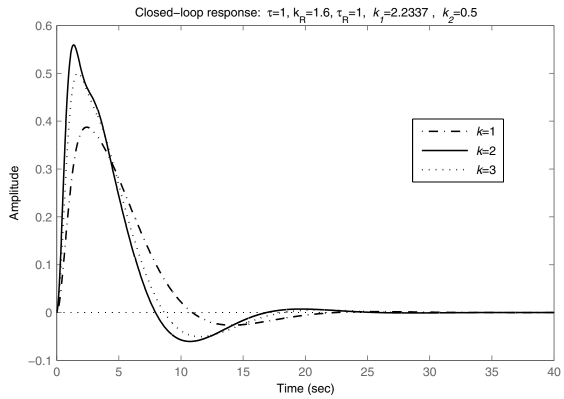

Fig. 5. Closed loop response of an uncertain recycle system to a step change inload by using the controller of set 3 in table 1.

the simulation result. The corresponding closed-loop step response, plotted in Fig. 5, clearly demonstrates that this PI controller achieves the requirement of robust stability. Finally, it should be noted that, for the same problem, the NLP problem formulated by Goh's method [2] has 14 decision variables while subjecting to 22 constraints (14 equality plus 8 inequality constraints). These constraints are overly stringent, which cause the proposed method to fail to satisfy the constraints and therefore there is no feasible solution for this recycle system. For Chen's approach [5], the formulated NLP problem consists of 6 decision variables and 12 constraints (4 equality and 8 inequality constraints). These constraints form a nonempty convex region, which thereby facilitates the solution of NLP. As far as our approach is concerned we require only 7 inequality constraints.

V. Conclusion

In this paper, a new approach is proposed for design of a robust controller for interval process systems in the presence of parametric uncertainties. The proposed method uses a necessary condition and sufficient condition stated in [1] for stability of interval polynomial. These conditions are used to derive a set of inequalities in terms of the compensator parameters which can be solved to obtain a robust controller. The proposed method is simple, involves less computational complexity and provides an easy method to obtain a robust compensator. The proposed method is applied to design a robust proportional-integral controller for a recycle system in the presence of process gain uncertainties. The results shows that for the entire range of process gain uncertainty k the designed ITAE PI controller preserves the robust stability.

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