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Improved Cascade Control Structure for Controlling Unstable and Integrating Processes

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Abstract— **Cascade control is an alternative to conventional single feedback control to improve the performance of a control system particularly in the presence of disturbances. However, so far published papers on cascade control strategies are for control of stable processes. In this paper, a new cascade control structure and controller design based on Standard forms are suggested for controlling unstable or integrating processes in a cascade control structure. Examples are given to illustrate the use of the proposed method and its superiority over some existing design methods.**

I. Introduction

Cascade control (CC), which was first introduced many years ago by Franks and Workey [1], is one of the strategies that can be used to improve the system performance particularly in the presence of disturbances. In conventional single feedback control, the corrective action for disturbances does not begin until the controlled variable deviates from the set point. A secondary measurement point and a secondary controller, G_{c2}, in cascade to the main controller, G_{c1}, as shown in Fig. 1, can be used to improve the response of the system to load changes, D_2.

Recent contributions on the tuning of PID controllers in cascade loops include [2]-[3] which have only concentrated on control of stable process transfer functions in both loops. More recently, Lee et al [4] suggested a cascade control structure for control of processes with unstable and integrating transfer functions.

In this paper, an improved cascade control structure, which is assumed that the inner loop includes stable plant transfer functions and the outer loop an unstable or integrating process transfer function, is proposed. It is assumed that the inner loop can be modelled by a first order plus dead time (FOPDT) and the outer loop by an unstable first order plus dead time (UFOPDT) or integrating plus first order plus dead time (IFOPDT) plant transfer function.

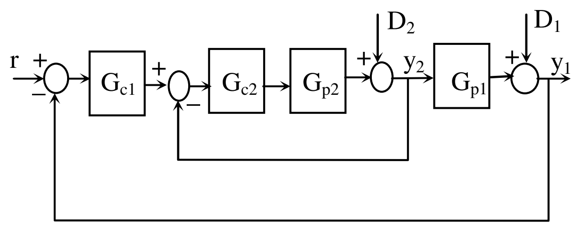

Fig. 1: Cascade Control System

In this paper, an improved cascade control structure, which is assumed that the inner loop includes stable plant transfer functions and the outer loop an unstable or integrating process transfer function, is proposed. It is assumed that the inner loop can be modelled by a first order plus dead time (FOPDT) and the outer loop by an unstable first order plus dead time (UFOPDT) or integrating plus first order plus dead time (IFOPDT) plant transfer function.

A cascade control strategy can be used to achieve better disturbance rejections. However, if a long time delay exists in the outer loop the cascade control may not give satisfactory closed loop responses for set point changes. In this case, a Smith predictor scheme can be used for a satisfactory set point response. The proposed improved cascade structure brings together above mentioned the best merits of the cascade control and Smith predictor scheme. Furthermore, a PI-PD structure, which is proved to give better closed loop performances for process transfer functions with large time constants, complex poles, unstable poles or an integrator [5]-[7], is used in the outer loop to improve the performance of the system even better. The outer loop PI-PD controllers' parameters are identified by the use of standard forms, which is a simple algebraic approach to controller design. Another advantage of the standard forms is that one can predict how good will be the performance of closed loop system. The

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inner loop controller is designed based on well-known Internal Model Controller [8].

II. STANDARD FORMS

The use of integral performance indices for control system design is well known. Many text books, such as [9], include short sections devoted to the procedure. For linear systems, the ISE can be evaluated efficiently on digital computers using the s-domain approach with Åström's recursive algorithm [10]. Thus for

$$J_0 = \int_0^{\infty} e^2(t)dt \quad (1)$$

the s-domain solution is given by

$$J_0 = \frac{1}{2\pi j}\int_0^{\infty} E(s)E(-s)ds \quad (2)$$

where $E(s)=B(s)/A(s)$, and $A(s)$ and $B(s)$ are polynomials with real coefficients, given by

$$A(s) = a_0 s^m + a_1 s^{m-1} + ... + a_{m-1}s + a_m$$

$$B(s) = b_1 s^{m-1} + ... + b_{m-1}s + b_m$$

Criteria of the form $J_n = \int_0^{\infty} [t^n e(t)]^2 dt$ can also be evaluated using this approach, since $L[tf(t)]=(-d/ds)F(s)$, where L denotes the Laplace transform and $L[f(t)] = F(s)$. Minimizing a control system using J_0, that is the ISE criterion, is well known to result in a response with relatively high overshoot for a step change. However, it is possible to decrease the overshoot by using a higher value of n and responses for $n=1$, that is the ISTE criterion. Therefore, in this paper results for the ISTE criterion only is given.

Another approach to optimization which has been little discussed for many years is the direct synthesis approach where the closed loop transfer function is synthesized to a standard form. Using this approach, it is possible to obtain the optimal parameters of a closed loop transfer function, which will provide a minimum value of the ISE. Tables of such all pole transfer functions were given many years ago [11] but are of limited use in design, because even with an all pole plant transfer function the addition of a typical controller produces a closed loop transfer function with a zero. Results with a single zero were also given so that the feedback loop would follow a ramp input with zero steady-state error but these expressions are not appropriate for step response design. For a closed loop transfer function with one zero it is easy to present results for these optimum transfer functions as the position of the zero varies [5]. Briefly, here, assuming a plant transfer function with no zero and a controller with a zero then a closed loop transfer function, T_{1j}, of the form

$$T_{1j} = \frac{c_1 s + 1}{s^j + d_{j-1}s^{j-1} + ... + d_1 s + 1} \quad (3)$$

is obtained, where the subscript '1' in T_{1j} indicates a zero in the numerator of the standard form and the subscript 'j' indicates the order of the denominator. Also, for a unit step set point, the error is obtained as

$$E_{1j} = \frac{s^{j-1} + d_{j-1}s^{j-2} + ... + (d_1 - c_1)}{s^j + d_{j-1}s^{j-1} + ... + d_1 s + 1} \quad (4)$$

Minimizing E_{1j} for the ISTE, the optimum values of the d's as functions of c_1 are shown in Fig. 4 for $T_{13}(s)$. Fig. 5 shows how J_1 (the minimum value for the ISTE criterion) varies as c_1 increases for $T_{13}(s)$. The figure illustrates that as c_1 increases the step response of the closed loop improves. Also, the step responses for the J_1 criterion for a few different c_1 values are shown in Fig. 6 for $T_{13}(s)$. It is seen that as c_1 increases the step responses are getting faster.

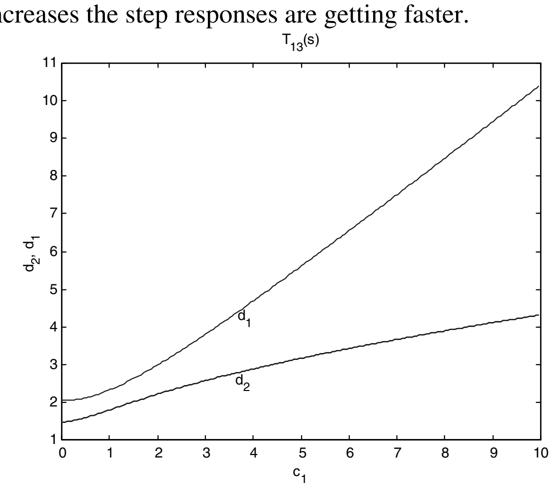

Fig. 4: Optimum values of d_1 and d_2 for varying c_1 values.

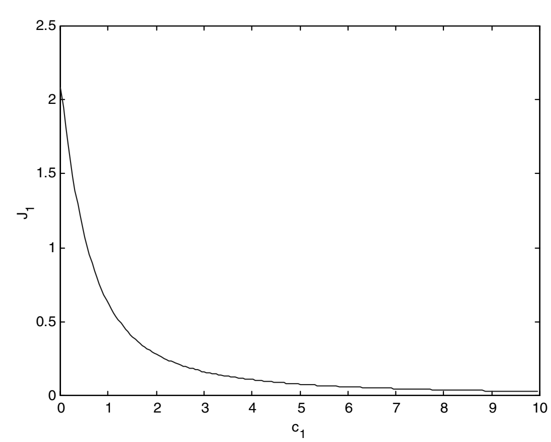

Fig. 5: Optimum values of J_1 for varying c_1 values

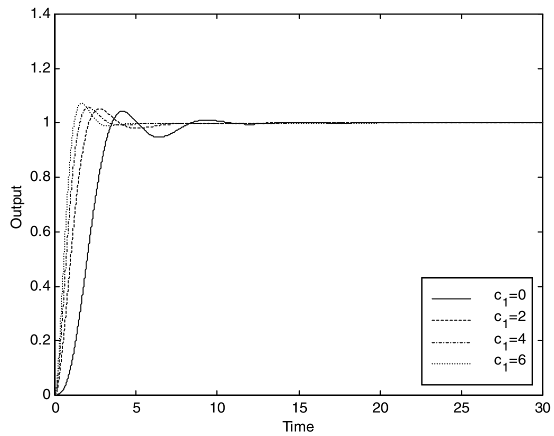

Fig. 6: Step responses for $T_{13}(s)$ and J_1 criterion

III. The New Cascade Control Structure and Design Method

The proposed cascade control structure is shown in Fig. 7. G_{c2} is used for stabilization of the inner loop while G_{c1} and G_{c3} are used for the outer loop stabilization. G_d is used for disturbance rejection. G_{p2m} and G_{pm} are the model transfer functions of the inner and outer loops respectively. Assuming that the plant transfer functions are known, then two loops can be tuned simultaneously. If the transfer functions are not known, then any existing modeling approach in the literature, such as [12], can be used to find the appropriate model transfer function.

In the next subsections, tuning rules for both loops are derived.

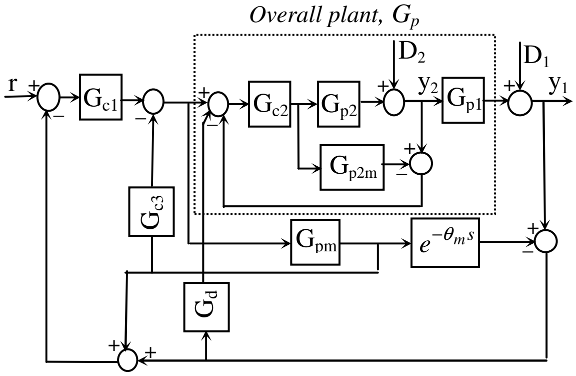

Fig. 7: Improved Cascade Control Structure

3.1 Designing inner loop controller (G_{c2}):

As stated in the introduction, the inner loop controller is designed based on IMC principles [8]. The details of design procedure are not given here, since; one can easily obtain them from the abovementioned references.

The outer loop plant transfer function is assumed to be a FOPDT

$$G_{p2}(s) = K_2 e^{-\theta_2 s} /(T_2 s + 1) \tag{5}$$

Using IMC principles, one can easily show that the inner loop controller is given by

$$G_{c2}(s) = (T_2 s + 1) / K_2 (\lambda s + 1) \tag{6}$$

where, λ is the only tuning parameter to be found.

The faster the inner loop than the outer loop, the better the performance of a cascade control system in the sense of a faster response. That is, the smaller the values of λ the better the performance of the cascade control system. Hence, as a rule of thumb λ can be chosen equal to inner loop time delay. If a faster response is requested, λ can be chosen as half the time delay of the inner loop, namely, $\lambda = \theta_2 / 2$, which is the value used throughout the paper.

3.2 Designing outer loop controllers (G_{c1}, G_{c3} and G_d):

The inner closed loop transfer function is given by

$$G_{pi}(s) = e^{-\theta_2 s} /(\lambda s + 1) \tag{7}$$

Then, the overall plant transfer function for the outer loop is

$$G_p(s) = G_{pi}(s) G_{p1}(s) = G_{pm}(s) e^{-\theta_m s} \tag{8}$$

where, $G_{pm}(s)$ is the delay free part of the overall plant transfer function and $\theta_m = \theta_1 + \theta_2$.

It can easily be shown that the closed loop transfer functions between y_1 and r, and y_1 and d_2, assuming a perfect matching for the sake of easiness of calculations, that is $G_p = G_{pm}(s) e^{-\theta_m s}$, is given by

$$T_r(s) = \frac{G_{pm}(s) G_{c1}(s) e^{-\theta_m s}}{1 + G_{pm}(s)[G_{c1}(s) + G_{c3}(s)]} \tag{9}$$

$$T_{d2}(s) = \frac{G_{p1}(1 - G_{c2} G_{p2})}{[1 + G_{pm}(s)(G_{c1}(s) + G_{c3}(s))]} \frac{[1 + G_{pm} G_{c3} + G_{pm}(s) G_{c1}(s)(e^{-\theta_m s} - 1)]}{(1 + G_d G_{pm} e^{-\theta_m s})} \tag{10}$$

Eqn. (9) reveals that the parameters of the two controllers $G_{c1}(s)$ and $G_{c3}(s)$ can be determined using the delay free part of the overall plant transfer function. Eqn. (10), on the other hand, shows that the controller $G_d(s)$ can be used to reject disturbances.

The outer loop controllers, $G_{c1}(s)$ and $G_{c3}(s)$, are assumed to have the forms

$$G_{c1}(s) = K_c(1+1/T_i s) \quad (11)$$

and

$$G_{c3}(s) = K_f + T_f s \quad (12)$$

The disturbance rejection controller $G_d(s)$ has the form

$$G_d(s) = K_d(1+T_d s) \quad (13)$$

The next sections give controller design for two different cases.

Case 1 (Design for an UFOPDT): It is assumed that the outer loop plant transfer function is unstable and given by

$$G_{p1}(s) = K_1 e^{-\theta_1 s}/(T_1 s - 1) \quad (14)$$

Therefore, the overall plant transfer function for the outer loop is

$$G_p(s) = \frac{K_1 e^{-(\theta_1+\theta_2)s}}{(\lambda s+1)(T_1 s-1)} \quad (15)$$

Eqn. (15) can be rearranged as

$$G_p(s) = \frac{k e^{-(\theta_1+\theta_2)s}}{s^2 + as + b} \quad (16)$$

where,

$$k = K_1/T_1\lambda \quad (17a)$$

$$a = (1/\lambda) - (1/T_1) \quad (17b)$$

$$b = -1/T_1\lambda \quad (17c)$$

Taking the delay free part of eqn. (16) equal to $G_{pm}(s)$ and using the delay free part eqn. (9) results in closed loop transfer function

$$T_{13}(s) = \frac{kK_c(T_i s+1)}{T_i s^3 + (a+kT_f)T_i s^2 + \ldots + (b + kK_f + kK_c)T_i s + kK_c} \quad (18)$$

Normalization of eqn. (18), assuming

$$s_n = s(T_i/kK_c)^{1/3} = s/\alpha \quad (19)$$

results in

$$T_{13}(s_n) = \frac{c_1 s_n + 1}{s_n^3 + d_2 s_n^2 + d_1 s_n + 1} \quad (20)$$

where,

$$c_1 = \alpha T_i \quad (21a)$$

$$d_2 = (a + kT_f)/\alpha \quad (21b)$$

$$d_1 = (b + kK_f + kK_c)/\alpha^2 \quad (21c)$$

In principle α can be selected by the choice of K_c and c_1 by the choice of T_i. Based on the value of c_1, the coefficients d_2 and d_1 can be found from Fig. 4 and then the value of T_f and K_f can be computed from eqns. (21b) and (21c) respectively.

Note that for a selected T_i, choosing larger K_c values results in larger α and c_1 values. This implies a faster closed loop system response. In practice, K_c will be constrained, possibly to limit the initial value of the control effort, so that the choice of K_c and T_i may involve a trade off between the values chosen for α and c_1.

For a satisfactory load disturbance rejection, the tuning parameters of the controller $G_d(s)$ have to be identified. Since, the roots of the factor $[1+G_{pm}(G_{c1}+G_{c3})]$ were placed properly by the earlier design, the controller $G_d(s)$ is designed on the basis of stabilization of the second part of the characteristic equation of eqn. (10)

$$1 + \frac{K_1 K_d(1+T_d s)e^{-\theta_m s}}{(\lambda s+1)(T_1 s-1)} = 0 \quad (22)$$

Choosing $T_d = \lambda$, eqn. (22) reduces to

$$1 + \frac{K_1 K_d e^{-\theta_m s}}{(T_1 s - 1)} = 0 \quad (23)$$

Using optimum phase margin criterion (De Paor and O'Malley, 1989), K_d can be found as

$$K_d = \sqrt{T_1/\theta_m K_1^2} \quad (24)$$

with the constraint $\theta_m/T_1 < 1$.

Case 2(Design for an IFPDT): In this case, it is assumed that the outer loop plant transfer function has an integrator and can be modeled by

$$G_{p1}(s) = K_1 e^{-\theta_1 s}/s \quad (25)$$

The overall plant transfer function for the outer loop is

$$G_p(s) = \frac{K_1 e^{-(\theta_1+\theta_2)s}}{s(\lambda s+1)} \quad (26)$$

Rearranging eqn. (26) gives

$$G_p(s) = \frac{k e^{-(\theta_1+\theta_2)s}}{s^2 + as} \quad (27)$$

where,

$$k = K_1/\lambda \quad (28a)$$

$$a = 1/\lambda \quad (28b)$$

Note that this is a special case of 'case 1'. Hence, the standard form in this case is again given by eqn. (20). The coefficients in the standard form are still given by eqns. (21a)-(21c), with $b=0$ in eqn. (21c).

As in case 1, α can be selected by the choice of K_c and c_1 by the choice of T_i. Based on the value of c_1, the coefficients d_2

and d_1 can be found from Fig. 4 and then the value T_f and K_f can be computed from eqns. (21b) and (21c), respectively.

Again, for a satisfactory load disturbance rejection, the tuning parameters of the controller $G_d(s)$ have to be identified. Applying Nyquist stability criterion to the characteristic equation of eqn. (10) gives

$$1+\frac{K_1 K_d (1+T_d s)e^{-\theta_m s}}{s(\lambda s+1)}=0 \qquad (29)$$

$T_d = \lambda$ was chosen so that

$$1+\frac{K_1 K_d e^{-\theta_m s}}{s}=0 \qquad (30)$$

Choosing K_d to give a phase margin of φ_m yields

$$K_d = (\pi/2-\varphi_m)/K_1\theta_m \qquad (31)$$

Extensive simulation examples show that a phase margin of $\varphi_m = 61.3065$, which corresponds to $\pi/2-\varphi_m=1/2$, yields a good disturbance rejection for integrating processes. Hence,

$$K_d = 1/2K_1\theta_m \qquad (32)$$

IV. SMULATION EXAMPLES

Two examples are given to illustrate the use of the proposed cascade control structure and design procedure.

Example 1: Consider $G_{p1}(s)=e^{-3s}/(10s-1)$, and $G_{p2}(s)=2e^{-2s}/(s+1)$.Taking $\lambda=1$, which is the half the inner loop time delay, gives $G_{c2}(s)=(s+1)/(2s+2)$. Therefore, from eqn. (15) $G_p(s)=e^{-5s}/(s+1)(10s-1)$, which results in $k=0.1$, $a=0.9$ and $b=-0.1$. Limiting K_c to *1.00* and choosing $T_i=0.10$ gives $\alpha=1.00$ and $c_1=0.10$. The standard form $T_{13}(s)$ to minimise J_1 for $c_1=0.10$ has $d_2=1.487$ and $d_1=2.046$, which then gives $T_f=5.867$ and $K_f=20.464$. The disturbance rejection controller parameters are $T_d=\lambda=1.0$ and $K_d=1.414$, from eqn. (24). Alternatively, choosing $K_c=0.50$ results in $\alpha=0.794$, $c_1=0.0794$, $d_2=1.483$ and $d_1=2.045$. The performance of the proposed design method, for both cases, with calculated controllers parameters is shown in Fig. 8 for a unit magnitude of set point change. For comparison, results for the design method proposed by Lee et al [4], which have $K_{pi}=0.278$, $T_{ii}=1.667$ and $T_{di}=0.400$ for the inner loop and $K_{po}=2.123$, $T_{io}=43.506$ and $T_{do}=2.865$ for the outer loop, are also given in the same figure. The pre-filters used by their method, for the inner and outer loops, respectively, are $q_{f1}=1/(40.490s+1)$ and $q_{f2}=1/(s+1)$. The proposed method when compared to the design method of Lee et al [4] gives a far superior response. To illustrate the robustness to parameter variations, a $\pm 10\%$ change in the outer loop time delay is assumed, as this normally has the most deteriorating effect on the system step response, and results for this case are given in Fig. 9. Fig. 10 illustrates responses for both design methods to a unit magnitude of a disturbance D_2.

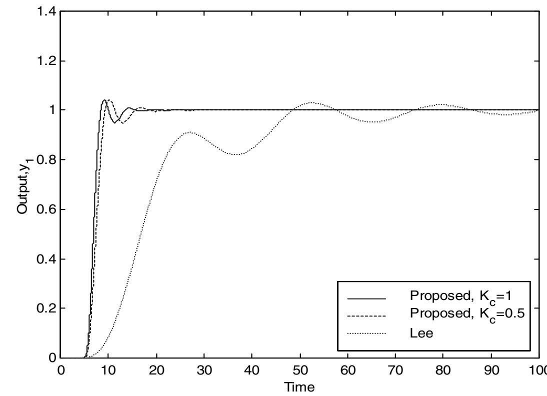

Fig. 8: Responses to a unity step set point change for example 1

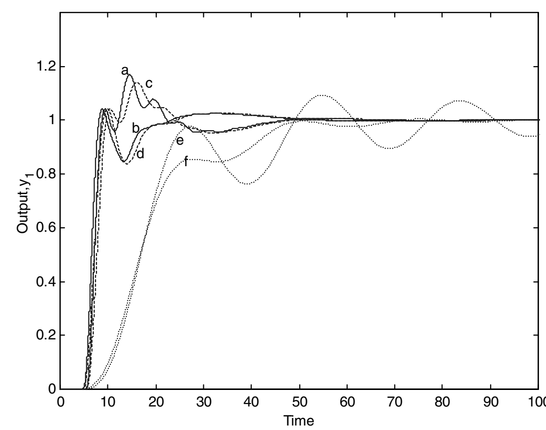

Fig. 9: Responses for example 1, a, c, e) +10% change in the time delay b, d, f) -10% change in the time delay (legends are as shown in Fig. 8)

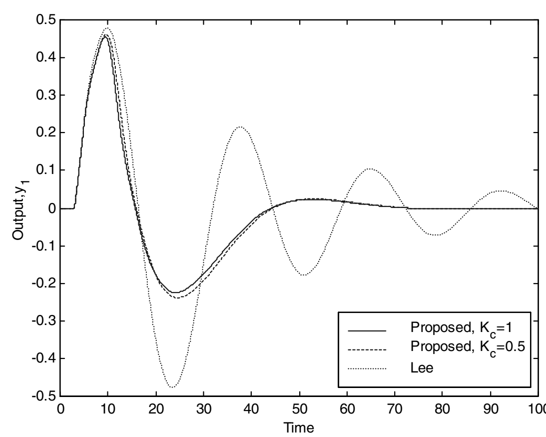

Fig. 10: Responses to disturbance D_2 with unity magnitude for example 1

Example 2: Let the outer and inner loop plant transfer functions be $G_{p1}(s) = 2e^{-2s} / s$, $G_{p2}(s) = 4e^{-s} /(s+1)$. Taking λ*=0.5,* results in $G_{c2}(s) = (s+1)/(2s+4)$. $G_p(s) = e^{-3s} / s(0.5s+1)$ is obtained from eqn. (27), which has $k = 4.0$ and $a = 2.0$. Limiting K_c to *0.1* and choosing $T_i=0.01$ gives $\alpha=6.325$ and $c_1=0.0632$. The standard form from Fig. 4 requires $d_2=1.481$ and $d_1=2.044$. These values can be obtained with $T_f=1.841$ from eqn. (21b) and $K_f=20.339$ from eqn. (21c). For the disturbance rejection controller $T_d = \lambda = 0.5$ and $K_d=0.0417$, from eqn. (32), were obtained. The design method proposed by Lee *et al*. [4] has $K_{pi}=0.222$, $T_{ii}=1.333$ and $T_{di}=0.250$ for the inner loop and $K_{po}=0.171$, $T_{io}=10.952$ and $T_{do}=1.232$ for the outer loop. The pre-filters for the inner and outer loops, respectively, are $q_{f1}=1/(40.490s+1)$ and $q_{f2}=1/(s+1)$. The performance of the proposed design method, together with the design method of Lee et al [4], are shown in Fig. 11. The proposed design method clearly gives a much better performance. Responses for both design methods to a disturbance D_2, with magnitude of 1, are shown in Fig. 12.

V. Conclusion

An improved cascade control structure and controller design method for controlling unstable and integrating processes have been introduced. A PI-PD Smith predictor scheme is used in the outer loop of the cascade control. This gained two advantages: First, the best merits of the cascade control and a Smith predictor scheme were combined in one structure. Second, the use of PI-PD controller accomplished to obtain improved performance for unstable and integrating processes. Several procedures for obtaining the parameters of the PI-PD controllers are possible, but one of the simplest approaches is to employ standard forms as this enables the design to be completed using simple algebra.

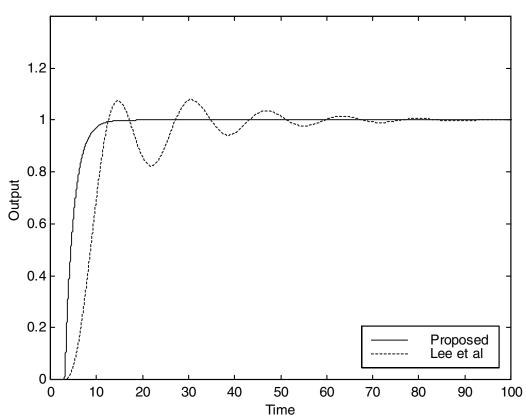

Fig. 11: Responses to a unity step set point change for example 2.

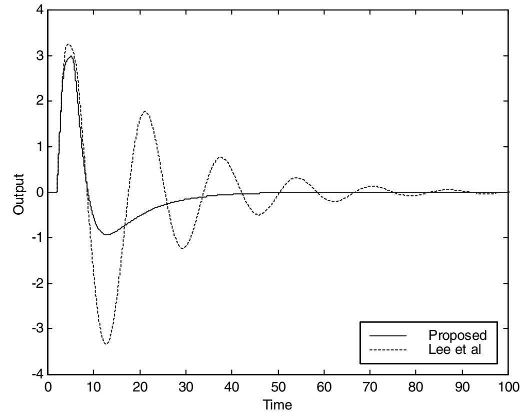

Fig.12: Responses to disturbance D_2 with unity magnitude for example 2.

References

[1] R.G. Franks and C.W. Worley, "Quantitive Analysis of Cascade Control", *Industrial and Engineering Chemistry*, vol. 48, no. 6, pp. 1074-1079, 1956.

[2] Y. Lee, S. Park and M. Lee, "PID Controller Tuning to Obtain Desired Loop Responses for Cascade Control Systems", *Ind. Eng. Chem. Res*. Vol. 37, pp. 1859-1865, 1998.

[3] I. Kaya, "Improving performance using cascade control and a Smith predictor", *ISA Trans.*, vol, 40, pp. 223-234, 200.

[4] Y. Lee, S. Oh and S. Park, "Enhanced Control with a General Cascade Control Structure", *Ind. Eng. Chem. Res*., vol. 41, pp. 2679-2688, 2002.

[5] D.P. Atherton and A.F. Boz, "Using Standard forms for controller design", *in Proc. of Control'98*, Sweansea, UK, 1998.

[6] S. Majhi, and D.P. Atherton, "Modified Smith predictor and controller for processes with time delay", *IEE Proc.-Control Theory Appl.*, vol. 146, pp. 359-366, 1999.

[7] I. Kaya, "A PI-PD Controller Design for Control of Unstable and Integrating Processes", *ISA Tans.*, vol. 42, pp. 111-121, 2003.

[8] M. Morari and E. Zafiriou, "Robust Process Control", Prentice-Hall, Englewood Cliffs, 1989.

[9] R.C. Dorf and R.H. Bishop, "Modern Control Systems", Addison-Wesley, Reading, 1995.

[10] K.J. Åström, "Introduction to Stochastic Control Theory", Academic, New York, 1970.

[11] D. Graham and R.C. Lathrop, "The synthesis of optimum response: criteria and standard forms" *II. Trans. AIME*, vol. 72, pp. 273-288, 1953.

[12] I. Kaya, "Relay feedback identification and model based controller design", Ph.D. Thesis, University of Sussex, U.K, 1999.