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Irrigation Multireaches Regulation Problem by Internal Model Boundary Control

V. DOS SANTOS and Y. TOURÉ

Abstract— **This paper deals with the regulation problem of irrigation canals and the multireaches case is considered. The control problem is stated as a boundary control of hyperbolic Saint-Venant Partial Differential Equations (pde).**

Regulation is done around an equilibrium state and the operator parameters spatial dependency is taken into account in the linearized model. The Internal Model Boundary Control (IMBC) used in a direct approach allows to make a control parameters synthesis by semigroup conservation properties, like the exponential stability. In this paper sufficient conditions are given more explicitely by the resolvent calculation using perturbations theory, in infinite dimensional Hilbert space, so the results can be used for more general hyperbolic systems.

Simulation and experimental results from Valence experimental micro-channel show that this approach shoud be suitable for more realistic situations.

Index Terms— **Shallow water equations, irrigation canal, infinite dimensional perturbation theory, multivariable internal model boundary control, hyperbolic systems.**

I. INTRODUCTION

Open surface hydraulic systems were studied by different approaches [8], [10] in modelling or control for mono and multireaches. The usual model is the Saint Venant equations with regard to the control. In this area, two approaches are currently used: indirect approach in finite dimensional (the pde's are approximated) and the direct one in infinite dimension (methods and tools directly relate to pde's).

This paper is located in the second approach, using directly partial differential equations for control synthesis [11], [12], [13], [14], [15].

The internal model boundary control is proposed for control synthesis for multireaches regulation. Internal model boundary control was introduced in [14] for dissipative parabolic and exponentially stable systems. It is extended here to the hyperbolic case. The spatial dependency of variables is taken into account. Conservation properties of semigroup stability give the control synthesis, using some previous perturbations theory results [9], [11], [12], [14].

In the first section, the non linear model for a rectangular canal is given in order to define a linear regulation model around an equilibrium state. Regulation problem is then defined for a canal with reaches in cascade, where hydraulics constructions are gates or overflows. Then, the control synthesis is studied.

V. DOS SANTOS is with the Laboratory of Mathematics and Applications, Mathematical Physic of Orléans, MAPMO, University of Orléans, France valerie.dos_santos@univ-orleans.fr

Y. TOURÉ is with the Laboratory of Vision and Robotic, LVR Bourges, University of Orléans, France youssoufi.toure@bourges.univ-orleans.fr

In the third section, the boundary control model is well posed to set up the essential properties of the open loop system to be conserved, due to the structural perturbation of the closed loop. In the fourth part, associated to a particular form of the well know internal model control structure (see [6], [15]), the extended system is represented by a closed operator, generator of an exponentially stable C_0-semigroup. The internal control law is taken as a multivariable integral controller or a proportional integral one. Then, synthesis parameters are obtained by a direct application of some previous results (see [9], [11], [14]), but in this paper the relation of the sufficient stability condition is explicitely given using resolvent operator.

In the last part, simulations and experimental results are given in multireaches case for water levels track, around equilibrium states.

II. THE CANAL REGULATION PROBLEM: A BOUNDARY CONTROL SYSTEM

A. Non Linear Multireaches Model

The considered class of open surface canals is constitued by several reaches in cascade (Fig. 1 reaches $p-2$, $p-1$) and a terminal reach (Fig. 1, e.g. p reaches followed by an overflow). Considering a reach, e.g. i^{th} one, the following notations are used:

- $Q(x,t)$ denotes the water-flow,
- $Z(x,t)$ is the water level in the canal,
- $L(i)$ is the i^{th} reach length , to be controlled between the upstream $x = x_{up} = 0_i$ and the downstream $x = x_{do} = L(i)$.
- $U_i(t)$ is the opening of the $(i+1)^{th}$ gate, $U_0(t)$ is the first one.

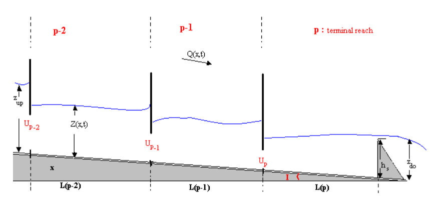

Fig. 1. Canal scheme: multireaches in cascade

Considering there are p reaches, then let

$$x \in \Omega = \cup_{i=1}^{p}]0_i, L(i)[\text{ and } \xi = \begin{pmatrix} Z & Q \end{pmatrix}^t \in X$$

$$X = \left[\prod_{i=1}^{p} L^2(0_i, L(i)) \times L^2(0_i, L(i)) \right]. \quad (1)$$

Reaches are supposed to have a sufficient length $L(i)$ such that a uniform movement can be assumed in the lateral direction. The shallow water's non linear pde for a rectangular canal can be written as follows [8], [10]:

$$\partial_t Z = -\partial_x \frac{Q}{b} \quad (2)$$

$$\partial_t Q = -\partial_x (\frac{Q^2}{bz} + \frac{1}{2} gbZ^2) + gbZ(I-J) \quad (3)$$

$$Z(x,0) = Z_0(x), \quad Q(x,0) = Q_0(x), \quad (4)$$

where b is the canal width, g the gravity constant, I the bottom slope, J the slope's rubbing expressed with the Manning-Strickler expression and R the hydraulic radius:

$$J = \frac{n^2 Q^2}{(bZ)^2 R^{4/3}}, \quad R = \frac{bZ}{b+2Z}. \quad (5)$$

The boundary conditions are stated for each reach. For all, hydraulic constructions are supposed submerged. Coefficients and functions depend on considered reach and considered hydraulic constructions. For example, let consider the i^{th} reach, $x_{up} = 0_i$ and $x_{do} = L(i)$, with an upstream gate. Then upstream boundary equation is given by:

$$Q(0_i,t) = U_{i-1}(t)\Psi_1(Z(0_i,t)). \quad (6)$$

In the same way, at downstream, when there is a gate, downstream boundary equation is:

$$Q(L(i),t) \quad = \quad U_i(t)\Psi_3(Z(L(i),t)), \quad (7)$$

For a terminal reach (p^{th}), generally the hydraulics construction is an overflow, so the downstream boundary condition is the overflow equation:

$$Z(L(p),t) = \Psi_2(Q(L(p),t)). \quad (8)$$

Ψ_j, $1 \le j \le 3$, are given by:

- $\Psi_1(Z) = K_{i-1}\sqrt{2g(z_{up}-Z)}$ with $Z < z_{up}$,
- $\Psi_3(Z) = K_i\sqrt{2g(Z-z_{do})}$ with $Z > z_{do}$, and
- $\Psi_2(Q) = (\frac{Q^2}{2gK_p^2})^{1/3} + h_s$, with $Z > h_s$,

where z_{up} and z_{do} are respectively the water levels before the upstream gate and after the downstream gate. K_i is the product of $(i)^{th}$ gate (or overflow) width and water-flow coefficient of the gate. The output to be controlled is the level at $x_{do} = L(i)$.

The control problem is the stabilization of the height and/or the water-flow, around an equilibrium behavior for each considered reach.

B. A Regulation Model

Let $(z_e(x), q_e(x))$ be an equilibrium state. A linearized model with variable coefficients can be involved to describe the variations around this equilibrium behavior.
This equilibrium state of the system checks the following equations:

$$\begin{aligned} \partial_x z_e &= gbz_e \frac{I + J_e + \frac{4}{3} J_e \frac{1}{1+2z_e/b}}{gbz_e - q_e^2/bz_e^2}, \\ \partial_x q_e &= 0 \end{aligned} \quad (9)$$

Remark 2.1: The fluvial case is assumed, i.e.:

$$\mathbf{z_e(x)} > \sqrt[3]{q_e^2/(gb^2)} = \mathbf{z_c}, \ \forall x \in \Omega. \quad (10)$$

Note that q_e is constant but z_e is space dependent.

Considering one equilibrium state for i^{th} reach, the linearized system around an equilibrium state $(z_{e,i}(x), q_{e,i})$ is, $\xi_i = \begin{pmatrix} z_i & q_i \end{pmatrix}^t \in X_i = L^2(0_i, L(i)) \times L^2(0_i, L(i))$:

$$\begin{aligned} \partial_t \xi_i(t) &= (\partial_t z_i(t) \ \ \partial_t q_i(t))^t \\ &= A_{1,i}(x)\partial_x \xi_i(x) + A_{2,i}(x)\xi_i(x) \quad (11) \\ \xi_i(x,0) &= \xi_{0,i}(x) \quad (12) \end{aligned}$$

Boundary limits are for an upstream gate:

$$\begin{aligned} q_i(0_i,t) - u_{i-1,e}\partial_z \Psi_1(z_{e,i}(0_i)) z_i(0_i,t) \\ = u_{i-1}(t)\Psi_1(z_{e,i}(0_i)) \end{aligned} \quad (13)$$

- for a downstream overflow:

$$z_i(L(i),t) - \partial_q \Psi_2(q_{e,i}) q_i(L(i),t) = 0 \quad (14)$$

- for a downstream gate:

$$\begin{aligned} q_i(L(i),t) - u_{i,e}\partial_z \Psi_3(z_{e,i}(L(i))) z_i(L(i),t) \\ = u_i(t)\Psi_3(z_{e,i}(L(i))) \end{aligned} \quad (15)$$

where $u_{i-1,e}$, $u_{i,e}$ are respectively the i^{th} gate upstream and downstream equilibrium state opening. u_{i-1}, u_i are the opening variations at upstream and downstream. Moreover

$$A_{1,i}(x) = \begin{pmatrix} 0 & -a_{1,i}(x) \\ -a_{2,i}(x) & -a_{3,i}(x) \end{pmatrix}, \quad (16)$$

$$A_{2,i}(x) = \begin{pmatrix} 0 & 0 \\ a_{4,i}(x) & -a_{5,i}(x) \end{pmatrix}, \quad (17)$$

with $a_{1,i}(x) = \frac{1}{b}$,
$a_{2,i}(x) = gbz_{e,i}(x) - \frac{q_{e,i}^2}{bz_{e,i}^2(x)}$, $a_{5,i}(x) = \frac{2gbJ_{e,i}(x)z_{e,i}(x)}{q_{e,i}}$,
$a_{3,i}(x) = \frac{2q_{e,i}}{bz_{e,i}(x)}$, $a_{4,i}(x) = gb(I + J_{e,i}(x) + \frac{\frac{4}{3}J_{e,i}(x)}{1+2z_{e,i}(x)/b})$.

The linearized system around an equilibrium state, through the p reaches, is written as:

$$\begin{aligned} \partial_t \xi(t) &= A_e(x)\partial_x \xi(x) + B_e(x)\xi(x) \quad (18) \\ \xi(x,0) &= \xi_0(x) \quad (19) \\ F(\xi, u_e) &= G(u(t)), \quad (20) \end{aligned}$$

where $\xi = (z_1 \ q_1 \ z_2 \ q_2 \dots z_p \ q_p)^t \in X$ (eq. 1), F and G numerize the boundary conditions (13)-(15), their coefficients are adapted according to the hydraulics construction used on each considered reach.

Operators $A_e(x)$ and $B_e(x)$ are the generalization of operators $A_{1,i}(x)$ and $A_{2,i}(x)$ respectively. Indeed:

$$A_e = \text{diag}(A_{1,i})_{1 \le i \le p} \text{ and } B_e = \text{diag}(A_{2,i})_{1 \le i \le p}. \quad (21)$$

Output variable y is the water levels variation around the equilibrium behaviour at each $x_j = L(j)$, $1 \le j \le p$,

$$y(t) = C\xi(t) \in Y = \mathbb{R}^p, \ \ t \ge 0$$

where C is a bounded operator (representation of the measurement):

$$C\xi = (diag(C_i))_{1\le i\le p}\xi dx, \quad \mu > 0,$$

and $C_i\xi = \begin{pmatrix} \frac{1}{2\mu}\int_{x_i-\mu}^{x_i+\mu}\mathbf{1}_{x_i\pm\mu} & 0 \end{pmatrix}\xi dx, \ \ \mu > 0,$ with $\mathbf{1}_{x_i\pm\mu}(x) = \mathbf{1}_{[x_i-\mu,x_i+\mu]}(x)$ the function such that equals 1 if $x\in\ [x_i-\mu, x_i+\mu]$, else 0, and $\mu > 0$.

The control is given by $u(t)\in U = \mathbb{R}^n$, $u\in C^{\alpha}([0,\infty],U)$ [1]. The control problem is to find the variations of the control vector $u(t)$ such that the water levels at each downstream reach $x = x_{L(i)}$ (i.e. the output variables) track reference signals $r_i(t)$, different for each reach.

The reference signal $r_i(t)$ is chosen, for all cases, constant or no persistent (a step stable response of a non oscillatory system).

III. Open Loop Characterization

The system is first written as a classical boundary control system. Associated to the internal model structure, the closed loop system is described as an open loop perturbation.

The control problem can be expressed as a stabilization problem around an equilibrium state, defined e.g. as $\partial_t\xi = 0$. The linearized boundary control model can be formulated as follows:

$$\begin{aligned}\partial_t\xi(t) &= A_d(x)\xi(t), \quad x\in\Omega, \quad t>0 && (22)\\ F_b\xi(t) &= B_b u(t), \text{ on } \Gamma = \partial\Omega, t>0 && (23)\\ \xi(x,0) &= \xi_0(x) && (24)\end{aligned}$$

where $A_d(x) = A_e(x)\partial_x + B_e(x)$ is an hyperbolic operator.

A. The Abstract Boundary Control System

The abstract boundary control system is obtained by change of variables and operators [7], [14] and the system (22)-(24) becomes:

$$\begin{aligned}\dot{\varphi}\,(t) &= A\varphi(t) - D\,\dot{u}\,(t), \quad \varphi(t)\in D(A), \quad t>0\\ \varphi(0) &= \xi(0) - Du(0) && (25)\end{aligned}$$

$$\text{where:} \qquad \varphi(t) = \xi(t) - Du(t) \quad \forall t\ge 0. \qquad (26)$$

D is a bounded operator from $U\to X$, such that:

$$Du\in D(A_d) \text{ and } F_b(Du(t)) = B_b u(t) \ \ \forall u(t)\in U$$

and $Im(D)\subset Ker(A_d))$, without lost of generality. So $D(A) = \{\varphi\in D(A_d) : F_b\varphi = 0\} = D(A_d)\cap Ker(F_b)$ and $A\varphi = A_d\varphi, \ \forall\varphi\in D(A)$ on X.

The classical solution of system (25) is:

$$\varphi(t) = T_A(t)\varphi_0 - \int_0^t T_A(t-s)D\,\dot{u}\,(s)ds$$

where $\dot{u}$ is assumed to be a continuous time function and A is an infinitesimal generator of a C_0-semigroup $T_A(t)$ such that the solution $\varphi(t) = T_A(t)\varphi_0$ exists and belongs to $D(A)$.

In this order, A has to be a closed, densely defined operator, generator of a C_0-semigroup. Following section purpose is to characterize its stability.

B. Open Loop Stability

The system is well defined i.e. it is a **densely defined** and closed operator, generator of a semigroup [3], [4]. Assume that $\Omega =]0,L[$ in order to simplify notations.

Now, the idea is to consider operator $A(x)$ as a perturbation of the operator $A_e(x)\partial_x$ by a bounded operator $B_e(x)$. Recall that the open loop system, without control is:

$$\dot{\varphi}(t) = A\varphi(t) \ \ t>0, \ \ x\in\Omega$$

$$\varphi(0) = \varphi_0 \ \ \in \ \ D(A(x))$$

and $\varphi(t) = T_A(t)\varphi_0$ is the open loop state, where $T_A(t)$ is the C_0-semigroup generated by $A(x) = A_e(x)\partial_x + B_e(x)$.

Proposition 3.1: Let assume that

$$\Re e(\sigma(A_e(x)\partial_x)) < 0, \ \forall x\in\Omega. \qquad (27)$$

Then, operator $A_e(x)\partial_x$ is generator of a C_0-semigroup exponentially stable.
Moreover, $\langle A_e(x)\partial_x\varphi,\varphi\rangle \le 0, \ \forall\varphi\in D(A_e(x)\partial_x)$.

Proof: Operator $A_e(x)\partial_x$ has a compact resolvent, as the operator ∂_x has a compact one and $A_e(x)$ is bounded [9]. So $A_e(x)\partial_x$ checks the spectral growth assumption [14], [16], and order w_0 of the reduced system $A_e(x)\partial_x$ verifies:

$$w_0 = w_\sigma = \sup\{Re(\lambda) : \lambda\in\sigma(A_e(x)\partial_x)\}.$$

Associated with the assumption $\Re e(\sigma(A_e(x)\partial_x)) < 0$, one get:

$$w_0 < 0.$$

So, $(sI-A)^{-1}\in H_\infty(L(X))$ (theorem 5.1.6 p223 [1]), and semigroup $T_{A_e(x)\partial_x}(t)$ generated by operator $A_e(x)\partial_x$ is exponentially stable (theorem 5.1.5 p222 [1]). So, it is dissipative and

$$\langle A_e(x)\partial_x\varphi,\varphi\rangle < 0, \ \forall\varphi\in X,$$
$$\|T_{A_e(x)\partial_x}(t)\| < \mathrm{e}^{-wt}, \ \forall w\ge 0. \qquad \square$$

Proposition 3.2: Let consider $A(x) = A_e(x)\partial_x + B_e(x)$, $x\in\Omega$ such that $A_e(x)\partial_x$ verifies $\Re e(\sigma(A_e(x)\partial_x)) < 0$ and $B_e(x)$ bounded, i.e. $B_e(x)\in\mathscr{L}(X), \ \forall x\in\Omega$.
Assume that:

i) $B_e(x)$ is semi-definite negative,
ii) $0\in\rho(A(x)) = \rho(A_e(x)\partial_x + B_e(x))$.

Then, $A(x)$ is generator of a C_0-semigroup exponentially stable.
Proof: First point $i)$ is equivalent to: $\langle B_e(x)\varphi,\varphi\rangle \le \ 0, \forall\varphi$.
For all $\varphi\in X$, one get:

$$\langle A(x)\varphi,\varphi\rangle = \langle B_e(x)\varphi,\varphi\rangle + \langle A_e(x)\partial_x\varphi,\varphi\rangle,$$

[1] Regularity coefficient is generally taken as $\alpha = 2$.

so proposition 3.1 and i) imply:

$$\langle A(x)\varphi, \varphi\rangle < 0,\ \forall \varphi \in X \text{ and } \forall x \in \Omega,$$

and if $0 \in \rho(A(x))$ then $\Re e(\sigma(A(x))) < 0$.

As $A(x)$ has a compact resolvent too, it verifies:

$$w_0 = \sup\{Re(\lambda) : \lambda \in \sigma(A_e(x)\partial_x + B_e(x))\} < 0.$$

Then, semigroup $T_A(t)$ generated by the operator $A(x) = A_e(x)\partial_x + B_e(x)$ is exponentially stable [1]. □

The operator of the open loop system, $A(x) = A_e(x)\partial_x + B_e(x)$ in (18)-(20), is generator of a C_0-semigroup exponentially stable. This operator verifies propositions 3.1 and 3.2 using fluvial condition (10), [2].

The control objective can be now achieved by a simple control law employed in the IMBC control structure.

IV. The IMBC Structure: Closed Loop

The Internal Model Boundary Control (IMBC) structure is a particular case of the classical IMC structure since it contains an internal feedback on the model. It allows to get best performances from closed loop added.

Tracking model M_r and low pass filter model M_f are stable

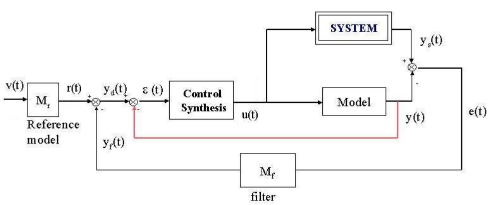

Fig. 2. IMBC structure

systems of finite dimension:

$$\dot{x}_r(t) = A_r x_r(t) + B_r v(t), \quad v(t) \in \mathbb{R}^p, r(t) \in \mathbb{R}^p, \qquad (28)$$
$$r(t) = C_r x_r(t) \text{ and } x_r(0) = 0,$$

$$\dot{x}_f(t) = A_f x_f(t) + B_f e(t), \quad e(t) \in \mathbb{R}^p, y_f(t) \in \mathbb{R}^p, \qquad (29)$$
$$y_f(t) = C_f x_f(t) \text{ and } x_f(0) = x_{f0}.$$

A multivariable proportional-integral feedback control is chosen for the control law:

$$\begin{aligned} u(t) &= \alpha_i \kappa_i \int \varepsilon(s)ds + \alpha_p \kappa_p \varepsilon(t) \\ &= \alpha_i \kappa_i \zeta(t) + \alpha_p \kappa_p \dot{\zeta}(t), \qquad (30) \end{aligned}$$

with $\dot{\zeta}(t) = \varepsilon(t)$.

Moreover, $\varepsilon(t) = y_d(t) - y(t)$ acts like an integrator compared to the "real" measured output, indeed:

$$\varepsilon(t) = r(t) - y(t) - y_f(t).$$

Signals $r(t)$ and $e(t)$ are supposed no persistent, i.e.:

$$\forall \varepsilon > 0, \exists\ t_0 > 0 : ||r(t) - r(t_0)|| < \varepsilon, \forall t > t_0, \qquad (31)$$

idem for $e(t)$.

Let $X_a(t) = \begin{pmatrix} x_r(t) & x_f(t) & x_a(t) \end{pmatrix}^t$ be the new state space with $x_a(t) = (\varphi(t) \ \ \zeta(t))^t$, and $\tilde{\kappa_p} = \alpha_p \kappa_p$, $\tilde{\kappa_i} = \alpha_i \kappa_i$, $\alpha = (\alpha_i, \alpha_p)$ then, the extended IMBC state space system is:

$$\begin{cases} \dot{X}_a(t) = \tilde{A}(\alpha) X_a(t) + \tilde{B}\begin{pmatrix} v(t) & e(t) \end{pmatrix}^t \\ X_a(0) = X_{a0} = \begin{pmatrix} 0 & x_{f0} & x_{a0} \end{pmatrix}^t, \end{cases} \qquad (32)$$

$$\text{where } \tilde{B} = \begin{pmatrix} B_r & 0 \\ 0 & B_f \\ \mathscr{C}(\alpha) C_r B_r & 0 \end{pmatrix},$$

$$\tilde{A}(\alpha) = \begin{pmatrix} A_r & 0 & 0 \\ 0 & A_f & 0 \\ \mathscr{B}(\alpha) C_r + \mathscr{C}(\alpha) C_r A_r & 0 & \mathscr{A}(\alpha) \end{pmatrix}$$

$$\mathscr{B}(\alpha) = \begin{pmatrix} -D\tilde{\kappa_i}(I - CDW\tilde{\kappa_p}) \\ (I - CDW\tilde{\kappa_p}) \end{pmatrix}, \quad \mathscr{C}(\alpha) = \begin{pmatrix} -D\tilde{\kappa_p} \\ 0 \end{pmatrix},$$

and where W is the left pseudo inverse of $(I + \alpha_p \kappa_p CD)$, such that $W(I + \alpha_p \kappa_p CD) = I$.
$\mathscr{A}(\alpha)$ can be viewed as a bounded perturbation of A:

$$\mathscr{A}(\alpha) = \mathscr{A}_e(\alpha) + \alpha_i \mathscr{A}_e^{(1)}(\alpha) + \alpha_i^2 \mathscr{A}_e^{(2)}(\alpha), \qquad (33)$$

where $\mathscr{A}_e(\alpha) = \begin{pmatrix} (I + D\tilde{\kappa_p}C)A & 0 \\ -(I - CDW\tilde{\kappa_p})C & 0 \end{pmatrix}$ contains open loop operator A, and $\mathscr{A}_e^{(1)}$ and $\mathscr{A}_e^{(2)}$ are bounded operators, as C, D and CD are bounded ones:

$$\mathscr{A}_e^{(1)}(\alpha) = \begin{pmatrix} D\kappa_i(I - CDW\kappa_p)C & 0 \\ 0 & -CDW\kappa_i \end{pmatrix},$$

$$\mathscr{A}_e^{(2)}(\alpha) = \begin{pmatrix} 0 & D\kappa_i CDW\kappa_i \\ 0 & 0 \end{pmatrix}.$$

Following the stability of both tracking and filter models (M_r and M_f), matrices A_r and A_f can be choosen as stable Hurwitz ones. So the stability study of $\tilde{A}(\alpha)$ in (32) is achieved by the stability of $\mathscr{A}(\alpha)$ in (33).

Remark 4.1 (Regulation): Let suppose that the system and its model verify the non persistent assumption (31). Then, if the closed loop system operator $\tilde{A}$ (32) is generator of an exponentially stable semigroup, the controlled system has the following asymptotic behaviour (Fig 2):

$$\lim_{t\to\infty} \varepsilon(t) = \lim_{t\to\infty} v(t) - y_s(t) = 0. \qquad (34)$$

A. Closed Loop Stability Results

Now the perturbation theory, from Kato's works [9], for control problem of infinite dimensional system [11], [12], [14] can be used.

Let recall the main results, where sufficient conditions are stated in order to preserve stability properties:
$*$**1** $rank(CD) = p$ [11],

$*\mathbf{2}$ $(I+\alpha_p\kappa_p CD)$ is (left) inversible,
$*\mathbf{3}$ $\kappa_p = [CD]^{\ddagger}$ [11], (‡ is the right pseudo inverse) and α_p:

$$0 \le \alpha_p < \alpha_{p,max} = \min_{\lambda\in\Gamma}(a\|R(\lambda;A)\| + b\|AR(\lambda;A)\|)^{-1}$$

with a and b such that $\|D\kappa_p Cx\| \le a\|x\| + b\|Ax\|$ for all $x \in D(A)$,
$*\mathbf{4}$ $\kappa_i = -(CD)^{\ddagger}$ [14], and:

$$\begin{cases} 0 \le \alpha_i < \alpha_{i,max} = \min_{\lambda\in\Gamma}(a\|R(\lambda;\mathscr{A}_e)\|+1)^{-1} \\ \mathbf{rg(CDW) = p} \\ \mathscr{R}\mathbf{e}(\sigma(\mathbf{CDW}\kappa_\mathbf{i})) < \mathbf{0}. \end{cases} \tag{35}$$

For the multireaches operator, those assumptions become [5]: - $\kappa_i = -\theta[CD]^{\ddagger}$, $0<\theta<1$,
- α_i checks the first condition of (35),
- $\kappa_p = [CD]^{\ddagger}$,
- $(I+\alpha_p\kappa_p CD)$ is inversible and its inverse is $W = k(I-\alpha_p\kappa_p CD)$, with $k=(1-\alpha_p^2)^{-1}$ and $a = \|D\kappa_p C\|$, such that:

$$0 \quad \le \quad \alpha_p < \alpha_{p,max} = (\sup_{\lambda\in\Gamma} a\|R(\lambda;A)\|)^{-1}. \tag{36}$$

The difficulty here is to get correct estimation of $\alpha_{i,max}$ and $\alpha_{p,max}$. Values given by simulations or experimentations are not revealing (by spectre simulation calculations e.g. [5]). So before numerical calculations, expressions of (35) and (36) must be expressed by a more explicit analytical expression of the resolvent operator.

B. Analytical Control Parameters Expression

In the particular case of hyperbolic operator, their resolvent can be explicitely calculated as their spectrum, their norm.

Indeed, let consider the operator $A(x) = A_e(x)\partial_x + B_e(x)$ and suppose that it is well defined, and generator of a C_0-semigroup exponentially stable such that $kv(0) = v(L)$, $k \neq Id$.
Let defined $\mu(x) = A_e^{-1}(x)(\lambda(x)Id - Be(x))$,
then $R(\lambda,A) = (\lambda Id - A)^{-1}$ is:

$$R(\lambda,A)v = \frac{e^{\mu(0)x}e^{\int_0^x \mu(s)ds}}{A_e(L)e^{\mu(0)L}e^{\int_0^L \mu(s)ds} - kA_e(0)}\mathscr{R}(\lambda,A)v, \tag{37}$$

$$\mathscr{R}(\lambda(x),A(x)) = kA_e(0)\int_0^x e^{-\mu(0)y}e^{-\int_0^y \mu(s)ds}A_e^{-1}(y)v(y)dy$$
$$+A_e(L)e^{\mu(0)L}e^{\int_0^L \mu(s)ds}\int_x^L e^{-\mu(0)y}e^{-\int_0^y \mu(s)ds}A_e^{-1}(y)v(y)dy$$

Relation (37) is not defined, only when

$$A_e(L)e^{\mu(0)L}e^{\int_0^L \mu(s)ds} = kA_e(0),$$

this allows to get explicitely the spectrum, reduced to discret spectrum for operator with compact resolvent.
When $B_e(x) = 0$, one get:

$$\sigma(A_e(x)\partial_x) = \{\lambda_n : \lambda_n(x) = \lambda(x) + \frac{2in\pi}{L}\theta(x)\}$$

with $\lambda : \Omega \to \mathbb{R}^- \setminus \{0\}$.

Similarly, using the stability of the semigroup $T_A(t)$, one get:

$$||R(\lambda,A)||_{L^2(\Omega)} = ||(\lambda Id - B_e)^{-1}||.$$

Notice that the resolvent norm of $\mathscr{A}_e$ is deduced from $||R(\lambda,A)||$:

$$R(\lambda,\mathscr{A}_e) = \begin{pmatrix} R(\lambda,A) & 0 \\ -\lambda^{-1}CR(\lambda,A) & \lambda^{-1} \end{pmatrix}. \tag{38}$$

So (37) and (38) use directly the open loop system operator to check sufficient conditions on control parameters synthesis.

V. Simulation and Experimental Results

Simulations were performed with Matlab and Simulink. They gave satisfactory results for a single reach (cf. [3]) and for the multireaches case, too. Then, the proposed control law was implemented on the Valence (France) experimental canal. This pilote canal is an experimental process (length=8 m, width=0.1 m) with a rectangular basis, a variable slope and with three gates (three reaches and an overflow). Rubbing are weak and the fluvial hypothesis (10) is realized thanks to the variable slope.

A. Simulation: Various Slopes

Multireaches cases is treated: two reaches are given with two different slopes

- Reach 1: length $L_1 = 70dm$, slope $I1 = 0.0016dm.dm^{-1}$, initial condition is $z_{e,1}(0) = 1.344dm$
- Reach 2: length $L_2 = 80dm$, slope $I2 = 0.001dm.dm^{-1}$, initial condition is $z_{e,2}(0) = 1.044dm$.

In both case, equilibrium flow equals $q_e = 2dm^3.s^{-1}$, reference is to track a water level of:

- Reach 1: $r_1(t) = 1.1 * r_{0,1} = 1.61dm$, with $r_{0,1} = 1.47dm$,
- Reach 2: $r_2(t) = 0.9 * r_{0,2} = 1.04dm$, with $r_{0,2} = 1.15dm$.

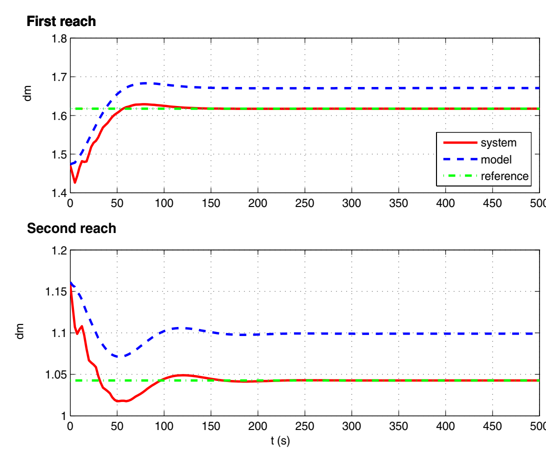

Fig. 3. Two reaches - two slopes: water level

Theorical values of α parameters are thus employed for those simulations, i.e. $\alpha_i = \alpha_{i,max} \simeq 0,65$ $\alpha_p = \alpha_{p,max} \simeq 0,5$.

Like this simulation, all simulations results obtained have shown the suitability of this approach, so experimentations have been realized on the experimental micro-canal.

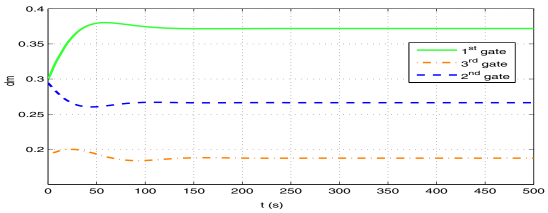

Fig. 4. Two reaches - two slopes: 3 gates control

B. Experimentation: two reaches

In this experimentation, one can notice that conditions (35) and (36) are sufficient but not necessary. Indeed, control parameters are $\alpha_i = 2$, $\alpha_p = 0$ with $\alpha_{i,max} \simeq 0,73$, $\alpha_{p,max} \simeq 0,65$. Initials conditions for this experimentation are:

$$q_e = 1dm^3.s^{-1},\ z_{e1}(0) = 1.22dm,\ z_{e2}(0) = 1.02dm.$$

Tracking references are for both reaches:
First reach, length equals $3.5dm$ and $r_0 = 1.28dm$:

$r(t) = r_0 dm$ for $0s \leq t \leq 85s$
$r(t) = 1.2 * r_0$ for $90s \leq t \leq 330s$
$r(t) = 0.9 * r_0$ for $330s \leq t \leq 475s$
$r(t) = 1.1 * r_0$ for $480s \leq t$.

Second reach, length equals $3.5dm$ and $r_L = 1.077dm$:

$r(t) = r_L dm$ for $0s \leq t \leq 160s$
$r(t) = 0.76 * r_L$ for $160s \leq t \leq 320s$
$r(t) = 0.9 * r_L$ for $325s \leq t$.

Even if $\alpha_i >> \alpha_{i,max}$ and variations levels are higher than $\pm 20\%$, error between the model and the system is less than 10%, the system tracks the references in both reaches.

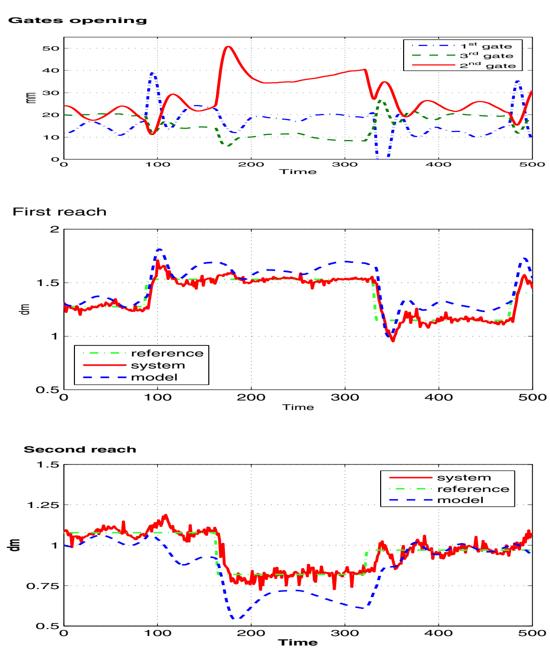

Fig. 5. Open gates & Water levels

Moreover, it seems that control parameters α decrease when the numbers of reaches increases until limit values.

All the experimental results show the suitability of this approach. Indeed, given an interval of $\pm 20\%$ around a given equilibrium state, results are still very satisfactory. However, if the variation asked is superior to $\pm 20\%$, the error between model and system increases dramatically.

Notice that three reaches cases has been experimented, but results where no really revealing as canal length is too small to consider that Saint-Venant assumptions are realized.

VI. Conclusion

The direct approach, which has been developped in this work, seems to be suitable for the regulation of canal irrigation. In addition the main theorical results in this paper (propositions 3.1 and 3.2) may concern more general hyperbolic systems of the form given by equations (22) to (24). Moreover, the fact that the spatial evolution of the model parameters has been taken into account allows to consider canals in real situation. Revealing results for various slopes simulations confirm this conclusion.

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