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LTR Design of Integral Controllers for Time-Delay Plants Using Disturbance Cancellation

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Abstract— **A design of integral controllers based on disturbance cancellation for time-delay plants is discussed. Since the extended plant used in the design is not stabilizable, the standard loop transfer recovery (LTR) method can not be applied. The new LTR method proposed by the authors to design disturbance cancellation controllers for lumped plants is extended to time-delay plants. It is shown that the new LTR technique provides a transparent two-step design.**

I. Introduction

THE design of an integral controller for a time-delay plant is often required in various application fields especially in process control. Among numerous design methods applicable for this class of controllers, the classical loop transfer recovery (LTR) method, *e.g.*, [1]-[3], is still attractive in its flexible two-step procedure for designing observer-based controllers. It is worth noting that a class of tuning methods for predictive controllers widely used in process control can be regarded as LTR procedure [5][6].

The early LTR results are restricted to the LQG design for lumped minimum phase plants. The method has been extended to lumped non-minimum phase plants [3][7] and time-delay plants [8][9][10] under the stabilizability and detectability assumption on the plant or the extended plant.

The standard method for introducing the integral action is to consider the extended system augmenting integrators to plant input or output. Recently, Wu et al. [11] have discussed the application of LTR method to design this type of integral controllers for time-delay plants. Another method for designing integral controllers is to use a disturbance cancellation technique [12]. In this case, the standard LTR procedure can not be applied since the design utilizes an unstabilizable extended system consisting of the plant and the disturbance model. To overcome the difficulty, Guo et al. [13][14] have proposed a new LTR procedure for the discrete-time case. Recently, Ishihara et al. [15] have extended the procedure to continuous-time non-minimum phase plants with finite unstable zeros. The extended procedure can be regarded as an extension of the partial LTR technique [4], the importance of which is sometimes overlooked in LTR literatures.

It is worth noting that the new LTR method of Guo et al. [13][14] differs from the conventional LTR method in the following points: First, the target state feedback controller includes a disturbance estimator based on the full state information. Second, the recovery procedure utilizes the parameter of the target controller. Despite the differences, the application of the new LTR method is straightforward once the target is determined as in the original LTR method.

In this paper we discuss the application of the new LTR technique to time-delay plants. The output feedback controller to be designed by the new LTR method cancels the disturbance by its predicted value generated by the combination of the predictor and the observer. The target of the LTR design is chosen as a predictor-based state feedback controller with a disturbance estimator based on the state information for the lumped part of the plant. Some useful properties of the target are discussed. The Riccati equation depending on the estimator gain matrix in the target is used to achieve the recovery. We show that the proposed LTR method provides a transparent design of a two-degree-of-freedom integral controller.

To discuss feedback properties, we use factorizations of sensitivity matrices instead of loop transfer matrices. The use significantly simplifies the discussion and the presentation.

This paper is organized as follows. In Section II, the plant and the disturbance are defined and some preliminary results are given. The output feedback controller using the disturbance cancellation to be designed by the new LTR method is given in Section III. The target of the LTR design is discussed in Section IV. The recovery procedure is given in Section V. Conclusions are summarized in Section VI.

II. Preliminaries

A. Plant Description

Consider a time-delay plant described by

$$\dot{x}(t) = Ax(t) + B[u(t-\tau) + d(t-\tau)],\\ y(t) = Cx(t), \tag{1}$$

where $\tau > 0$ is a time delay, $x(t) \in R^n$ is a state vector, $u(t) \in R^m$ is a control input, $y(t) \in R^m$ is an output and $d(t) \in R^m$ is a step disturbance vector satisfying

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$$\dot{d}(t-\tau) = 0 . \tag{2}$$

For notational convenience, define

$$G(s) \triangleq C(sI - A)^{-1} B . \tag{3}$$

We make the following assumptions:

A1: (A, B, C) is a minimal realization and $G(s)$ is non-singular for almost all s.
A2: (A, B, C) is minimum phase.
A3: (A, B, C) has no zero at $s = 0$.

B. Observer for the Extended Plant

From (1) and (2), we can construct an extended plant as

$$\dot{\xi}(t) = \Phi\xi(t) + \Gamma u(t-\tau), \;\; y(t) = H\xi(t), \tag{4}$$

where

$$\xi(t) \triangleq \begin{bmatrix} d'(t-\tau) & x'(t) \end{bmatrix}', \tag{5}$$

$$\Phi \triangleq \begin{bmatrix} 0 & 0 \\ B & A \end{bmatrix}, \; \Gamma \triangleq \begin{bmatrix} 0 \\ B \end{bmatrix}, \; H \triangleq \begin{bmatrix} 0 & C \end{bmatrix}. \tag{6}$$

It can easily be checked that the pair (H, Φ) is observable but (Φ, Γ) is not stabilizable under the A1 and A3.

By the observability of (H, Φ), it is possible to construct an observer for estimating the state vector (5). A full order observer is given by

$$\dot{\hat{\xi}}(t) = \Phi\hat{\xi}(t) + \Gamma u(t) + K[y(t) - H\hat{\xi}(t)], \tag{7}$$

where

$$\hat{\xi}(t) \triangleq \begin{bmatrix} \hat{d}'(t-\tau) & \hat{x}'(t) \end{bmatrix}' \tag{8}$$

is the estimate of $\xi(t)$ and K is an observer gain matrix. The Kalman filter theory can be used to determine the observer gain matrix K by introducing a stochastic model

$$\begin{aligned} \dot{\xi}(t) &= \Phi\xi(t) + \Gamma u(t-\tau) + \bar{\Gamma} w(t), \\ y(t) &= H\xi(t) + v(t), \end{aligned} \tag{9}$$

where $v(t)$ and $w(t)$ are mutually independent zero-mean white noise processes with the covariance matrices given by $V > 0$ and $\sigma I > 0$ and the matrix $\bar{\Gamma}$ is chosen such that the pair $(\Phi, \bar{\Gamma})$ is controllable. Then the observer gain matrix can be determined by solving the Riccati equation which has a unique positive definite solution.

C. Predictor for the Extended Plant

To construct a disturbance cancellation controller, we need the prediction of the disturbance as well as the that of the state vector. The observer (7) generate the estimate $\hat{d}(t-\tau)$ and $\hat{x}(t)$ at time t. Define the prediction of $d(\lambda-\tau)$ ($\lambda \ge t$) based on the observation up to time t as $\hat{d}(\lambda-\tau \mid t)$ It is obvious from (2) that

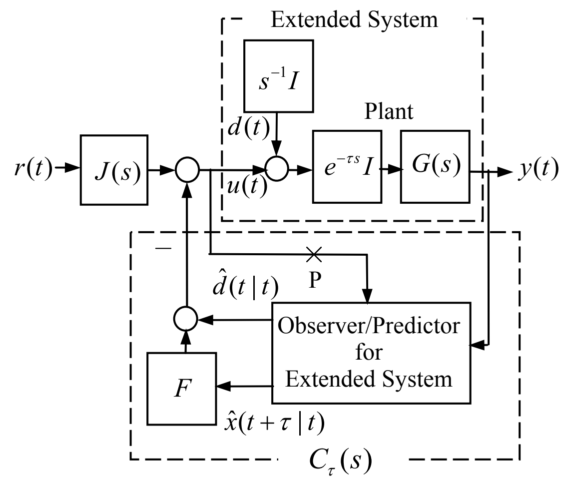

Fig. 1. The output feedback control system

$$\hat{d}(\lambda-\tau \mid t) = \hat{d}(t-\tau), \;\; \lambda \ge t . \tag{10}$$

The prediction of $x(t+\tau)$ based on the observation up to time t is given by

$$\begin{aligned} \hat{x}(t+\tau \mid t) &= e^{A\tau}\hat{x}(t) + \int_t^{t+\tau} e^{A(t+\lambda-\sigma)} B \\ &\quad \times \left[u(\lambda-\tau) + \hat{d}(\lambda-\tau \mid t) \right] d\lambda \\ &= e^{A\tau}\hat{x}(t) + \int_{t-\tau}^{t} e^{A(t-\lambda)} Bu(\lambda)\, d\lambda \\ &\quad + \left(\int_{t-\tau}^{t} e^{A(t-\lambda)} B\, d\lambda \right) \hat{d}(t-\tau). \end{aligned} \tag{11}$$

III. DISTURBANCE CANCELLATION CONTROLLER

A. Disturbance Cancellation as an LQ Problem

First, we show that the disturbance cancellation controller can be obtained as a solution of an LQ problem. Assume that the state vector (5) for the extended plant (4) with $\tau = 0$ is perfectly measurable. Consider the performance index

$$J_\xi \triangleq \int_0^\infty \{ y'(t)y(t) + [d(t)+u(t)]' R [d(t)+u(t)] \} dt, \tag{12}$$

where $R > 0$. Note that the state $d(t)$ is uncontrollable and constant. The control input $u(t)$ should cancel the constant disturbance $d(t)$, otherwise the performance index (12) diverges. Consequently, $u(t)$ must have the form

$$u(t) = u_x(t) - d(t) , \tag{13}$$

where $u_x(t)$ is an input component depending only on the state $x(t)$. The performance index (12) is minimized if and only if $u_x(t)$ minimizes the standard performance index

$$J_x \triangleq \int_0^\infty [y'(t)y(t) + u_x'(t) R u_x(t)] dt . \tag{14}$$

The optimal control input for the performance index (12) is given as

$$u(t) = -Fx(t) - d(t)\,, \tag{15}$$

where F is an optimal state feedback gain matrix for (14).

B. Predictor-Based Output Feedback Controller

The separation principle is used to extend the disturbance cancellation control law (15) to the output feedback control of the time-delay plant (1). Using the predictions given by (10) and (11), we can obtain the disturbance cancellation controller with the reference input term as

$$u(t) = -F\hat{x}(t+\tau \mid t) - \hat{d}(t \mid t) + J(s)r(t)\,, \tag{16}$$

where $r(t)$ is a reference input with the pre-compensator $J(s)$. The structure of the output feedback control system is shown in Fig. 1.

The Laplace transform of (16) is given by

$$u(s) = -F\hat{x}_\tau(s) - \hat{d}_\tau(s) + J(s)r(s)\,, \tag{17}$$

where

$$\hat{x}_\tau(s) \triangleq \mathcal{L}\left[\hat{x}(t+\tau \mid t)\right],\;\; \hat{d}_\tau(s) \triangleq \mathcal{L}\left[\hat{d}(t \mid t)\right]. \tag{18}$$

Define the partition of the gain matrix K as

$$K = \begin{bmatrix} K_d \\ K_x \end{bmatrix}. \tag{19}$$

From (6), (7) and (19), we have

$$\hat{d}_\tau(s) = s^{-1}K_d\left[y(s) - C\hat{x}(s)\right], \tag{20}$$

$$\begin{aligned}\hat{x}(s) &= (sI - A + K_xC)^{-1} \\ &\quad\times\left\{B\left[e^{-\tau s}u(s) - \hat{d}_\tau(s)\right] + K_x y(s)\right\}.\end{aligned} \tag{21}$$

Note that

$$\begin{aligned}&\mathcal{L}\left[\int_{t-\tau}^{t} e^{A(t-\lambda)}Bu(\lambda)d\lambda\right] \\ &\qquad = \left(I - e^{-(sI-A)\tau}\right)(sI-A)^{-1}Bu(s)\end{aligned} \tag{22}$$

holds. It follows from (11) that

$$\begin{aligned}\hat{x}_\tau(s) &= e^{A\tau}\hat{x}(s) + \left(I - e^{-(sI-A)\tau}\right)(sI-A)^{-1}B \\ &\quad\times\left[u(s) + \hat{d}(s)\right],\end{aligned} \tag{23}$$

Substituting (23) in (17), we have

$$\begin{aligned}u(s) &= -Fe^{A\tau}\hat{x}(s) \\ &\quad-\left[I + F\left(I - e^{-(sI-A)\tau}\right)(sI-A)^{-1}B\right]\hat{d}_\tau(s) \\ &\quad-F\left(I - e^{-(sI-A)\tau}\right)(sI-A)^{-1}Bu(s) \\ &\quad+J(s)r(s).\end{aligned} \tag{24}$$

From (20), (21) and (24), we can obtain the controller transfer function matrix. Let $C_\tau(s)$ denote the transfer function matrix from the measured output $y(s)$ to the control input $u(s)$. After straightforward matrix compu- tations, we can express $C_\tau(s)$ as

$$C_\tau(s) = M_\tau^{-1}(s)N_\tau(s)\,, \tag{25}$$

where

$$\begin{aligned}M_\tau(s) \triangleq &\left\{\left[I + F\left(I - e^{-(sI-A)\tau}\right)(sI-A)^{-1}B\right]\right. \\ &\quad\times\left[I - \left(1 - e^{-\tau s}\right)s^{-1}K_dC(sI - A + K_xC)^{-1}B\right] \\ &\quad\left.-Fe^{-(sI-A)\tau}(sI - A + K_xC)^{-1}B\right\} \\ &\quad\times\left[I + s^{-1}K_dC(sI - A + K_xC)^{-1}B\right]^{-1},\end{aligned} \tag{26}$$

$$\begin{aligned}N_\tau(s) \triangleq &-Fe^{A\tau}(sI - A + K_xC)^{-1} \\ &\times\left\{I - B\left[I + s^{-1}K_dC(sI - A + K_xC)^{-1}B\right]^{-1}\right. \\ &\left.s^{-1}K_dC(sI - A + K_xC)^{-1}\right\}K_x \\ &-Fe^{A\tau}(sI - A + K_xC)^{-1}B \\ &\times\left[I + s^{-1}K_dC(sI - A + K_xC)^{-1}B\right]^{-1}s^{-1}K_d \\ &-\left[I + F\left(I - e^{-(sI-A)\tau}\right)(sI-A)^{-1}B\right] \\ &\times\left[I + s^{-1}K_dC(sI - A + K_xC)^{-1}B\right]^{-1} \\ &\times s^{-1}K_d\left[I - C(sI - A + K_xC)^{-1}K_x\right].\end{aligned} \tag{27}$$

Using the above results, we can prove the following proposition.

Proposition 1: Let $S_\tau(s)$ denote the sensitivity matrix at the plant input. The sensitivity matrix can be factored as

$$S_\tau(s) = S(s)M_\tau(s), \tag{28}$$

where

$$S(s) \triangleq \left[I + F(sI-A)^{-1}B\right]^{-1}, \tag{29}$$

and $M_\tau(s)$ is defined in (26).

Proof: The sensitivity matrix at the plant input is defined as

$$S_\tau(s) \triangleq \left[I - C_\tau(s)G(s)e^{-\tau x}\right]^{-1}, \tag{30}$$

where $G(s)$ is defined in (3). From (25) and (30), we have

$$S_\tau(s) = \left[M_\tau(s) - e^{-\tau s}N_\tau(s)G(s)\right]^{-1}M_\tau(s). \tag{31}$$

Straightforward matrix computations using (26) and (27) yields

$$M_\tau(s) - e^{-\tau s}N_\tau(s)G(s) = I + F(sI-A)^{-1}B\,. \tag{32}$$

The factorization (28) follows from (31) and (32). ■

The factor $[I+s^{-1}K_d C(sI-A+K_x C)^{-1}B]^{-1}$ in (26) has a zero at $s=0$ if $K_d \neq 0$ under the assumptions A1 ~ A3. Consequently, the sensitivity matrix (28) has a zero at $s=0$, which shows the controller has the integral action.

The design of the above integral controller requires to determine F, K_x and K_d such that design specifications are satisfied. The task is not easy because the effect of the three matrices on the feedback properties is not simple as seen from the expression (26) of the matrix $M_\tau(s)$. For the efficient design, we propose to extend the new LTR technique [13]-[15] to time-delay plants.

IV. TARGET DESIGN

A. Target Controller

The target of the conventional LTR design for the plant input side is a state feedback controller. For our design, we choose a predictor-based state feedback controller including a disturbance estimator.

First, we construct the disturbance estimator on the assumption that the state vector $x(t)$ in (1) is measurable. Note that (1) and (2) can be rearranged as

$$\begin{aligned} \dot{d}(t-\tau) &= 0, \\ \dot{x}(t)-Ax(t)-Bu(t-\tau) &= Bd(t-\tau). \end{aligned} \tag{33}$$

Let $\bar{d}(t-\tau\,|\,t)$ denote the estimate of $d(t-\tau)$ based on the measured state information up to t. From (33), the disturbance estimator generating $\bar{d}(t-\tau\,|\,t)$ can be constructed as

$$\begin{aligned} \dot{\bar{d}}(t-\tau\,|\,t) = L_d\big[\,&\dot{x}(t)-Ax(t)-Bu(t-\tau) \\ &-\bar{d}(t-\tau\,|\,t)\big], \end{aligned} \tag{34}$$

where L_d is an estimator gain matrix. From (33) and (34), the estimation error

$$\tilde{d}(t-\tau\,|\,t) \triangleq d(t-\tau)-\bar{d}(t-\tau\,|\,t) \tag{35}$$

satisfies

$$\dot{\tilde{d}}(t-\tau\,|\,t) = -L_d B\,\tilde{d}(t-\tau\,|\,t)\ . \tag{36}$$

It follows from (36) that the estimation error converges to zero if and only if the gain matrix L_d is chosen such that the matrix $-L_d B$ is stable.

Let $\bar{d}(\lambda-\tau\,|\,t)$ ($\lambda \geq t$) denote the prediction of $d(\lambda-\tau)$ based on the estimate $\bar{d}(t-\tau\,|\,t)$. It is obvious from (2) that

$$\bar{d}(\lambda-\tau\,|\,t) = \bar{d}(t-\tau\,|\,t),\ \ \lambda \geq t\ . \tag{37}$$

Denote the prediction of $x(t+\tau)$ based on the measured state information up to time t by $\hat{x}(t+\tau\,|\,t)$. Then the prediction $\hat{x}(t+\tau\,|\,t)$ can be expressed as

$$\begin{aligned} \bar{x}(t+\tau\,|\,t) = e^{A\tau}x(t) &+ \int_{t-\tau}^{t} e^{A(t-\lambda)}Bu(\lambda)\,d\lambda \\ &+\left(\int_{t-\tau}^{t} e^{A(t-\lambda)}B\,d\lambda\right)\bar{d}(t-\tau\,|\,t). \end{aligned} \tag{38}$$

The predictions given by (34) and (35) are used to achieve the disturbance cancellation. The control input is given by

$$u(t) = -F\bar{x}(t+\tau\,|\,t)-\bar{d}(t\,|\,t)+J(s)r(t)\,, \tag{39}$$

where the matrix F is a stabilizing state feedback matrix and $r(t)$ is a reference input with the pre-compensator $J(s)$.

Note that the matrices L_d and $L_d B$ are $m\times n$ and $m\times m$, respectively. Since $n>m$ in most cases, the matrix L_d has redundant elements to determine $L_d B$.

The structure of the target control system is shown in Fig. 2.

B. Target Feedback Property

The Laplace transform of the control input (39) is given by

$$u(s) = -F\bar{x}_\tau(s)-\bar{d}_\tau(s)+J(s)r(s)\,, \tag{40}$$

where

$$\bar{x}_\tau(s) \triangleq \mathcal{L}\big[\bar{x}(t+\tau\,|\,t)\big],\ \ \bar{d}_\tau(s) \triangleq \mathcal{L}\big[\bar{d}(t\,|\,t)\big]. \tag{41}$$

From (38), we can express $\bar{x}_\tau(s)$ as

$$\begin{aligned} \bar{x}_\tau(s) = e^{A\tau}x(s) &+ \left(I-e^{-(sI-A)\tau}\right)(sI-A)^{-1}B \\ &\times\big[u(s)+\bar{d}_\tau(s)\big]. \end{aligned} \tag{42}$$

In addition, we can write the Laplace transform of (34) as

$$\bar{d}_\tau(s) = \left(sI+L_d B\right)^{-1}L_d[(sI-A)x(s)-e^{-\tau s}Bu(s)]. \tag{43}$$

From (40), (42) and (43), we can expressed the control input (40) as

$$u(s) = C_x^*(s)x(s)+C_r^*(s)r(s)\ . \tag{44}$$

where

$$\begin{aligned} C_x^*(s) = -&\left[I-e^{-s\tau}\left(sI+L_d B\right)^{-1}L_d B\right]^{-1} \\ &\times\Big\{\left[I+F\left(I-e^{-(sI-A)\tau}\right)(sI-A)^{-1}B\right]^{-1} \\ &\times Fe^{A\tau}+\left(sI+L_d B\right)^{-1}L_d\left(sI-A\right)\Big\}, \end{aligned} \tag{45}$$

$$\begin{aligned} C_r^*(s) = -&\left[I-e^{-s\tau}\left(sI+L_d B\right)^{-1}L_d B\right]^{-1} \\ &\times\left[I+F\left(I-e^{-(sI-A)\tau}\right)(sI-A)^{-1}B\right]^{-1}J(s). \end{aligned} \tag{46}$$

From (44) and (45), we can obtain the following result by straightforward matrix computations.

Proposition 2: For the target control system consisting of the plant (1) and the controller (39), the sensitivity matrix at the plant input

$$S_\tau^*(s) \triangleq [I-C_x^*(s)(sI-A)^{-1}Be^{-\tau s}]^{-1} \tag{47}$$

can be factored as

$$S_\tau^*(s) = S(s)\Theta_\tau(s)Q(s)\Xi_\tau(s), \tag{48}$$

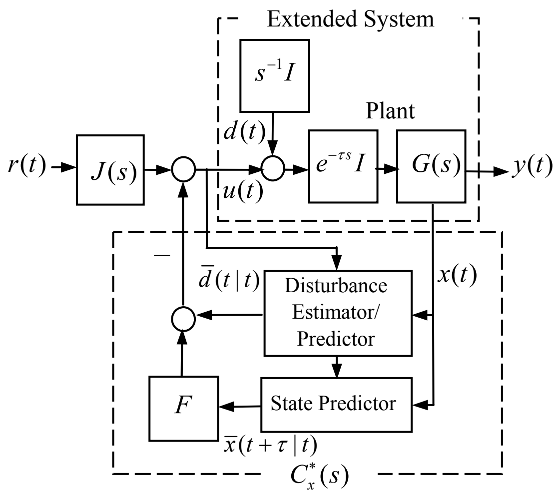

Fig. 2. The target control system

where

$$\Theta_\tau(s) \triangleq I + F\left(I - e^{-(sI-A)\tau}\right)(sI-A)^{-1}B, \tag{49}$$

$$Q(s) \triangleq s(sI + L_d B)^{-1}, \tag{50}$$

$$\Xi_\tau(s) \triangleq I + s^{-1}(1 - e^{-\tau s})L_d B\ . \tag{51}$$

and $S(s)$ is defined in (29). □

For the delay free case ($\tau = 0$), (48) is simplified to

$$S_0^*(s) = S(s)Q(s)\,, \tag{52}$$

since $\Theta_0(s) = I$ and $\Xi_0(s) = I$. Note that $S(s)$ is the sensitivity matrix for the standard regulator and $Q(s)$ can be regarded as the sensitivity matrix for the disturbance estimator. For $\tau > 0$, the matrices $\Theta_\tau(s)$ and $\Xi_\tau(s)$ can be interpreted as the effect of the time-delay τ on the regulator sensitivity and the estimator sensitivity, respectively.

The matrix $Q(s)$ has a zero at $s = 0$, which means that the target controller has the integral action introduced by the disturbance estimator. In addition, the estimator gain matrix L_d appears in (50) and (51) in the form of $L_d B$.

From the factorization (48), it can easily be shown that the factors of the sensitivity matrix have the following asymptotic properties with respect to the design parameters.

Proposition 3: Assume that the feedback gain matrix F is determined by the performance index (14) with the weight $R = rI$. If the cheap control, *e.g.*, [2], is used,

$$\begin{aligned} S(s)\Theta_\tau(s) \to & \left[C(sI-A)^{-1}B\right]^{-1} \\ & \times C\left(I - e^{-(sI-A)\tau}\right)(sI-A)^{-1}B, \end{aligned} \tag{53}$$

as $r \downarrow 0$. On the other hand, if the estimator gain matrix L_d satisfying $L_d B = -\rho I$ exists for any positive scalar ρ , then

$$Q(s)\Xi_\tau(s) \to \left(1 - e^{-\tau s}\right)I, \tag{54}$$

as $\rho \to \infty$. □

The above result shows that the zero sensitivity property can not be achieved for the time-delay plants.

The target controller has the two-degree-of-freedom structure, which is explicitly shown by the following result.

Proposition 4: For the target control system consisting of the plant (1) and the controller (39), the transfer function matrices from the disturbance $d(s)$ to the plant output $y(s)$ is given by

$$G_{yd}^*(s) \triangleq e^{-\tau s}C(sI - A + BF)^{-1}B\Theta_\tau(s)Q(s)\Xi_\tau(s), \tag{55}$$

which depends on F and L_d . On the other hand, the transfer function matrix from the reference $r(s)$ to the plant output $y(s)$ is given by

$$G_{yr}^*(s) \triangleq e^{-\tau s}C(sI - A + BF)^{-1}BJ(s)\,, \tag{56}$$

and is independent of L_d . □

V. RECOVERY PROCEDURE

In this section, we show that the new LTR procedure developed by Guo *et al.*,[13][14] can be used for our purpose. As in [13][14], we adopt the Riccati equation formalism as the recovery procedure. More general eigenstructure assignment method, *e.g.*, [3] can be used with the asymptotic behavior determined by the zero structure assigned here for the Riccati equation.

The stochastic model (9) is used for the computation of the observer gain matrix K to achieve the asymptotic recovery. We choose the matrix $\bar{\Gamma}$ as

$$\bar{\Gamma} \triangleq \begin{bmatrix} L_d B \\ B \end{bmatrix}, \tag{57}$$

where L_d is the estimator gain matrix used in the target.

The following assumption on L_d is introduced:

A4: The matrix L_d is chosen such that the matrix $-L_d B$ is stable.

For the choice of the matrix (57), the transfer function matrix from the white noise $w(t)$ to the output $y(t)$ is given by

$$H(sI - \Phi)^{-1}\bar{\Gamma} = G(s)\left(I + s^{-1}L_d B\right). \tag{58}$$

Noting that the zeros of $(I + s^{-1}L_d B)$ coincide with the eigenvalues of $-L_d B$, we can easily prove the following result.

Lemma 1: Assume the conditions A1 ~ A4. Then the pair $(\Phi,\ \bar{\Gamma})$ is stabilizable and $(H,\ \Phi)$ is detectable. In addition, the invariant zeros of the realization $(\Phi,\ \bar{\Gamma},\ H)$ are in the open left plane. □

For the stochastic model (9) with (57), the Kalman filter gain matrix is given by

$$K(\sigma) \triangleq \Pi H' V^{-1}, \tag{59}$$

where Π is a solution of the Riccati equation

$$\Pi\Phi' + \Phi\Pi - \Pi H' V^{-1} H\Pi + \sigma\bar{\Gamma}(\bar{\Gamma})' = 0. \quad (60)$$

The existence of a stabilizing solution of the Riccati equation (60) is guaranteed by Lemma 1. In addition, Lemma 1 allows us to use the standard result for the asymptotic behaviour of the Kalman filter gain matrix, *e.g.*, [2]. The result is stated as follows.

Lemma 2: Assume the conditions A1 ~ A4. The Kalman filter gain matrix (59) satisfies the asymptotic relation

$$\lim_{\sigma\to\infty} \sigma^{-1/2} K(\sigma) V^{1/2} = \bar{\Gamma} \quad (61)$$

where $\bar{\Gamma}$ is defined by (57). □

Using the above lemma, we can show the recovery as follows.

Proposition 5: Assume the conditions A1 ~ A4. Consider the output feedback controller (16) with the observer (7), the predictors (10) and (11). Assume that the observer gain matrix $K(\sigma)$ is determined by using the Riccati equation (60) with (57). Then, as the scalar parameter σ tends to the infinity, the sensitivity matrix $S_\tau(s)$ given by (28) approaches the target sensitivity matrix $S_\tau^*(s)$ given by (48).

Proof: From Lemma 2, we can write $K(\sigma)$ for sufficiently large σ as

$$K(s) = \begin{bmatrix} K_d(\sigma) \\ K_x(\sigma) \end{bmatrix} = \begin{bmatrix} \sigma^{1/2} L_d B V^{-1/2} \\ \sigma^{1/2} B V^{-1/2} \end{bmatrix}. \quad (62)$$

Using the above expressions, we can obtain the asymptotic expressions of the two transfer function matrices in $M_\tau(s)$ defined in (26):

$$\begin{aligned} &[sI - A + K_x(\sigma)C]^{-1} B \\ &\quad = (sI-A)^{-1} B [I + \sigma^{1/2} V^{-1/2} C (sI-A)^{-1} B]^{-1} \\ &\quad \to 0 \;\; (\sigma \to \infty) \end{aligned} \quad (63)$$

$$\begin{aligned} &K_d(s) C [sI - A + K_x(\sigma)C]^{-1} B \\ &\quad = \sigma^{1/2} L_d B V^{-1/2} C (sI-A)^{-1} B \\ &\quad\quad \times [I + \sigma^{1/2} V^{-1/2} C (sI-A)^{-1} B]^{-1} \\ &\quad \to L_d B \;\; (\sigma \to \infty) \end{aligned} \quad (64)$$

Using (63) and (64) in (26), we can show that

$$M_\tau(s) \to \Theta_\tau(s) Q(s) \Xi_\tau(s) \;\; (\sigma \to \infty), \quad (65)$$

which implies that $S_\tau(s) \to S_\tau^*(s) \;\; (\sigma \to \infty)$. ■

The following observations regarding the control input signal fed back to the observer are worth noting.

1) In the conventional LTR method for LQG controllers, the asymptotic property similar to (63) is sufficient for the recovery. This property has been used by several researchers, *e.g.*, [3], to achieve the conventional LTR with a simplified controller structure utilizing the observer without the control input feedback. However, this is not the case for the disturbance cancellation which requires the condition (64) as well as (63). The control input signal at the point P in Fig. 1 can not be disconnected to achieve the proposed LTR.
2) It is obvious that, if the control input signal to the observer is disconnected at the point P in Fig. 1, the controller can not distinguish the reference input from the disturbance. The control input signal fed back to the observer is indispensable to track the reference signal as well as to cancel the disturbance.

VI. Conclusion

The new LTR technique has been applied to design integral controllers for time-delay plants based on the disturbance cancellation. Although the recovery procedure of the proposed method utilizes the fictitious disturbance gain matrix used in the target design, its application is straightforward once the target is determined. Therefore, the new LTR method retains the two-step structure of the original LTR method.

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