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Robust Rotor Flux and Speed Control of Induction Motors Using On-Line Time-Varying Rotor Resistance Adaptation

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Abstract— **A novel scheme is proposed for direct speed and flux control of induction motors using on-line estimation of the rotor resistance. The stator current/voltage and the rotor mechanical angular speed are assumed to be measured. The design is based on the variable structure theories. The overall controller combined with on-line rotor resistance adaptation is implemented by using rotor flux and electromagnetic torque observers and the exponential convergence of these observers is demonstrated. The originality of the work is that it considers the unknown rotor resistance as time-varying. The convergence of the estimated rotor resistance is achieved using a Lyapunov-like technique and this guarantees the stability of the rotor speed and flux tracking. Simulation results show the quick convergence of both estimated rotor resistance and tracking of the rotor speed and flux. The robustness analysis revealed that the direct speed and flux tracking schemes are insensitive to the stator/rotor resistance variations (up to 50% for R_s and 100% for R_r) and to stator currents measurements noises. The proposed method can also be applied to failure and fault detection since it is easily implementable in real-time.**

Index Terms— **Time-varying parameter, parameter estimation/identification, sliding mode observer, equivalent control, adaptive control, induction motor.**

I. INTRODUCTION

A perfect knowledge of the induction motor (IM) rotor resistance is a key point for achieving its high performance control when flux measurements are not available. In fact, flux observers and both direct and indirect field oriented controls (DFOC and IFOC) rely on a good knowledge of the rotor resistance of the IM. Whatever method is used, an important problem is to estimate correctly the uncertain or time-varying parameter that affects the control performance. The rotor and stator resistances may vary up to 100% and 50%, respectively, due to rotor/stator heating and can be hardly recovered using thermal models temperature sensors. Consequently, any variation of the rotor/stator resistances should be tracked as it occurs. Although different approaches have been recently developed ([2], [3], [6], [7]) for IM parameters estimation, only partial and quite weak results have been obtained in term of robustness with respect to time-varying parameter. As a consequence, many contributions concerning IM controllers presented in the literature assumed that the electrical parameters are constant and are available or properly estimated.

In [2], an on-line method, based on the least square (LS) technique, is presented to identify the rotor resistance using the transient state under the speed sensorless control of IM. In [3], an adaptive observer using an ad hoc procedure is proposed to perform on-line estimation of both state and electrical parameters of IM, on the basis of the stator currents/voltages and rotor speed measurements when persistency of excitation condition is satisfied. The rotor magnetic flux is also recovered. In [6], a sophisticated method for on-line identification of the stator and rotor resistances is presented under the assumption that both parameters are constant during the estimation process. The proposed method provides on-line exponentially convergent estimates of both rotor and stator resistances, when persistency of excitation condition is satisfied and the stator currents integrals are bounded, on the basis of rotor speed and stator currents/voltages measurements. Rotor flux is also asymptotically recovered. In [7], a theoretically solution based on the model reference adaptive system (MRAS) approach is proposed for tuning both stator/rotor resistances of IM using stator currents/voltages measurements and their filtered parts. This method is based on a very simplified model of an IM. In [8], direct torque and flux regulation of an IM in sensorless control is proposed but IM electrical parameters are assumed to be known or properly estimated.

In this article, a novel on-line technique is proposed for IM rotor resistance estimation under rotor flux and speed control and the hypothesis of linear magnetic circuit and balanced operating conditions. Following the same concepts developed in [1] for nonlinear systems identification and control with time-varying parameter perturbation, the proposed method is derived using a new formulation of the parameter adaptation law. The proposed design is based on the variable structure theories assuming that the stator current/voltage and the rotor mechanical angular speed are available. The overall controller combined with on-line rotor resistance adaptation with the stator resistance variation is implemented by using rotor flux and electromagnetic torque observers. The exponential convergence of these observers is also demonstrated. The convergence of the estimated rotor resistance is achieved using a Lyapunov-like technique and this guarantees the stability of the rotor speed and flux tracking. The robustness analysis revealed that the direct speed

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and flux tracking schemes are insensitive to the stator/rotor resistance variations (up to 50% for R_s and 100% for R_r) and to stator currents measurements noises.

The paper is organized as follows. The IM model and the control statements are introduced in Section II. In Section III, the dynamics equations for the rotor speed/flux tracking are presented as well as the observers for the rotor flux and electromagnetic torque and the rotor resistance tuning procedure. In Section IV, simulation results are reported to show the performance of the proposed algorithm in different operating conditions. Finally, Section V is devoted to the conclusion of the paper.

II. MODEL DESCRIPTION AND CONTROL STATEMENTS

Assuming linear magnetic circuits and balanced three-phase windings, the classical dynamic model of an IM, expressed in the stator frame is the following ([5], [7]):

$$\frac{d\omega}{dt} = \mu i_s^T J\lambda_r - \frac{\alpha\omega}{m} - \frac{T_L}{m} \quad (1)$$

$$\frac{d\lambda_r}{dt} = (-\frac{R_r}{L_r}I + n_p\omega J)\lambda_r + \frac{R_r}{L_r}Mi_s \quad (2)$$

$$\frac{di_s}{dt} = -\frac{M}{\sigma L_s L_r}(-\frac{R_r}{L_r}I + n_p\omega J)\lambda_r - \frac{1}{\sigma L_s}(R_s + \frac{M^2R_r}{L_r^2})i_s + \frac{1}{\sigma L_s}v_s \quad (3)$$

where

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix},$$
$$i_s = \begin{pmatrix} i_{sd} \\ i_{sq} \end{pmatrix}, \quad v_s = \begin{pmatrix} v_{sd} \\ v_{sq} \end{pmatrix}, \quad \lambda_r = \begin{pmatrix} \lambda_{rd} \\ \lambda_{rq} \end{pmatrix}.$$

In (1), (2) and (3), ω denotes the angular speed of the rotor; λ_r, i_s and v_s are the rotor flux, the stator current and voltage, respectively. R_s, L_s, R_r and L_r are the stator resistance and inductance, rotor resistance and inductance, respectively; n_p is the number of pole pairs; $\sigma = 1 - \frac{M^2}{L_s L_r}$ is the leakage parameter; M is the mutual inductance; m is the total motor and load moment of inertia; $T_e = m\mu i_s^T J\lambda_r$ is the electromagnetic torque; T_L is the external load torque; α is the damping gain and $\mu = \frac{3}{2}\frac{n_p M}{m L_r}$.

The following assumptions will be considered until further notice:

Assumption 1. It is assumed that the stator current and voltage are continuous and bounded with time-derivatives bounded piecewise-continuous.

Assumption 2. It is also assumed that stator and rotor resistances as well as the rotor speed changes significantly slower relative to the rotor flux. Thus the rotor speed, the stator/rotor resistances can be considered as a constant parameter with respect to the rotor flux.

Assumption 3. It is also assumed that the rotor resistance $R_r \in \Omega_{R_r}$ which is a compact set of $\mathbb{R}$.

The first control goal is for the rotor speed ω to follow the reference value ω_{ref}

$$\lim_{t\to\infty}(\omega - \omega_{ref}) = 0. \quad (4)$$

To achieve this goal, the squared norm of the rotor flux is to be kept at a certain level.

Thus, the second control goal is

$$\lim_{t\to\infty}(||\lambda_r||^2 - F_{ref}) = 0, \quad (5)$$

where F_{ref} is the squared norm of the rotor flux reference value.

It is assumed that the reference values $\omega_{ref}, F_{ref} \in C^1$. Furthermore, the value of F_{ref} should be selected by taking into account constraint on the voltage and current signals ([8]). From the fact that the rotor flux is not measurable, in the control scheme, only its estimated value can be used. Thus the problem of the estimation of the rotor flux arises. This can be achieved by construction of an observer for the rotor flux. Since parameters of an IM may vary during its operation and their exact values are keys points for achieving high performance motion control, the problem of on-line estimation of the motor parameters also arises. In particular, it is of great interest that a high performance control scheme of an IM utilizes on-line estimation of the rotor resistance whose value may vary up to 100%.

This paper deals with the design of the rotor flux and speed controllers of an IM based on the construction of the electromagnetic torque and rotor flux observers and on the on-line estimation of the rotor resistance assuming that the remaining IM parameters are known or properly estimated.

III. ROTOR SPEED AND FLUX CONTROL

In this section the rotor speed and flux control schemes are designed. The new scheme is partially based on the dynamics equations for the electromagnetic torque and rotor flux regulation developed in [8].

A. Dynamics equations for the rotor speed and flux control

In order to derive the dynamic equation for the rotor speed control, let us consider the derivative of $\frac{d\omega}{dt}$

$$\frac{d^2\omega}{dt^2} = \frac{1}{m}\frac{dT_e}{dt} - \frac{\alpha}{m}\frac{d\omega}{dt} - \frac{1}{m}\frac{dT_L}{dt}. \quad (6)$$

The derivative of the electromagnetic torque T_e is given by

$$\frac{dT_e}{dt} = m\mu\big(i_s^T J\frac{d\lambda_r}{dt} - \lambda_r^T J\frac{di_s}{dt}\big). \quad (7)$$

Multiplying both sides of (2) and (3) by $i_s^T J$ and $\lambda_r^T J$, respectively, and then combining the results with (6) and (7), it follows that

$$\frac{d^2\omega}{dt^2} = -\frac{k}{m}T_e - n_p\omega\mu\big(\frac{M}{\sigma L_s L_r}||\lambda_r||^2 + \lambda_r^T i_s\big) + \frac{m\mu}{\sigma L_s}\lambda_r^T Jv_s - \frac{\alpha}{m}\frac{d\omega}{dt} - \frac{1}{m}\frac{dT_L}{dt}, \quad (8)$$

$$\text{with} \quad k = \frac{R_s}{\sigma L_s} + \frac{R_r}{\sigma L_r}.$$

By taking into account the property of the matrix operator J[1] and multiplying both sides of (2) by λ_r^T, the following

[1] $\lambda_r^T J\lambda_r = 0$

expression for the derivative of $||\lambda_r||^2$ is obtained:

$$\frac{d(||\lambda_r||^2)}{dt} = -2\frac{R_r}{L_r}||\lambda_r||^2 + 2\frac{R_r}{L_r}Mi_s^T\lambda_r. \quad (9)$$

The control scheme for the rotor flux can not be derived from (9) since it does not contain the voltage as an input. To make the voltage appear explicitly as an input in (9), the following transformations are used. First, computing the dynamic equation for the fictitious variable $i_s^T\lambda_r$ by multiplying both sides of (2) and (3) by i_s^T and λ_r^T, respectively, and summing them up, yields

$$\frac{d(i_s^T\lambda_r)}{dt} = -ki_s^T\lambda_r + \frac{R_rM}{L_r}||i_s||^2 + \frac{M}{\sigma L_sL_r}\frac{R_r}{L_r}||\lambda_r||^2 + + \frac{n_p\omega}{m\mu}T_e + \frac{1}{\sigma L_s}\lambda_r^Tv_s. \quad (10)$$

Secondly, differentiating (9) and using (10), the dynamic equation for the rotor flux containing the control input v_s is obtained as follows

$$\frac{d^2(||\lambda_r||^2)}{dt^2} = -2\frac{R_r}{L_r}\Big[-\frac{d(||\lambda_r||^2)}{dt} + M\Big(-ki_s^T\lambda_r + + \frac{R_rM}{L_r}||i_s^2|| + \frac{M}{\sigma L_sL_r}\frac{R_r}{L_r}||\lambda_r||^2 + + \frac{n_p\omega}{m\mu}T_e + \frac{1}{\sigma L_s}\lambda_r^Tv_s\Big)\Big]. \quad (11)$$

B. Design of the rotor speed and flux controllers

To design the rotor speed and flux controllers we now consider the following decomposition of the stator voltage:

$$v_s = \frac{\lambda_r}{||\lambda_r||}v_{sd} + \frac{J\lambda_r}{||\lambda_r||}v_{sq}. \quad (12)$$

Let the rotor speed and flux tracking errors be $e_\omega = \omega - \omega_{ref}$ and $e_{\lambda_r} = ||\lambda_r||^2 - F_{ref}$, respectively. By taking into consideration decomposition (12), the dynamics equations for the rotor speed and flux (8) and (11) become

$$\frac{d^2\omega}{dt^2} = -\frac{k}{m}T_e - n_p\omega\mu\big(\frac{M}{\sigma L_sL_r}||\lambda_r||^2 + \lambda_r^Ti_s\big) + \frac{m\mu}{\sigma L_s}||\lambda_r||v_{sq} - \frac{\alpha}{m}\frac{d\omega}{dt} - \frac{1}{m}\frac{dT_L}{dt}, \quad (13)$$

$$\frac{d^2(||\lambda_r||^2)}{dt^2} = -2\frac{R_r}{L_r}\big[-\frac{d(||\lambda_r||^2)}{dt} + M\big(-ki_s^T\lambda_r + \frac{R_rM}{L_r}||i_s^2|| + \frac{M}{\sigma L_sL_r}\frac{R_r}{L_r}||\lambda_r||^2 + \frac{n_p\omega}{m\mu}T_e + \frac{1}{\sigma L_s}||\lambda_r||v_{sd}\big)\big]. \quad (14)$$

To achieve both control goal (4) and (5), let us consider the following variable structure control ([9]):

$$v_{sq} = \hat{v}_{sq} - k_{v_{sq}}sign(s_\omega) \quad (15)$$
$$v_{sd} = \hat{v}_{sd} - k_{v_{sd}}sign(s_{\lambda_r}) \quad (16)$$

where $\hat{v}_{sq}$, $\hat{v}_{sd}$ are the unique solutions of $\dot{s}_\omega = 0$ and $\dot{s}_{\lambda_r} = 0$, respectively, and the sliding manifolds s_ω and s_{λ_r} are given by

$$s_\omega = (\frac{d}{dt} + c_\omega)e_\omega$$
$$s_{\lambda_r} = (\frac{d}{dt} + c_{\lambda_r})e_{\lambda_r}. \quad (17)$$

Following the above considerations, the expressions for $\hat{v}_{sq}$ and $\hat{v}_{sd}$ are derived

$$\hat{v}_{sd} = \frac{k_7}{||\lambda_r||}\Big[(c_{\lambda_r} - \frac{2R_r}{L_r})(||\lambda_r||^2 - Mi_s^T\lambda_r) + k_2i_s^T\lambda_r + R_r(k_3i_s^T\lambda_r - k_4||i_s||^2 - k_5||\lambda_r||^2) - \omega k_6T_e\Big] \quad (18)$$

$$\hat{v}_{sq} = \frac{k_0}{||\lambda_r||}\Big[(\frac{\alpha}{m} - c_\omega)(T_e - \alpha\omega - T_L) + m(\frac{d^2\omega_{ref}}{dt^2} + c_\omega\frac{d\omega_{ref}}{dt}) + \frac{1}{M}(k_2 + R_rk_3)T_e + \frac{dT_L}{dt} + n_p\omega m\mu(k_1||\lambda_r||^2 + \lambda_r^Ti_s)\Big], \quad (19)$$

where $k_i, i = 0\cdots7$ are the IM parameters expressed as

$$k_0 = \frac{\sigma L_s}{m\mu} \quad k_1 = \frac{M}{\sigma L_sL_r} \quad k_2 = \frac{R_sM}{\sigma L_s}$$
$$k_3 = \frac{M}{\sigma L_r} \quad k_4 = \frac{M^2}{L_r} \quad k_5 = \frac{M}{L_r}k_1$$
$$k_6 = \frac{n_pM}{m\mu} \quad \text{and} \quad k_7 = \frac{R_s}{k_2}.$$

Remark 4. The presence of the additive terms with the derivative of the reference rotor speed and the derivative of the load torque guarantees that the rotor speed converges to the reference value for non-constant ω and for non-constant load torque. In this work the magnitude of the rotor flux is assumed to be constant. In the case of non-constant magnitude of the rotor flux, the control input (18) will contain the term with the derivative of its reference value and the control scheme will also work.

Remark 5. In the above controllers (18) and (19) the rotor flux magnitude $||\lambda_r||$ and the electromagnetic torque T_e are not available for measurements. Moreover the rotor resistance is assumed to be time-varying. Consequently, observers for $||\lambda_r||$, T_e and an on-line adaptation law for R_r are required to achieve the convergence of the above controllers.

By taking into account Remark 5, (13) and (14) can be rewritten as

$$\ddot{\omega} = f_\omega + b_\omega v_{sq} \quad (20)$$
$$||\ddot{\lambda_r}|| = f_{\lambda_r} + b_{\lambda_r}v_{sd} \quad (21)$$

where $f_\omega = -\frac{k}{m}T_e - n_p\omega\mu\big(\frac{M}{\sigma L_sL_r}||\lambda_r||^2 + \lambda_r^Ti_s\big) - \frac{\alpha}{m}\frac{d\omega}{dt} - \frac{1}{m}\frac{dT_L}{dt}$, $b_\omega = \frac{m\mu}{\sigma L_s}||\lambda_r||$,

$$f_{\lambda_r} = -2\frac{R_r}{L_r}\big[-\frac{d(||\lambda_r||^2)}{dt} + M\big(-ki_s^T\lambda_r + \frac{R_rM}{L_r}||i_s^2|| + \frac{M}{\sigma L_sL_r}\frac{R_r}{L_r}||\lambda_r||^2 + \frac{n_p\omega}{m\mu}T_e\big)\big]$$

and $b_{\lambda_r} = \frac{1}{\sigma L_s}||\lambda_r||$.

In (20) and (21), the dynamics f_ω and f_{λ_r} are time-varying and are not exactly known, but estimated as $\hat{f}_\omega$ and $\hat{f}_{\lambda_r}$, respectively. The estimation errors on f_ω and f_{λ_r} are assumed to be bounded by some known functions F_ω and F_{λ_r}:

$$|\hat{f}_j - f_j| \leq F_j, \quad j = \omega, \lambda_r. \tag{22}$$

Also note that the control gains b_ω and b_{λ_r} are unknown but of known bounds:

$$0 < b_{jmin} \leq b_j \leq b_{jmax}, \quad j = \omega, \lambda_r. \tag{23}$$

In (15) and (16), $\hat{v}_{sq}$, $\hat{v}_{sd}$ are actually the best approximation of the continuous control law that would achieve $\dot{s}_\omega = 0$ and $\dot{s}_{\lambda_r} = 0$, respectively, since we now have $\hat{v}_{sq} = \hat{v}_{sq}(\hat{f}_\omega, \hat{b}_\omega)$ and $\hat{v}_{sd} = \hat{v}_{sd}(\hat{f}_{\lambda_r}, \hat{b}_{\lambda_r})$. In order to satisfy the sliding condition

$$\frac{1}{2}\frac{d}{dt}s^2 = -\eta|s| \tag{24}$$

despite uncertainties on f_j and b_j, $j = \omega, \lambda_r$, one can easily show that the control discontinuity $k_{v_{sq}}$ and $k_{v_{sd}}$ in (15) and (16) must verify:

$$k_{sq} \geq \frac{F_\omega + \eta_\omega}{b_{\omega_{min}}} + \left|1 - \frac{b_{\omega_{max}}}{b_{\omega_{min}}}\right| |\hat{v}_{sq}| \tag{25}$$

$$k_{sd} \geq \frac{F_{\lambda_r} + \eta_{\lambda_r}}{b_{\lambda_{r_{min}}}} + \left|1 - \frac{b_{\lambda_{r_{max}}}}{b_{\lambda_{r_{min}}}}\right| |\hat{v}_{sd}|. \tag{26}$$

Note that η is a strictly positive arbitrary constant which formally reflects the time to reach the sliding surface.

C. Observers construction and rotor resistance estimation algorithm

To complete the design of the above controllers, an adaptation law for the rotor resistance has to be selected and observers for the rotor flux and electromagnetic torque should be constructed. The choice of the rotor resistance estimation rather than the stator resistance adaptation is motivated by the fact that the rotor resistance always varies more than the stator resistance. Therefore, it is more interesting to combine the controllers of the rotor speed and flux to the on-line adaptation of the rotor resistance and to verify the robustness of these controllers with respect to the variations of the stator resistance. However, the method proposed in this section can also be applied to estimate both rotor and stator resistances at the cost of more complicated transformations and calculations.

1) Rotor resistance estimation algorithm: In order to derive the rotor resistance estimation dynamic, the rotor flux is eliminated in (3) by taking into consideration that Assumption 2 is satisfied. Using this assumption and differentiating (3) combined with (2), the following expression is obtained (for more details see [4]):

$$\frac{d^2 i_s}{dt^2} = f_0 + R_r f_1 \tag{27}$$

where f_0 and f_1 are given as follows

$$f_0 = \gamma_1\Big(\frac{dv_s}{dt} - R_s\frac{di_s}{dt}\Big) + \omega J\Big(\beta_2 i_s + \beta_3 v_s + \beta_1\frac{di_s}{dt}\Big)$$
$$f_1 = \gamma_2 v_s - \gamma_3 i_s - \gamma_4\frac{di_s}{dt}$$

and β_1, β_2, β_3, γ_1, γ_2, γ_3, γ_4 are IM parameters expressed as

$$\beta_1 = n_p \quad \beta_2 = \frac{n_p R_s}{\sigma L_s} \quad \beta_3 = \frac{-n_p}{\sigma L_s}$$
$$\gamma_1 = \frac{1}{\sigma L_s} \quad \gamma_2 = \frac{1}{\sigma L_s L_r} \quad \gamma_3 = \frac{R_s}{\sigma L_s L_r} \text{ and } \gamma_4 = \frac{1}{\sigma L_r}.$$

The rotor resistance is assumed to be time-varying as reported in Remark 5 and can therefore be expressed as

$$R_r = R_{rn} + \Delta R_r \quad \text{with} \quad |\dot{R}_r| = |\Delta \dot{R}_r| \leq \mu \tag{28}$$

where μ is a known positive number.

Note that f_0 and f_1 are expressed as functions of the derivatives of the stator current and voltage which can not be obtained directly using numerical differentiation due to the presence of noise. To estimate these stator current and voltage derivatives let us consider the following high-gain observer (HGO):

$$\dot{\hat{x}}_i = -\Gamma_i(\hat{x}_i - x_i) = -\Gamma_i e_i, i = 1d, 1q, \tag{29}$$
$$\dot{\hat{x}}_l = -\Gamma_l(\hat{x}_l - x_l) = -\Gamma_l e_l, l = 2d, 2q, \tag{30}$$

where $e_i = \hat{x}_i - x_i$, $e_l = \hat{x}_l - x_l$ are the observer errors; Γ_i and Γ_l are the gains of the HGO, $x_1 = i_s$ and $x_2 = v_s$. Therefore, (27) can be rewritten as follows

$$\frac{d\tilde{x}_1}{dt} = \tilde{f}_0 + R_r \tilde{f}_1 \tag{31}$$

where $\tilde{f}_0$ and $\tilde{f}_1$ are the estimates of f_0 and f_1, respectively, computed by replacing the stator current and voltage derivatives by their estimates $\tilde{x}_1 = \dot{\hat{x}}_1$ and $\tilde{x}_2 = \dot{\hat{x}}_2$ derived from HGO (29) and (30).

Assuming that (31) is the reference model for the rotor resistance identification, let us consider the following tuning model:

$$\frac{d\hat{\tilde{x}}_1}{dt} = \tilde{f}_0 + \hat{R}_r \tilde{f}_1 + u_{\tilde{x}_1} \tag{32}$$

where $\hat{\tilde{x}}_1$, $\hat{R}_r$ are the adaptive observer of $\tilde{x}_1$ and estimate of R_r, respectively; $u_{\tilde{x}_1} = -K_{\tilde{x}_1} Sign(\hat{\tilde{x}}_1 - \tilde{x}_1)$ is the input of the variable structure observer (32).

The function $\text{Sign}(.) : \mathbb{R}^2 \to \mathbb{R}^2$ is defined as

$$\text{Sign}(x^T) = \big(\text{sign}(x_1)\ \text{sign}(x_2)\big)$$

$$\text{with} \quad x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad \text{and} \quad \text{sign}(x_i) = \begin{cases} 1 & \text{if } x_i > 0 \\ [-1,1] & \text{if } x_i = 0 \\ -1 & \text{if } x_i < 0 \end{cases}$$

$K_{\tilde{x}_1}$ is a square 2×2 diagonal matrix with positives elements. Let $diag(A)$, be the column vector whose elements are the diagonal elements of a given square diagonal matrix A. The following notation is also used:

$$|x^T|_G = \big(|x_1|\ |x_2|\big).$$

By defining $e_{\tilde{x}_1} = \hat{\tilde{x}}_1 - \tilde{x}_1$, the vector of the observer error and $e_{R_r} = \hat{R}_r - R_r$, the parameter estimation error and taking into consideration (31) and (32), it follows that the dynamic equation of the observer error is given by

$$\dot{e}_{\tilde{x}_1}=\tilde{f}_1 e_{R_r} + u_{\tilde{x}_1}. \tag{33}$$

By taking into account Assumption 1, (29) and (30), it follows that $\tilde{f}_1$ is bounded. Moreover since R_r is bounded and assuming that $\hat{R}_r \in \Omega_{R_r}$ (Assumption 3), if the elements of the matrix gain $diag(K_{\tilde{x}_1})$ are chosen sufficiently large

$$diag(K_{\tilde{x}_1})_i > (|\tilde{f}_1 e_{R_r}|_G)_{imax}, i = d, q \tag{34}$$

a sliding mode regime occurs on the surface $\dot{e}_{x_1} = e_{x_1} = 0$ ([9], [11]). Therefore, (33) can be rewritten as

$$u_{\tilde{x}_{1eq}}=-\tilde{f}_1 e_{R_r} \tag{35}$$

where $u_{\tilde{x}_{1eq}}$ is the equivalent control.

Remark 6. From a practical point of view, it is not possible to implement $u_{\tilde{x}_{1eq}}$ because the time-varying parameter $R_r(t)$ is assumed to be not available. To overcome this problem we used the average control vector as approximation of the equivalent control $u_{\tilde{x}_{1eq}}$ ([10]).

If the IM operates in its stable region, one has $||\tilde{f}_1||^2 > 0$. Therefore by taking into consideration the above remark and assuming that the IM operates in its stable region, the expression of the parameter estimation error can be derived as follows

$$e_{R_r}=-\frac{\tilde{f}_1^T u_{\tilde{x}_{1eq}}}{||\tilde{f}_1||^2}. \tag{36}$$

We can then consider the following adaptive parameter identifier

$$\dot{\hat{R}}_r=-k_{R_r}\text{sign}(-\frac{\tilde{f}_1^T u_{\tilde{x}_{1eq}}}{||\tilde{f}_1||^2}) = -k_{R_r}\text{sign}(e_{R_r}) \tag{37}$$

where k_{R_r} is a suitable chosen positive number. Now the dynamic of the parameter estimation error can be deduced as

$$\dot{e}_{R_r}=-k_{R_r}\text{sign}(-\frac{\tilde{f}_1^T u_{\tilde{x}_{1eq}}}{||\tilde{f}_1||^2}) - \dot{R}_r. \tag{38}$$

The main result of this part can be summarized as follows.

Theorem 7. *Consider the system described by (27), the model reference (31) and the variable structure identifier (32) assuming that the mechanical angular speed measurement ω is available. Under the assumptions 1 through 2 and 3 and assuming that conditions (28) and (34) hold, then the estimate of the rotor resistance $\hat{R}_r$ given by (37) converges to the true value R_r in finite time.*

The proof of the above theorem is given in Appendix I.

2) Observers construction: Assuming that $\hat{R}_r$ converges to R_r in finite time as reported in Theorem 7, let us now consider the following observers for T_e and λ_r:

$$\frac{d\hat{\lambda}_r}{dt}=(-\frac{\hat{R}_r}{L_r}I + n_p\omega J)\hat{\lambda}_r + \frac{\hat{R}_r}{L_r}Mi_s \tag{39}$$

$$\hat{T}_e=m\mu i_s^T J\hat{\lambda}_r \tag{40}$$

Let the rotor flux and the electromagnetic torque observer errors be $e_{\hat{\lambda}_r} = \hat{\lambda}_r - \lambda_r$ and $e_{T_e} = \hat{T}_e - T_e$, respectively. By taking into account (1) and (2) and from the fact that $T_e = m\mu i_s^T J\lambda_r$ and $\hat{R}_r \rightarrow R_r$ in finite time, the observer errors dynamics equations may be computed as follows

$$\dot{e}_{\hat{\lambda}_r}=-(\frac{\hat{R}_r}{L_r}I - n_p\omega J)e_{\hat{\lambda}_r} \tag{41}$$

$$e_{T_e}=m\mu i_s^T J e_{\hat{\lambda}_r}. \tag{42}$$

Moreover, under the assumption that the IM operates in its stable region, one has $(\frac{\hat{R}_r}{L_r}I - n_p\omega J) > 0$. Therefore, the observer errors $e_{\hat{\lambda}_r}$ and e_{T_e} converge exponentially to 0 which immediately implies that $\hat{\lambda}_r$ and $\hat{T}_e$ converge exponentially to λ_r and T_e, respectively.

Finally, the overall controller combined with the above observers and rotor resistance adaptation law has the form:

$$\dot{\hat{R}}_r=-k_{R_r}\text{sign}(-\frac{\tilde{f}_1^T u_{\tilde{x}_{1eq}}}{||\tilde{f}_1||^2}) \tag{43}$$

$$\frac{d\hat{\lambda}_r}{dt}=(-\frac{\hat{R}_r}{L_r}I + n_p\omega J)\hat{\lambda}_r + \frac{\hat{R}_r}{L_r}Mi_s \tag{44}$$

$$\hat{T}_e=m\mu i_s^T J\hat{\lambda}_r \tag{45}$$

$$v_s=\frac{\hat{\lambda}_r}{||\hat{\lambda}_r||}v_{sd} + \frac{J\hat{\lambda}_r}{||\hat{\lambda}_r||}v_{sq} \tag{46}$$

$$\begin{aligned}\hat{v}_{sd}=&\frac{k_7}{||\hat{\lambda}_r||}\Big[(c_{\lambda_r} - \frac{2\hat{R}_r}{L_r})(||\hat{\lambda}_r||^2 - Mi_s^T\hat{\lambda}_r) + k_2 i_s^T\hat{\lambda}_r \\ &+\hat{R}_r(k_3 i_s^T\hat{\lambda}_r - k_4||i_s||^2 - k_5||\hat{\lambda}_r||^2) - \omega k_6\hat{T}_e\Big]\end{aligned} \tag{47}$$

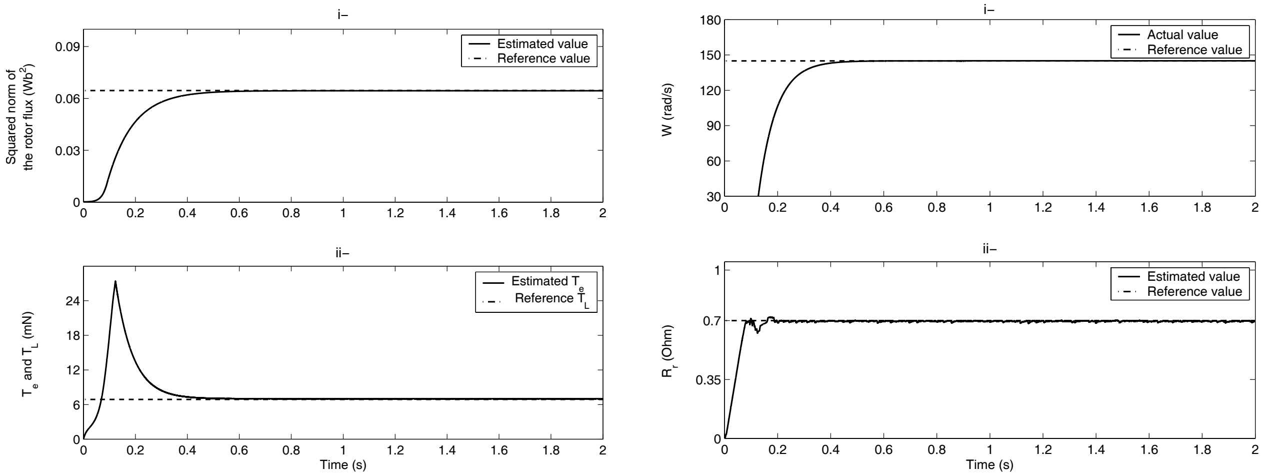
$$\begin{aligned}\hat{v}_{sq}=&\frac{k_0}{||\hat{\lambda}_r||}\Big[(\frac{\alpha}{m} - c_\omega)(\hat{T}_e - \alpha\omega - T_L) \\ &+m(\frac{d^2\omega_{ref}}{dt^2} + c_\omega\frac{d\omega_{ref}}{dt}) + \frac{1}{M}(k_2 + \hat{R}_r k_3)\hat{T}_e \\ &+\frac{dT_L}{dt} + n_p\omega m\mu(k_1||\hat{\lambda}_r||^2 + \hat{\lambda}_r^T i_s)\Big]\end{aligned} \tag{48}$$

where v_{sq} and v_{sd} are computed from (15) and (16) respectively.

IV. Simulation results

The effectiveness of the proposed controllers combined with the rotor resistance estimation has been verified by computer simulations in Matlab/Simulink environments software in different operating conditions. During simulation, the sampling time has been set to $106\mu s$. The availability of cheap computation now allow high sampling rates and thus real-time implementation of the proposed algorithm can be achieved using the currently available commercial DSPs. Induction motor data are given in Appendix II.

Parameters of the controllers are given in Table I. The gain of the high-gain observer is 31000 and the parameters of the rotor resistance identifier are given as follows.
$K_{\tilde{x}_{1d}} = K_{\tilde{x}_{1q}} = 150000$ and $k_{R_r} = 9.5$. The equivalent control $u_{\tilde{x}_{1eq}}$ has been approximated as in section III-C (see *Remark 6*) using first order low pass-pass filter with time-constant of $1ms$. Actual simulation results are performed using a constant squared norm of the rotor flux reference



(a) (b)

Fig. 1. Simulation results with $\hat{R}_r(0) = 0$ when the rotor speed and resistance references values and the load torque are assumed to be constant. (a): rotor flux and electromagnetic estimation results. i- F_{ref}: dash-dot line and $||\hat{\lambda}_r||^2$: solid line. ii- $\hat{T}_e$: solid line and T_L: dash-dot line. (b): rotor speed regulation and rotor resistance estimation results. i- ω_{ref}: dash-dot line and ω: solid line. ii- $\hat{R}_r$: solid line and R_{r-ref}: dash-dot line.

TABLE I
PARAMETERS OF THE CONTROLLERS

$k_{v_{sd}}$	c_{λ_r}	$k_{v_{sq}}$	c_ω
5	10	50	15

equal to $F_{ref} = 0.0645 Wb^2$. Results relative to constant reference of the mechanical rotor speed, rotor resistance and constant load torque equal to $\omega_{ref} = 145\ rad/s$, $R_{r-ref} = 0.7\Omega$ and $T_L = 6.89\ mN$, respectively, are reported in Fig. 1. The small transient phases of the estimates reported in this figure confirm the quick convergence of the method proposed.

In order to implement the algorithm in real time, the stator current has been contaminated with the white noise with the rotor resistance reference and load torque assumed to vary sinusoidally. The magnitude of the white noise reaches about 8% of the maximum value of the stator current. Fig. 2 shows the results of the estimates relative to a sinusoidal variation of 4% of the rotor speed reference, 100% of the rotor resistance reference, 50% of the stator resistance and 50% of the load torque. Simulation results, reported in this figure, show that the proposed method is capable to achieve rotor speed/flux tracking when the stator/rotor resistances change, e.g. due to the rotor heating, and under noisy condition.

Assumption 2 does not seem to be verified both in Fig. 1 and Fig. 2. This assumption is limited to steady states.

V. CONCLUSIONS AND FUTURE WORKS

A novel scheme for direct speed and flux control of induction motors using on-line estimation of the rotor resistance is presented. It requires the stator current/voltage and the mechanical angular speed measurements. The overall controller combined with on-line rotor resistance adaptation has been implemented by using rotor flux and electromagnetic torque observers and the exponential convergence of these observers has been demonstrated. The design is based on the sliding mode observer and the quick convergence of both the estimated rotor resistance and rotor speed/flux tracking to their true values is demonstrated via simulation results. The originality of this work is that it considers unknown rotor resistance as time-varying. This feature distinguishes the proposed scheme from the known ones. The robustness analysis revealed that the direct speed and flux tracking schemes are insensitive to the rotor/stator resistance variations (up to 100% for R_r and 50% for R_s) and to stator currents measurements noises. This demonstrated the robustness of the control that could result in better efficiency of an electric drive as compared to the commonly used IFO scheme. The other interesting feature of the work is that it is easily implementable in real-time. Therefore the proposed algorithm can be applied for diagnostics and fault detection.

It would be meaningful in the future works to implement in real-time the proposed algorithm in order to verify its robustness with respect to the discretization effects, parameter uncertainties (e.g. inaccuracies on motor inductance values) and modeling inaccuracies.

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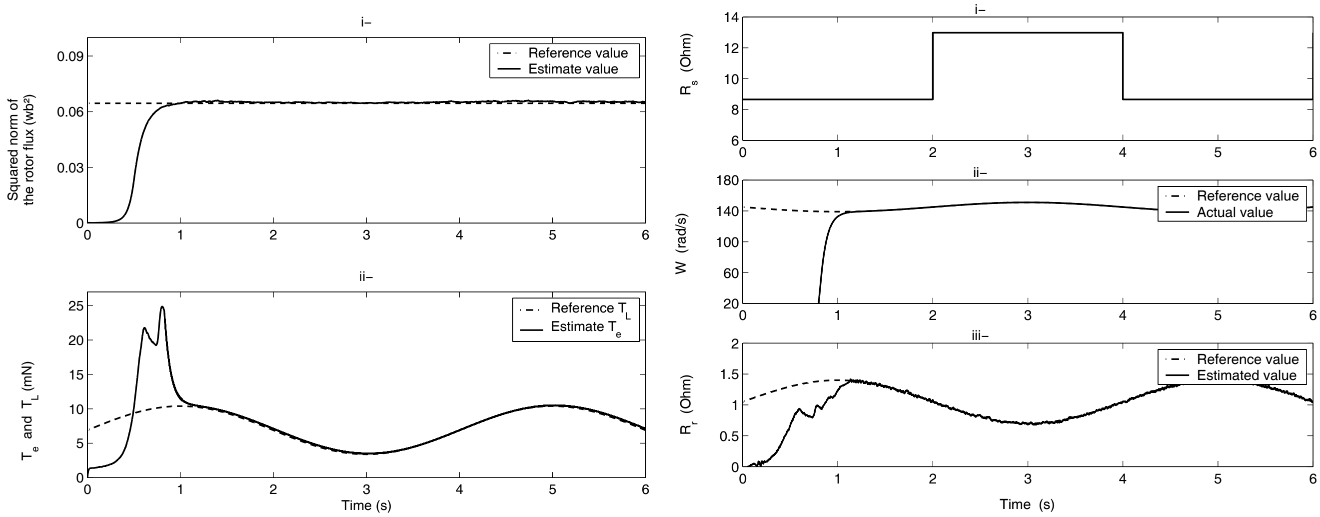
APPENDIX I
PROOF OF THEOREM 7.

Let us rewrite the dynamic of the rotor resistance estimation error (see (38) of Section III-C)

$$\begin{aligned}\dot{e}_{R_r} &= -k_{R_r}\text{sign}\left(-\frac{\tilde{f}_1^T u_{\tilde{x}_{1eq}}}{||\tilde{f}_1||^2}\right) - \dot{R}_r \\ &= -k_{R_r}\text{sign}(e_{R_r}) - \dot{R}_r. \qquad \text{(I.49)}\end{aligned}$$



(a) (b)

Fig. 2. Simulation results with stator and rotor resistance and rotor speed variation equal to $\Delta R_s = 50\%$ and $\Delta R_{r-ref} = 100\%$ and $\Delta\omega_{ref} = 4\%$, respectively, under noisy condition. The stator current is contaminated with white noise whose magnitude reached about 8% of the stator current maximum value. (a): rotor flux and electromagnetic estimation results. i- F_{ref}: dashed line and $||\hat{\lambda}_r||^2$: solid line. ii- $\hat{T}_e$: solid line and T_L: dash-dot line with sinusoidal variation. (b): rotor speed tracking and rotor resistance estimation results. i- stator resistance R_s with squared variation occurred at time $t = 2s$. ii- ω_{ref} with sinusoidal variation: dash-dot line and ω: solid line. iii- $\hat{R}_r$: solid line and R_{r-ref} (dash-dot line) with sinusoidal variation.

Using the following Lyapunov candidate function

$$V = \frac{1}{2} e_{R_r}^2 \qquad \text{(I.50)}$$

and computing its time-derivative along the solution of (I.49), we obtain

$$\dot{V} = e_{R_r}\dot{e}_{R_r} = e_{R_r}(\dot{\hat{R}}_r - \dot{R}_r). \qquad \text{(I.51)}$$

Substituting (32) in (I.51), it follows that

$$\dot{V} = e_{R_r}(-k_{R_r}\text{sign}(e_{R_r}) - \dot{R}_r) = -|e_{R_r}|k_{R_r} - e_{R_r}\dot{R}_r.$$

From the fact that $|\dot{R}_r| \leq \mu_{R_r}$, we can deduce the following inequality

$$\dot{V} \leq -|e_{R_r}|(k_{R_r} - \mu_{R_r}).$$

If the gain of the adaptive parameter identifier (32) is chosen such that $k_{R_r} > \mu_{R_r}$, then the rotor resistance estimation error e_{R_r} converges to 0 and the estimated parameter $\hat{R}_r$ converges to the true value R_r in finite time.

APPENDIX II
INDUCTION MOTOR DATA

Rated power	1000 W.
Rated speed	1385 rpm.
Rated torque	6.89 mN.
Rated frequency	50 Hz.
Excitation current	6.4 A.
Rated current	2.5 A.
Stator resistance	$R_{sn} = 8.65\ \Omega$.
Rotor resistance	$R_{rn} = 0.7\ \Omega$.
Stator inductance	$L_{sn} = 0.828\ H$.
Rotor inductance	$L_{rn} = 0.082\ H$.
Mutual inductance	$M_n = 0.246\ H$.
Number of pole pairs	$n_p = 2$.
Motor-load inertia	$m = 0.0112\ kg\ m^2$

REFERENCES

[1] T. Ahmed-Ali, F. Floret and F. Lamnabhi-Lagarrigue, Robust identification and control with time-varying parameter perturbation, *Journal of Mathematical and Computer Modelling of Dynamical Systems*, vol. 10, Number 3-4, 2004, pp. 201–216.

[2] K. Akatsu and A. Kawamura, On-line rotor resistance estimation using the transient state under the speed sensorless control of induction motor, *IEEE Transaction On Power Electronics*, vol. 15, N0. 3, 2000, pp. 553–560.

[3] P. Castaldi, W. Geri, M. Montanari and A. Tilli, A new adaptive approach for on-line parameter and state estimation of induction motors, *Control Engineering Practice*, vol. 13, N0. 1, 2005, pp. 81–94.

[4] G. Kenné, "Méthodes d'identification pour des systèmes non linéaires avec paramètres variant dans le temps: Application aux machines tournantes à induction", *Thèse de Doctorat de l'Université Paris XI Orsay, L2S-SUPELEC–C.N.R.S*, France, Nov. 2003.

[5] W. Leonhard, Control of electric drives, Springer Verlag, 1984.

[6] R. Marino, S. Peresada and P. Tomei, On-line stator and rotor resistance estimation for induction motors, *IEEE Transaction On Control Systems Technology*, vol. 8, N0. 3, 2000, pp. 570–579.

[7] A.V. Pavlov and A.T. Zaremba, "Real-time rotor and stator resistances estimation of an induction motor", *Proceedings of NOLCOS-01*, St-Petersbourg, 2001.

[8] A.V. Pavlov and A.T. Zaremba, "Direct torque and flux regulation in sensorless control of an induction motor", *Proceedings of the American Control Conference*, Arlington, VA June 25-27, 2001, pp. 137–142.

[9] J.J.E. Slotine and W. Li, Applied Nonlinear Control, Prentice-Hall, International Editions, Englewood Cliffs, 1991.

[10] V.I. Utkin, Sliding modes in optimization and control, Springer-Verlag, 1992.

[11] V.I. Utkin, Sliding mode control design principles and applications to electric drives, *IEEE Transactions On Industrial Electronics.*, vol. 40, 1993, pp. 26–36.