Proceedings of the
44th IEEE Conference on Decision and Control, and
the European Control Conference 2005
Seville, Spain, December 12-15, 2005

$\mathscr{H}_\infty$ Stochastic Stabilization of Active Fault Tolerant Control Systems: Convex Approach

Samir Aberkane, Dominique Sauter, Jean Christophe Ponsart and Didier Theilliol

Abstract— **This paper deals with the problem of $\mathscr{H}_\infty$ and *robust* $\mathscr{H}_\infty$ control, via dynamic output feedback, of continuous time Active Fault Tolerant Control Systems with Markovian Parameters (AFTCSMP) subject to *structured* parameter uncertainties. The above problematic are addressed under a convex programming approach. Indeed, the fundamental tool in the analysis is an LMI (*Linear Matrix Inequalities*) characterization of dynamical compensators that *stochastically* (*robustly*) *stabilize* the closed loop system and ensure $\mathscr{H}_\infty$ and robust $\mathscr{H}_\infty$ constraints.**

I. Introduction

Active fault tolerant control systems are feedback control systems that reconfigure the control law in real time based on the response from an automatic fault detection and identification (FDI) scheme. The dynamic behaviour of active fault tolerant control systems (AFTCS) is governed by stochastic differential equations (because the failures and failure detection occur randomly) and can be viewed as a general hybrid system[16]. A major class of hybrid systems is jump linear systems (JLS). In JLS, a single jump process is used to describe the random variations affecting the system parameters. This process is represented by a finite state Markov chain and is called the plant regime mode. The theory of stability, optimal control and $\mathscr{H}_2/\mathscr{H}_\infty$ control, as well as important applications of such systems, can be found in several papers in the current literature, for instance in [3], [5], [6], [7], [8], [9].

To deal with AFTCS, another class of hybrid systems was defined, denoted as active fault tolerant control systems with Markovian parameters (AFTCSMP), two random processes are defined: the first random process represents system components failures and the second random process represents the FDI process used to reconfigure the control law. This model was proposed by Srichander and Walker [16]. Necessary and sufficient conditions for stochastic stability of AFTCSMP were developed for a single component failure (actuator failures). In [10], the authors proposed a dynamical model that takes into account multiple failures occurring at different locations in the system, such as in control actuators and plant components. The authors derived necessary and sufficient conditions for the stochastic stability in the mean square sens. The problem of stochastic stability of AFTCSMP in the presence of noise, parameter uncertainties, detection errors, detection delays and actuator saturation limits has also been investigated in [10], [12], [13]. Another issue related to the synthesis of fault tolerant control laws was also addressed by [11], [14], [15]. In [11], the authors designed an optimal control law for AFTCSMP using the matrix minimum principle to minimize an equivalent deterministic cost function. The problem of $\mathscr{H}_\infty$ and robust $\mathscr{H}_\infty$ control (in the presence of structured parameter uncertainties) was treated in [14], [15] for both continuous and discret time AFTCSMP. The authors showed that the state feedback control problem can be solved in terms of the solutions of a set of coupled Riccati inequalities.

Convex analysis has shown to be a powerful tool to derive numerical algorithms for several important control problems. The first problematic we consider in this paper is the dynamic output feedback stabilization (robust stabilization) of AFTCSMP affected by both plant components and actuator failures (the same analysis can be done in the additional presence of sensor failures). It is shown that the necessary and sufficient conditions for the exponential (robust exponential) stability in the mean square sense can be written in terms of an LMI problem. Having obtained this result, we can move on the control problems and write the dynamic output feedback $\mathscr{H}_\infty$ and robust $\mathscr{H}_\infty$ control problems of continuous time AFTCSMP in terms of LMI optimization problems. The convex approach naturally leads to powerful numerical algorithms to solve these problematic.

This paper is organized as follows: section II describes the dynamical model of the system with appropriately defined random processes. A brief summary of basic stochastic terms, results and definitions are given in section III. Section IV derives the necessary and sufficient conditions for the stochastic (robust) exponential stability in the mean square, and the LMI characterization of the dynamic compensators. Section V and VI consider, respectively, the $\mathscr{H}_\infty$ and robust $\mathscr{H}_\infty$ control problems for the output feedback system via LMI optimization problems. Finally, a conclusion is given in section VII.

Notations. The notations in this paper are quite standard. The notation $X \geq Y$ ($X > Y$, respectively), where X and Y are symmetric matrices, means that $X - Y$ is positive semi-definite (positive definite, respectively); I is the identity matrix; $\mathscr{E}\{\cdot\}$ denotes the expectation operator with respect to some probability measure P; $L^2[0,\infty)$ stands for the space of square-integrable vector functions over the interval $[0,\infty)$; $\|\cdot\|$ refers to either the Euclidean vector norm or the matrix norm, which is the operator norm induced by the standard vector norm; $\|\cdot\|_2$ stands for the norm in $L^2[0,\infty)$; while $\|\cdot\|_{\mathscr{E}_2}$ denotes the norm in $L^2((\Omega,\mathscr{F},P),[0,\infty))$; $(\Omega,\mathscr{F},P)$ is a probability space.

The authors are with Université Henri Poincaré, Nancy 1, CRAN – CNRS UMR 7039, BP 239, F-54506 Vandœuvre-lès-Nancy Cedex samir.aberkane@cran.uhp-nancy.fr

II. Dynamical Model Of AFTCSMP with Parameter Uncertainties

The system under normal operation (φ) can be described by:

$$\varphi : \begin{cases} \dot{x}(t) = (A + \Delta A(t))x(t) + Bu(t) + Ew(t) \\ y(t) = C_2 x(t) + D_2 w(t) \\ z_\infty(t) = C_1 x(t) + D_1 u(t) \end{cases} \quad (1)$$

where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times r}$, $E \in \mathbb{R}^{n\times m}$, $C_1 \in \mathbb{R}^{p\times n}$, $C_2 \in \mathbb{R}^{q\times n}$, $D_1 \in \mathbb{R}^{p\times r}$, $D_2 \in \mathbb{R}^{q\times m}$, $x(t) \in \mathbb{R}^n$ is the system state, $u(t) \in \mathbb{R}^r$ is the system input, $y(t) \in \mathbb{R}^q$ is the system measured output, $z_\infty(t) \in \mathbb{R}^p$ is the controlled output, $w(t) \in \mathbb{R}^m$ is the disturbance input which belongs to $L^2[0,\infty)$ and $\Delta A(t)$ is a real, time-varying matrix function representing a norm-bounded parameter uncertainty. For the synthesis of the control action $u(t)$, we introduce a dynamical compensator (φ_c) of the form:

$$\varphi_c : \begin{cases} \dot{v}(t) = A_c v(t) + B_c y(t) \\ u(t) = C_c v(t) \end{cases} \quad (2)$$

where $A_c \in \mathbb{R}^{n\times n}$, $B_c \in \mathbb{R}^{n\times q}$, $C_c \in \mathbb{R}^{r\times n}$.
In this paper, we will consider that the system is subject to both plant components and actuator failures. The random changes affecting plant components are represented by a homogeneous Markov process $\xi(t)$ with the finite state space $Z = \{1,2,...,z\}$, and the random changes that occur in actuators are represented by another homogeneous Markov process $\eta(t)$ with the finite state space $S = \{1,2,...,s\}$. In practice, these random variations are not directly measurable but rather can only be monitored by an FDI scheme. Let $\psi(t)$ denote the state of the FDI process which monitors the states $\xi(t)$ and $\eta(t)$ of the random processes describing the failures. The process $\psi(t)$ is a finite state stochastic process whose random behaviour is conditioned on the failures processes states $\eta(t)$ and $\xi(t)$. The state space of the FDI process $\psi(t)$ is also finite and is denoted by $R = \{1,2,...,r\}$. In AFTCS, we consider that the control law is only a function of the mesurable FDI process $\psi(t)$. Therefore, the linear AFTCSMP with parameter uncertainties can be described as:

$$\varphi : \begin{cases} \dot{x}(t) = [A(\xi(t)) + \Delta A(\xi(t))]x(t) + B(\eta(t))u(y(t),\psi(t),t) + E(\xi(t),\eta(t))w(t) \\ y(t) = C_2 x(t) + D_2(\xi(t),\eta(t))w(t) \\ z_\infty(t) = C_1 x(t) + D_1(\eta(t))u(y(t),\psi(t),t) \end{cases} \quad (3)$$

$$\varphi_c : \begin{cases} \dot{v}(t) = A_c(\psi(t))v(t) + B_c(\psi(t))y(t) \\ u(t) = C_c(\psi(t))v(t) \end{cases} \quad (4)$$

where $A(\xi(t))$, $B(\eta(t))$, $E(\xi(t),\eta(t))$, $D_2(\xi(t),\eta(t))$, $D_1(\eta(t))$, $A_c(\psi(t))$, $B_c(\psi(t))$ and $C_c(\psi(t))$ are properly dimensioned matrices which depends on random parameters, $\Delta A(\xi(t))$ represents the norm-bounded parameter uncertainty for each $\xi(t) = i \in Z$ and $z_\infty(t)$ belongs to $L^2((\Omega,\mathscr{F},P),[0,\infty))$ *i.e.*:

$$\| z_\infty \|_{\mathscr{E}_2} = \mathscr{E}\left\{ \int_0^\infty z_\infty^T(t) z_\infty(t) dt \right\}^{1/2} < \infty.$$

The system φ coupled with φ_c can be written as follows:

$$\begin{cases} \dot{\chi}(t) = [\Lambda(\xi(t),\eta(t),\psi(t)) + \Delta\Lambda(\xi(t))]\chi(t) + \bar{E}(\xi(t),\eta(t),\psi(t))w(t) \\ \bar{y}(t) = \bar{C}_2(\psi(t))\chi(t) + \bar{D}_2(\xi(t),\eta(t))w(t) \\ z_\infty(t) = \bar{C}_1(\eta(t),\psi(t))\chi(t) \end{cases} \quad (5)$$

where: $\chi(t) = [x(t), v(t)]^T$, $\bar{y}(t) = [y(t), u(t)]^T$,
$\Lambda(\xi(t),\eta(t),\psi(t)) = \begin{pmatrix} A(\xi(t)) & B(\eta(t)C_c(\psi(t)) \\ B_c(\psi(t))C_2 & A_c(\psi(t)) \end{pmatrix}$,
$\Delta\Lambda(\xi(t)) = \begin{pmatrix} \Delta A(\xi(t)) & 0 \\ 0 & 0 \end{pmatrix}$,
$\bar{E}(\xi(t),\eta(t),\psi(t)) = \begin{pmatrix} E(\xi(t),\eta(t)) \\ B_c(\psi(t))D_2(\xi(t),\eta(t)) \end{pmatrix}$,
$\bar{C}_2(\psi(t)) = \begin{pmatrix} C_2 & 0 \\ 0 & C_c(\psi(t)) \end{pmatrix}$,
$\bar{D}_2(\xi(t),\eta(t)) = \begin{pmatrix} E(\xi(t),\eta(t)) \\ 0 \end{pmatrix}$,
$\bar{C}_1(\eta(t),\psi(t)) = \begin{pmatrix} C_1 & D_1(\eta(t))C_c(\psi(t)) \end{pmatrix}$

A. The FDI and the Failure Processes

$\xi(t)$, $\eta(t)$ and $\psi(t)$ being homogeneous Markov processes with finite state spaces, we can define the transition probability of the plant components failure process as [13], [16]:

$$\begin{cases} p_{ij}(\Delta t) = \pi_{ij}\Delta t + o(\Delta t) & (i \neq j) \\ p_{ii}(\Delta t) = 1 - \sum\limits_{i\neq j} \pi_{ij}\Delta t + o(\Delta t) & (i = j) \end{cases} \quad (6)$$

The transition probability of the actuator failure process is given by:

$$\begin{cases} p_{kl}(\Delta t) = \nu_{kl}\Delta t + o(\Delta t) & (k \neq l) \\ p_{kk}(\Delta t) = 1 - \sum\limits_{k\neq l} \nu_{kl}\Delta t + o(\Delta t) & (k = l) \end{cases} \quad (7)$$

where π_{ij} is the plant components failure rate, and ν_{kl} is the actuator failure rate. Given that $\xi = k$ and $\eta = l$, the conditional transition probability of the FDI process $\psi(t)$ is:

$$\begin{cases} p_{iv}^{kl}(\Delta t) = \lambda_{iv}^{kl}\Delta t + o(\Delta t) & (i \neq v) \\ p_{ii}^{kl}(\Delta t) = 1 - \sum\limits_{i\neq v} \lambda_{iv}^{kl}\Delta t + o(\Delta t) & (i = v) \end{cases} \quad (8)$$

Here, λ_{iv}^{kl} represents the transition rate from i to v for the Markov process $\psi(t)$ conditioned on $\xi = k \in Z$ and $\eta = l \in S$.

B. The Model of Parameter Uncertainties

To study the robust stabilization (and the robust $\mathscr{H}_\infty$ control) of the uncertain AFTCSMP, we will assume in this work that the admissible structured parameter uncertainties have a norm bounded uncertainty (*NBU*) form. This is the most adopted form in robust stability analysis [13]. In this form, the admissible structured parameter uncertainties are modelled as:

$$\Delta A(\xi(t)) = H(\xi(t))F(\xi(t))G(\xi(t)) \quad (9)$$

where $H(\xi(t)) \in \mathbb{R}^{n\times p_u}$, $F(\xi(t)) \in \mathbb{R}^{p_u\times q_u}$ and $G(\xi(t)) \in \mathbb{R}^{q_u\times n}$. $H(\xi(t))$, $G(\xi(t))$ are known constant matrices, and $F(\xi(t))$ is a Lipschitz measurable matrix function satisfying the condition

$$F^T(\xi(t))F(\xi(t)) \leq I_{p_u}, \forall t \geq 0, \xi(t) = i \in Z \quad (10)$$

For notational simplicity, we will denote $A(\xi(t)) = A_i$, $H(\xi(t)) = H_i$, $F(\xi(t)) = F_i$ and $G(\xi(t)) = G_i$ when $\xi(t) = i \in Z$, $B(\eta(t)) = B_j$ and $D_1(\eta(t)) = D_{1j}$ when $\eta(t) = j \in S$, $E(\xi(t),\eta(t)) = E_{ij}$ and $D_2(\xi(t),\eta(t)) = D_{2ij}$ when $\xi(t) =$

$i \in Z, \eta(t) = j \in S$ and $A_c(\psi(t)) = A_{ck}$, $B_c(\psi(t)) = B_{ck}$, $C_c(\psi(t)) = C_{ck}$ when $\psi(t) = k \in R$. We also denote $x(t) = x_t$, $y(t) = y_t$, $z_\infty(t) = z_{\infty t}$, $w(t) = w_t$, $\xi(t) = \xi_t$, $\eta(t) = \eta_t$, $\psi(t) = \psi_t$ and the initial conditions $x(t_0) = x_0$, $\xi(t_0) = \xi_0$, $\eta(t_0) = \eta_0$ and $\psi(t_0) = \psi_0$.

III. Basic Definitions

Under the assumption that the system (φ) coupled with (φ_c) satisfies the global Lipchitz condition, the solution χ_t determines a family of unique continuous stochastic processes, one for each choice of the random variable χ_0. The joint process $\{\chi_t, \xi_t, \eta_t, \psi_t\}$ is a Markov process.

For system (5), when the uncertainties are equal to zero for all $t \geq 0$, we have the following definitions

Definition 1 *System (5) is said to be*

(i) stochastically stable (SS) if there exists a finite positive constant $K(\chi_0, \xi_0, \eta_0, \psi_0)$ such that the following holds for any initial conditions $(\chi_0, \xi_0, \eta_0, \psi_0)$:

$$\mathscr{E}\left\{\int_0^\infty \| \chi_t \|^2 \, dt\right\} < K(\chi_0, \xi_0, \eta_0, \psi_0) \tag{11}$$

(ii) internally exponentially stable in the mean square sense if it is exponentially stable in the mean square sense for $w_t = 0$, i.e. for any ξ_0, η_0, ψ_0 and some $\gamma(\xi_0, \eta_0, \psi_0)$, there exists two numbers $a > 0$ and $b > 0$ such that when $\|\chi_0\| \leq \gamma(\xi_0, \eta_0, \psi_0)$, the following inequality holds $\forall t \geq t_0$ for all solution of (5) with initial condition χ_0:

$$\mathscr{E}\left\{\|\chi_t\|^2\right\} \leq b\|\chi_0\|^2 \exp\left[-a(t-t_0)\right]. \tag{12}$$

Definition 2 *System (5) is said to be*

(i) robustly stochastically stable (RSS) if there exists a finite positive constant $K(\chi_0, \xi_0, \eta_0, \psi_0)$ such that the condition (11) holds for any initial conditions $(\chi_0, \xi_0, \eta_0, \psi_0)$ and for all admissible uncertainties.

(ii) internally robustly exponentially stable in the mean square sense if it is robustly exponentially stable in the mean square sense for $w_t = 0$, i.e. for any ξ_0, η_0, ψ_0 and some $\gamma(\xi_0, \eta_0, \psi_0)$, there exists two numbers $a > 0$ and $b > 0$ such that when $\|\chi_0\| \leq \gamma(\xi_0, \eta_0, \psi_0)$, the condition (12) holds $\forall t \geq t_0$ for all solution of (5) with initial condition χ_0 and for all admissible uncertainties.

Lemma 1 [13] Let G, M, N be real matrices of appropriate dimensions. Then, for any $\gamma > 0$, and for all the functional matrices satisfying $M^T(t)M(t) \leq I$, we have

$$2x^T PGM(t)Nx \leq \gamma x^T PGG^T Px + (1/\gamma)x^T N^T Nx. \tag{13}$$

IV. Stochastic Stabilization

In this section, we will first derive a necessary and sufficient condition for the internal exponential (robust exponential) stability in the mean square of the system (5), and then we will give an LMI characterization of dynamical compensators (φ_c) that internally stabilize (robustly stabilize) the closed-loop system in the mean square sense.

Proposition 1: a necessary and sufficient condition for internal exponential stability in the mean square of the system (5) is that there exist symmetric positive-definite matrices P_{ijk}, $i \in Z$, $j \in S$ and $k \in R$ such that

$$\tilde{\Lambda}_{ijk}^T P_{ijk} + P_{ijk}\tilde{\Lambda}_{ijk} + \sum_{\substack{h\in Z\\ h\neq i}} \pi_{ih} P_{hjk} + \sum_{\substack{l\in S\\ l\neq j}} \nu_{jl} P_{ilk} + \sum_{\substack{v\in R\\ v\neq k}} \lambda_{kv}^{ij} P_{ijv} < 0 \tag{14}$$

$\forall i \in Z$, $j \in S$ and $k \in R$, where

$$\tilde{\Lambda}_{ijk} = \Lambda_{ijk} - 0.5I\left(\sum_{\substack{h\in Z\\ h\neq i}} \pi_{ih} + \sum_{\substack{l\in S\\ l\neq j}} \nu_{jl} + \sum_{\substack{v\in R\\ v\neq k}} \lambda_{kv}^{ij}\right) \tag{15}$$

Proof See [1]. ■

Proposition 2 gives a LMI characterization of dynamical compensators (φ_c) that internally stabilize the closed-loop system in the mean square sense.

Proposition 2: a necessary and sufficient condition for internal exponential stability in the mean square of the system (5) is that the following matrix inequalities

$$\begin{bmatrix} \tilde{A}_{ijk}Y_{ijk} + Y_{ijk}\tilde{A}_{ijk}^T + F_{ijk}^T B_j^T + B_j F_{ijk} & R_{ijk}(Y) \\ R_{ijk}(Y)^T & S_{ijk}(Y) \end{bmatrix} < 0 \tag{16}$$

$$\tilde{A}_{ijk}^T X_{ijk} + X_{ijk}\tilde{A}_{ijk} + C_2^T H_{ijk}^T + H_{ijk}C_2 + \sum_{\substack{h\in Z\\ h\neq i}} \pi_{ih} X_{hjk} + \sum_{\substack{l\in S\\ l\neq j}} \nu_{jl} X_{ilk} + \sum_{\substack{v\in R\\ v\neq k}} \lambda_{kv}^{ij} X_{ijv} < 0 \tag{17}$$

$$\begin{bmatrix} Y_{ijk} & I \\ I & X_{ijk} \end{bmatrix} > 0 \tag{18}$$

where

$$\begin{cases} R_{ijk} = [R1_{ijk}, R2_{ijk}, R3_{ijk}] \\ R1_{ijk} = \left[\alpha_{i1}Y_{ijk}, \ldots \alpha_{i(i-1)}Y_{ijk}, \alpha_{i(i+1)}Y_{ijk}, \ldots, \alpha_{iz}Y_{ijk}\right] \\ R2_{ijk} = \left[\beta_{j1}Y_{ijk}, \ldots \beta_{j(j-1)}Y_{ijk}, \beta_{j(j+1)}Y_{ijk}, \ldots, \beta_{js}Y_{ijk}\right] \\ R3_{ijk} = \left[\gamma_{k1}Y_{ijk}, \ldots \gamma_{k(k-1)}Y_{ijk}, \gamma_{k(k+1)}Y_{ijk}, \ldots, \gamma_{kr}Y_{ijk}\right] \\ \alpha_{il} = \sqrt{\pi_{il}}; \beta_{jl} = \sqrt{\nu_{jl}}; \gamma_{kl} = \sqrt{\lambda_{kl}^{ij}} \\ S_{ijk} = -\mathrm{diag}[S1_{ijk}, S2_{ijk}, S3ijk] \\ S1_{ijk} = [Y_{1jk}, \ldots, Y_{(i-1)jk}, Y_{(i+1)jk}, \ldots, Y_{zjk}] \\ S2_{ijk} = [Y_{i1k}, \ldots, Y_{i(j-1)k}, Y_{i(j+1)k}, \ldots, Y_{isk}] \\ S3_{ijk} = [Y_{ij1}, \ldots, Y_{ij(k-1)}, Y_{ij(k+1)}, \ldots, Y_{ijr}] \\ \tilde{A}_{ijk} = A_i - 0.5I\sum_{\substack{h\in Z\\ h\neq i}} \pi_{ih} - 0.5I\sum_{\substack{l\in S\\ l\neq j}} \nu_{jl} - 0.5I\sum_{\substack{v\in R\\ v\neq k}} \lambda_{kv}^{ij} \end{cases}$$

$\forall i \in Z$, $j \in S$ and $k \in R$.

have feasible solutions $X_{ijk} = X_{ijk}^T$, $Y_{ijk} = Y_{ijk}^T$, H_{ijk}, and F_{ijk}. The corresponding compensator (φ_c) is given by

$$\begin{aligned} A_{cijk} = {} & (X_{ijk} - Y_{ijk}^{-1})^{-1}\Big[\tilde{A}_{ijk}^T + X_{ijk}\tilde{A}_{ijk}Y_{ijk} + X_{ijk}B_jF_{ijk} + H_{ijk}C_2Y_{ijk} \\ & + \Big(\sum_{\substack{h\in Z\\ h\neq i}} \pi_{ih}Y_{hjk}^{-1} + \sum_{\substack{l\in S\\ l\neq j}} \nu_{jl}Y_{ilk}^{-1} + \sum_{\substack{v\in R\\ v\neq k}} \lambda_{kv}^{ij}Y_{ijv}^{-1}\Big)Y_{ijk}\Big]Y_{ijk}^{-1} \\ & + 0.5I\Big(\sum_{\substack{h\in Z\\ h\neq i}} \pi_{ih} + \sum_{\substack{l\in S\\ l\neq j}} \nu_{jl} + \sum_{\substack{v\in R\\ v\neq k}} \lambda_{kv}^{ij}\Big) \end{aligned} \tag{19}$$

$$B_{cijk} = (Y_{ijk}^{-1} - X_{ijk})^{-1}H_{ijk} \tag{20}$$

$$C_{cijk} = F_{ijk}Y_{ijk}^{-1} \tag{21}$$

Proof see [1]. ■

Remark 1 (*Perfect FDI performance assumption*) In this case, the FDI process is assumed to be able to instantaneously detect and always correctly identify failures. In addition if we do not take into account the location and the nature of the faulty components, then the two failure processes and the FDI process will share the same state space. This situation is similar to the one considered by JLS. Indeed, in this case, proposition 2 reduces to a result obtained in [6].

The following proposition gives a necessary and sufficient condition for internal robust exponential stability in the mean square sense for the system (5).

Proposition 3: a necessary and sufficient condition for internal robust exponential stability in the mean square of the system (5) is that there exist symmetric positive-definite matrices P_{ijk} and some positive constants γ_i, $i \in Z$, $j \in S$ and $k \in R$ such that

$$\begin{aligned}\tilde{\Lambda}_{ijk}^T P_{ijk} + P_{ijk}\tilde{\Lambda}_{ijk} + \sum_{\substack{h\in Z\\ h\neq i}} \pi_{ih} P_{hjk} + \sum_{\substack{l\in S\\ l\neq j}} \nu_{jl} P_{ilk} + \sum_{\substack{v\in R\\ v\neq k}} \lambda_{kv}^{ij} P_{ijv} \\ + \gamma_i P_{ijk}\bar{D}_i P_{ijk} + (1/\gamma_i)\bar{K}_i < 0 \end{aligned} \tag{22}$$

$\forall i \in Z,\ j \in S$ and $k \in R$.

Where: $\bar{D}_i = \bar{H}_i \bar{H}_i^T, \quad \bar{H}_i = \begin{bmatrix} H_i \\ 0_{n\times p_u}\end{bmatrix}$,

and: $\bar{K}_i = \bar{G}_i^T \bar{G}_i, \quad \bar{G}_i = \begin{bmatrix} G_i & 0_{q_u \times n}\end{bmatrix}$.

Proof The proof of this proposition follows essentially the same lines as in [2]. ■

Proposition 4 gives a LMI characterization of of dynamical compensators (φ_c) that internally robustly stabilize the closed-loop system in the mean square sense.

Proposition 4: a necessary and sufficient condition for internal robust exponential stability in the mean square of the system (5) is that the following matrix inequalities

$$\begin{bmatrix} \phi_{ijk} & Y_{ijk}G_i^T & R_{ijk}(Y) \\ G_i Y_{ijk} & -\gamma_i I & 0 \\ R_{ijk}(Y)^T & 0 & S_{ijk}(Y)\end{bmatrix} < 0 \tag{23}$$

$$\begin{bmatrix} \theta_{ijk} & X_{ijk}H_i \\ H_i^T X_{ijk} & -(1/\gamma_i) I\end{bmatrix} < 0 \tag{24}$$

$$\begin{bmatrix} Y_{ijk} & I \\ I & X_{ijk}\end{bmatrix} > 0 \tag{25}$$

where

$$\begin{cases} \phi_{ijk} = \tilde{A}_{ijk}Y_{ijk} + Y_{ijk}\tilde{A}_{ijk}^T + F_{ijk}^T B_j^T + B_j F_{ijk} + \gamma_i H_i H_i^T \\ \theta_{ijk} = \tilde{A}_{ijk}^T X_{ijk} + X_{ijk}\tilde{A}_{ijk} + C_2^T L_{ijk}^T + L_{ijk}C_2 \\ \qquad +(1/\gamma_i) G_i^T G_i + + \sum\limits_{\substack{h\in Z\\ h\neq i}} \pi_{ih} X_{hjk} + \sum\limits_{\substack{l\in S\\ l\neq j}} \nu_{jl} X_{ilk} + \sum\limits_{\substack{v\in R\\ v\neq k}} \lambda_{kv}^{ij} X_{ijv} \end{cases}$$

$\forall i \in Z,\ j \in S$ and $k \in R$.

have feasible solutions $X_{ijk} = X_{ijk}^T$, $Y_{ijk} = Y_{ijk}^T$, L_{ijk}, F_{ijk} and $\gamma_i > 0$. The corresponding compensator (φ_c) is given by

$$B_{cijk} = (Y_{ijk}^{-1} - X_{ijk})^{-1} L_{ijk} \tag{26}$$

$$C_{cijk} = F_{ijk} Y_{ijk}^{-1} \tag{27}$$

$$\begin{aligned} A_{cijk} = (X_{ijk} - Y_{ijk}^{-1})^{-1}\Big[\tilde{A}_{ijk}^T + X_{ijk}\tilde{A}_{ijk}Y_{ijk} + X_{ijk}B_j F_{ijk} + L_{ijk}C_2 Y_{ijk} \\ + \gamma_i X_{ijk} H_i H_i^T + \Big(\sum_{\substack{h\in Z\\ h\neq i}} \pi_{ih} Y_{hjk}^{-1} + \sum_{\substack{l\in S\\ l\neq j}} \nu_{jl} Y_{ilk}^{-1} + \sum_{\substack{v\in R\\ v\neq k}} \lambda_{kv}^{ij} Y_{ijv}^{-1} \\ + (1/\gamma_i) G_i^T G_i\Big) Y_{ijk}\Big] Y_{ijk}^{-1} + 0.5 I \Big(\sum_{\substack{h\in Z\\ h\neq i}} \pi_{ih} + \sum_{\substack{l\in S\\ l\neq j}} \nu_{jl} + \sum_{\substack{v\in R\\ v\neq k}} \lambda_{kv}^{ij}\Big) \end{aligned} \tag{28}$$

Proof The proof of this proposition follows the same arguments as for the proof of proposition 2. ■

Remark 2 It is obvious that the inequality (23) is linear on Y_{ijk}, F_{ijk} and γ_i, while the inequality (24) is linear on X_{ijk}, L_{ijk} and $(1/\gamma_i)$, but (23) and (24) are not jointly linear on γ_i or $(1/\gamma_i)$. A possible approach to dealing with the three LMIs in proposition 4 is to first solve the LMI (23) for Y_{ijk}, F_{ijk} and γ_i, and then use the obtained Y_{ijk} and γ_i to solve the LMIs (24)-(25) for X_{ijk} and L_{ijk}.

V. $\mathscr{H}_\infty$ CONTROL

In this section, we deal with the design of controllers that stochastically stabilizes the closed-loop system and guarantees the disturbance rejection, with a certain level $\mu > 0$, in a convex optimization framework. Mathematically, we are concerned with the LMI characterization of the dynamical compensators φ_c that stochastically stabilizes the system (5) and guarantees the following for all $w \in L^2[0,\infty)$:

$$\| z_\infty \|_{\mathscr{E}_2} = \mathscr{E}\left\{\int_0^\infty z_{\infty t}^T z_{\infty t} dt\right\}^{1/2} < \mu \left[\| w \|_2^2 + a(\chi_0,\xi_0,\eta_0,\psi_0)\right]^{1/2} \tag{29}$$

where $\mu > 0$ is a prescribed level of disturbance attenuation to be achieved and $a(\chi_0,\xi_0,\eta_0,\psi_0)$ is a constant that depends on the initial conditions $(\chi_0,\xi_0,\eta_0,\psi_0)$.

Proposition 5 If the system (5) is internally exponentially stable in the mean square sense, then it is stochastically stable.

Proof Since the system (5) is internally exponentially stable in the mean square sense, it follows from proposition 1 that there exist symmetric positive-definite matrices P_{ijk}, $i \in Z$, $j \in S$ and $k \in R$ such that

$$\tilde{\Lambda}_{ijk}^T P_{ijk} + P_{ijk}\tilde{\Lambda}_{ijk} + \sum_{\substack{h\in Z\\ h\neq i}} \pi_{ih} P_{hjk} + \sum_{\substack{l\in S\\ l\neq j}} \nu_{jl} P_{ilk} + \sum_{\substack{v\in R\\ v\neq k}} \lambda_{kv}^{ij} P_{ijv} = \Xi_{ijk} < 0 \tag{30}$$

$\forall i \in Z,\ j \in S$ and $k \in R$.

It is easy to show that there exists $\alpha > 0$, such that

$$\Xi_{ijk} + \alpha P_{ijk}^2 < 0. \tag{31}$$

$\forall i \in Z,\ j \in S$ and $k \in R$.

Let us consider the quadratic stochastic Lyapunov function

$$\vartheta(\chi_t,\xi_t,\eta_t,\psi_t) = \chi_t^T P(\xi_t,\eta_t,\psi_t)\chi_t \tag{32}$$

Then

$$\mathscr{L}\vartheta(\chi_t,\xi_t,\eta_t,\psi_t) = \chi_t^T \{\Xi(\xi_t,\eta_t,\psi_t)\}\chi_t + 2\chi_t^T P(\xi_t,\eta_t,\psi_t)\bar{E}(\xi_t,\eta_t,\psi_t) w_t. \tag{33}$$

Using lemma 1, it follows from (33) that

$$\begin{aligned}\mathscr{L}\vartheta(\chi_t,\xi_t,\eta_t,\psi_t) \le -\chi_t^T \Gamma(\xi_t,\eta_t,\psi_t)\chi_t \\ + \alpha^{-1} w_t^T \bar{E}(\xi_t,\eta_t,\psi_t)^T \bar{E}(\xi_t,\eta_t,\psi_t) w_t\end{aligned} \tag{34}$$

where

$$\Gamma(\xi_t,\eta_t,\psi_t) = -\Xi(\xi_t,\eta_t,\psi_t) - \alpha P^2(\xi_t,\eta_t,\psi_t) \tag{35}$$

From Dynkin's formula, we have

$$\begin{aligned} &\mathscr{E}\{\vartheta(\chi_T,\xi_T,\eta_T,\psi_T)\} - \vartheta(\chi_0,\xi_0,\eta_0,\psi_0) \\ &= \mathscr{E}\left\{\int_0^T \mathscr{L}\vartheta(\chi_\tau,\xi_\tau,\eta_\tau,\psi_\tau)d\tau\right\} \\ &\le -\mathscr{E}\left\{\int_0^T \chi_\tau^T \Gamma(\xi_\tau,\eta_\tau,\psi_\tau)\chi_\tau d\tau\right\} \\ &+ \alpha^{-1}\mathscr{E}\left\{\int_0^T w_\tau^T \bar{E}(\xi_\tau,\eta_\tau,\psi_\tau)^T \bar{E}(\xi_\tau,\eta_\tau,\psi_\tau) w_\tau d\tau\right\} \\ &\le -\mathscr{E}\left\{\int_0^T \lambda_{\min}\Gamma(\xi_\tau,\eta_\tau,\psi_\tau)\chi_\tau^T \chi_\tau d\tau\right\} \\ &+ \alpha^{-1}\mathscr{E}\left\{\int_0^T \lambda_{\max}(\bar{E}(\xi_\tau,\eta_\tau,\psi_\tau)^T \bar{E}(\xi_\tau,\eta_\tau,\psi_\tau)) w_\tau^T w_\tau d\tau\right\} \end{aligned} \tag{36}$$

From (36), we get

$$\lim_{T\to\infty}\left\{\mathscr{E}\left\{\chi_T^T P(\xi_T,\eta_T,\psi_T)\chi_T\right\}+\min_{i,j,k}\{\lambda_{\min}\Gamma(i,j,k)\}\mathscr{E}\left[\int_0^T \chi_\tau^T\chi_\tau d\tau\right]\right\}$$
$$\leq \{\chi_0^T P(\xi_0,\eta_0,\psi_0)\chi_0\}+\alpha^{-1}\max_{i,j,k}\{\lambda_{\max}\bar{E}^T(i,j,k)\bar{E}(i,j,k)\}\mathscr{E}\left[\int_0^\infty w_\tau^T w_\tau d\tau\right] \quad (37)$$

From (37), and knowing that $\mathscr{E}\left[\chi_t^T P(\xi_t,\eta_t,\psi_t)\chi_t\right]\geq 0$ and $w(\cdot)\in L^2[0,\infty)$, then the system (5) is stochastically stable. ■

Proposition 6 If there exist symmetric positive-definite matrices P_{ijk}, $i\in Z$, $j\in S$ and $k\in R$ such that

$$\tilde{\Lambda}_{ijk}^T P_{ijk}+P_{ijk}\tilde{\Lambda}_{ijk}+\bar{C_{1jk}}^T\bar{C_{1jk}}+\mu^{-2}P_{ijk}\bar{E_{ijk}}\bar{E_{ijk}}^T P_{ijk}$$
$$+\sum_{\substack{h\in Z\\ h\neq i}}\pi_{ih}P_{hjk}+\sum_{\substack{l\in S\\ l\neq j}}\nu_{jl}P_{ilk}+\sum_{\substack{v\in R\\ v\neq k}}\lambda_{kv}^{ij}P_{ijv}=\Pi_{ijk}<0 \quad (38)$$

$\forall i\in Z,\ j\in S$ and $k\in R$.
then the system (5) (when the uncertainties are equal to zero) is stochastically stable and satisfies

$$\|z_\infty\|_{\mathscr{E}_2}<\left[\mu^2\|w\|_2^2+\chi_0^T P(\xi_0,\eta_0,\psi_0)\chi_0\right]^{1/2}. \quad (39)$$

Proof Since $\bar{C_{1jk}}^T\bar{C_{1jk}}+\mu^{-2}P_{ijk}\bar{E_{ijk}}\bar{E_{ijk}}^T P_{ijk}\geq 0$, it follows from (38) and proposition 1, that the system (5) is internally exponentially stable in the mean square sense. Using proposition 5, it follows that (5) is stochastically stable.
Let us know prove that (39) is verified. We begin by defining the following function

$$\mathscr{J}_T=\mathscr{E}\left\{\int_0^T\left[z_{\infty t}^T z_{\infty t}-\mu^2 w_t^T w_t\right]dt\right\} \quad (40)$$

Then, to prove (39), it suffices to establish that

$$\mathscr{J}_\infty\leq\chi_0^T P(\xi_0,\eta_0,\psi_0)\chi_0$$

Let a quadratic stochastic Lyapunov function as defined in (32), then:

$$\mathscr{L}\vartheta(\chi_t,\xi_t,\eta_t,\psi_t)=\chi_t^T\{\Xi(\xi_t,\eta_t,\psi_t)\}\chi_t+2\chi_t^T P(\xi_t,\eta_t,\psi_t)\bar{E}(\xi_t,\eta_t,\psi_t)w_t. \quad (41)$$

adding and subtracting $\mathscr{E}\left\{\int_0^T\mathscr{L}\vartheta(\chi_t,\xi_t,\eta_t,\psi_t)dt\right\}$ to (40), we get

$$\mathscr{J}_T=\mathscr{E}\Big\{\int_0^T\chi_t^T\Big[\Xi(\xi_t,\eta_t,\psi_t)+\bar{C}_1(\eta_t,\psi_t)^T\bar{C}_1(\eta_t,\psi_t)\Big]\chi_t$$
$$+\mu^{-2}\chi_t^T P(\xi_t,\eta_t,\psi_t)\bar{E}(\xi_t,\eta_t,\psi_t)\bar{E}(\xi_t,\eta_t,\psi_t)^T P(\xi_t,\eta_t,\psi_t)\chi_t$$
$$-\mu^2(w_t-\mu^{-2}\bar{E}(\xi_t,\eta_t,\psi_t)^T P(\xi_t,\eta_t,\psi_t)\chi_t)^T$$
$$(w_t-\mu^{-2}\bar{E}(\xi_t,\eta_t,\psi_t)^T P(\xi_t,\eta_t,\psi_t)\chi_t)dt\Big\}-\mathscr{E}\Big\{\int_0^T\mathscr{L}\vartheta(\chi_t,\xi_t,\eta_t,\psi_t)dt\Big\}$$
$$=\mathscr{E}\Big\{\int_0^T\chi_t^T\Big[\Pi(\xi_t,\eta_t,\psi_t)\Big]\chi_t-\mu^2(w_t-\mu^{-2}\bar{E}(\xi_t,\eta_t,\psi_t)^T P(\xi_t,\eta_t,\psi_t)\chi_t)^T$$
$$(w_t-\mu^{-2}\bar{E}(\xi_t,\eta_t,\psi_t)^T P(\xi_t,\eta_t,\psi_t)\chi_t)dt\Big\}-\mathscr{E}\Big\{\int_0^T\mathscr{L}\vartheta(\chi_t,\xi_t,\eta_t,\psi_t)dt\Big\} \quad (42)$$

From Dynkin's formula, we have

$$\mathscr{E}\{\vartheta(\chi_T,\xi_T,\eta_T,\psi_T)\}-\vartheta(\chi_0,\xi_0,\eta_0,\psi_0)=\mathscr{E}\left\{\int_0^T\mathscr{L}\vartheta(\xi_t,\eta_t,\psi_t)dt\right\} \quad (43)$$

Since $\Pi(\xi_t,\eta_t,\psi_t)<0$ and $\mathscr{E}\{\vartheta(\chi_T,\xi_T,\eta_T,\psi_T)\}\geq 0$, it follows from (42) and (43) that

$$\mathscr{J}_T\leq\vartheta(\chi_0,\xi_0,\eta_0,\psi_0)$$

Which yields $\mathscr{J}_\infty\leq\chi_0^T P(\xi_0,\eta_0,\psi_0)\chi_0$. Hence the proof is complete. ■

The $\mathscr{H}_\infty$ constraints (39) can be rephrased in LMI form. This is illustrated by proposition 7, which gives a LMI characterization of output feedback dynamical compensators (φ_c) that stochastically stabilize the AFTCSMP and ensures (39).

Proposition 7 The $\mathscr{H}_\infty$ constraints (38) are equivalent to (42)-(46)

$$\begin{bmatrix}\bar{\phi}_{ijk} & (C_1Y_{ijk}+D_{1j}F_{ijk})^T & R_{ijk}(Y)\\ (C_1Y_{ijk}+D_{1j}F_{ijk}) & -I & 0\\ R_{ijk}(Y)^T & 0 & S_{ijk}(Y)\end{bmatrix}<0 \quad (44)$$

$$\begin{bmatrix}\bar{\theta}_{ijk} & (X_{ijk}E_{ij}+L_{ijk}D_{2ij})\\ (X_{ijk}E_{ij}+L_{ijk}D_{2ij})^T & -\mu^2 I\end{bmatrix}<0 \quad (45)$$

$$\begin{bmatrix}Y_{ijk} & I\\ I & X_{ijk}\end{bmatrix}>0 \quad (46)$$

where

$$\begin{cases}\bar{\phi}_{ijk}=\tilde{A}_{ijk}Y_{ijk}+Y_{ijk}\tilde{A}_{ijk}^T+F_{ijk}^T B_j^T+B_jF_{ijk}+\mu^{-2}E_{ij}E_{ij}^T\\ \bar{\theta}_{ijk}=\tilde{A}_{ijk}^T X_{ijk}+X_{ijk}\tilde{A}_{ijk}+C_2^T L_{ijk}^T+L_{ijk}C_2\\ \quad +C_1^T C_1+\sum\limits_{\substack{h\in Z\\ h\neq i}}\pi_{ih}X_{hjk}+\sum\limits_{\substack{l\in S\\ l\neq j}}\nu_{jl}X_{ilk}+\sum\limits_{\substack{v\in R\\ v\neq k}}\lambda_{kv}^{ij}X_{ijv}\end{cases}$$

$\forall i\in Z,\ j\in S$ and $k\in R$.
The corresponding compensator (φ_c) is given by

$$B_{cijk}=(Y_{ijk}^{-1}-X_{ijk})^{-1}L_{ijk} \quad (47)$$

$$C_{cijk}=F_{ijk}Y_{ijk}^{-1} \quad (48)$$

$$A_{cijk}=(X_{ijk}-Y_{ijk}^{-1})^{-1}\Big[\tilde{A}_{ijk}^T+X_{ijk}\tilde{A}_{ijk}Y_{ijk}+X_{ijk}B_jF_{ijk}+L_{ijk}C_2Y_{ijk}$$
$$+C_1^T(C_1Y_{ijk}+D_{1j}F_{ijk})+\mu^{-2}(X_{ijk}E_{ij}+L_{ijk}D_{2ij})$$
$$+\Big(\sum_{\substack{h\in Z\\ h\neq i}}\pi_{ih}Y_{hjk}^{-1}+\sum_{\substack{l\in S\\ l\neq j}}\nu_{jl}Y_{ilk}^{-1}+\sum_{\substack{v\in R\\ v\neq k}}\lambda_{kv}^{ij}Y_{ijv}^{-1}\Big)Y_{ijk}\Big]Y_{ijk}^{-1}$$
$$+0.5I(\sum_{\substack{h\in Z\\ h\neq i}}\pi_{ih}+\sum_{\substack{l\in S\\ l\neq j}}\nu_{jl}+\sum_{\substack{v\in R\\ v\neq k}}\lambda_{kv}^{ij}) \quad (49)$$

Proof The proof of this proposition follows essentially the same lines as for the proof of Proposition 2. ■

Remark 3 As for the internal stochastic stabilization problematic, if we make the assumption of perfect FDI performance and if we do not take into account the location and the nature of the faulty components (this situation is similar to the one considered by JLS) then proposition 7 reduces to result obtained in [6].

VI. ROBUST $\mathscr{H}_\infty$ CONTROL

In this section, we investigate the robust $\mathscr{H}_\infty$ control of AFTCSMP subject to structured parameter uncertainties ((NBU) uncertainties). It is shown that the above problematic can be recast as a convex optimization problem characterized by LMI, providing thus a characterization of output feedback dynamic compensators that robustly stochastically stabilize the AFTCSMP and ensures (39) for all admissible uncertainties.

Proposition 8 If the system (5) is internally robustly exponentially stable in the mean square sense, then it is robustly stochastically stable.

Proof The proof of this proposition follows the same arguments as for the proof of Proposition 5. ∎

Proposition 9 gives sufficient conditions for the robust $\mathscr{H}_\infty$ control problem.

Proposition 9 If there exist symmetric positive-definite matrices P_{ijk} and some positive constants γ_i, $i \in Z$, $j \in S$ and $k \in R$ such that

$$\tilde{\Lambda}_{ijk}^T P_{ijk} + P_{ijk}\tilde{\Lambda}_{ijk} + \bar{C}_{1jk}^T \bar{C}_{1jk} + \mu^{-2} P_{ijk}\bar{E}_{ijk}\bar{E}_{ijk}^T P_{ijk} + \gamma_i P_{ijk}\bar{D}_i P_{ijk} + \gamma_i^{-1}\bar{K}_i + \sum_{\substack{h\in Z\\ h\neq i}} \pi_{ih} P_{hjk} + \sum_{\substack{l\in S\\ l\neq j}} \nu_{jl} P_{ilk} + \sum_{\substack{v\in R\\ v\neq k}} \lambda_{kv}^{ij} P_{ijv} < 0 \quad (50)$$

$\forall i \in Z,\ j \in S$ and $k \in R$.

then the system (5) is robustly stochastically stable and satisfies

$$\| z_\infty \|_{\mathscr{E}_2} < \left[\mu^2 \| w \|_2^2 + \chi_0^T P(\xi_0, \eta_0, \psi_0)\chi_0\right]^{1/2}. \quad (51)$$

for all admissible uncertainties.

Proof The proof of this proposition follows the same lines as for the proof of Proposition 6. ∎

We will now give a LMI characterization of output feedback dynamical compensators (φ_c) that robustly stochastically stabilize the AFTCSMP and ensures (51) for all admissible uncertainties.

Proposition 10 The constraints (50) are equivalent to (52)-(54)

$$\begin{bmatrix} \Upsilon_{ijk} & (C_1 Y_{ijk} + D_{1j}F_{ijk})^T & Y_{ijk}G_i^T & R_{ijk}(Y) \\ (C_1 Y_{ijk} + D_{1j}F_{ijk}) & -I & 0 & 0 \\ G_i Y_{ijk} & 0 & -\gamma_i I & 0 \\ R_{ijk}(Y) & 0 & 0 & S_{ijk}(Y) \end{bmatrix} < 0 \quad (52)$$

$$\begin{bmatrix} \Omega_{ijk} & (X_{ijk}E_{ij} + L_{ijk}D_{2ij}) & X_{ijk}H_i \\ (X_{ijk}E_{ij} + L_{ijk}D_{2ij})^T & -\mu^2 I & 0 \\ H_i^T X_{ijk} & 0 & -(1/\gamma_i)I \end{bmatrix} < 0 \quad (53)$$

$$\begin{bmatrix} Y_{ijk} & I \\ I & X_{ijk} \end{bmatrix} > 0 \quad (54)$$

where

$$\begin{cases} \Upsilon_{ijk} = \tilde{A}_{ijk}Y_{ijk} + Y_{ijk}\tilde{A}_{ijk}^T + F_{ijk}^T B_j^T + B_j F_{ijk} + \mu^{-2}E_{ij}E_{ij}^T + \gamma_i H_i H_i^T \\ \Omega_{ijk} = \tilde{A}_{ijk}^T X_{ijk} + X_{ijk}\tilde{A}_{ijk} + C_2^T L_{ijk}^T + L_{ijk}C_2 \\ \quad + C_1^T C_1 + (1/\gamma_i)G_i^T G_i + \sum_{\substack{h\in Z\\ h\neq i}} \pi_{ih}X_{hjk} + \sum_{\substack{l\in S\\ l\neq j}} \nu_{jl}X_{ilk} + \sum_{\substack{v\in R\\ v\neq k}} \lambda_{kv}^{ij}X_{ijv} \end{cases}$$

$\forall i \in Z,\ j \in S$ and $k \in R$.

The corresponding compensator (φ_c) is given by

$$B_{cijk} = (Y_{ijk}^{-1} - X_{ijk})^{-1}L_{ijk} \quad (55)$$

$$C_{cijk} = F_{ijk}Y_{ijk}^{-1} \quad (56)$$

$$\begin{aligned} A_{cijk} = (X_{ijk} - Y_{ijk}^{-1})^{-1}\Big[&\tilde{A}_{ijk}^T + X_{ijk}\tilde{A}_{ijk}Y_{ijk} + X_{ijk}B_jF_{ijk} + L_{ijk}C_2Y_{ijk} \\ &+ \gamma_i X_{ijk}H_iH_i^T + C_1^T(C_1Y_{ijk} + D_{1j}F_{ijk}) + \mu^{-2}(X_{ijk}E_{ij} + L_{ijk}D_{2ij}) \\ &+ \Big(\sum_{\substack{h\in Z\\ h\neq i}} \pi_{ih}Y_{hjk}^{-1} + \sum_{\substack{l\in S\\ l\neq j}} \nu_{jl}Y_{ilk}^{-1} + \sum_{\substack{v\in R\\ v\neq k}} \lambda_{kv}^{ij}Y_{ijv}^{-1} + (1/\gamma_i)G_i^TG_i\Big)Y_{ijk}\Big]Y_{ijk}^{-1} \\ &+ 0.5I\Big(\sum_{\substack{h\in Z\\ h\neq i}} \pi_{ih} + \sum_{\substack{l\in S\\ l\neq j}} \nu_{jl} + \sum_{\substack{v\in R\\ v\neq k}} \lambda_{kv}^{ij}\Big) \end{aligned} \quad (57)$$

Proof The proof of this proposition follows the same arguments as for the proof of Proposition 2. ∎

VII. Conclusion

This paper has introduced an LMI approach to the $\mathscr{H}_\infty$ and robust $\mathscr{H}_\infty$ control for linear uncertain continuous time AFTCSMP under a dynamic output feedback control. We have derived some linear matrix inequalities whose solutions indicate the achievability of the desired control problems; *i.e.* we have shown that the $\mathscr{H}_\infty$ and robust $\mathscr{H}_\infty$ control problematic can be recast as a convex optimization problem under constraints of LMIs which can be solved effectively using the recently developed LMI tool. Then, based on these LMIs, we have given a simple procedure to construct the required output feedback stabilizing controllers.

References

[1] S. Aberkane, J.C. Ponsart and D. Sauter, Output Feedback Stochastic Stabilization of Active Fault Tolerant Control Systems: LMI Formulation, *16th IFAC World Congress, Prague, Czech Republic*, 2005.

[2] S. Aberkane, J.C. Ponsart and D. Sauter, Robust Output Feedback Stochastic Stabilization of Active Fault Tolerant Control Systems, *Joint 2005 International Symposium on Intelligent Control and 13th Mediterranean Conference on Control and Automation, ISIC-MED'05, Limassol, Cyprus*, June 27-29, 2005.

[3] E. K. Boukas, Stabilization of Stochastic Nonlinear Hybrid Systems, *Int. J. Innovative Computing, Information and Control*, Vol.1, No.1, pp.131-141, 2005.

[4] S. Boyd, L. El Ghaoui and E. Feron and V. Balakrishnan, Linear Matrix Inequalities in System and Control Theory, Philadelphia: SIAM; 1994.

[5] O. L. V. Costa, J. B. R. do Val and J. C. Geromel, Continuous-time state-feedback H_2-control of Markovian jump linear systems via convex analysis, *Automatica*, Vol 35, 1999, pp 259-268.

[6] D. P. de Farias, J. C. Geromel, J. B. R. do Val and O. L. V. Costa, Output Feedback Control of Markov Jump Linear Systems in Continuous-Time, *IEEE Transaction on automatic control*, Vol 45, 2000, pp 944-949.

[7] C. E. de Souza and M. D. Fragoso, $\mathscr{H}_\infty$ Control For Linear Systems With Markovian Jumping Parameters, *Control Theory and Advanced Technology*, Vol 9, No. 2, 1993, pp 457-466.

[8] Y. Ji and H. J. Chizeck, Controllability, stabilizability, and continuous-time Markovian jump linear quadratic control, *IEEE Transaction on automatic control*, Vol 35, 1990, pp 777-788.

[9] Y. Ji and H. J. Chizeck, Jump linear quadratic Gaussian control in continuous time, *IEEE Transaction on automatic control*, Vol 37, 1992, pp 1884-1892.

[10] M. Mahmoud, J. Jiang and Y. Zhang, Analysis of the Stochastic Stability for Active Fault Tolerant Control Systems, *Proceedings of the* 38^{th} *IEEE Conference on Decision & Control, Phoenix, Arizona USA*, 1999, pp 3188-3193.

[11] M. Mahmoud, J. Jiang and Y. Zhang, Optimal Control Law for Fault Tolerant Control Systems, *Proceedings of the* 39^{th} *IEEE Conference on Decision & Control, Sydney, Australia*, 2000, pp 4126-4131.

[12] M. Mahmoud, J. Jiang and Y. Zhang, Stochastic Stability Analysis of Active Fault-Tolerant Control Systems in the Presence of Noise, *IEEE Transaction on automatic control*, Vol 46, 2001, pp 1810-1815.

[13] M. Mahmoud, J. Jiang and Y. Zhang, Active Fault Tolerant Control Systems: Stochastic Analysis and Synthesis, *Springer*, 2003.

[14] P. Shi and E. K. Boukas, H_∞-Control for Markovian Jumping Lunear Systems with Parametric Uncertainty, *Journal of optimization theory and applications*, Vol 95, 1997, pp 75-99.

[15] P. Shi, E. K. Boukas, S. K. Nguang and X. Guo, Robust disturbance attenuation for discrete-time active fault tolerant control systems with uncertainties, *Optimal control applications and methods*, Vol 24, 2003, pp 85-101.

[16] R. Srichander and B. K. Walker, Stochastic stability analysis for continuous-time fault tolerant control systems, *International journal of control*, Vol 57, 1993, pp 433-452.

[17] Y. Zhang and J. Jiang, Bibliographical review on reconfigurable fault-tolerant control systems, *IFAC SAFEPROCESS*, 2003, pp 265-276.