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Hierarchical Tracking Control of Car-Like Mobile Robots

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***Abstract*—A hierarchical tracking controller for a car-like mobile robot subject to nonholonomic constraints is designed in this paper. The special structure of the derived equations of motion makes it possible to separate the design into three levels: motion planning, kinematic, and dynamic. In the proposed scheme, a fuzzy inference engine in the kinematic level is used to change the desired trajectory computed from the motion planning level. A sliding mode controller is adopted to track the new reference values of privileged coordinates in the dynamic level, which subsequently drives the non-privileged coordinates to the desired values. From the simulation results, it is shown that such hierarchical tracking control which simultaneously takes kinematics and dynamics into consideration indeed effectively solve the tracking problem. All the variables can be steered to their desired values asymptotically, which is assured by the skew-symmetric property of the reduced Appell equation.**

I. Introduction

To deal with the motion planning of a nonholonomic system, the subject of nonholonomic motion planning (NMP) has been studied extensively [1], in which various algorithms [2], [3] have been developed to generate a feasible trajectory satisfying the nonholonomic constraints. However, ignoring the mass and the moment of inertia of the system, which appear in the dynamic equations, may lead to the inability of following the generated trajectory effectively. Alternatively, the kinematic equations are sometimes treated as a control problem and one tries to design the so-called kinematic controller, in which discontinuous feedback [4], [5], time-varying feedback [6], [7], or hybrid feedback [8] may be used. Such a controller is not directly realizable through actual control mechanism in the car-like mobile robot system.

In practical situations, control torque command enters through the dynamical equation, and it is thus desired to design the controller based upon the combined kinematic and dynamic equations. An intrinsic, feasible and practical methodology of controller design which appropriately accommodates the dynamical equations and the kinematical constraints simultaneously is still in demand, cf. [9, 10], and the hierarchical tracking control algorithm proposed here suits such a need. In this paper, we start with a fundamental principle in mechanics, the Principle of Virtual Power, to deal with the constraints and dynamics of a car-like mobile robot.

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Among many fundamental equations in mechanics, such as the D'Alembert-Lagrange equation, the Gibbs-Appell equation, and the Boltzmann-Hamel equations, cf. [11], the Jourdain variational equation [12] in terms of virtual velocities is most suitable to be used to treat the kinematic (velocity) constraints. In a recent paper [13], the variational equation is applied to derive systematically the Appell equation, which is analogous to the equation obtained from Kane's approach [14]. In this method, the system variables are divided into two sets, one for privileged coordinates, and the other for non-privileged coordinates. It was further noted that if the system is reducible, in the sense that the non-privileged variables do not appear in the Appell equation, the reduced dynamical equation is decoupled from the kinematic equation. Extending the results of [13], we establish a systematic and structural procedure to derive the reduced dynamics in matrix form, which is decoupled from the kinematics and is used in our dynamic controller design. For complex multi-body systems, this approach reduces the complexity significantly over conventional methods, such as Lagrangian formulism. Moreover, we may design a controller based on the reduced dynamics to steer the privileged coordinates without interlacing with the kinematics, and then drive the non-privileged variables to the desired trajectory through the kinematic relation. This is the basic idea of the proposed scheme for the design of a tracking controller.

The design is composed of three levels. On the top, according to mission requirements, NMP methods are employed to generate a feasible path that no collision is possible and the kinematic constraints are satisfied. The desired trajectories of the privileged variables are however changed in the middle level by a kinematic compensator based on the current configuration of the system. The compensated values are then fed into the dynamic controller in the bottom level, where the reference values for the privileged variables may not be the same as those in the top level. Detailed description is given below after the physical problem being stated.

II. Problem Description

The practical problem to be solved in this paper is the tracking of a desired trajectory for a four-wheeled mobile robot, as depicted in Fig. 1, moving on a horizontal plane. The system may be modeled by a platform with mass m_p, width w and length l, attached by four rolling without slipping wheels. To simplify the analysis, assume that the two wheels on each axle (front or rear) can be treated as a single wheel with mass m_w and radius a centered at the midpoint of the axle (Q_f or Q_r, respectively), cf. Fig. 1. This approximated model is the so-called car-like mobile robot, including a platform (body p),

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a front wheel (body f), and a rear wheel (body r). The masses of the rim of each wheel and platform are denoted by m'_w and m'_p, respectively. Let ρ_f, ρ_r be the distances from the points Q_f, Q_r to the mass center of platform C_p, respectively.

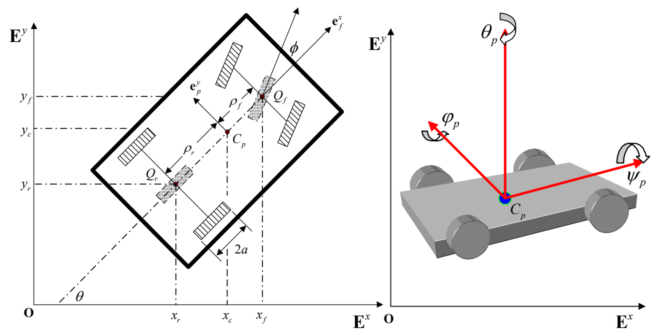

Fig. 1. The configuration of the car-like mobile robot.

The translational motion of body i ($i = p, f, r$) may be described by the position of its mass center, which is expressed in the inertial frame $\{\mathbf{E}^x, \mathbf{E}^y, \mathbf{E}^z\}$ as

$$\mathbf{r}_i^C = x_i\mathbf{E}^x + y_i\mathbf{E}^y + z_i\mathbf{E}^z. \quad (1)$$

To describe the rotational motion, the type (3-1-2) Eulerian angles are used to rotate the inertial frame to $\{\mathbf{i}_i', \mathbf{j}_i', \mathbf{k}_i' = \mathbf{E}^z\}$ by the heading angle θ_i, and then to $\{\mathbf{i}_i'' = \mathbf{i}_i', \mathbf{j}_i'', \mathbf{k}_i''\}$ by the camber angle ψ_i, and finally to the pre-specified body frame $\{\mathbf{e}_i^x, \mathbf{e}_i^y = \mathbf{j}_i'', \mathbf{e}_i^z\}$ of body i by the spin angle φ_i. If the rotation is time-varying, the rates of change of the Eulerian angles are related to the angular velocity $\boldsymbol{\omega}_i$ by:

$$\boldsymbol{\omega}_i = \dot{\theta}_i\mathbf{E}^z + \dot{\psi}_i\mathbf{i}_i' + \dot{\varphi}_i\mathbf{j}_i''. \quad (2)$$

The six variables $(x_i, y_i, z_i, \psi_i, \varphi_i, \theta_i)$ are adopted here to describe the configuration of body i, and therefore there are eighteen variables to be specified for the system. However, due to physical constraints, the number required may be reduced. By the assumption that the motion is horizontal, we have (i) $z_r = h_w$, (ii) $z_f = h_w$, (iii) $z_p = h_p$. The platform is assumed to be kept horizontal as well so that (iv) $\varphi_p = 0$ and (v) $\psi_p = 0$, and hence the triad $\{\mathbf{e}_p^x, \mathbf{e}_p^y, \mathbf{e}_p^z\}$ coincides with $\{\mathbf{i}_p', \mathbf{j}_p', \mathbf{k}_p'\}$ and $\{\mathbf{i}_p'', \mathbf{j}_p'', \mathbf{k}_p''\}$. If the wheels are properly aligned, the camber angles for the wheels vanish, (vi) $\psi_r = 0$, (vii) $\psi_f = 0$, so that $\{\mathbf{i}_i'', \mathbf{j}_i'', \mathbf{k}_i''\}$ coincides with $\{\mathbf{i}_i', \mathbf{j}_i', \mathbf{k}_i'\}$, for $i = r, f$, respectively. Moreover, if the vehicle is FWD(front-wheel-driven), we have (viii) $\theta_r = \theta_p (\equiv \theta)$. From the geometry of the interconnected bodies (Fig. 1), we have the last four geometric constraints: (ix) $x_p = x_r + \rho_r\cos\theta$, (x) $y_p = y_r + \rho_r\sin\theta$, (xi) $x_f = x_r + \rho\cos\theta$, (xii) $y_f = y_r + \rho\sin\theta$, where $\rho = \rho_f + \rho_r$. Finally, the condition that the wheels roll without slipping is realized by the following constraints:

$$\dot{x}_r = a\dot{\varphi}_r\cos\theta, \quad \dot{y}_r = a\dot{\varphi}_r\sin\theta,$$

$$\dot{x}_f = a\dot{\varphi}_f\cos(\theta+\phi), \quad \dot{y}_f = a\dot{\varphi}_f\sin(\theta+\phi),$$

where ϕ is the steering angle of front wheel $(\theta_f = \theta + \phi)$. Applying the previous geometric constraints, the four kinematic constraints can be converted to the independent ones:

(xiii) $\dot{x}_r\sin\theta - \dot{y}_r\cos\theta = 0$, (xiv) $\dot{x}_r\cos\theta + \dot{y}_r\sin\theta = a\dot{\varphi}_r$,

(xv) $\dot{x}_r\sin(\theta+\phi) - \dot{y}_r\cos(\theta+\phi) - \rho\dot{\theta}\cos\phi = 0$,

(xvi) $\dot{x}_r\cos(\theta+\phi) + \dot{y}_r\sin(\theta+\phi) + \rho\dot{\theta}\sin\phi = a\dot{\varphi}_f$.

Based on these constraints, the angular velocities of the bodies in (2) can be simplified as

$$\boldsymbol{\omega}_p = \dot{\theta}\mathbf{E}^z, \ \boldsymbol{\omega}_r = \dot{\theta}\mathbf{E}^z + \dot{\varphi}_r\mathbf{j}_r'', \ \boldsymbol{\omega}_f = (\dot{\theta}+\dot{\phi})\mathbf{E}^z + \dot{\varphi}_f\mathbf{j}_f''. \quad (3)$$

It is desired to control the steering angle and the spin of the rear wheel by exerting torques $\tau_i, (i = 1, 2)$, respectively, such that the system is able to track a reference trajectory which satisfies the constraints. Due to the twelve geometric ones, the dimension of the system becomes six, and we may choose $(x_r, y_r, \theta, \varphi_r, \varphi_f, \phi)$ as the generalized coordinates. The desired trajectory may be then specified by ($x_{rd}(t)$, $y_{rd}(t)$, $\theta_d(t)$, $\varphi_{rd}(t)$, $\varphi_{fd}(t)$, $\phi_d(t)$), $t \in (0, t_f)$. Due to the four nonholonomic constraints, the degree of freedom of the system is dropped to two. The moments of inertia of the three bodies are given by

$$\mathbf{J}_i = \frac{1}{2}m_w'a^2\mathbf{e}_i^x\mathbf{e}_i^x + m_w'a^2\mathbf{j}_i''\mathbf{j}_i'' + \frac{1}{2}m_w'a^2\mathbf{e}_i^z\mathbf{e}_i^z, \quad (i = r, f) \quad (4)$$

$$\mathbf{J}_p = \frac{1}{3}m_p'w^2\mathbf{i}_p''\mathbf{i}_p'' + \frac{1}{3}m_p'l^2\mathbf{j}_p''\mathbf{j}_p'' + \frac{1}{3}m_p'(w^2+l^2)\mathbf{k}_p''\mathbf{k}_p''. \quad (5)$$

These parameters shall be accommodated in the proposed controller design process through dynamical equations.

III. Reduced Appell's Equation of Motion

A. Reduced Appell's Equation for Coupled Rigid Bodies

Consider a mechanical system consisting of N interconnected rigid bodies. The motion of body j with mass m_j may be described by the translational motion of its center of mass C_j, denoted by $\mathbf{r}_j^C(t) = [x_j(t) \quad y_j(t) \quad z_j(t)]^T \in R^3$, $j = 1, 2, \cdots, N$, and a rotation about C_j, represented by $\Phi_j(t) \in SO(3)$, the special orthogonal group. Let the angular velocity of body j be denoted by $\boldsymbol{\omega}_j$. In dyadic notation, we have

$$\dot{\Phi}_j = \boldsymbol{\omega}_j \times \Phi_j = (\mathbf{1} \times \boldsymbol{\omega}_j) \cdot \Phi_j. \quad (6)$$

The principle of virtual power and the corresponding variational equation in virtual velocity may be applied to find the equations of motion for a system of coupled rigid bodies as discussed in [13]. For a system subject to K geometric independent constraints in the form of:

$$g_s(\mathbf{r}^C, \boldsymbol{\Phi}) = 0, \quad (s = 1, 2, \cdots, K) \quad (7)$$

where $\mathbf{r}^C = [\mathbf{r}_1^C \quad \mathbf{r}_2^C \quad \cdots \quad \mathbf{r}_N^C]$, $\boldsymbol{\Phi} = [\Phi_1 \quad \Phi_2 \quad \cdots \quad \Phi_N]$, the dimension of the system becomes $n = 6N - K$. By solving the geometric conditions (7), we may be able to express the position and attitude of each body in terms of the generalized coordinates $\mathbf{q}=[q_1, q_2, \ldots, q_n]^T$ as

$$\mathbf{r}_j^C = \bar{\mathbf{r}}_j^C(\mathbf{q}), \ \Phi_j = \bar{\Phi}_j(\mathbf{q}). \quad (8)$$

Differentiation of the above equations yields

$$\dot{\mathbf{r}}_j^C = \sum_{r=1}^{n}\frac{\partial\bar{\mathbf{r}}_j^C(\mathbf{q})}{\partial q_r}\dot{q}_r, \ \dot{\Phi}_j = \sum_{r=1}^{n}\frac{\partial\bar{\Phi}_j(\mathbf{q})}{\partial q_r}\dot{q}_r. \quad (9)$$

If the system is further constrained by L nonholonomic constraints, the generalized velocities $\dot{q}_r$, $r = 1,2,\ldots,n$, are related by the following L equations

$$\sum_{r=1}^{n} D_{sr}(\bar{\mathbf{r}}^C(\mathbf{q}), \bar{\boldsymbol{\Phi}}(\mathbf{q}))\dot{q}_r = 0 \quad (s = 1, 2, \cdots, L) \cdot \quad (10)$$

Here, D_{sr} denotes the (s,r)-component of the matrix $\mathbf{D}$. Since the constraints are assumed to be independent, the matrix $\mathbf{D} \in R^{L\times n}$ is of full rank, so that the degree of freedom of the system becomes $m = n - L$. One may now choose m independent generalized velocities from (10), say $\dot{y}_k$, $k = 1,...,m$, called the privileged velocities, so that some column operations may be performed to decompose $\mathbf{D}$ into two parts: a nonsingular matrix $\mathbf{D}' \in R^{L\times L}$ and another matrix $\mathbf{D}'' \in R^{L\times m}$. Eq. (10) can be re-written as

$$\dot{z}_h = \sum_{k=1}^{m} B_{kk}\dot{y}_k,\ \ (h = 1,2,\cdots,L), \text{with } B_{hk} = -\sum_{s=1}^{L}\left(\mathbf{D}'\right)^{-1}_{hs} D''_{sk}. \quad (11)$$

where $\dot{z}_h$, $h = 1,...,L,$ are the non-privileged velocities in the set of generalized velocities. Under this setting, we have

$$\dot{q}_r = \sum_{k=1}^{m} A_{rk}\dot{y}_k,\ \ (r = 1,2,\cdots,n), \quad (12)$$

where $\mathbf{A} \in R^{n\times m}$ contains the components of $\mathbf{B}$. By substituting (12) into (9), we obtain

$$\dot{\mathbf{r}}_j^C = \sum_{k=1}^{m} \boldsymbol{\gamma}_{jk}(\mathbf{y},\mathbf{z})\dot{y}_k,\ \ \boldsymbol{\omega}_j = \sum_{k=1}^{m} \varpi_{jk}(\mathbf{y},\mathbf{z})\dot{y}_k, \quad (13)$$

where $$\boldsymbol{\gamma}_{jk} = \sum_{r=1}^{n}\frac{\partial \overline{\mathbf{r}}_j^C(\mathbf{q})}{\partial q_r}A_{rk},\ \ \varpi_{jk} = \sum_{r=1}^{n}\frac{\partial \overline{\Phi}_j(\mathbf{q})}{\partial q_r}A_{rk}, \quad (14)$$

are functions of $\mathbf{y}$, $\mathbf{z}$ in general. Taking the time derivative of (13), the accelerations $\ddot{\mathbf{r}}_j^C$ and $\dot{\boldsymbol{\omega}}_i$ can be expressed as

$$\ddot{\mathbf{r}}_j^C = \sum_{k=1}^{m}\left(\dot{\boldsymbol{\gamma}}_{jk}\dot{y}_k + \boldsymbol{\gamma}_{jk}\ddot{y}_k\right),\ \ \dot{\boldsymbol{\omega}}_j = \sum_{k=1}^{m}\left(\dot{\varpi}_{jk}\dot{y}_k + \varpi_{jk}\ddot{y}_k\right). \quad (15)$$

Now we invoke the Appell equation derived in [13] to establish the equations of motion for privileged coordinates,

$$\sum_{j=1}^{N}\left(m_j\ddot{\mathbf{r}}_j^C(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{y}}) - \mathbf{F}_j^a\right)\cdot\boldsymbol{\gamma}_{jk}(\mathbf{q}) + \left(\mathbf{J}_j\cdot\dot{\boldsymbol{\omega}}_j + \boldsymbol{\omega}_j\times\mathbf{J}_j\cdot\boldsymbol{\omega}_j - \mathbf{M}_j^a\right)(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{y}})\cdot\varpi_{jk}(\mathbf{y}) = 0.$$

$$(k = 1, 2,\dots, m) \quad (16)$$

Here, the $\mathbf{M}_j^a$ is the total applied moment about C_j, and $\mathbf{J}_j$ is the moment of inertial dyadic about C_j. If (16) contains only the privileged variables, the system is called *reducibl*e for which the set of reduced Appell's equations is decoupled from that of kinematic equation.

B. Dynamic Equation in matrix form

Substituting the expression (15) into (16), the equation for $\mathbf{y}$ can be re-written in the following form:

$$\mathbf{M}(\mathbf{y})\ddot{\mathbf{y}} + \mathbf{C}(\mathbf{y},\dot{\mathbf{y}})\dot{\mathbf{y}} = \mathbf{T}(\mathbf{y}), \quad (17)$$

where

$$M_{rs} = \sum_{j=1}^{N} m_j\boldsymbol{\gamma}_{jr}\cdot\boldsymbol{\gamma}_{js} + \varpi_{jr}\cdot\mathbf{J}_j\cdot\varpi_{js}, T_r = \sum_{j=1}^{N}\mathbf{F}_j^a\cdot\boldsymbol{\gamma}_{jr} + \sum_{j=1}^{N}\mathbf{M}_j^a\cdot\varpi_{jr}, \quad (18)$$

$$C_{rs} = \sum_{j=1}^{N}(m_j\dot{\boldsymbol{\gamma}}_{js}\cdot\boldsymbol{\gamma}_{jr} + \dot{\varpi}_{js}\cdot\mathbf{J}_j\cdot\varpi_{jr} + \sum_{k=1}^{m}\varpi_{jr}\cdot(\varpi_{jk}\times\mathbf{J}_j\cdot\varpi_{js})\dot{y}_k). \quad (19)$$

For this dynamic system, the following lemmas can be proved

Lemma 1: $\mathbf{M}(\mathbf{y})$ is an $(m\times m)$ positive-definite matrix.

Lemma 2: $\dot{\mathbf{M}}(\mathbf{y}) - 2\mathbf{C}(\mathbf{y},\dot{\mathbf{y}})$ is skew-symmetric.

If some parameters in (17) are uncertain, which form an uncertain parameter vector $\boldsymbol{\Theta} \in R^{\ell}$, the left side of (17) may be further expressed in the following linear parametric form

$$\mathbf{M}(\mathbf{y})\ddot{\mathbf{y}} + \mathbf{C}(\mathbf{y},\dot{\mathbf{y}})\dot{\mathbf{y}} = \boldsymbol{\Delta}(\mathbf{y},\dot{\mathbf{y}},\ddot{\mathbf{y}})\boldsymbol{\Theta}, \quad (20)$$

where $\boldsymbol{\Delta}(\cdot) \in R^{m\times\ell}$ is termed the regressor matrix, cf. [17]. This form shall be used to design the adaptive control law in the dynamic level.

C. Equations of Motion of Car-Like Mobile Robots

We shall now derive the reduced Appell equations for the system of car-like mobile robot depicted in Fig. 1. The four nonholonomic conditions (xiii), (xiv), (xv) and (xvi) may be re-written in the form of (10) with the (4×6) coefficient matrix $\mathbf{D}$ given by

$$\mathbf{D} = \begin{bmatrix} \sin\theta & -\cos\theta & 0 & 0 & 0 & 0 \\ \sin(\theta+\phi) & -\cos(\theta+\phi) & -\rho\cos\phi & 0 & 0 & 0 \\ \cos\theta & \sin\theta & 0 & -a & 0 & 0 \\ \cos(\theta+\phi) & \sin(\theta+\phi) & \rho\sin\phi & 0 & -a & 0 \end{bmatrix}. \quad (21)$$

The velocity corresponding to the zero column, i.e. $\dot{\phi}$, must be chosen as a privileged velocity, otherwise $\mathbf{D}'$ shall become singular. For a car driven by the rear wheels, one may naturally choose φ_r as the other privileged coordinate. Based on this structure, Eq. (11) for the non-privileged velocities $\dot{\mathbf{z}} = \left(\dot{x}_r, \dot{y}_r, \theta, \dot{\varphi}_f\right)$ can be derived, which leads to (12) with coefficient matrix $\mathbf{A}$ given by

$$\mathbf{A} = \begin{pmatrix} a\cos\theta & a\sin\theta & \eta\tan\phi & 1 & \sec\phi & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}^T, \text{where } \eta = \frac{a}{\rho}. \quad (22)$$

Substituting the (22) into (14), the vectors $\boldsymbol{\gamma}_{ik}$ are found as

$$\boldsymbol{\gamma}_{r\varphi_r} = a\cos\theta\mathbf{E}^x + a\sin\theta\mathbf{E}^y,\ \ \boldsymbol{\gamma}_{r\phi} = \boldsymbol{\gamma}_{f\phi} = \boldsymbol{\gamma}_{p\phi} = 0,$$

$$\boldsymbol{\gamma}_{f\varphi_r} = (a\cos\theta - a\sin\theta\tan\phi)\mathbf{E}^x + (a\sin\theta + a\cos\theta\tan\phi)\mathbf{E}^y,$$

$$\boldsymbol{\gamma}_{p\varphi_r} = (a\cos\theta - \eta\rho_r\sin\theta\tan\phi)\mathbf{E}^x + (a\sin\theta + \eta\rho_r\cos\theta\tan\phi)\mathbf{E}^y.$$

and ϖ_{ik} are given by

$$\varpi_{r\varphi_r} = (\eta\tan\phi\mathbf{k}''_r + \mathbf{j}''_r),\ \ \varpi_{r\phi} = 0,\ \ \varpi_{f\varphi_r} = (\eta\tan\phi\mathbf{k}''_f + \sec\phi\mathbf{j}''_f),$$

$$\varpi_{f\phi} = \mathbf{k}''_f,\ \ \varpi_{p\varphi_r} = \eta\tan\phi\mathbf{k}''_p,\ \ \varpi_{p\phi} = 0,$$

in which the index k for the privileged velocity is changed to the original variable to enhance the readability. From the physical description of the system in Section II, the applied forces and torques acting on each rigid body are expressed as $\mathbf{F}_r^a = \mathbf{F}_f^a = -m_w g\mathbf{E}^z, \mathbf{F}_p^a = -m_p g\mathbf{E}^z, \mathbf{M}_r^a = \tau_r\mathbf{j}''_r, \mathbf{M}_f^a = \tau_f\mathbf{k}''_f, \mathbf{M}_p^a = 0.$ Now we are ready to apply the Appell equation (16). In matrix form, we have

$$\mathbf{M}(\mathbf{y})\ddot{\mathbf{y}} + \mathbf{C}(\mathbf{y},\dot{\mathbf{y}})\dot{\mathbf{y}} = \mathbf{B}(\mathbf{y})\boldsymbol{\tau} \quad (23)$$

where $$\mathbf{M}(\mathbf{y}) = \begin{bmatrix} \eta^2 I_1\tan^2\phi + I_2 + I_w\sec^2\phi & 0.5\eta I_w\tan\phi \\ 0.5\eta I_w\tan\phi & 0.5 I_w \end{bmatrix},$$

$$\mathbf{C}(\mathbf{y},\dot{\mathbf{y}}) = \begin{bmatrix} \left(\eta^2 I_1 + I_w\right)\dot{\phi}\sec^2\phi\tan\phi & 0 \\ 0.5\eta I_w\dot{\phi}\sec^2\phi & 0 \end{bmatrix},\ \mathbf{B}(\mathbf{y}) = \begin{bmatrix} 1 & \eta\tan\phi \\ 0 & 1 \end{bmatrix},$$

$$I_1 = (I_p + m_p\rho_r^2 + m_w\rho^2 + I_w),\ I_2 = (m_p a^2 + 2m_w a^2 + I_w),$$

$$I_w = m'_w a^2, I_p = m'_p(w^2 + l^2)/3.$$

Moreover, by choosing the unknown vector of parameters as

$$\boldsymbol{\Theta} = \begin{bmatrix}\eta^2 I_1 & I_2 & I_w\end{bmatrix}^T, \quad (24)$$

the corresponding regression matrix $\mathbf{\Delta}$ in (20) is given by

$$\begin{bmatrix} \ddot{\varphi}_r \tan^2\phi + \dot{\varphi}_r\dot{\phi}\sec^2\phi\tan\phi & \ddot{\varphi}_r & 0.5\eta\ddot{\phi}\tan\phi + \ddot{\varphi}_r\sec^2\phi + \dot{\varphi}_r\dot{\phi}\sec^2\phi\tan\phi \\ 0 & 0 & 0.5\eta(\ddot{\varphi}_r\tan\phi + \dot{\varphi}_r\dot{\phi}\sec^2\phi) + 0.5\ddot{\phi} \end{bmatrix}.$$

It is noted that $\mathbf{z}$ does not appear in (23), and hence the system of mobile robot is reducible so that the idea of the hierarchical control proposed in this paper is applicable.

IV. Design of Hierarchical Controller

A. General Methodology

As described before, the dynamics of a reducible mechanical system may be separated into two parts: the reduced Appell equation (16) and the kinematic equation (11). Since the reduced equations for the privileged variables $\mathbf{y}$ are decoupled from the kinematic equation, a controller to steer $\mathbf{y}$ to the desired $\mathbf{y}_d(t)$ may be designed independently. On the other hand, if the initial condition is not perfectly on the desired track or there are some disturbances during the motion, the desired $\mathbf{z}_d(t)$ may not be tracked. It is necessary to invoke (11) to fulfill the control objective. A hierarchical tracking controller is proposed with its block diagram shown in Fig. 2.

On the top level, the motion planner produces the desired $\mathbf{q}_d(t)$ based on task requirements and the conditions of constraints. The tracking of $\mathbf{z}_d(t)$ is achieved by changing the desired privileged velocities $\mathbf{u}_d (= \dot{\mathbf{y}}_d)$ to $\mathbf{u}_c = \mathbf{u}_d + \Delta\mathbf{u}$, where $\Delta\mathbf{u}$ is computed through a kinematic compensator in the middle level, in which the current configuration of the system is provided by the estimator. The updated desired privileged coordinates are then found, $\mathbf{y}_c = \mathbf{y}_d + \Delta\mathbf{y}$, where $\Delta\mathbf{y} = \int \Delta\mathbf{u}$, which is fed along with $\mathbf{u}_c$ to the dynamical controller in the bottom level, which in turn drives the mechanical system.

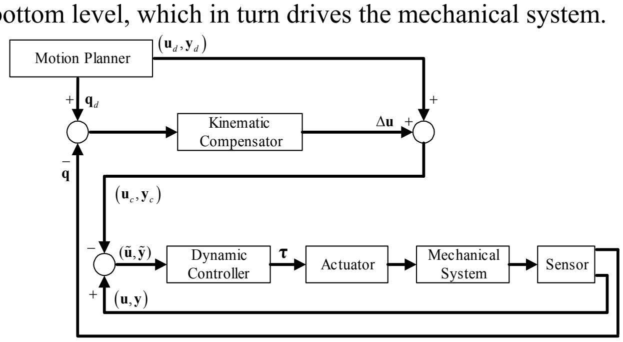

Fig. 2. The block diagram of hierarchical control design.

We now apply the concept of the hierarchical control to the car-like mobile robot, with a feasible and desired trajectory $\mathbf{q}_d = \left(x_{rd}(t), y_{rd}(t), \theta_d(t), \varphi_{rd}(t), \varphi_{fd}(t), \phi_d(t)\right)$ given by a motion planner. The system can be classified as a differential flatness system with two output variables (x_r, y_r), so that if ($x_{rd}(t)$, $y_{rd}(t)$) is given, the rest variables can be determined. In our design, the kinematic compensator is realized by a fuzzy logic system with drivers' experience to infer the compensated values for the $\left(\Delta\dot{\varphi}_r, \Delta\dot{\theta}\right)$ based on the deviation of the current position from the desired one. Since $\phi = \tan^{-1}(\rho\dot{\theta}/a\dot{\varphi}_r)$, a new set of reference values for the privileged variables,

$$\mathbf{u}_c = \left(\Delta\dot{\varphi}_r + \dot{\varphi}_{rd}, \frac{d}{dt}\left(\tan^{-1}\frac{\rho\left(\Delta\dot{\theta} + \dot{\theta}_d\right)}{a(\Delta\dot{\varphi}_r + \dot{\varphi}_{rd})}\right)\right), \mathbf{y}_c = \left(\Delta\varphi_r + \varphi_{rd}, \left(\tan^{-1}\frac{\rho\left(\Delta\dot{\theta} + \dot{\theta}_d\right)}{a(\Delta\dot{\varphi}_r + \dot{\varphi}_{rd})}\right)\right). \quad (25)$$

are then computed, which is fed into a sliding mode controller to track the corrected desired values. This in turn steers $\mathbf{z}$ to the desired trajectory.

B. Fuzzy Logic Compensator in Kinematic Level

From a feasible and desired trajectory of a car-like mobile robot described by the position of the midpoint of the rear axle $x_{rd}(t)$, $y_{rd}(t)$, the reference velocities can be found as

$$\dot{\varphi}_{rd} = \sqrt{\dot{x}_{rd}^2 + \dot{y}_{rd}^2}\Big/a, \; \theta_d = \tan^{-1}\left(\dot{y}_{rd}/\dot{x}_{rd}\right), \; \phi_d = \tan^{-1}\left(\rho\dot{\theta}_d/a\dot{\varphi}_{rd}\right). \quad (26)$$

Due to possible initial drifts or some disturbances during the motion, only the dynamic controller on the privileged variables is insufficient to solve the tracking problem. Let the deviation of the current posture from the reference one be represented by, cf. Fig. 3,

$$\begin{bmatrix} \xi_e \\ \eta_e \\ \theta_e \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{rd} - x_r \\ y_{rd} - y_r \\ \theta_d - \theta \end{bmatrix}. \quad (27)$$

Denote the distance from the current position to the desired position by $d_e = \sqrt{\xi_e^2 + \eta_e^2}$, and the corresponding orientation angle by $\vartheta_e = \tan^{-1}\left(\eta_e/\xi_e\right)$, which shall be termed the *path angle*. With these information, the kinematic compensator is established which adopt the idea of fuzzy logic system. The sets of input and output variables of the fuzzy system are chosen as $\left(d_e, \vartheta_e, \theta_e\right)$ and $\left(\Delta\dot{\varphi}_r, \Delta\dot{\theta}\right)$, respectively.

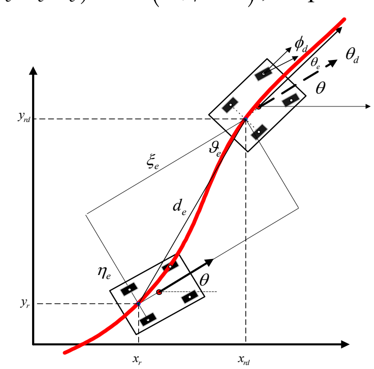

Fig. 3. Current vs. desired configuration of the mobile robot

It is remarked that both the heading error θ_e and the path angle error ϑ_e are taken into account simultaneously in the fuzzy system. If only θ_e is considered, the path angle cannot be adjusted and the vehicle may trace a trajectory parallel to the reference one. On the other hand, if only ϑ_e is used, the compensated value $\Delta\dot{\theta}$ may exhibit a chattering phenomenon, which may lead to zig-zag motion of the vehicle.

Adopting this concept, the fuzzy rules are designed as

R_i: *IF* d_e is A_i *and* θ_e is B_i *and* ϑ_e is C_i *THEN* $\Delta\dot{\varphi}_r$ is D_i, $\Delta\dot{\theta}$ is E_i,

where A_i, B_i, C_i, D_i and E_i are fuzzy sets for *i*-th rule with their linguistic values given by, $A_i \in$ {Z, PS, PM, PB}, $B_i \in$ {P, Z, N }, $C_i \in$ {PB, PM, PS, Z, NS, NM, NB}, $D_i \in$ {ZE, PS, PM, PB}, and E_i = {PB, PM, PS, ZE, NS, NM, NB}. Here, Z, P, N, S, M, B represent for Zero, Positive, Negative, Small, Medium, Big, respectively. The corresponding membership functions of the linguistic values are shown in Fig. 4, where the triangle and singleton are respectively used to describe the antecedent (input variables) and consequent (output variables) fuzzy sets. A total of 28×3 rules are used to find the intermediate values of $\Delta\dot{\varphi}_r$ and $\Delta\dot{\theta}$ as listed in Table I.

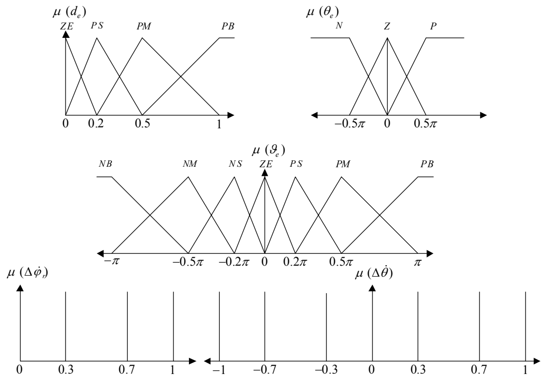

Fig. 4. The membership function of fuzzy sets A_i, B_i, C_i, D_i, and E_i.

TABLE I

TABLES OF FUZZY RULES FOR KINEMATIC COMPENSATOR

θ_e		d_e							
Z / P / N		ZE		PS		PM		PB	
		$\Delta\dot{\varphi}_r$	$\Delta\dot{\theta}$	$\Delta\dot{\varphi}_r$	$\Delta\dot{\theta}$	$\Delta\dot{\varphi}_r$	$\Delta\dot{\theta}$	$\Delta\dot{\varphi}_r$	$\Delta\dot{\theta}$
ϑ_e	PB	ZE / ZE / ZE	ZE / PS / NS	ZE / ZE / ZE	PM / PM / PM	ZE / ZE / ZE	PS / PS / PS	ZE / ZE / ZE	ZE / ZE / ZE
	PM	ZE / ZE / ZE	ZE / PS / NS	PS / ZE / ZE	PM / PM / PM	PS / ZE / ZE	PS / PS / PS	PM / PS / PS	ZE / ZE / ZE
	PS	ZE / ZE / ZE	ZE / PS / NS	PS / PS / PS	PS / PS / PS	PM / PS / PS	ZE / ZE / ZE	PB / PM / PM	ZE / ZE / ZE
	ZE	ZE / ZE / ZE	ZE / PS / NS	PS / PS / PS	ZE / ZE / ZE	PM / PM / PM	ZE / ZE / ZE	PB / PB / PB	ZE / ZE / ZE
	NS	ZE / ZE / ZE	ZE / PS / NS	PS / PS / PS	NS / NS / NS	PM / PS / PS	ZE / ZE / ZE	PB / PM / PM	ZE / ZE / ZE
	NM	ZE / ZE / ZE	ZE / PS / NS	PS / ZE / ZE	NM / NM / NM	PS / ZE / ZE	NS / NS / NS	PM / PS / PS	ZE / ZE / ZE
	NB	ZE / ZE / ZE	ZE / PS / NS	ZE / ZE / ZE	NM / NM / NM	ZE / ZE / ZE	NS / NS / NS	ZE / ZE / ZE	ZE / ZE / ZE

C. Adaptive Sliding Mode Control in Dynamic Level

With the compensated desired values $\dot{\varphi}_{rc}$ and $\dot{\phi}_c$ obtained in the middle level, it is now desired to track the new set of reference privileged variables through the reduced dynamics (24), which may be rewritten as following,

$$\dot{\mathbf{y}} = \mathbf{u},\ \ \mathbf{M}(\mathbf{y})\dot{\mathbf{u}} + \mathbf{C}(\mathbf{y},\dot{\mathbf{y}})\mathbf{u} = \mathbf{B}(\mathbf{y})\boldsymbol{\tau}. \qquad (28)$$

To accommodate the uncertainties in the knowledge of system parameters, an adaptive sliding mode controller is adopted to design $\boldsymbol{\tau}$ such that the tracking errors $\tilde{\mathbf{u}}(t) \equiv \mathbf{u}(t) - \mathbf{u}_c(t) \to 0$ and $\tilde{\mathbf{y}}(t) \equiv \mathbf{y}(t) - \mathbf{y}_c(t) \to 0$ as t→∞. Applying the idea of sliding mode [16], the sliding surface is given by $\mathbf{s} = 0$ where

$$\mathbf{s} = \begin{bmatrix} s_1 & s_2 \end{bmatrix}^T = \tilde{\mathbf{u}} + \boldsymbol{\Lambda}\tilde{\mathbf{y}}, \qquad (29)$$

with $\boldsymbol{\Lambda}$ being a positive definite matrix. Let $\mathbf{s} = \mathbf{u} - \mathbf{u}_s$, where $\mathbf{u}_s = [\dot{\varphi}_{rs} \quad \dot{\phi}_s]^T = \mathbf{u}_c - \boldsymbol{\Lambda}\tilde{\mathbf{y}}$. The linear parametric form in term of $\mathbf{u}_a$ analogous to (20) with $\boldsymbol{\Theta}$ is found to be

$$\mathbf{M}(\mathbf{y})\dot{\mathbf{u}}_s + \mathbf{C}(\mathbf{y},\dot{\mathbf{y}})\mathbf{u}_s = \boldsymbol{\Delta}(\mathbf{y},\dot{\mathbf{y}},\mathbf{u}_s,\dot{\mathbf{u}}_s)\boldsymbol{\Theta}, \qquad (30)$$

$$\boldsymbol{\Delta}(\mathbf{y},\dot{\mathbf{y}},\mathbf{u}_s,\dot{\mathbf{u}}_s) = \begin{bmatrix} \ddot{\varphi}_{rs}\tan^2\phi + \dot{\varphi}_{rs}\dot{\phi}\sec^2\phi\tan\phi & \ddot{\varphi}_{rs} & 0.5\eta\ddot{\phi}_s\tan\phi + \ddot{\varphi}_{rs}\sec^2\phi + \dot{\varphi}_{rs}\dot{\phi}\sec^2\phi\tan\phi \\ 0 & 0 & 0.5\eta(\ddot{\varphi}_{rs}\tan\phi + \dot{\varphi}_{rs}\dot{\phi}\sec^2\phi) + 0.5\ddot{\phi}_s \end{bmatrix}.$$

From (29), it is noted that if the system is initially on the sliding surface $\mathbf{s} = 0$, then $\tilde{\mathbf{y}}(t) \to \mathbf{0}$, since $\tilde{\mathbf{u}} = \dot{\tilde{\mathbf{y}}}$. Hence it is desired to drive the system toward the sliding surface, i.e. the sliding variable $\mathbf{s}$→0. From (28) and (30), the dynamics of the sliding variable is governed by

$$\mathbf{M}(\mathbf{y})\dot{\mathbf{s}} + \mathbf{C}(\mathbf{y},\dot{\mathbf{y}})\mathbf{s} = \mathbf{B}(\mathbf{y})\boldsymbol{\tau} - \boldsymbol{\Delta}(\mathbf{y},\dot{\mathbf{y}},\mathbf{u}_s,\dot{\mathbf{u}}_s)\boldsymbol{\Theta}. \qquad (31)$$

The objective is then converted to finding $\boldsymbol{\tau}$ such that $\mathbf{s}$→0. However, the control torque must depend on the unknown parameter vector $\boldsymbol{\Theta}$, and hence an estimation scheme for $\boldsymbol{\Theta}$ is needed. By integrating these ideas, we have

***Theorem* 1.** Using the control law:

$$\boldsymbol{\tau} = \mathbf{B}^{\#}(\mathbf{y})\left[\boldsymbol{\Delta}(\mathbf{y},\dot{\mathbf{y}},\mathbf{u}_s,\dot{\mathbf{u}}_s)\hat{\boldsymbol{\Theta}} - \mathbf{K}_s\mathbf{s}\right], \qquad (32)$$

where $\mathbf{B}^{\#}$ is the left inverse of $\mathbf{B}$, $\mathbf{K}_s$ is a positive-definite matrix, and $\hat{\boldsymbol{\Theta}}$ is obtained from the parametric adaptive law,

$$\dot{\hat{\boldsymbol{\Theta}}} = -\boldsymbol{\Gamma}^{-1}\boldsymbol{\Delta}(\mathbf{y},\dot{\mathbf{y}},\mathbf{u}_s,\dot{\mathbf{u}}_s)^T\mathbf{s}, \qquad (33)$$

with $\boldsymbol{\Gamma}$ being a positive-definite matrix, the tracking problem for the system described by (31) with unknown parameter vector $\boldsymbol{\Theta}$ can be solved.

Proof: Let $\tilde{\boldsymbol{\Theta}} = \hat{\boldsymbol{\Theta}} - \boldsymbol{\Theta}$ be the estimation error. With the control law (32), the closed loop system equation for $\mathbf{s}$ becomes

$$\mathbf{M}(\mathbf{y})\dot{\mathbf{s}} + \mathbf{C}(\mathbf{y},\dot{\mathbf{y}})\mathbf{s} = -\mathbf{K}_s\mathbf{s} + \boldsymbol{\Delta}(\mathbf{y},\dot{\mathbf{y}},\mathbf{u}_s,\dot{\mathbf{u}}_s)\tilde{\boldsymbol{\Theta}}. \qquad (34)$$

Consider the following Lyapunov candidate function,

$$V(\mathbf{s},\tilde{\mathbf{y}},\tilde{\boldsymbol{\Theta}}) = \frac{1}{2}\mathbf{s}^T\mathbf{M}(\mathbf{y})\mathbf{s} + \tilde{\mathbf{y}}^T\boldsymbol{\Lambda}^T\mathbf{K}_s\tilde{\mathbf{y}} + \frac{1}{2}\tilde{\boldsymbol{\Theta}}^T\boldsymbol{\Gamma}\tilde{\boldsymbol{\Theta}}. \qquad (35)$$

By using ***Lemma* 2** and Barbalat's lemma, cf. [17], it can be shown that $\mathbf{u} \to \mathbf{u}_c$, $\mathbf{y} \to \mathbf{y}_c$. □

Theorem 1 shows that the privileged variables can be steered to their desired values by using the adaptive control law (32) and (33), independent of the non-privileged variables, which are taken care by the fuzzy machine in the kinematic level. The effectiveness of the proposed scheme is demonstrated by the following simulation results.

V. SIMULATION RESULTS

Consider a car-like mobile robot (cf. Fig. 1) with the following system parameters $a = 0.3$m, $\rho_r = 0.5$m, $\rho_f = 0.75$m, $l = 1.75$m, $w = 1.5$m, $m_c = 20$kg, $m_c' = 6$kg, $m_w = 1$kg, $m_w' = 2$kg, b=0.5m. The desired trajectory ($x_{1d}(t)$, $y_{1d}(t)$) is obtained by finding a cubic *B*-spline function (cf. [15]) passing through 12 intermediate points for $(x_{rd}(t), y_{rd}(t))$, {(-15,-5), (-13,4), (-10, 11), (-8,13.5), (-5,15), (-2,13), (-0.5,10), (2,6), (7,8), (7,15), (2,17), (-2,13)}. Next, the desired privileged variables $(\varphi_{rd}(t), \phi_d(t))$ are computed from (26). In the fuzzy inference engine, the scaling factors for $\Delta\dot{\varphi}_r$ and $\Delta\dot{\theta}$ are chosen to be 10 and 20 respectively. The control parameters in the control law (32) and adaptive scheme (33) are chosen by testing as $\mathbf{K}_s$ = *diag* {50,50}, $\boldsymbol{\Lambda}$= *diag*{4,4}, and $\boldsymbol{\Gamma}$ = *diag* {10,10,10,10} to assure appropriate convergence rates. Since the system parameters listed above are unknown to the controller, the adaptive law (33) with initial condition $\hat{\boldsymbol{\Theta}}(0) = [0\ 0\ 0]^T$ is used, which is different from the true value $\boldsymbol{\Theta}_{\text{true}}$=[1 2.16 0.18]^T. It is assumed that initially $x_r(0) = y_r(0) = -5, \theta(0) = 0^o, \varphi_r(0) = 0^o, \phi(0) = 0^o$ which is away from the desired ones $(-15, -5, 45^o, 2^o, -42^o)$.

The simulation results are shown in Figs. 5 to 7. In Fig. 5, the solid line is the desired *B*-spline curve, and shaded block line indicates the tracking performance. It shows that while the initial condition is significantly away from the desired configuration in both position and heading, the proposed hierarchical controller can indeed steer the mobile robot back to the desired trajectory, with the tracking errors of (x_r, y_r, θ) and $(\dot{\varphi}_r, \phi)$ being plotted in Fig. 6 and Fig. 7, respectively. The same set of control parameters has also been used to steer the mobile robot along several type desired trajectories. If the initial condition is not away from the desired one significantly, the tracking can be efficiently fulfilled for all the cases.

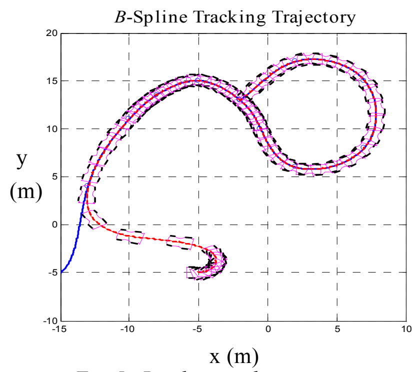

Fig. 5. B-spline tracking trajectory

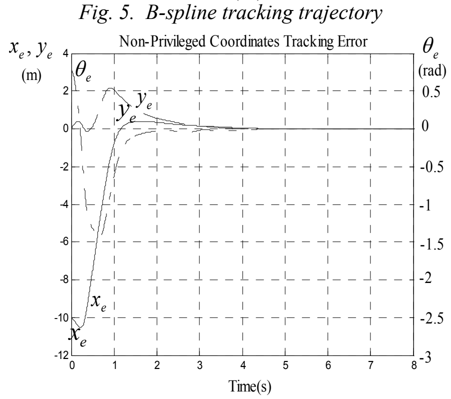

Fig. 6. Tracking error of non-privileged coordinates

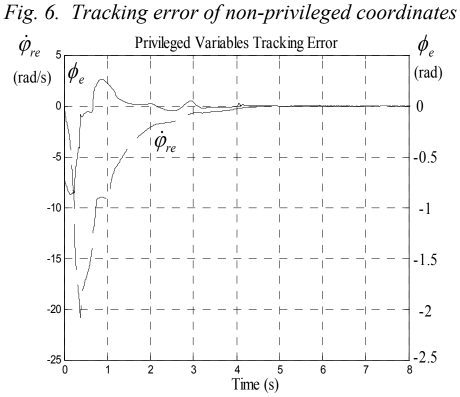

Fig. 7. Tracking error of privileged variables

VI. Conclusions

In this paper, a hierarchical tracking controller was designed to steer all the variables of a car-like mobile robot with some unknown system parameters to their respective desired trajectories. The separation of coordinates in the early stage such that the dynamics is decoupled from the kinematics makes it possible to deal with the privileged coordinates by the dynamic controller. The non-privileged ones are tracked using the kinematic compensator, in which the practical driving experience is incorporated in the design of fuzzy rules. Based on our design, it is possible to run the controller and the compensator at different sampling rates, which may not be feasible for the previous designs of two-level controller such as [9,10,18]. Moreover, the structured form of Appell equation is more suitable to be applied to complicated systems than using Lagrange's formulism. The skew-symmetric property of the reduced Appell equation assures that the privileged variables can be asymptotically tracked by the selected controller.

References

[1] I. Kolmanovsky, and N. H. McClamroch, "Developments in Non -holonomic Control Problems", *IEEE Control Systems Magazine*, vol. 15, no. 6, pp. 20-36, 1995.

[2] R. M. Murray, and S. S. Sastry, "Nonholonomic Motion Planning: Steering Using Sinusoids", *IEEE Trans. on Automatic Control*, vol. 38, no. 5, pp. 700-716, 1993.

[3] G. Paolo, and G. Alessandro, "A Technique to Analytically Formulate and to Solve the 2-Dimensional Constrained Trajectory Planning Problem for a Mobile Robot", *Journal of Intelligent and Robotic Systems*, vol. 27, pp. 237-262, 2000.

[4] A. Astolfi, "Exponential Stabilization of a Car-Like Vehicle", *IEEE Int. Conf. on Robotics and Automation*, pp. 1391-1396, 1995.

[5] J. Guldner, and V. I. Utkin, "Sliding Mode Control for Gradient Tracking and Robot Navigation Using Artificial Potential Fields", *IEEE Trans. on Robots and Automation*,vol.11,no.2, pp.247-254,1995.

[6] C. Samson, "Control of Chained System: Application to Path Following and Time-Varying Point-Stabilization of Mobile Robots", *IEEE Trans. on Automatic Control*, vol. 40, no. 1, pp. 64-77, 1995.

[7] Z.-P. Jiang, and H. Nijmeijer, " A Recursive Technique for Tracking Control of Nonholonomic Systems in Chained Form", *IEEE Trans. on Automatic Control*, vol. 44, no. 2, pp. 265-279, 1999.

[8] O. J. Sørdalen, and O. Egeland, "Exponential Stabilization of Nonholonomic Chained Systems", *IEEE Trans. on Automatic Control*, vol. 40, no. 1, pp. 35-49, 1995.

[9] W. Dong, and W. Huo, "Adaptive Stabilization of Uncertain Dynamic Non-holonomic Systems", *Int. J. of Control*, vol. 72, no. 18, pp. 1689-1700, 1999.

[10] S.-S. Ge, and G.-Y. Zhou, "Adaptive Robust Stabilization of Dynamic Nonholonomic Chained Systems", *Journal of Robotic System*, vol. 18, no. 3, pp. 119-133, 2001.

[11] T. R. Kane, and J. Levinson, "Formulation of Equations of Motion for Complex Spacecraft", *J. of Guidance and Control*, vol. 3, no. 2, pp. 99-112, 1980.

[12] P. E. B. Jourdain, "Note on an Analogue of Gauss' Principle of Least Constraint," *Q. J. Pure Appl. Math.* (8), pp. 153-157, 1909.

[13] L.-S. Wang, and Y.-H. Pao, "Jourdain's Variational Equation and Appell's Equation of Motion for Nonholonomic Dynamical Systems," *American J. of Physics,* Vol. 71, No. 1, pp.72-82, 2003.

[14] T. R. Kane, "Dynamics of Nonholonomic Systems," *J. Applied Mechanics*, vol. 83, pp. 574-578, 1961.

[15] P.-S. Tsai, L.-S. Wang, F.-R. Chang and T.-F. Wu, "Systematic Backstepping Design for *B*-spline Trajectory Tracking Control of the Mobile Robot in Hierarchical Model", *IEEE Int. Conf. on Networking, Sensing and Control*, pp. 713-718, 2004.

[16] V. I. Utkin, "Variable Structure Systems with Sliding Modes", *IEEE Trans. on Automatic Control*, vol. 22, no. 2, pp.212-222, 1977.

[17] K.-Y. Lian, L.-S. Wang, L.-C Fu, "Globally Valid Adaptive Controllers of Mechanical Systems", *IEEE Trans. on Automatic Control*, vol. 42, no. 8, pp. 1149-1154, 1997.

[18] Oya, M., C. Y. Su, R. Katoh, "Robust Adaptive Motion/Force Tracking Control of Uncertain Nonholonomic Mechanic Systems," *IEEE Trans. on Robotics and Automation*, 19 (1), pp. 175-181, 2003.